Unit 5 : Magnetic Force

5.1 Introduction to magnetic fields

A bar magnet is a source of a magnetic field $\vec{B}$ as can be readily demonstrated by moving a compass near the magnet. The compass needle will line up along the direction of the magnetic field produced by the magnet, as depicted in Figure 5.1. A bar magnet consists of two poles known as the **north** (N) and the **south pole** (S). Magnetic fields are strongest at the poles. Magnetic field lines leave from the north pole and enter the south pole. When holding two bar magnets close to each other, the like poles will repel each other while the opposite poles attract (Figure 5.2).

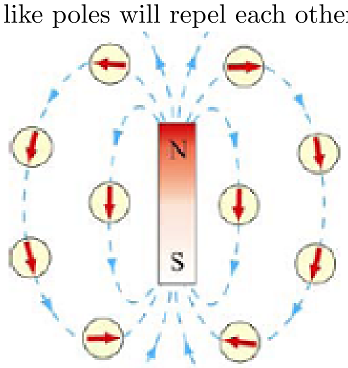

Fig. 5.1 Magnetic field of a bar magnet

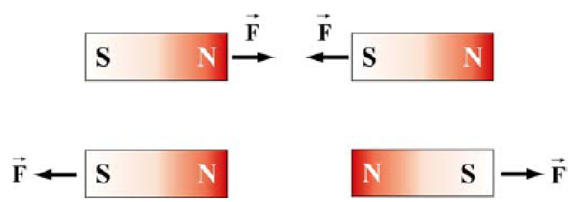

Figure 5.2 Magnets attracting and repelling

Unlike electric charges which can be isolated, the two magnetic poles always come in a pair. When you break the bar magnet, two new bar magnets are obtained, each with a north pole and a south pole (Figure 5.3). In other words, magnetic "monopoles" do not exist in isolation, although they are of theoretical interest.

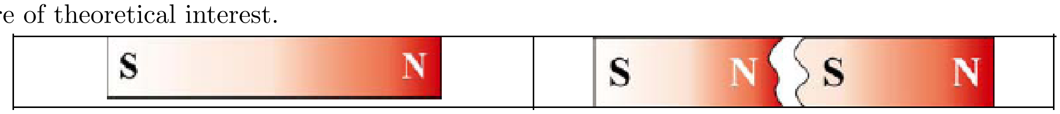

Figure 5.3 Magnetic monopoles do not exist in isolation

How do we define the magnetic field $\vec{B}$? In the case of an electric field $\vec{E}$, we have already seen that the field is defined as the force per unit charge:

$$\vec{E} = \frac{\vec{F}}{q}$$

Due to the absence of magnetic monopoles, $\vec{B}$ must be defined in a different way.

5.2 The Definition of a Magnetic Field

To define the magnetic field at a point, consider a particle of charge q and moving at a velocity $\vec{v}$. Experimentally we have the following observations:

(1) The magnitude of the magnetic force $\vec{F_B}$ exerted on the charge is proportional to both v and q.

(2) The magnitude and direction of $\vec{F_B}$ depends on $\vec{v}$ and $\vec{B}$

(3) $\vec{F_B}$ vanishes when $\vec{v}$ is parallel to $\vec{B}$. However, when $\vec{v}$ makes an angle θ with $\vec{B}$, the direction of $\vec{F_B}$ is $\perp$ to the plane formed by $\vec{v}$ and $\vec{B}$, the magnitude of $\vec{F_B}$ is proportional to $\sin\theta$.

(4) When the sign of the charge of the particle is switched from positive to negative (or vice versa), the direction of the magnetic force also reverses.

The above observations can be summarized with the following equation:

$$\vec{F_B} = q\vec{v} \times \vec{B} \quad (1)$$

The magnitude of $\vec{F_B}$ is given by

$$F_B = |q|vB\sin\theta \quad (2)$$

The SI unit of magnetic field is the tesla (T):

$$1\ tesla = 1\ T = 1\frac{Newton}{(Coulomb)(meter/second)} = 1\frac{N}{C\cdot m/s} = 1\frac{N}{A\cdot m}$$

Another commonly used non-SI unit for $\vec{B}$ is the gauss (G), where $1\ T\ =\ 10^4 G$.

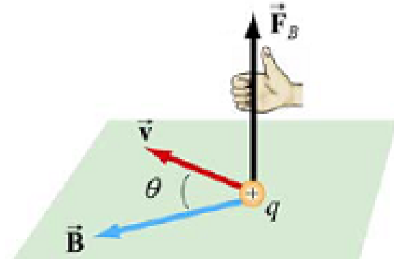

Fig.5.4 The direction of the magnetic force

The direction of $\vec{F_B}$ is obtained using **right-hand rule**. **If the four fingers of a right hand are curled(rotated)from the direction of velocity to the direction of magnetic field, the stretched thumb of the right hand points the direction of the magnetic force.**

Note that $\vec{F_B}$ is always perpendicular to $\vec{v}$ and $\vec{B}$ and cannot change the particle's speed v (and thus the kinetic energy). In other words, magnetic force cannot speed up or slow down a charged particle. Consequently, $\vec{F_B}$ can do no work on the particle:

$$dW\ =\ \vec{F_B}\cdot d\vec{s} = q(\vec{v}\times\vec{B})\cdot\vec{v}dt\ =\ q(\vec{v}\times\vec{v})\cdot\vec{B}dt\ =\ 0$$

The direction of $\vec{v}$, however, can be altered by the magnetic force.

5.3 Magnetic Force on a Current-Carrying Wire

We have just seen that a charged particle moving through a magnetic field experiences a magnetic force $\vec{F_B}$. Since electric current consists of a collection of charged particles in motion, when placed in a magnetic field, a current-carrying wire will also experience a magnetic force.

Consider a segment of wire of length l and cross-sectional area A placed in a magnetic field directed into the page as represented with crosses ($\times$) as shown in Figure 5.6. The charges move at an average drift velocity $\vec{v_d}$. Since the total amount of charge in this segment is $Q_{tot}\ =\ q(nAl)$, where n is the number of charges per unit volume, the total magnetic force on the segment is

$$\vec{F_B} = Q_{tot}\vec{v}_d\times\vec{B}\ =\ qnAl(\vec{v}_d\times\vec{B})\ =\ I(\vec{l}\times\vec{B}) \quad (3)$$

$I = nqv_dA$, and $\vec{l}$ is a length vector with l magnitude and directed along the direction of current. For a wire of arbitrary shape, the magnetic force can be obtained by summing over the forces acting on the small segments that make up the wire. Let the differential segment be denoted as $d\vec{s}$(Fig.5.7) The magnetic force acting on the segment is

$$d\vec{F}_B = Id\vec{s} \times \vec{B} \tag{4}$$

Thus, the total force is

$$\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B} \tag{5}$$

where a and b represent the endpoints of the wire.

As an example, consider a curved wire carrying a current I in a uniform magnetic field $\vec{B}$, as shown in Figure 5.8.

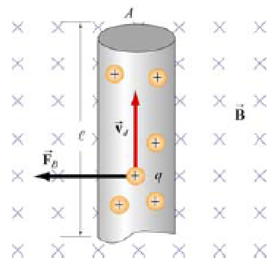

Figure 5.6 Magnetic force on a conducting wire

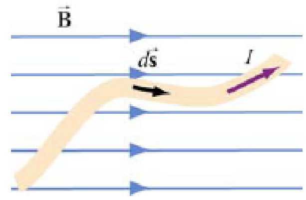

Figure 5.7 Current-carrying wire placed in a magnetic field

Using the last equation we obtain, the magnetic force on the wire is given by

$$\vec{F}_B = I(\int_a^b d\vec{s}) \times \vec{B} = I\vec{l} \times \vec{B}$$

where $\vec{l}$ is the length vector directed from a to b. However, if the wire forms a closed loop of arbitrary shape (Figure 5.9), then the force on the loop becomes

$$\vec{F}_B = I(\oint d\vec{s}) \times \vec{B} \tag{6}$$

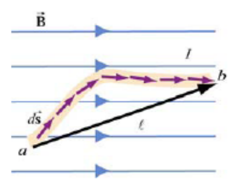

Figure 5.8 A curved wire carrying a current *I*.

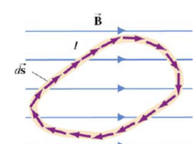

Figure 5.9 A closed loop carrying a current *I* in a uniform magnetic field.

Since the set of differential length elements $d\vec{s}$ form a closed polygon, and their vector sum is zero, i.e., $\oint d\vec{s} = 0$. The net magnetic force on a closed loop is $\vec{F}_B = 0$.

5.4 Torque on a Current Loop

What happens when we place a rectangular loop carrying a current I in the xy plane and switch on a uniform magnetic field $\vec{B} = B\hat{i}$ which runs parallel to the plane of the loop, as in Figure 5.10(a)? From the equation written below, we see the magnetic forces acting on sides 1 and 3 vanish because the length vectors $\vec{l}_1 = -b\hat{\hat{i}}$ and $\vec{l}_3 = b\hat{i}$ are parallel and anti-parallel to $\vec{B}$ and their cross products vanish. On the other hand, the magnetic forces acting on segments 2 and 4 are non-vanishing:

$$\vec{F}_2 = I(-a\hat{j}) \times (B\hat{i}) = IaB\hat{k} \qquad \vec{F}_4 = I(a\hat{j}) \times (B\hat{i}) = -IaB\hat{k}$$

with $\vec{F}_2$ pointing out of the page and $\vec{F}_4$ into the page. Thus, the net force on the loop is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

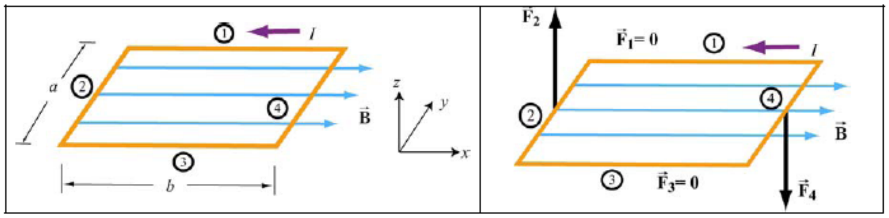

Figure 5.10 (a) A rectangular current loop placed in a uniform magnetic field. (b) The magnetic forces acting on sides 2 and 4.

as expected. Even though the net force on the loop vanishes, the forces $\vec{F}_2$ and $\vec{F}_4$ will produce a torque which causes the loop to rotate about the y-axis (Figure 5.11). The torque with respect to the center of the loop is

$$\vec{\tau} = (\frac{-b\hat{i}}{2}) \times \vec{F}_2 + (\frac{b\hat{i}}{2}) \times \vec{F}_4 = (\frac{-b\hat{i}}{2}) \times (IaB\hat{k}) + (\frac{b\hat{i}}{2}) \times (-IaB\hat{k}) = (\frac{IabB}{2} + \frac{IabB}{2})\hat{j} = IabB\hat{j} = IAB\hat{j}$$

where $A = ab$ represents the area of the loop and the positive sign indicates that the rotation is clockwise about the y-axis. It is convenient to introduce the area vector $\vec{A} = A\hat{n}$ where $\hat{n}$ is a unit vector in the direction normal to the plane of the loop. The direction of the positive sense of is set by the conventional right-hand rule. In our case, we have $\hat{n} = +\hat{k}$. The above expression for torque can then be rewritten as

$$\vec{\tau} = I\vec{A} \times \vec{B} \tag{7}$$

For a loop consisting of N turns, the magnitude of the toque is

$$\tau = NIAB\sin\theta \tag{8}$$

The quantity $NI\vec{A}$ is called the magnetic dipole moment $\vec{\mu}$:

$$\vec{\mu} = NI\vec{A} \tag{9}$$

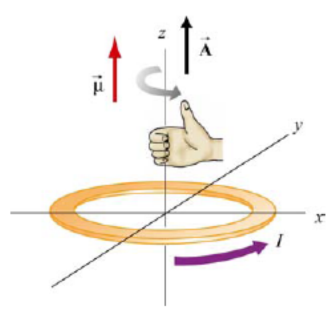

Figure 5. 12 Right-hand rule for determining the direction of $\vec{\mu}$

The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule (Figure 5.12). The SI unit for the magnetic dipole moment is $ampere \cdot meter^2$ $(A \cdot m^2)$. Using the expression for $\vec{\mu}$, the torque exerted on a current-carrying loop can be rewritten as

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (10)$$

The above equation is analogous to $\vec{\tau} = \vec{p} \times \vec{E}$ as we have seen in unit one, the torque exerted on an electric dipole moment $\vec{p}$ in the presence of an electric field $\vec{E}$.

If an external agent does a work in rotating an electric dipole in a magnetic field, the work done is stored in the form of potential energy which can be calculated as

$$U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B} \quad (11)$$

5.4 Motion of Charged Particles in a Uniform Magnetic Field

If a charged particle enters to a region of magnetic field, it will experience a magnetic force $F_m = qvB\sin\theta$. The field exerts maximum magnetic force given as $F_m = qvB$ if the charge enters in the direction perpendicular to the field. This maximum magnetic force acting on the charge will provide the necessary centripetal force $F_c = \frac{mv^2}{r}$ for the charge to perform circular motion in the field as shown in Figure 5.13. With

$$qvB = \frac{mv^2}{r}$$

the radius of the circle is found to be

$$r = \frac{mv}{qB} \quad (12)$$

The period T (time required for one complete revolution) is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB} \quad (13)$$

Similarly, the angular speed (cyclotron frequency) ω the particle can be obtained as

$$\omega = 2\pi f = \frac{v}{r} = \frac{qB}{m} \quad (14)$$

If the initial velocity of the charged particle has a component parallel to the magnetic field $\vec{B}$ instead of a circle, the resulting trajectory will be a helical path, as shown in Figure 5.14.

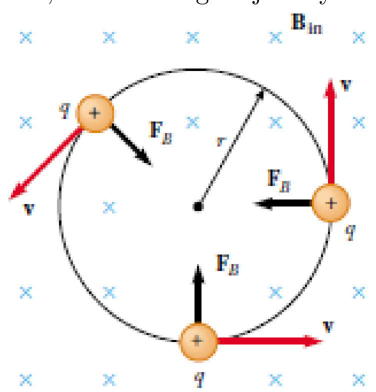

Figure 5.13 Circular motion of a charge in a uniform magnetic field

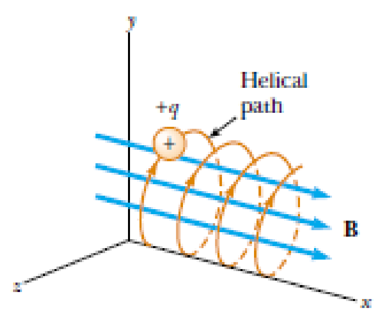

Figure 5.14 Helical motion of a charge in a magnetic field