Unit 2 : Electric potential

2. Electric Potential

a) Electrical Potential energy(U)

It is the amount of work that must be done to bring or assemble a system of point charges or a charge from infinity to a given point and form some configuration.

i.e. $U = W$

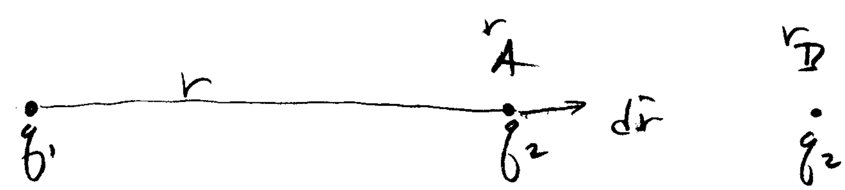
The total electrostatic potential energy of a system of n point charges is equal to the sum of pair-wise terms and is given by

$$U = W = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{kq_i q_j}{rij}$$

For instance

1) For a system of two charges q_1 and q_2 the total electrostatic potential energy will be

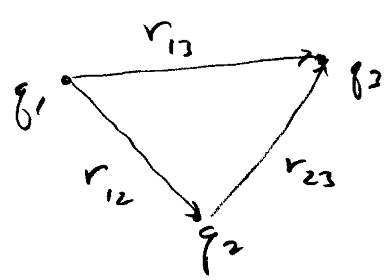
$$U(r_{12}) = \frac{kq_1 q_2}{r_{12}}$$



This claim can be verified by computing the work done by the electrostatic force as two point charges are brought from infinite separation to a separation r_{12}.

2) For a system of three charges the total electrostatic potential energy is given by

$$U = \frac{kq_1 q_2}{r_{12}} + \frac{kq_1 q_3}{r_{13}} + \frac{kq_2 q_3}{r_{23}}$$



b) Electric Potential, or voltage due to a point charge

(i) Definition of Electric Potential

Electric Potential (V) at a point in space is defined as "the work done (equals the potential energy stored in the field) to bring a charge from infinite separation to the vicinity of another charge divided by the charge that is moved.'

Electric Potential is also defined as the potential energy per unit charge

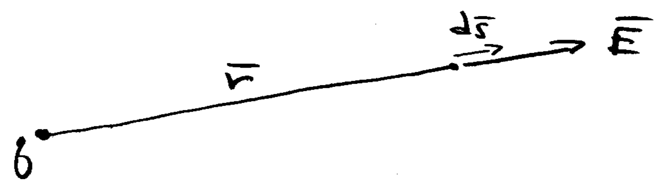
$$V = \frac{U}{q_o}$$

<u>Note</u>

The SI unit of electrostatic potential is the Volt (V). $1V = \frac{J}{C}$. In terms of voltage, the magnitude of the electric field is $\frac{V}{m}$. In this connection, we can define a new unit of energy, the *electron Volt* (*eV*). $1eV = e \cdot 1V = 1.6 \times 10^{-19} J$. It's the energy gained by an electron when experiencing a voltage change of one Volt.

(ii) Electric potential due to a point charge

The electric potential at a given point a distance r from a single point charge q as shown in the figure below is given by $V = \frac{k_e q}{r}$



(iii) Electric potential due to a system of discrete charges

For a system of point charges, the total electric potential at a given point is equal to **"the scalar sum of the electric potentials of the individual point charges at a given point."**

i.e. $V = \sum \frac{k_e q_i}{r_i}$, where r_i is the distance from q_i to the *field point*, the point in space at which we are computing the potential

iv) Electric potential difference (ΔV) in a uniform E-field

The electric potential difference between any two points (ΔV) is " **the work needed to carry a positive test charge from an initial point to a final point per unit test charge**."

$$\Delta V = \frac{\Delta U}{q_o} = \frac{\Delta W}{q_o}$$

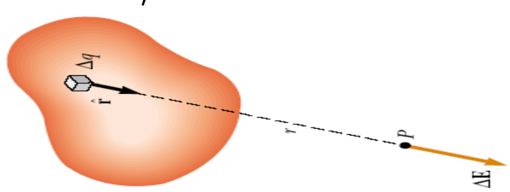
<u>**NOTE**</u>

In the region where there is a <u>uniform</u> electric field, $\overline{E}$, the electric potential difference between the points is given by $\Delta V = E.d$ where d is the distance between the two points.

c) Electric potential due to a continuous charge distribution

For a continuous charge distribution, first we have to calculate the potential due to an infinitesimal charge dq at the given point as $dV = \frac{Kdq}{r}$ and then we have to integrate, in order to find the total electric potential.

i.e $V = \int \frac{Kdq}{r}$



Note

$V = k\int \frac{\lambda ds}{r}$ for line or a ring of charge.

$V = k\int \frac{\sigma da}{r}$ for a surface or layer of charge.

$V = \mathrm{k}\int \frac{\rho \mathrm{dv}}{\mathrm{r}}$ for a volume charge distribution.

d) Relations between electric field (E)and electric potential (V)

- ✓ If the electric field(E) between any two points along the path connecting the points is known , then we can calculate the electric potential difference between the two points as $\Delta V = V_b - V_a = -\int_a^b \vec{E}.d\vec{s}$

  Note

  If we choose $a = \infty$ **then** $V_a = 0$ and $V_b = V$, The above equation will be

  $$V = -\int_{\infty}^{b} \vec{E}.d\vec{s}$$

- ✓ If the electric potential(V) over a certain region of space is known , we can calculate the vector electric filed (E) over the same region a

  $$\overline{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = -\nabla V$$

Where $\nabla = -\frac{\partial}{\partial x}\hat{i} - \frac{\partial}{\partial y}\hat{j} - \frac{\partial}{\partial z}\hat{k}$ is called the del or gradient operator.

e) Equipotential lines/surfaces and their properties

Equipotential lines/surfaces are family of curves or surfaces connecting points having the same value of electric potential.
i.e. An Equipotential is a curve or surface in space along which the electric potential has the same value.

Properties of Equipotential Surfaces

Equipotential Surfaces have the following additional properties:

i) The potential difference between any two points on an Equipotential surface is always zero. i.e. $\Delta V = V_b - V_a = 0$

ii) There is no work done in moving a test charge between any two points on an Equipotential surface.

iii) Equipotential surfaces are always perpendicular to the electric field

iv) Equipotential lines or surfaces never cross each other—if they did so it would imply that there were two values of potential at the same point in space.