Unit 7 : Electromagnetic Induction

7.1 Magnetic Flux

The flux associated with a magnetic field is defined in a manner similar to that used to define electric flux. Consider an element of area dA on an arbitrarily shaped surface, as shown in Figure 7.1. If the magnetic field at this element is B, the magnetic flux through the element is $\vec{B} \cdot d\vec{A}$, where $d\vec{A}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dA. Therefore, the total magnetic flux ϕ_B through the surface is

$$\phi_B = \int \vec{B} \cdot d\vec{A} \qquad (1)$$

Consider the special case of a plane of area A in a uniform field B that makes an angle θ with $d\vec{A}$. The magnetic flux through the plane in this case is

$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta \qquad (2)$$

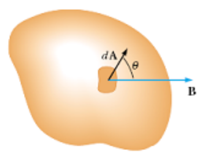


Fig. 7.1

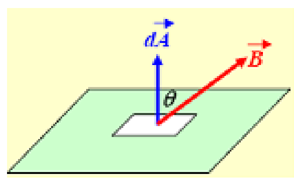


Fig. 7.2

∗ SI Unit for magnetic flux is $T \cdot m^2 = Wb$ (1 Weber).

7.2 Gausss Law in Magnetism

In unit 1 we found that the electric flux through a closed surface surrounding a net charge is proportional to that charge (Gausss law). In other words, the number of electric field lines leaving the surface depends only on the net charge within it. This property is based on the fact that electric field lines originate and terminate on electric charges.

The situation is quite different for magnetic fields, which are continuous and form closed loops. In other words, magnetic field lines do not begin or end at any pointas illustrated in Figures 7.3. Figure 7.3 shows the magnetic field lines of a bar magnet. Note that for any closed surface, such as the one outlined by the dashed line in Figure 7.3, the number of lines entering the surface equals the number leaving the surface; thus, the net magnetic flux is zero. In contrast, for a closed surface surrounding one charge of an electric dipole (Fig. 7.4), the net electric flux is not zero.

Gausss law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\int \vec{B} \cdot d\vec{A} = 0 \qquad (3)$$

Equation (3) is called **Gauss's Law in magnetism**.

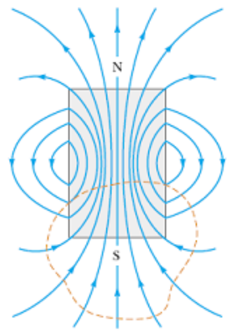


Figure 7.3 The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through a closed surface surrounding one of the poles (or any other closed surface) is zero. (The dashed line represents the intersection of the surface with the page.)

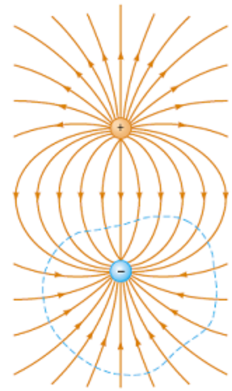


Figure 7.4 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge. The electric flux through a closed surface surrounding one of the charges is not zero.

This statement is based on the experimental fact that isolated magnetic poles (monopoles) have never been detected and perhaps do not exist. Nonetheless, scientists continue the search because certain theories that are otherwise successful in explaining fundamental physical behavior suggest the possible existence of monopoles.

7.3 Faraday's Law of Induction

Experiments conducted by Michael Faraday in England in 1831 and independently by Joseph Henry in the United States that same year showed that an emf can be induced in a circuit by a changing magnetic field. The results of these experiments led to a very basic and important law of electromagnetism known as Faradays law of induction. An emf (and therefore a current as well) can be induced in various processes that involve a change in a magnetic flux.

- A magnet moving into/out of a loop of wire induces a current

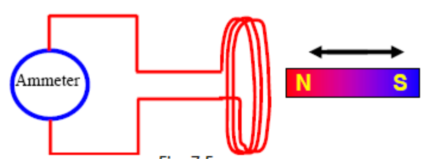


Fig. 7.5

- Starting/stopping current through loop in one coil (primary) causes pulse of current in coil (secondary) linked by iron coil - this is a transformer

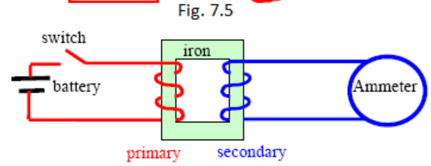


Fig. 7. 6

The experiments shown in Figures 7.5 and 7.6 have one thing in common: in each case, an emf is induced in the circuit when the magnetic flux through the circuit changes with time.

In general, **the emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.**

This statement, known as **Faradays law of induction**, can be written

$$\varepsilon \;=\; -\frac{d\phi_B}{dt} \tag{4}$$

where $\phi_B \;= \int \vec{B}\cdot d\vec{A}$ is the magnetic flux through the circuit. If the circuit is a coil consisting of N loops all of the same area and if ϕ_B is the magnetic flux through one loop, an emf is induced in every loop. The loops are in series, so their $emfs$ add; thus, the total induced emf in the coil is given by the expression

$$\varepsilon \;=\; -N\frac{d\phi_B}{dt} \tag{5}$$

The negative sign in Equations (4) and (5) is of important physical significance, Lenz's law gives its significance.

Sources of time-varying magnetic flux $\Phi_B(t)$

- look at loop of area A in field $\vec{B}$ with angle θ between $\vec{B}$ and normal $\vec{A}$
  - magnetic flux is: $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$

SO Faraday's law is $\varepsilon = -\dfrac{d(BA\cos\theta)}{dt}$

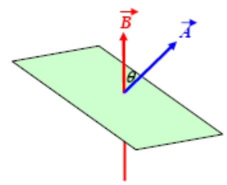
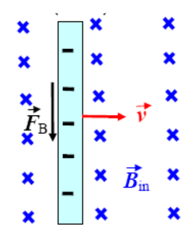


Fig. 7.7

Can induce emf by:
- varying magnitude B
- varying magnitude A
- varying angle θ (i.e. rotating coil as in a generator)
- combination of above.

7.4 emf **Induced by motion of conductor through magnetic field**

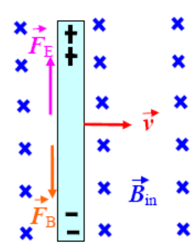
1st: **Look at straight conductor:**
- Conductor $\perp$ to $\vec{B}$; $\vec{v}$ $\perp$ to $\vec{B}$; $\vec{v}\perp$ to length of conductor
- Magnetic force on free electrons in conductor is:
  $\vec{F} = -e\vec{v}\times\vec{B} = evB$ (down)
  - lower end → more negative; upper end → more positive
- electrons move **UNTIL** resulting electric and magnetic forces balance



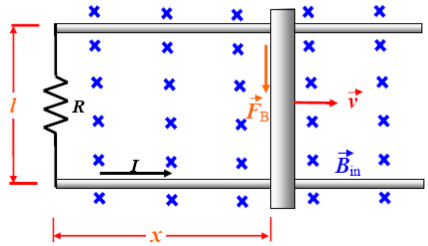
- Balances when $|F_E| = |F_B|$ so that $eE = evB$
- Magnitude of potential difference between conductor ends:

$$|\Delta V| = El = Blv$$

- As long as $\vec{v}$ is constant
- Top of conductor is positive (here)



2nd: **Complete circuit:** Bar sliding on conducting rails connected by resistance R

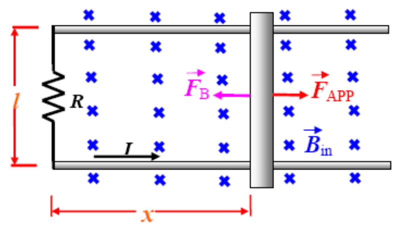


- Pull bar to right with force $\vec{F}_{APP}$
- Can get direction of I from picture in previous section
- What is magnitude of I ?

Use Faraday's Law:

- Magnetic flux through area bounded by circuit is $\Phi_B = Blx$
- Induced emf is $\varepsilon = -\dfrac{d\Phi_B}{dt} = -Bl\dfrac{dx}{dt} = -Blv$
- Magnitude of current is $I = \dfrac{|\varepsilon|}{R} = \dfrac{Blv}{R}$

Sliding Bar: Power dissipated (as heat) in resistance is $P = I^2 R$



- Where does this energy come from?

Look at force needed to pull bar to right at constant speed?

Current flows up in bar – causes magnetic force on bar : $\vec{F}_B = I\vec{l} \times \vec{B} = IlB\,(\text{left})$

For constant speed, net force is zero.

- So applied force is: $\vec{F}_{APP} = IlB\,(\text{right})$

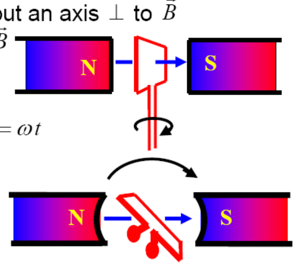
Power delivered by applied force is $P = F_{APP}v = IlBv$

- But from $I = \dfrac{Blv}{R}$ can write $v = \dfrac{IR}{Bl}$

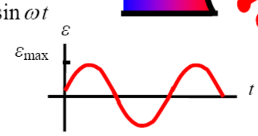
So Power delivered by applied force is $P = IlB\dfrac{IR}{Bl} = I^2 R$

- Same as power dissipated in resistor – CONSERVATION OF ENERGY

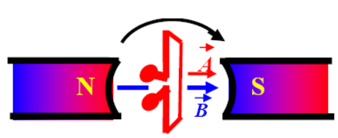
Alternating Current Generator: A loop rotating about an axis $\perp$ to $\vec{B}$



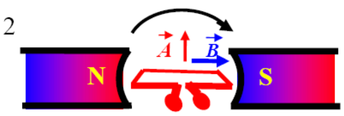
- Flux through loop depends on angle θ between $\vec{B}$ and loop normal vector
  - $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$
- If loop is rotating with angular speed ω, then $\theta(t) = \omega t$
  - Then $\Phi_B(t) = BA\cos\omega t$
- If there are N turns in the loop then induced emf is
  - $\varepsilon = -N\dfrac{d\Phi_B}{dt} = -NAB\dfrac{d\cos\omega t}{dt} = NAB\omega\sin\omega t$
- Result is AC voltage: $\varepsilon = NAB\omega\sin\omega t$
  - Amplitude is $\varepsilon_{max} = NAB\omega$



- Induced emf $\varepsilon = 0$ when $\theta(t) = \omega t = 0$
  - i.e. when loop normal is parallel to $\vec{B}$
  - at this orientation, edges of loop are not cutting any $\vec{B}$ field lines



- Induced emf $\varepsilon = \varepsilon_{\max} = NAB\omega$ when $\theta(t) = \omega t = \pi/2$
  - i.e. when loop normal is perpendicular to $\vec{B}$
  - at this orientation, edges of loop are cutting $\vec{B}$ field lines



- For real AC generator, need "slip rings" to connect to rotating coil
  - AC is OK for heat, lights, etc.
- To get DC voltage, need a "rectifier"

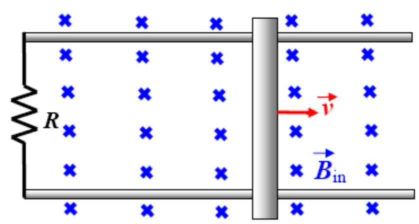
7.5 Lenz's Law

Faradays law (Eq.4) indicates that the induced emf and the change in flux have opposite algebraic signs. This has a very real physical interpretation that has come to be known as **Lenzs law: The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.**

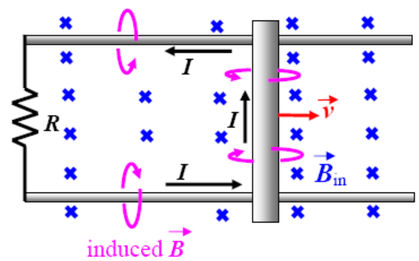
⋆ Lenz's law gives direction of induced current when flux through loop changes.

⋆ Rule for "polarity" of induced emf in loop:

- Example: Bar moving to right **increases** magnetic flux through loop
  - Lenz's law → resulting current should cause field tending to cancel increased flux in loop
  - **SO:** $\vec{B}$ from induced current should be out.

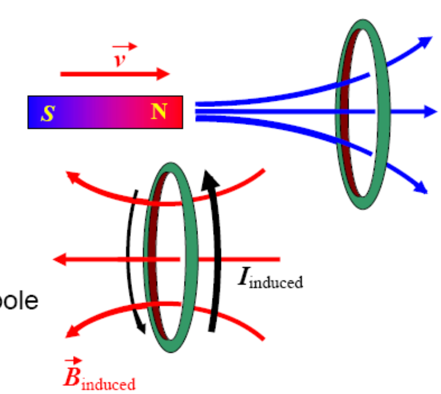


- Means that induced current should be ccw
  - Will generate $\vec{B}$ out of page/screen **INSIDE** the loop (by RH-rule)

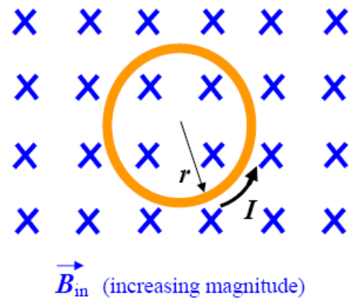


EXAMPLE:

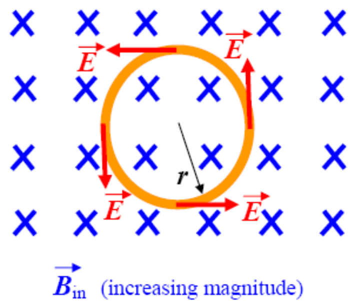
- Move bar magnet right toward loop
  - Increases flux through loop
- By Lenz's Law, current induced in loop should oppose increase in flux through loop
  - Means that induced current should generate field pointing to left
- Result: Current in loop generates magnetic dipole pointing to left
  - Acts like magnet with north pole to left



7.6 Induced emf and Electric Field



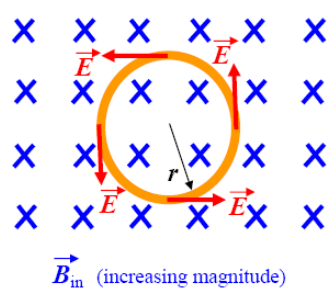
- Look at a conducting loop of radius r in a region of magnetic field $\vec{B}$ pointing into the page/screen
  - The magnetic flux through the loop is $\Phi_B = BA = B\pi r^2$
  - If B is changing, there is an emf around the loop given by Faraday's Law
    - $\varepsilon = -\dfrac{d\Phi_B}{dt} = -\pi r^2 \dfrac{dB}{dt}$
    - Picture shows direction of current for increasing B



- The induced current implies a tangential electric field $\vec{E}$ in the conductor

IMPORTANT INSIGHT:

- Changing magnetic flux **ALWAYS** induces an electric field $\vec{E}$ **EVEN IF NO CHARGES PRESENT**



Connects $\vec{E}$ to $\dfrac{d\vec{B}}{dt}$

- work to move charge q around a loop is

$$W = q \oint_{\substack{\text{around}\\\text{loop}}} \vec{E}\cdot d\vec{s} = q\varepsilon = -q\frac{d\Phi_B}{dt}$$

- Implies: $\displaystyle\oint_{\substack{\text{around}\\\text{loop}}} \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
  - integral is around closed path
  - Φ_B is the flux through the area enclosed by the path
- This is one of **MAXWELL'S EQUATIONS**
  - $\vec{E}$ field in this equation **NOT** electrostatic – does not end on charges