Quasicrystals and Prime Numbers

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This section is the abstract. We describe a pair of transformations (exponential and Fourier) of the distribution of prime numbers. We establish a few properties of the distribution of primes as it relates to scattering. We calculate the scattering amplitude function as a volume in the space of amplituhedrons

I. INTRODUCTION

Consider a scattering potential and it's Fourier transform:

$$\mathcal{F}\left\{\sum_{\gamma_n \in X} \delta_D(\gamma - \gamma_n)\right\} = \sum_{k_m \in X^*} F_m \delta_D(k - k_m), \quad (1)$$

According to Dyson's quasicrystal hypothesis, if we can show the distribution of the prime numbers (2, 3, 5, 7, 11, 13, 17, 19, 23, 29,...) has a quasicrystalline structure, meaning the scattering spectrum of the distribution is a Cantor set of zero measure (point-like), then we can identify the 1-dimensional quasicrystal whose corresponding spectrum is the zeros of the famed Riemann Zeta Function (RZF).

The prime numbers present a special challenge for discussions of scattering, because scattering, and the associated spectral patterns, is understood with the mathematical tool of the Fourier transform, which allows to change variables between real-space coordinates (x) and the coordinates of reciprocal momentum space representation, (k). We suggest first applying a 1-dimensional change of variables from x to $\frac{1}{\ln x}$.

The boundaries of the amplituhedron, A, are where the volume form logarithmically diverges, so in computing scattering amplitudes, it's natural to consider such a logarithmic change of variables.

$$x \rightarrow \frac{x}{\ln(x)} \quad (2)$$

before applying the Fourier transform

$$F(k) = \int_{-\infty}^{\infty} f(x) \exp^{2\pi i k x} \, dx \quad (3)$$

In terms of the

For certain spiral distributions[?] , related to the Fibonacci sequence, it has been shown that the spectrum may be a Cantor set of measure zero.

II. AMPLITUHEDRON

Instead of considering triangulations as in[?] , we consider decompositions into hyper-rectangular volumes as the basic building block

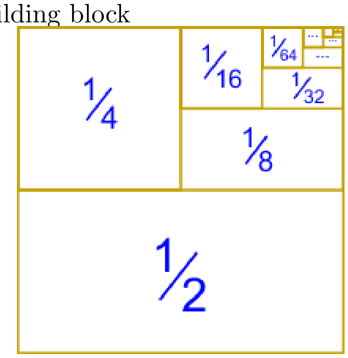

FIG. 1: The sum of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

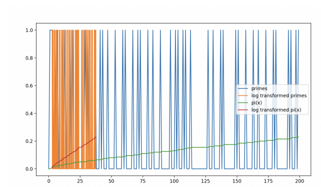

FIG. 2: The prime and exponentially squished prime scattering potentials and associated prime counting functions

III. CONCLUSION

This is the conclusion of the document.

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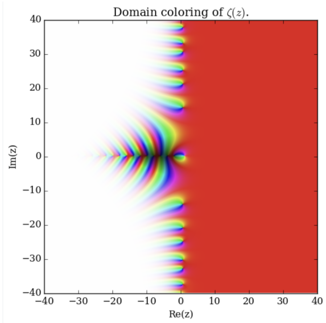

FIG. 3: The zeros of the Riemann zeta function are the tips of the feathers extending in the vertical direction.

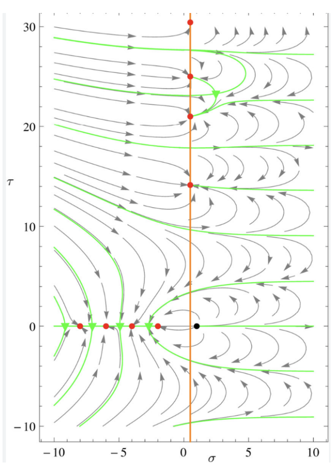

FIG. 4: Newton flow vector representation of the RZF