Brief Article

The Author

August 3, 2023

Abstract

We carry out scattering calculations on a one-dimensional point like arrangement of atoms, $\chi(\mathbf{x})$, related to the distribution of prime numbers by a shift operation making the atomic density approximately constant. We show how the Riemann Zeta Function (RZF) naturally parameterizes the analytic structure of the scattering amplitude. Inspired by Dyson's suggestion **??**Dyson] we present a simple proof that the non-trivial zeros of the RZF all lie along the line $Re = 1/2$ in the complex plane.

1 Introduction

1.1 Fourier Transform

The Fourier transform of $V(x)$ is $\hat{V}(k)$

$$\hat{V}(k) = \int_{-\infty}^{\infty} V(x) e^{-i2\pi kx} dx \tag{1}$$

A one-dimensional scattering potential may be of the form

$$V(x) = \sum_{x_n \in X} \delta_D(x - x_n) \tag{2}$$

where the x_n are elements of a countable set of real numbers. Then $V(x)$ is called a tempered distribution. For certain $V(x)$ it is the case that it's Fourier transform, $\hat{V}(k)$, is also contain a tempered distribution.

$$\mathcal{F}\left\{V(x)\right\} = \mathcal{F}\left\{\sum_{x_n \in X} \delta_D(x - x_n)\right\} = \hat{h}(k) + \sum_{k_m \in X^*} V_m \delta_D(k - k_m), \tag{3}$$

If $\hat{h}(k) = 0$ everywhere, then $V(x)$ is called a quasicrystal. By applying the Fourier transform a second time to $\hat{V}(k)$, it is evident that $\hat{V}(k)$ is also a quasicrystal when $V(x)$ is a quasicrystal.

In general, $\hat{V}(k) = \hat{g}(k) + \hat{h}(k)$ may have a discrete component, $\hat{g}(k)$ and a continuous component,$\hat{h}(k)$.

$$\mathcal{F}\left\{\sum_{x_n \in X} \delta_D(x - x_n)\right\} = h(k) + \sum_{k_m \in X^*} F_m \delta_D(k - k_m), \tag{4}$$

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1.2 Wave scattering

Consider a scattering potential, V(x), which is distribution of Dirac delta functions along the positive real line, one at each prime number. The delta functions are located at integers and so they have spacing at least 1.

1.3 The Quasicrystal $\chi and Wave Scattering$

Following **??** we define a specific quasicrystalline arrangement of atoms suitable for scattering calculations. Consider a scattering potential, V(x), which is distribution of Dirac delta functions along the positive real line, one at each prime number. The delta functions are located at integers and so they have spacing at least 1. It is well known that the prime counting function, $\pi(x)$, is approximately $\frac{x}{log(x)}$

Figure 1: default

V(k) is a function on the complex plane, $k = k_{Re} + ik_{Im}$ If V(k) is holomorphic, we can compute it by contour integration from V(x).

2 Method

3 Results

4 Proof that all nontrivial zeros of the RZF lie on the line $Re = \frac{1}{/}2$

5 Conclusion