ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY

DEPARTMENT OF PHYSICS

ELECTRICITY AND MAGNETISM (Phys 206) LECTURE NOTES

Compiled by: Nika Bekri

Unit 1: Electric field

1. Electric field

1.1. Properties of Electric Charges

a. Definition of an electric Charge(q)

The physical quantity we call *electric charge* is a property of matter that causes it to produce and experience electric and magnetic properties/effects.

The particles of which all material objects are made have inertia (mass) and electric charge, among other properties. In contrast to mass, however, electric charge occurs in two kinds, which are called positive (+) and negative (-). The *net charge* on an object is simply the sum of all the individual +/- charges present. Charged objects (objects with non-zero net charge) exert equal and opposite forces on each other.

- An electric charge is a scalar and derived physical quantity.
- The SI unit of charge is the Coulomb (C).

b. properties of electric charge

An electric charge has the following fundamental properties:

1) There are two kinds of electric charges in nature or An electric charge has a polarity; that is, it is either positive or negative.
    - A negative charge is a charge with excess number of electrons but deficiency of protons.
    - A positive charge is a charge with excess number of protons but deficiency of electrons

2) Like /similar/ charges repel each other, and opposite/unlike/ charges attract each other.
3) An electric charge is quantized.
    - an electric charge always exists in discrete packets.
    - an electric charge is always observed to occur as an integer multiple of the charge carried by an elementary particle(e) ,Where $e = 1.60 \times 10^{-19}$ coulomb(C). The charge on an electron is −e and on a proton is +e.

    i.e $q=ne$

    where $e = 1.60 \times 10^{-19}$ coulomb(C)

    q= the total amount of charge contained in a charged body

    $n=0, \pm 1, \pm 2, \pm 3$………………
4) Electric charge is conserved

    i.e. an electric charge is neither created nor destroyed but it transfers from one body to another body. This is called the law of conservation of electric charge.

1.2. Electrostatic field force and Coulomb's "Law"

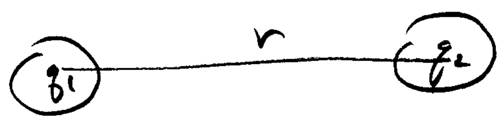
When two stationary/rest charged bodies are placed separated by some distance, the first charged body will exert a force on the second one and the vice versa and this force is called electrostatic force.

a. Coulomb's law

Coulomb's law is a law that governs the electrostatic interaction between two charged bodies. The force that acts between two point charges is described by Coulomb's "Law".

Coulomb's "Law" states that "the magnitude of the electrostatic force between any two stationary charges q1 and q2 separated by distance r is inversely proportional to the square of the distance b/n the charges along the line that joins them and directly proportional to the product of the magnitudes of the charges, q1 and q2.

i.e. $F_e \propto q_1 q_2$ and $F_e = \frac{1}{r^2}$



Mathematically the electric force on q_1 by q_2 is given by $\overline{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}$; the direction is along the radius joining the two charges.

where

$\hat{r}$ is the unit vector in the radial direction.

$k = \frac{1}{4\pi\epsilon_0}$ =9.0 × $10^9 Nm^2/C^2 = Coulomb'sconstant$

ϵ_0 is a permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-11}\ C^2/N \cdot m^2$

The constant k is called the Coulomb constant; in SI units $k = 9.0 \times 10^9 \frac{N \cdot m^2}{C^2}$.

Note

The electrostatic force between any two point charges at rest has the following properties:

1. $F_e \propto q_1 q_2$ and $F_e = \frac{1}{r^2}$
2. The direction of the electrostatic force on each charge is along the radius/line joining the centers of the two charges.

3. Since charge comes in (+) and (-) kinds, the electrostatic force may be either attractive or repulsive. i.e. If the charges are of Like charges the force is repulsive and for unlike charges the force will be attractive.

b. Superposition principle

If more than two charged particles exist in a system, then the Coulomb force exerted on one particle is the summation of all of the Coulomb forces between that particle and the rest of the particles in the ensemble (known as the principle of superposition.)

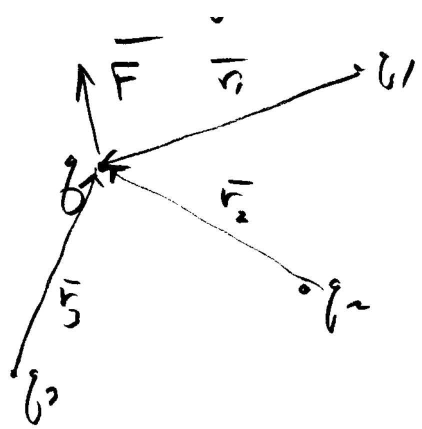
Principle of superposition states that "the total electrostatic force on q_1 exerted by a number of other point charges is the vector sum of the forces by the individual charges."

If there are N charges q_1 , q_2, q_3 , q_4................ q_N, the resultant force due to these charges on the charge q_1 is given by

$$\overline{F}_{1i} = \sum \frac{kq_1q_i}{r_{1i}^2}\hat{r}_{1i}$$

For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\vec{F_1} = \vec{F_{21}} + \vec{F_{31}} + \vec{F_{41}}$$



$$\overline{F}_{1} = \sum \frac{kq_1q_i}{r_{1i}^2}\hat{r}_{1i}$$

Naturally, this vector sum is performed using the force components along the axes of a selected reference frame.

1.3. Electric Field due to a point charge

a. Definition of electric field

An electric field is a region around which electrostatic force is being experienced by stationary charges.
The concept of an *electric field* is a means of representing the influence of a charge on other, distant charges. The *source charge* gives rise to an electric field that extends throughout space. Other charges in the field experience an electrostatic force.

b) Electric field lines
An electric field is represented pictorially using *electric field lines*. Electric field lines are a set of imaginary lines which are used to visualize the electric field strength at a given point. These are lines drawn according to the following rules:

i) The electric field at any point is tangent to the electric field line.
ii) The spacing, or density, of the field lines is proportional to the magnitude of the electric field. i.e. where the magnitude is greater, the lines are more closely spaced.
iii) Field lines originate from (+) charges and end on (-) charges.

Quantitatively, the electric field at a point is space is derived from the net electric force on a small positive test charge, q_o, located at that point.
Hence analytically the electric field vector **E** at a point in space is defined as ' the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the test charge':

$$E = \frac{F_e}{q_o}$$

The vector **E** has the SI units of Newton's per coulomb (N/C).

Note

E is the field produced by some charge or charge distribution *separate from* the test charge—it is not the field produced by the test charge itself. Also, note that the existence of an electric field is a property of its source—the presence of the test charge is not necessary for the field to exist. The test charge serves as a *detector* of the electric field. The Equation electric field can be rearranged as

$$F_e = E\ q_o$$

where we have used the general symbol q for a charge. This equation gives us the force on a charged particle placed in an electric field. If q is positive, the force is in the same direction as the field. If q is negative, the force and the field are in opposite directions (as shown in the following figure).

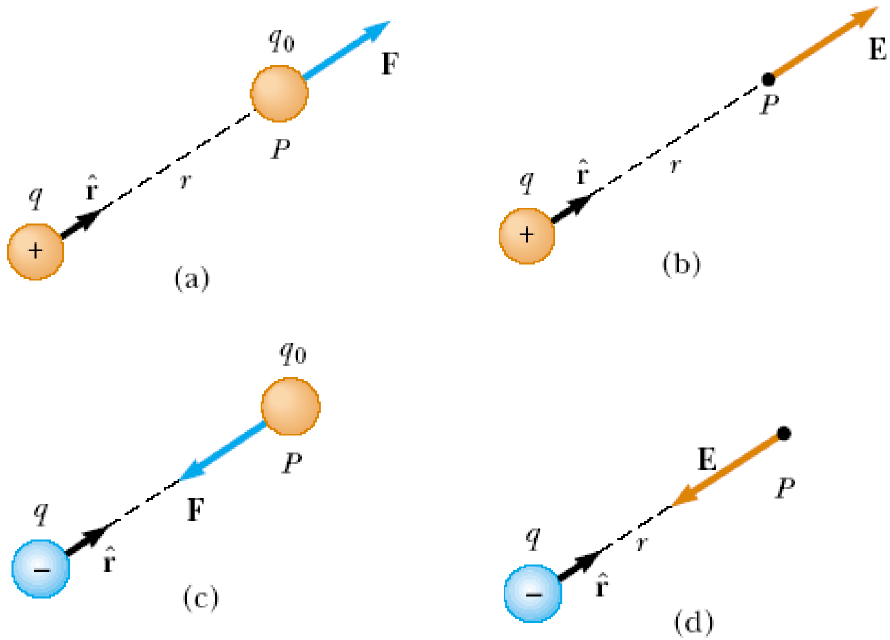


Figure (1)

b) Electric field due to a point charge

To determine the electric field due to a single point charge at a given point , consider a point charge $\boldsymbol{q}$ as a source charge as shown in Figure 1-a . A test charge $\boldsymbol{q_0}$ is placed at point P, a distance $\mathbf{r}$ from the source charge.
According to Coulomb's law, the force exerted by q on the test charge is

$$F_e = K_e \frac{q\, q_o}{r^2} \hat{r}$$

$$\because \; E = \frac{F_e}{q_o}$$

$$\therefore \; E = K_e \frac{q}{r^2} \hat{r}$$

where $\mathbf{\hat{r}}$ is a unit vector directed from q toward q_0.

If the source charge q is positive, Figure 1-b shows the situation with the test charge removed—the source charge sets up an electric field at point P, directed away from q. If q is negative, as in Figure 1-c, the force on the test charge is toward the source charge, so the electric field at P is directed toward the source charge, as in Figure 1-d.

c) Electric field due to a system of discrete charges

The resultant/total electric field strength at any point *P* due to a group of source charges equals the vector sum of the electric fields of all the charges. Thus, the electric field at point *P* due to a group of source charges can be expressed as the vector sum

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

where $\mathbf{r_i}$ is the distance from the **i th** source charge $\boldsymbol{q_i}$ to the point *P* and $\hat{r}_i$ is a unit vector directed from qi toward *P*.

<u>Example 1</u> (**Electric Field Due to Two Charges)**
A charge q_1= 7.0 µC is located at the origin, and a second charge $q2$ = -5.0 µC is located on the x axis, 0.30 m from the origin (Figure 2). Find the electric field at the point *P*, which has coordinates (0, 0.40) m.

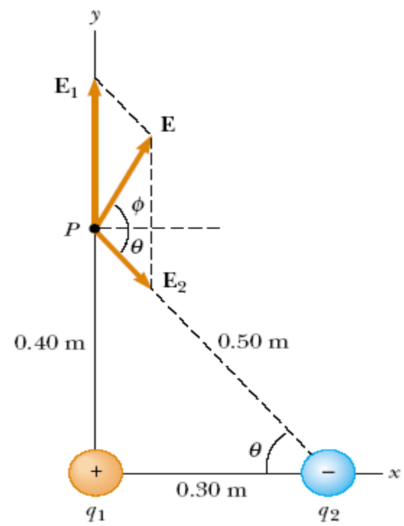


Figure (2)

Solution

First, let us find the magnitude of the electric field at *P* due to each charge. The fields $\mathbf{E_1}$ due to the 7.0 µC charge and $\mathbf{E_2}$ due to the - 5.0 µC charge are shown in Figure 2. Their magnitudes are

$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(7.0 \times 10^{-6}\ \mathrm{C})}{(0.40\ \mathrm{m})^2}$$
$$= 3.9 \times 10^5\ \mathrm{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.0 \times 10^{-6}\ \mathrm{C})}{(0.50\ \mathrm{m})^2}$$
$$= 1.8 \times 10^5\ \mathrm{N/C}$$

The vector $\mathbf{E_1}$ has only a $\boldsymbol{y}$ component. The vector $\mathbf{E_2}$ has an $\boldsymbol{x}$ component given by E_2 Cos $\theta = \frac{3}{5}$ $\boldsymbol{E_2}$ and a negative y component given by E_2 Sin $\theta = \frac{4}{5}$ $\boldsymbol{E_2}$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}} \text{ N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

The resultant field **E** at P is the superposition of $\mathbf{E_1}$ and $\mathbf{E_2}$:

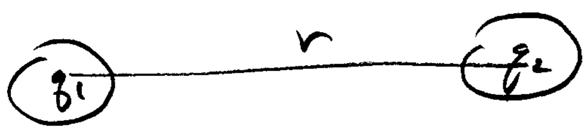
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

From this result, we find that **E** makes an angle ϕ of 66° with the positive x axis and has a magnitude of 2.7×10^5 N/C

1.4. Electric filed due to a dipole

(i) electric dipole

- It is a system of equal and opposite charges separated by a small distance.
- It is the arrangement or configuration of a pair of equal and opposite charges separated by a small distance d.



An electric dipole

(ii) Dipole moment of a dipole

The electrical properties of an electric dipole are characterized completely by its electric dipole moment ($\vec{P}$)

The strength and orientation of an electric dipole are described by its dipole moment ($\vec{P}$)

The electric dipole moment ($\vec{P}$) of an electric dipole is defined as " the product of the magnitude of the charge q making the dipole and the separation d between them."

i.e. $\vec{P}$=qd

<u>**Note**</u>

The direction of $\vec{P}$ is always from the negative charge to the positive charge.

The SI unit of $\vec{P}$ is coulomb meter (Cm).

<u>Example 2</u> (Electric Field of a Dipole)

An electric dipole is defined as a positive charge **+ q** and a negative charge **- q** separated by a distance **2*a***. For the dipole shown in Figure 3, find the electric field **E** at *P* due to the dipole, where *P* is a distance **y >> a** from the origin.

Solution

At *P*, the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ due to the two charges are equal in magnitude because *P* is equidistant from the charges. The total field is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where

$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

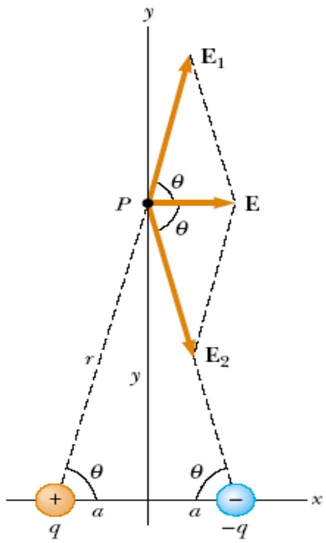


Figure (3)

The ***y*** components of $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel each other, and the ***x*** components are both in the positive ***x*** direction and have the same magnitude. Therefore, **E** is parallel to the ***x*** axis and has a magnitude equal to 2 E_1 Cos θ. From Figure 3 we see that

$$Cos\ \theta = \frac{a}{r} = \frac{a}{\left(y^2 + a^2\right)^{\frac{1}{2}}}$$

Therefore

$$E = 2E_1 \cos\theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}}$$

$$= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}$$

Because $y >> a$, we can neglect a^2 compared to y^2 and write

$$E \approx \boxed{k_e \frac{2qa}{y^3}}$$

1.5. Electric Field of a Continuous Charge Distribution

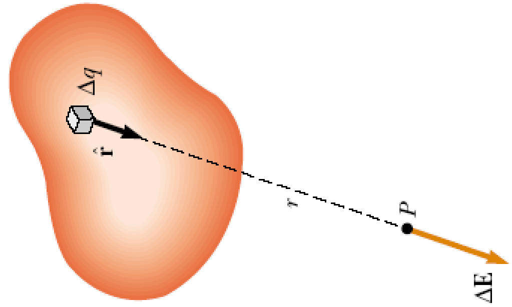


Figure (4)

To evaluate the electric field created by a continuous charge distribution, we use the following procedure: first, we divide the charge distribution into small elements, each of which contains a small charge $0q$, as shown in Figure 4. Next, we use the following Equation to calculate the electric field due to one of these elements at a point *P*.

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

Finally, we evaluate the total electric field at *P* due to the charge distribution by summing the contributions of all the charge elements.

The electric field at *P* due to one charge element carrying charge Δq is

$$\Delta\mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

where **r** is the distance from the charge element to point *P*. The total electric field at *P* due to all elements in the charge distribution is approximately

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

where the index ***i*** refers to the ***i*** **th** element in the distribution. Because the charge distribution is modeled as continuous, the total field at *P* in the limit $\boldsymbol{\Delta q_i} \to 0$ is

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

where the integration is over the entire charge distribution. We illustrate this type of calculation with several examples, in which we assume the charge is uniformly distributed on a line, on a surface, or throughout a volume. When performing such calculations, it is convenient to use the concept of a *charge density* along with the following notations:

Volume charge density

If a charge Q is uniformly distributed throughout a volume V, the volume charge density is ρ defined by

$$\rho \equiv \frac{Q}{V}$$

where ρ has units of coulombs per cubic meter (C/m^3).

Surface charge density

If a charge Q is uniformly distributed on a surface of area A, the surface charge density σ is defined by

$$\sigma \equiv \frac{Q}{A}$$

where σ has units of coulombs per square meter (C/m^2).

Linear charge density

If a charge Q is uniformly distributed along a line of length L , the linear charge density λ is defined by

$$\lambda \equiv \frac{Q}{\ell}$$

where λ has units of coulombs per meter (C/m).

If the charge is non uniformly distributed over a volume, surface, or line, the amounts of charge *dq* in a small volume, surface, or length element are

$$dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$$

<u>Example 3</u> (The Electric Field Due to a Charged Rod)

A rod of length L has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance $\boldsymbol{a}$ from one end (Figure 5).

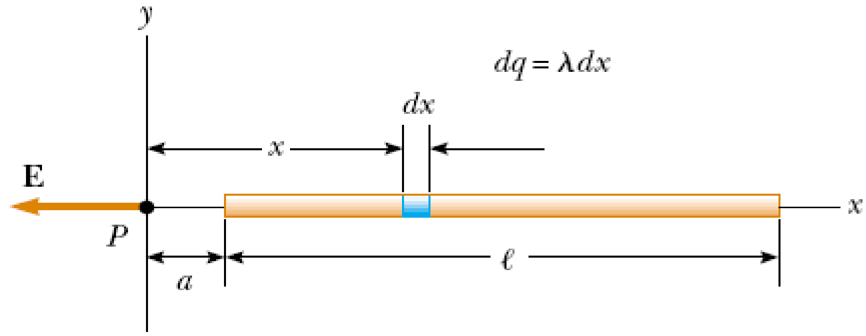


Figure (5)

Solution

Let us assume that the rod is lying along the ***x*** axis, that ***dx*** is the length of one small segment, and that ***dq*** is the charge on that segment. Because the rod has a charge per unit length λ, the charge ***dq*** on the small segment is ***dq*** = λ ***dx***.

The field ***d*E** at *P* due to this segment is in the negative ***x*** direction (because the source of the field carries a positive charge), and its magnitude is

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

$$E = \int_a^{a+L} K \frac{dq}{x^2} = \int_a^{a+L} K \frac{\lambda\, dx}{x^2}$$

where the limits on the integral extend from one end of the rod ($x = a$) to the other ($x=L+a$). The constants k_e and λ can be removed from the integral to yield

$$E = K\lambda \int_a^{a+L} \frac{dx}{x^2} = K\lambda \left[-\frac{1}{x} \right]_a^{a+L}$$

$$E = K\lambda \left[\frac{1}{a} - \frac{1}{a+L} \right] = K\lambda \left[\frac{L}{(a+L)a} \right]$$

<u>Example 4</u> (The Electric Field of a Uniform Ring of Charge)

A ring of radius ***a*** carries a uniformly distributed positive total charge ***Q***. Calculate the electric field due to the ring at a point *P* lying a distance ***x*** from its center along the central axis perpendicular to the plane of the ring (Figure 6 - a).

Solution

The magnitude of the electric field at ***P*** due to the segment of charge ***dq*** is

$$dE = K_e \frac{dq}{r^2}$$

This field has an ***x*** component $dE_x = dE \cos\theta$ along the ***x*** axis and a component $dE_\perp$ perpendicular to the ***x*** axis. As we see in **Figure 6-b**, however, the resultant field at *P* must lie along the *x* axis because the perpendicular components of all the various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r = \sqrt{x^2 + a^2}$ and **Cos θ =** $\boldsymbol{x/r}$, we find that

$$dE = K\frac{dq}{r^2} = K\frac{dq}{a^2 + x^2}$$

$$dE_x = dE \;\; Cos\;\theta$$

$$E_x = \int dE\cos\theta = \int\left(K_e\frac{dq}{a^2+x^2}\right)\left(\frac{x}{\left(a^2+x^2\right)^{1/2}}\right)$$

$$= \frac{K_e\; x}{\left(a^2+x^2\right)^{1/2}} \int dq$$

$$E = K_e \frac{q\;x}{\left(a^2+x^2\right)^{3/2}}$$

<u>**This result shows that the field is zero at *x* = 0. Does this finding surprise you?**</u>

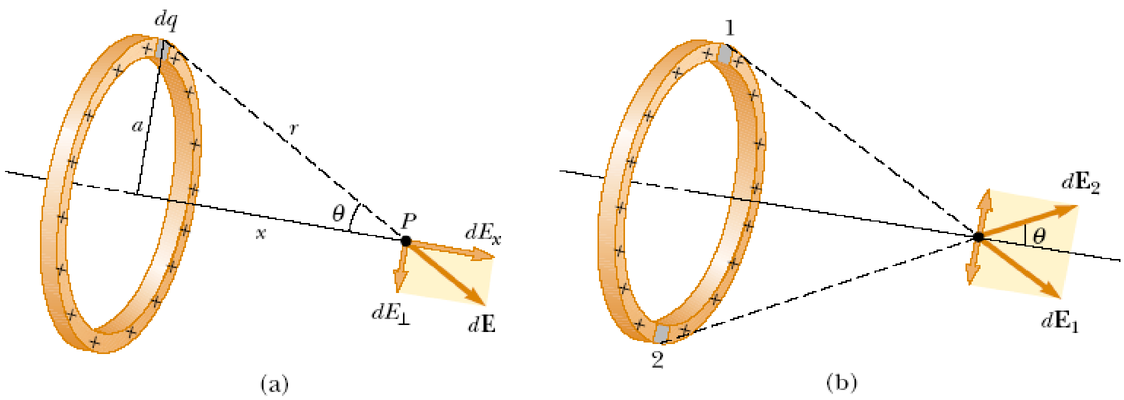


Figure (6)

<u>**Example 5**</u> **(The Electric Field of a Uniformly Charged Disk)**

A disk of radius $\boldsymbol{R}$ has a uniform surface charge density σ. Calculate the electric field at a point $\boldsymbol{P}$ that lies along the central perpendicular axis of the disk and a distance x from the center of the disk (Figure 7).

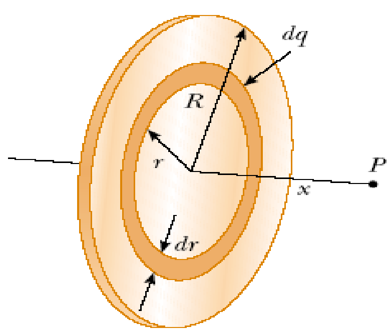


Figure (7)

Solution

If we consider the disk as a set of concentric rings, we can use our result from Example 4 - which gives the field created by a ring of radius *a* - and sum the contributions of all rings making up the disk. By symmetry, the field at an axial point must be along the central axis.

The ring of radius *r* and width *dr* shown in Figure 7 has a surface area equal to **2 π r dr**. The charge *dq* on this ring is equal to the area of the ring multiplied by the surface charge density: ***dq* = 2 π σ r dr**. Using this result in the equation given for *Ex* in Example 4 (with ***a*** replaced by **r**), we have for the field due to the ring

$$dE_x = K_e \; \frac{x}{\left(x^2 + r^2\right)^{3/2}} \; (2\pi \;\; \sigma r \; dr)$$

To obtain the total field at *P*, we integrate this expression over the limits **r = 0** to **r = R**, noting that ***x*** is a constant.

$$E_x = K_e \;\; x \;\; \pi \;\; \sigma \int_0^R \frac{2rdr}{\left(x^2 + r^2\right)^{3/2}}$$

$$= K_e 2 \;\; \pi \;\; x \;\; \sigma \int_0^R \left(x^2 + r^2\right)^{-3/2} d\left(r^2\right)$$

$$= \frac{1}{2 \;\pi \;\varepsilon_o} \;\; 2 \;\; \pi \;\; x \;\; \sigma \int_0^R \left(x^2 + r^2\right)^{-3/2} d\left(r^2\right)$$

$$= \left(\frac{x\sigma}{4\varepsilon_0}\right) \int_0^R \left(x^2 + r^2\right)^{-3/2} \;\; d\left(r^2\right)$$

$$= \left(\frac{x \;\; \sigma}{4 \;\; \varepsilon_0}\right) \left[\frac{\left(\; x^2 \; + \; r^2\right)^{-1/2}}{-1/2} \right]_0^R$$

Then

$$E_x = \left(\frac{\sigma}{2\varepsilon_0}\right) \left[1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}} \right]$$

When $x \to 0$ and $R \to \infty$

$$E_x \; = \; \frac{\sigma}{2 \;\; \varepsilon_0}$$

1.6. Motion of Charged Particles in a Uniform Electric Field

When a particle of charge *q* and mass *m* is placed in an electric field **E**, the electric force exerted on the charge is *q***E**. If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law. Thus,

$$F_e \; = \; q \; E = \; m \; a$$

The acceleration of the particle is therefore

$$a = \frac{q\ E}{m}$$

If **E** is uniform (that is, constant in magnitude and direction), then the acceleration **a** is constant. If the particle has a positive charge, its acceleration is in the direction of the electric field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Example 6 (An Accelerating Positive Charge)

A positive point charge q of mass m is released from rest in a uniform electric field **E** directed along the x axis, as shown in Figure 11. Describe its motion.

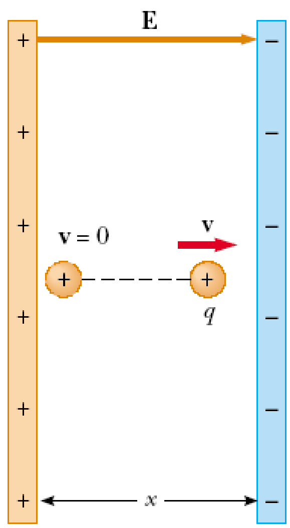


Figure (11)

Solution

The acceleration is constant and is given by

$$a = \frac{q\ E}{m}$$

The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one dimension

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$v_f = v_i + at$$

$$v_f^{\,2} = v_i^{\,2} + 2a(x_f - x_i)$$

Choosing the initial position of the charge as $x_i = 0$ and assigning $v_i = 0$ because the particle starts from rest, the position of the particle as a function of time is

$$x_f = \frac{1}{2}at^2 = \frac{qE}{2m}\,t^2$$

The speed of the particle is given by

$$v_f = at = \frac{qE}{m}\,t$$

The third kinematic equation gives us

$$v_f^{\,2} = 2ax_f = \left(\frac{2qE}{m}\right)x_f$$

from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_f - x_i$:

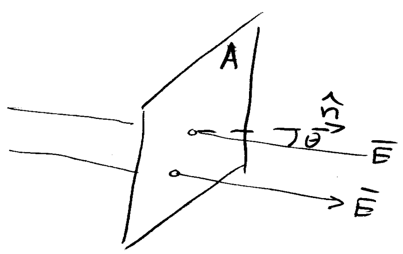
$$K = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{2qE}{m}\right)\Delta x = qE\Delta x$$

1.7. Electric flux (Ψ) and Gauss's law

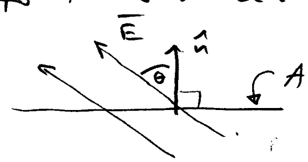
a) Electric flux (Ψ)

An electric flux(Ψ) is a measure of the number of E-field lines that crosses a given area.

Imagine that electric field lines represent the "flow" of something from one place to another. Now, consider field lines passing through a possibly imaginary surface of area A.



The field lines make an angle θ with a normal, $\hat{n}$, to the surface:



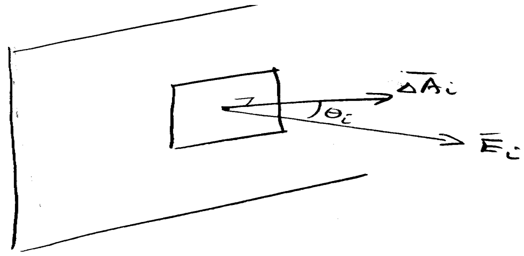
If the electric field lines make an angle, θ, with the surface normal, $\hat{n}$. We define the *electric flux* to be $\Psi = EA\cos\theta = \overline{E}\cdot A\hat{n}$.

Note

- For an E-field whose lines penetrate a cross-sectional area A perpendicular to surface the electric flux will be given by $\Psi = EA$

- If the electric field may not be uniform over the entire surface, so we break the surface into surface elements, and integrate. Thus the electric flux will be given by

$$\Psi = \sum \overline{E}_i \cdot \Delta A_i \hat{n}_i \rightarrow \int_{surface} \overline{E} \cdot d\overline{A}$$



- If the surface is a closed surface, then unless there are source charges inside the surface, the flux entering the closed surface equals the flux leaving, and the net flux through the closed surface is zero.

b) Gauss's law

One of the most important law's in all of electromagnetism is Gauss's law

Gauss's law states that "The net electric flux Ψ_c through any closed surface is equal to the net enclosed electric charge q divided by the permittivity of free space ε_o."

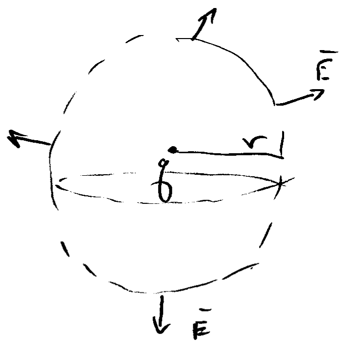
i.e. $\Psi_e = \int_{surface} \overline{E} \cdot d\overline{A} = \dfrac{q}{\varepsilon_o}$

where q is nothing more than the total net electric charge enclosed by the surface. The constant $\varepsilon_o = 8.85 \times 10^{-12} \dfrac{C^2}{N \cdot m^2}$ is called the *permittivity* of free space (or of the vacuum).

Note

- Gauss's law states that the total electric flux out of a closed surface is proportional to the charge enclosed which then yields the expression for the electric field strength around the charged object
- While using gauss's law we need to use a closed hypothetical surface called **Gaussian surface**.
- Gauss' "Law" is particularly useful when the source charge distribution is highly symmetric in shape. That such a case, the flux is easy to compute, and the E-field can be obtained without carrying out an integral over the source charge distribution.

For instance, we can demonstrate the validity of Gauss' "Law" for a point charge source.



We know that $\bar{E}$ is everywhere $\perp$ to spherical surface, so

$$\sum \bar{E}_i \cdot \Delta \bar{A}_i = EA = E\, 4\pi r^2$$

On the other hand, from Coulomb's law $E = \dfrac{kq}{r^2}$

$$\psi = E\,A$$

$$\psi = \frac{kq}{r^2} \cdot 4\pi r^2$$

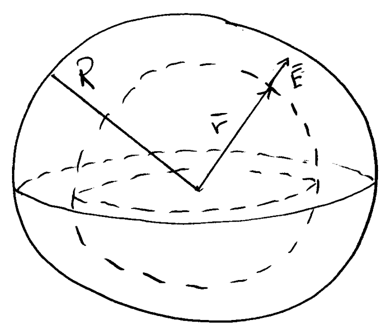
$$\psi = 4\pi k q = \frac{q}{\varepsilon_o} \quad .$$

ta-dah

We use this to find E, given a shape + Q.

c) Applications of Gauss's law

i) Uniform spherical distribution of charge



$$\oint \bar{E} \cdot d\bar{a} = \frac{Q}{\varepsilon_o}$$

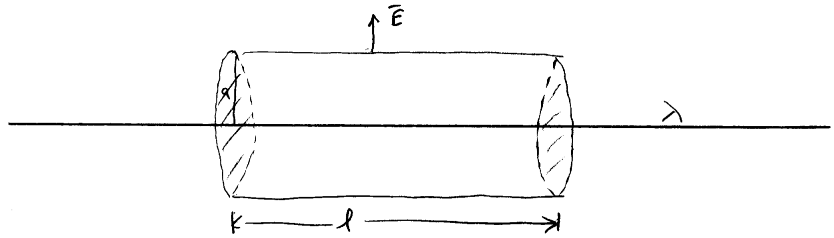
$$E\, 4\pi\, R^2 = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{4\pi\varepsilon_o\, R^2}$$

This seems almost too easy. It requires that we realize that the electric field will be the same magnitude, and directed outward (or inward) all around at a specified distance from the spherically shaped, uniform charge distribution.

ii) line of charge

Example   Infinite line of charge



We know that $\bar{E}$ is entirely $\perp$ to the line. So, there will be no flux through the ends of the cylindrical surface. We know, too, that the magnitude of E depends on a.

$$\psi = 0 \cdot \pi a^2 \cdot 2 + E\, 2\pi a l$$

$$\psi = 2\pi a l E.$$

By Gauss' law

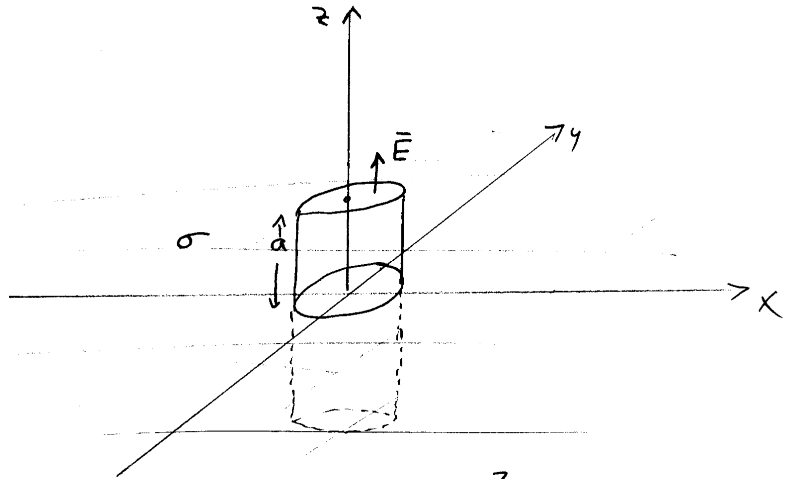
$$\psi = \frac{Q}{\epsilon_0}, \quad \text{with } Q = \lambda l.$$

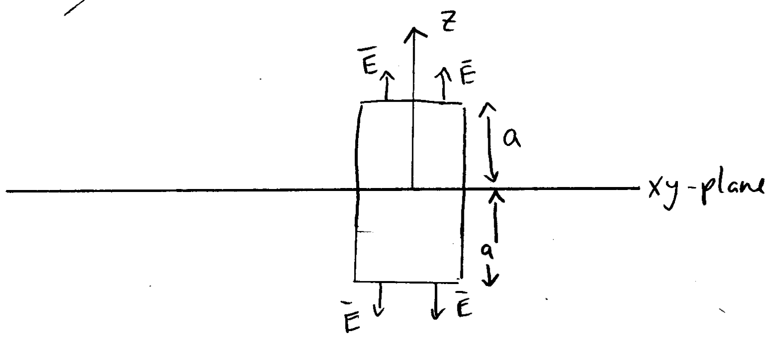
So we obtain

$$2\pi a l E = \frac{\lambda l}{\epsilon_0}, \quad \text{solve for } E$$

$$E = \frac{\lambda}{2\pi \epsilon_0 a} = \frac{2\lambda}{4\pi \epsilon_0 a} = \frac{2k\lambda}{a} \; !$$

iii) infinite sheet of charge





$$\bar{E} = E\hat{z}$$

$$\Phi_E = 2E\pi r^2 = \frac{q_{enclosed}}{\epsilon_0}$$

$$2E\pi r^2 = \frac{\sigma \pi r^2}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

Notice that the electric field points upward <u>and downward</u> from the sheet of charge. Therefore the flux goes out both the top and bottom of the cylindrical surface.