# Removing Projective Distortion from Images

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## 1 Homogenous Coordinates

Similar to cartesian coordinates, homogenous coordinates are a way to represent points in a given dimension.

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

(a) Cartesian $\mathbb{R}^2$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

(b) Homogenous $\mathbb{R}^2$

Figure 1: Representing coordinates as homogenous and cartesian

The only practical difference between cartesian and homogenous representations for the purposes of this assignment is that homogenous coordinates contain an extra dimension value that represents vector scaling. This allows a point to be further from the origin on its component vector without altering the other coordinates.

## 2 Projective Transformations

When a photo is taken of a scene, the camera applies a homography to the original scene, creating a new perspective captured by the photo. An original scene $\mathbf{x}$ manipulated by a photo becomes $\mathbf{x}'$ through the application of projective homography $H$.

$$\mathbf{x}' = H\mathbf{x}$$

Figure 2: Homography application

This equation is notably of the form $Ax = B$, which will allow us to solve for the homography matrix easily.

This means that in order to restore the original image's perspective, we need to determine $H^{-1}$. Once we have $H^{-1}$ we can apply it to every pixel location in the image and determine its location in a new image.

In order for a program to apply these inverse transformations, 8 coordinates within $\mathbb{R}^2$ have to be defined. 4 representing the corners of a box within the supplied image and 4 representing the destination location of those points within the new image. We'll choose those locations as the corners of a rectangle within the image.

$$\begin{pmatrix} x^{ll} \\ x^{lr} \\ x^{ul} \\ x^{ur} \end{pmatrix}$$

(a) Image points

$$\begin{pmatrix} x'^{ll} \\ x'^{lr} \\ x'^{ul} \\ x'^{ur} \end{pmatrix}$$

(b) World points

Figure 3: Image and world point definitions

For i in all 4 corners:

$$\begin{pmatrix} x^i & y^i & 1 & 0 & 0 & 0 & -x^i x'^i & -x^i y'^i \\ 0 & 0 & 0 & x^i & y^i & 1 & -y'^i x^i & -y'^i y^i \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{pmatrix} = \begin{pmatrix} x'^i \\ y'^i \end{pmatrix} \tag{1}$$

Figure 4: Homography application

$$
\begin{pmatrix}
x^{ll} & y^{ll} & 1 & 0 & 0 & 0 & -x^{ll}{x'}^{ll} & -x^{ll}{y'}^{ll} \\
0 & 0 & 0 & x^{ll} & y^{ll} & 1 & -{y'}^{ll}x^{ll} & -{y'}^{ll}y^{ll} \\
x^{lr} & y^{lr} & 1 & 0 & 0 & 0 & -x^{lr}{x'}^{lr} & -x^{lr}{y'}^{lr} \\
0 & 0 & 0 & x^{lr} & y^{lr} & 1 & -{y'}^{lr}x^{lr} & -{y'}^{lr}y^{lr} \\
x^{ul} & y^{ul} & 1 & 0 & 0 & 0 & -x^{ul}{x'}^{ul} & -x^{ul}{y'}^{ul} \\
0 & 0 & 0 & x^{ul} & y^{ul} & 1 & -{y'}^{ul}x^{ul} & -{y'}^{ul}y^{ul} \\
x^{rl} & y^{rl} & 1 & 0 & 0 & 0 & -x^{rl}{x'}^{rl} & -x^{rl}{y'}^{rl} \\
0 & 0 & 0 & x^{rl} & y^{rl} & 1 & -{y'}^{rl}x^{rl} & -{y'}^{rl}y^{rl}
\end{pmatrix}
\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{pmatrix}
= \tag{2}
$$

Figure 5: Homography application

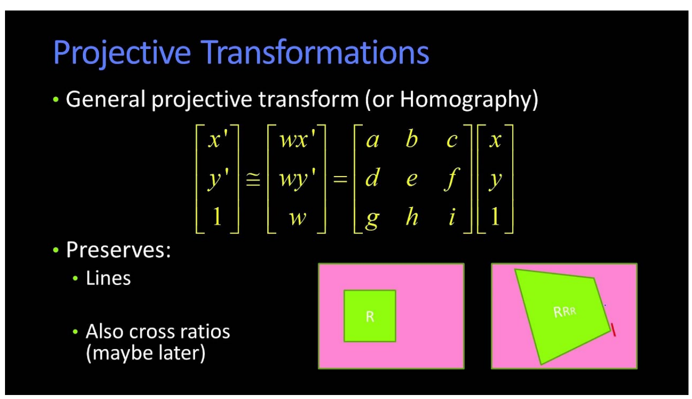


Figure 6: Explanation of projective transforms