CHAPTER FOUR: THE THEORY OF PRODUCTION

In the previous chapter, we focused on the demand side of the market – the preferences and the behavior of consumers. Now we turn to the supply side of the market and examine the behavior of producers. Consider some of the problems that a company like General Motors faces regularly. How much assembly-line machinery and how much labor should it use in its new automobile plants? If it wants to increase production, should it hire more workers, or should it also construct new plants? Does it make more sense for one automobile plant to produce different models, or should each model manufactured in a separate plant? These questions apply to not only to business firms, but also to other producers of goods and services, such as governments and nonprofit agencies. In this chapter, therefore, we will see how firms can organize their production efficiently.

4.1. Concepts of Production and Production Function

In simple words, **production** refers to the process of transferring inputs into outputs. An input (factor of production) is a good or service that goes into the production of another good or service. In other words, an input is simply anything which the firm buys for use in its production process. Inputs include labor, land, capital and entrepreneurial talent. The end products of the production process are outputs which could be tangible (goods) or intangible (services).

Simple examples of production:

- A furniture-producing firm combines workers labor time, machineries, his organizational skills and various raw materials like wood, metal, *etc* to produce sofas for sale to its customers.
- A high school uses teachers, books, educational materials (aids), class rooms, the available technology (like plasma tv), *etc* to provide educational services to students.

The theory of production explains and formalizes the nature of relationships between factors (inputs) used and output. As you might have noticed, the production process does not necessarily involve physical conversion of raw materials into tangible goods. Besides teachers, lawyers, doctors, social workers, consultants, hair-dressers, *etc* are all engaged in producing intangible goods.

Production function is a technical relationship between inputs and outputs. It shows the maximum output that can be produced with a fixed amount of inputs and the existing technology. A production function may take the form of an algebraic equation, table or graph.

A general equation for production function could for instance be described as:

$$Q = f(X_1, X_2, X_3, \ldots, X_n) \qquad (4.1)$$

where Q is the maximum output produced; and $X_1, X_2, X_3, \ldots, X_n$ are different types of inputs.

To illustrate, suppose a wheat-producing firm uses labor (L), capital (K), land (S) and entrepreneurship (E). Other inputs such as seeds, fertilizers, insecticides, and the like may be included in one of these large groups of factors of production. The production function for wheat may then be expressed as:

$$\text{Quantity of Wheat} = f(L, K, S, E) \qquad (4.2)$$

Note that we must assume that the production of Q tons of wheat is realized in the most efficient way possible. If it, for instance, is possible to produce 20 tons of wheat using a certain combination of L, K, S and E, it is also possible to produce only 19 tons with the same combination. So, the second technology has to be abandoned as it is not efficient (it is wasting resources that could be used for the production of one more ton).

In sum, production functions describe what is technically feasible when the firm operates efficiently: that is, when the firm uses each combination of inputs as effectively as possible.

Fixed Vs Variable Inputs

Normally, firms employ inputs whose amount does not change for some time and others whose quantity varies according to the amount of production (output). One can hence categorize inputs as fixed and variable.

Fixed inputs are those inputs whose quantity can not readily be changed when market conditions indicate that an immediate change in output is required. In fact no input is ever absolutely fixed, but may be fixed during an immediate requirement. For example, if the demand for Beer shoots up suddenly in a week, the brewery factories cannot plant additional machinery over a night to respond to the increased demand. It takes long time to buy new machineries, to plant them and use for production. Thus, the quantity of machinery is fixed for some times such as a weak. Buildings, plot of land and machineries are examples of fixed inputs because their quantity cannot be manipulated easily in short time periods.

Variable inputs, on the other hand, are those inputs whose quantity can be changed almost instantaneously in response to desired changes in output. That is, their quantity can easily be diminished when the market demand for the product decreases and vise versa. The best example of variable input includes unskilled labor and raw materials.

The Short Run Vs the Long Run

Depending on the nature of economic adjustment in a firm to changing economic environment, the production period is divided into short-run and long-run.

The **short run** refers to a period of time in which one or more factors of production cannot be changed. In this case the firm can alter its level of output by increasing or decreasing the use of variable inputs. A firm's capital, for example, usually requires time to change – a new factory must be planned and built, machinery and other equipment must be ordered and delivered, all of which can take a given time period.

The **long run**, on the other hand, is the amount of time needed to make all inputs variable. Here the period is long enough to allow changes in the level of all inputs.

Note that the supply of fixed inputs in the short-run is *inelastic* while the supply of variable inputs in the short-run is elastic.

How long should the long-run be? A month? A year? 2-3 years?10 years? …What do you think? If you attempt to put figures, that is wrong. Sorry, there is no precise answer for the question. In short, it differs from industry to industry and more specifically from firm to firm. In some industries, such as groceries, short-run may be a few weeks or while in some other industries like electricity and telecommunications, short-run may mean 4 or more years. Similarly, long-run may be 2 or 3 years while in other industries it might be 10 or more years. Therefore, long-run and short-run do not refer to any fixed period of time. There is no hard and fast rule that specifies how short is short-run or how long is long-run.

Based on these classifications and concepts, we can see the short-run and the long-run production functions in the sub-sections that follow.

4.2. The Short-Run Production Function: Production with One Variable Input

The majority of production decisions of firms are made in the short-run in which the quantity of at least one factor of production changes with output. Hence, the short-run production function shows the relationship between the maximum product and the level of the variable input. In more general expression, a short-run production could take the following form, for Q output and X1 variable input quantities:

$$Q = f(X_1) \qquad (4.3)$$

Imagine, for example, that you are managing a clothing plant. You have a fixed amount of equipment, but you can hire more labor or less to sew and to run the machines. You have to decide how much labor to hire and how much clothing to produce. To make the decision, you will need to know how the amount of output Q increases (if at all), as the input of labor L increases. Therefore, the short-run production function of cloth could be expressed as:

$$\text{Quantity of Cloth} = f\,(\text{Labor}) \qquad (4.4)$$

Here, the short-run level of cloth produced is supposed to depend on labor, the only variable input. Since other factors are assumed to be fixed in the short-term, we do not include them in the production function. This, however, does not mean that they are not used in the production process.

4.2.1. Total, Average and Marginal Products

The contribution that variable input makes to the production process can be described in terms of the total, average and marginal product.

Total Product (TP)

It refers to the total output (say cloth) produced by a given amount of a variable input (say labor) keeping the quantity of other inputs fixed (say machines). In almost all real world production processes, TP in the short-run follows a certain trend: it initially increases at an increasing rate, then increases at a decreasing rate, reaches a maximum point but eventually falls with a rise in the quantity of the variable input. That is, initially, as we combine more and more units of the variable input with the fixed input output continues to increase. But eventually, increasing the unit of the variable input may not help output increase. Even as we employ more and more unit of the variable input beyond the carrying capacity of a fixed input, out put may tends to decline. Thus increasing the variable input can increase the level of output only up to a certain point, beyond which the total product tends to fall as more and more of the variable input is utilized. This tells us what shape a total product curve assumes. The shape of the total product curve is nearly S-shape (see fig 4.1 Panel A)

Average Product (AP)

Average product of an input is the level of output that each unit of input produces, on the average. It tells us the mean contribution of each variable input in the total product. Mathematically, AP is total product divided by the amount of variable input used to produce that product. The average product of labor (AP_L), for instance, is given by:

$$AP_L = \frac{Total\ Product}{Total\ Labor} = \frac{TP}{L} \tag{4.5}$$

Like that of TP, AP_L first increases, gets its maximum value and eventually falls continuously afterwards.

<u>Marginal Product (MP)</u>

We may, at times, be interested in knowing the extra output brought about by the extra employment of a variable input. In terms of labor, we may ask "how much has the last laborer added to total product?" These issues are explained by the marginal concept.

Marginal product is the extra or additional output obtained when one extra unit of a variable input is entered in production while other factors remain fixed. Simply, marginal product is a change in the amount of total product divided by a change in the amount of variable input used. For instance, marginal product of labor (MP_L) is given by:

$$MP_L = \frac{Change\ in\ Total\ Product}{Change\ in\ Labor} = \frac{\Delta TP}{\Delta L} = \frac{\partial Q}{\partial L} \tag{4.6}$$

The last term in this equation (read as the partial derivative of *Q* with respect to *L*) is applied when a continuous production function (i.e. an algebraic equation) is given for output *Q*.

Thus, MPL measures the slope of the total product curve at a given point. In the short run, the MP of the variable input first increases reaches its maximum and then tends to decrease to the extent of being negative. That is, as we continue to combine more and more of the variable inputs with the fixed input, the marginal product of the variable input increases initially and then declines.

Tabular and Graphical representation of the short run product curves

Table 4.1 contains the total product (TP) and other product types for various amounts of labor units (e.g. number of workers) and a constant amount of capital (e.g. 5 machineries). The values in column 3 of the table show total output. If you carefully have a look at the values of the TP, they initially increase at an increasing rate as labor units rise from 0 to 4. TP still increases but at a decreasing rate when labor units are between 4 and 8. The maximum total output of 119 units is realized by combining 8 units of labor and 5 units of capital at the cheaply available technology. This could be interpreted as the total output maximizing level of labor employment is 8. Finally, the total product begins to decline when the amount of the variable input exceeds 8 units.

Table 4.1 Production with One Variable Input

Amount of Labor, L	*Amount of Capital, K*	*Total Product TP (Q)*	*Average Product AP(=TP/L)*	*Marginal Product MP(=ΔTP/ΔL)*	*Stage of Production*
0	5	*0*	-	-	*Stage I*
1	5	*10*	*10*	*10*	
2	5	*32*	*16*	*22*	
3	5	*63*	*21*	*31*	
4	5	*84*	*21*	*21*	*Stage II*
5	5	*100*	*20*	*16*	
6	5	*111*	*18.5*	*11*	
7	5	*119*	*17*	*8*	
8	5	*119*	*14.9*	*0*	
9	5	*117*	*13*	*-2*	*Stage III*
10	5	*110*	*11*	*-7*	

Now, copy the values of the 1st and 3rd columns of *Table 4.1* on a separate piece of paper. Then, use *equation (4.5)* to compute the corresponding values of AP_L (for each value of labor unit). Compare your results with those in the 4th column. What trend have you observed for the AP_L?

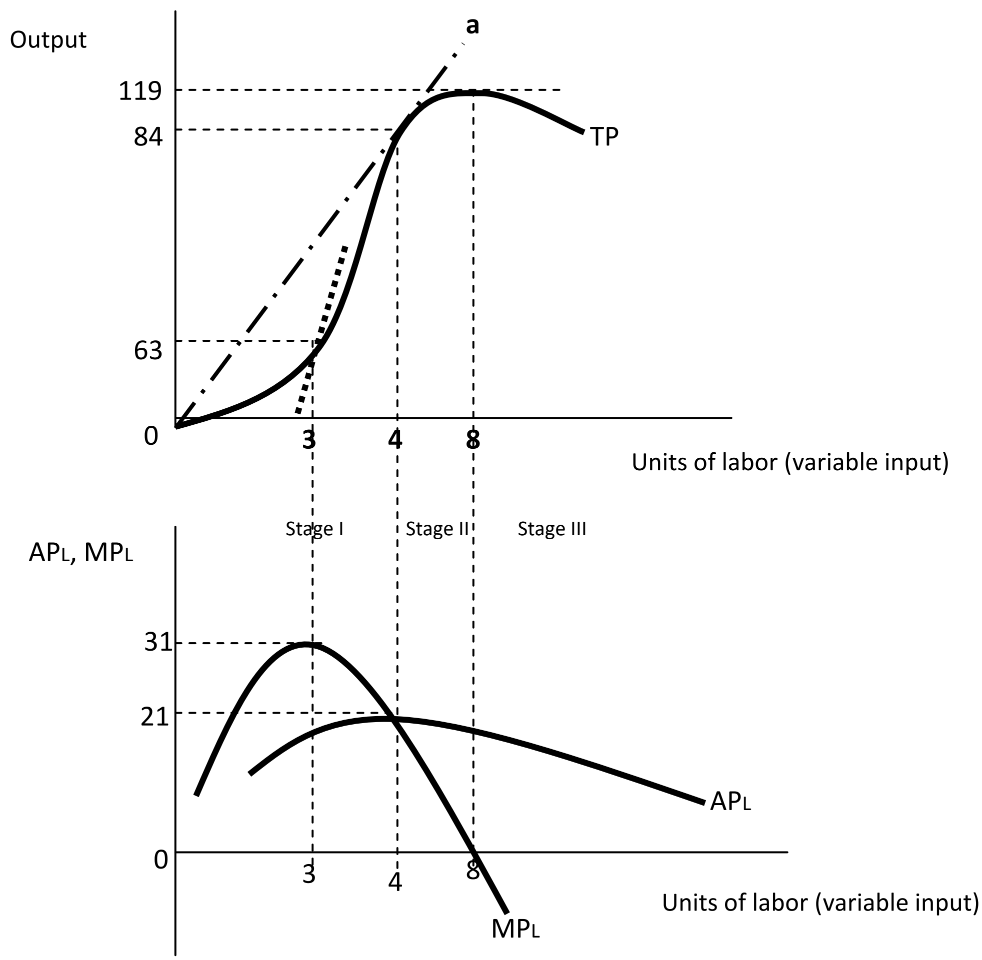
We can see that the behavior of AP is almost similar to that of TP. Like that of TP, AP_L first increases up to 4 labor units, gets its maximum value of 21 output units at the 4th labor unit and eventually falls continuously afterwards.

Now also, copy the values of the 1st and 3rd columns of *Table 4.1* on a separate piece of paper. Then, use *equation (4.6)* to compute the corresponding values of MP_L (for each value of labor unit). Compare your results with those in the fifth column.

We observe an almost similar behavior for MP_L as AP_L and TP. Hence, MP_L first rises up to the 3rd labor unit, gets its maximum value of 31 output units for the 3rd labor unit and eventually falls afterwards. MP_L becomes zero when total product gets its maximum. This means that the last factor of production adds nothing to total output. Further employment of labor beyond the zero MP_L point even makes it negative implying that total production is falling from its maximum value. In our illustration, this happens after the employment of 8 units of labor. Workers hired after 8th laborer contribute negatively.

These phenomenons's can also be depicted in graph as shown in figure 4.1. below.

Figure 4.1 Total, Average and Marginal Product Curves



As the number of the labor hired increases (capital being fixed), the TP curve first rises, reaches its maximum when 8 amount of labor is employed, beyond which it tends to decline. Assuming that this short run production curve represents a certain cloth manufacturing firm, it implies that 8 numbers of workers are required to efficiently run the machineries. If the numbers of workers fall below 8, the machine is not fully operating, resulting in a fall in TP below 119. On the other hand, increasing the number of workers above 8 will do nothing for the production process because only 8 numbers of workers can efficiently run the machine. Increasing the number of workers above 8, rather results in lower total product because it results in overcrowded and unfavorable working environment.

Marginal product curve increases until 3 amount of laborer is employed and then it tends to fall. The MPL is zero at 8 amount of labor (when the TP is maximal); beyond which its value assumes zero indicating that each additional worker above 8 tends to create over crowded working condition and reduces the total product. Thus, in the short run (where some inputs are fixed), the marginal product of successive units of labor hired increases initially, but not continuously, resulting in the limit to the total production. Geometrically, the MP curve measures the slope of the TP. The slope of the TP curve increases (MP increases) up to 3 amounts of labor units, it decreases from 3 to 8 and it becomes negative beyond 8.

The average product curve increases up to labor unit of 4, beyond which it continuously declines. The AP curve can be measured by the slope of rays originating from the origin to a point on the TP curve. For example, the AP_L at 4 is the ratio of 84 level of output to 4 labor units. This is identical to the slope of ray a.

Relationship between Average Product and Marginal Product

From the previous discussions and figurative presentations, we can notice specific relationships between AP_L and $MP_{L.}$

If we just have a close look at *Table 4.1* and *Figure 4.1*, particularly at AP_L and MP_L, we easily come up with the following:

- When $MP_L > AP_L$, AP_L keeps on increasing. This is what you observe in the entire area labeled as stage I.
- When $MP_L = AP_L$, AP_L is already at its maximum. One can locate this at the point where MP_L and AP_L intersect (end of stage I and beginning of stage II).
- When $MP_L < AP_L$, AP_L keeps on decreasing. You see such a scenario in what we labeled as stages II and III.

A simpler way to understand the relationship between average and marginal product is to think of it in terms of grades. Suppose you took only two courses so far and your average score for the two courses is 90 ($=AP_L$). If your score for an additional (marginal) course is 93 ($=MP_L>AP_L$), your new average will be 91. But, had your additional (marginal) score been 87 ($=MP_L<AP_L$), your new average would be 89. Thus, your marginal score pushes up or down your average product depending on whether the marginal score is above or below the average respectively. The same relationship holds true for AP_L and MP_L.

As we know, in addition to tables and graphs, equations are important tools in economics to understand relationships.

<u>Numerical Illustration:</u>

Suppose that the short-run production function for cut-flower by a certain Ethiopian firm is given by:

$$Q = 4KL - 0.6K^2 - 0.1L^2$$

where Q - represents the annual quantity of cut-flower produced.

K - annual capital input; suppose K=5.

L - annual labor input.

a) Determine the average product of labor (AP_L) function.

b) At what level of labor does the total output of cut-flower reach the maximum?

c) What will be the maximum achievable amount of cut-flower production?

<u>Solution:</u>

a) $$AP_L = \frac{Q}{L} = \frac{4KL - 0.6K^2 - 0.1L^2}{L} = 4K - \frac{0.6K^2}{L} - 0.1L = 20 - \frac{15}{L} - 0.1L = \underline{\underline{\frac{20L - 15 - 0.1L^2}{L}}}$$

b) *We know that when total product (Q) is maximum, MP will be zero. And* $MP_L = \frac{\partial Q}{\partial L}$.

That is, partially differentiating[1] *Q with respect only to L and equating it to zero:*

$$MP_L = \frac{\partial Q}{\partial L} = \frac{\partial(4KL - 0.6K^2 - 0.1L^2)}{\partial L} = 4K - 0.2L = 0$$

$$\Rightarrow 20 - 0.2L = 0 \ \Rightarrow \ L = \frac{20}{0.2} = \underline{\underline{100}}$$

(Q – cut-flower level of output will be the maximum if the firm employs 100 units of labor.)

c) *Substituting the optimal values of labor (L=100) and capital (K=5) into the original production function (Q) gives the maximum level of cut-flower production:*

$$Q_{max} = 4KL - 0.6K^2 - 0.1L^2 = 4*5*100 - 0.6*5^2 - 0.1*100^2 = \underline{\underline{985}}$$

4.2.2. Efficient Region of Production in the Short-Run

We are now not in a position to determine the specific number of the variable input (labor) that the firm should employ because this depends on several other factors than the productivity of

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[1] *As you may recall from your calculus, the partial derivatives of the multivariate function* $Z = aX^b + cY^d + eXY$ *with respect to X and Y (a-e are constants), respectively, are:*

$$\frac{\partial Z}{\partial X} = baX^{b-1} + eY \quad \textit{and} \quad \frac{\partial Z}{\partial Y} = dcY^{d-1} + eX$$

labor such as the price of labor, the structure of input and output markets, the demand for output, etc. However, it is possible to determine ranges over which the variable input (labor) be employed.

To do best with this, based on the relationship between TP, MP and AP, economists have defined three stages of production. For visual observation of the discussions that follow, refer back to figure 4.1.

Stage I: This stage of production includes the range of variable input levels at which the average product (AP_L) continues to increase. Stage I goes from the origin to the point where the AP_L is maximum, which is the equality of MP_L and AP_L. In terms of *Figure 4.1*, this goes to 4 level of labor employment. At 4, the AP_L is at its maximum value. A unit increase in labor initially has an impact of increasing output at an increasing rate (up to the 3 unit of labor). That is why this stage of production also called the stage of *increasing marginal returns*.

Two explanations may be given for the presence of increasing marginal returns:

i. There is plenty of fixed input supply compared to the variable input. Therefore, as more and more units of the variable input are added to the fixed input, the fixed input will be more intensively and effectively exploited. Hence, the efficiency of capital or land will increase in proportion to the additional units of variable input, labor for instance.

ii. Another explanation: As more and more units of the variable input is used, there will be division of labor which will result in specialization. Specialization leads to higher productivity.

However, this will not persist indefinitely as average output begins to fall and marginal output continues falling which marks the commencement of the second stage in production.

Stage II: This stage of production covers that range of variable input used at which MP_L is less than AP_L and is positive. In this stage, both the AP_L and MP_L of the variable input (labor) are diminishing but positive. According to *Figure 4.1*, this starts from labor unit of 4 and goes up to 8. In other words, stage II goes from the point where AP_L is at its maximum (MP_L=AP_L) to the point where MP_L is zero. Here, as the input increases by one unit, output still increases but at a decreasing rate, i.e., each increment of labor generates a smaller increase in output than the last. This continues until output reaches its maximum at the 8 unit of labor. Due to this, the second stage of production is termed the stage of *diminishing marginal returns*.

The reason for decreasing average and marginal products is due to the scarcity of the fixed factor. Once the optimum capital-labor combination is achieved, employment of additional unit of the variable input will cause the output to increase at a slower rate. As a result, the marginal product diminishes. The additional labor will have less and less of the fixed input to work with.

This reasoning has led to the emergence of the law of diminishing marginal product, which will be our focus in the upcoming sub-section.

Any further additional labor unit after 8 will result in a decline in output and such a situation happens at the third stage of production.

Stage III: At this stage, an increase in the variable input results in the decline of the total product. Hence, the total product curve slopes downwards and the marginal product of labor become negative. This happens after labor unit of 8. The stage is also known as the stage of *negative marginal returns* to the variable input.

The cause of negative marginal returns is the fact that the volume of the variable inputs is quite excessive to the fixed input to the extent that they get in each others' way – creating a problem of overcrowding. As a result, the total product declines and results in negative marginal product. The saying "too many cooks spoil the broth" seems to well fit this stage of production.

Out of the three stages of production, we can identify the stage which is most efficient at which a rational producer should produce.

Obviously, a firm should not operate in stage III because in this stage additional units of variable input are contributing negatively to the total product (MP of the variable input is negative) because of overcrowded working environment i.e., the fixed input is over utilized.

Stage I is also not an efficient region of production though the MP of variable input is positive. The reason is that the variable input (the number of workers) is too small to efficiently run the fixed input; so that the fixed input is underutilized (not efficiently utilized)

Thus, the efficient region of production is stage II. At this stage additional inputs are contributing positively to the total product and MP of successive units of variable input is declining (indicating that the fixed input is being optimally used). Hence, the efficient region of production is over that range of employment of variable input where the marginal product of the variable input is declining but positive.

4.2.3. The Law of Diminishing Marginal Product (LDMP)

The LDMP is the major factor behind the relationship between TP, MP, and AP. It states that as the number of units of the variable input increases, other inputs held constant, the marginal product of the variable input declines after a certain point. The law is sometimes called the law of variable proportions.

For instance, as you add more labor to a garden plot of fixed size, the marginal increase in vegetable may increase. Nonetheless, a point is reached where an increase in the use of the variable input yields progressively less and less additional (marginal) product. Each additional unit has, on average, fewer units of the fixed input with which to work.

4.3. The Long-Run Production Function: Productio4.3. The Long-Run Production Function: Production with Two Variable Inputs

We have completed our analysis of the short-run production function in which the firm uses one variable input (labor) and one fixed input (capital). Now we turn to the long run analysis of production. Remember that long run is a period of time (planning horizon) which is sufficient for the firm to change the quantity of all inputs. For the sake of simplicity, assume that the firm uses two inputs (labor and capital) and both are variable. This can be expressed in equation form as:

$$Q = f(L, K) \qquad (4.7)$$

The firm can now produce its output in a variety of ways by combining different amounts of labor and capital. With both factors variable, a firm can usually produce a given level of output by using a great deal of labor and very little capital or a great deal of capital and very little labor or moderate amount of both. In this section, we will see how a firm can choose among combinations of labor and capital that generate the same output. To do so, we make the use of isoquant and isocosts. So it is necessary to first see what is meant by isoquants and isocosts and their properties.

Isoquants and Isocosts

For the analysis of a production function with two variables factors in the long-run, we make use of a concept known as isoquants. And adding the concept of isocosts to isoquants, we will be able to analyze the equilibrium situation of a producing firm in the long-run.

(A) Isoquant

An isoquant is a curve that shows the different technically efficient combination of the two inputs that can produce the same level of output. It is also called equal-product curve or product indifference curve.

Since an isoquant represents those combinations of two inputs which will capable of producing an equal quantity of output the producer will be indifferent between them. The various combinations of labor and capital could be presented, as usual, in tables called isoquant tables or schedules, graphs called isoquant curves and equations. Due to their simplicity and visual expression, we prefer isoquant curves to others.

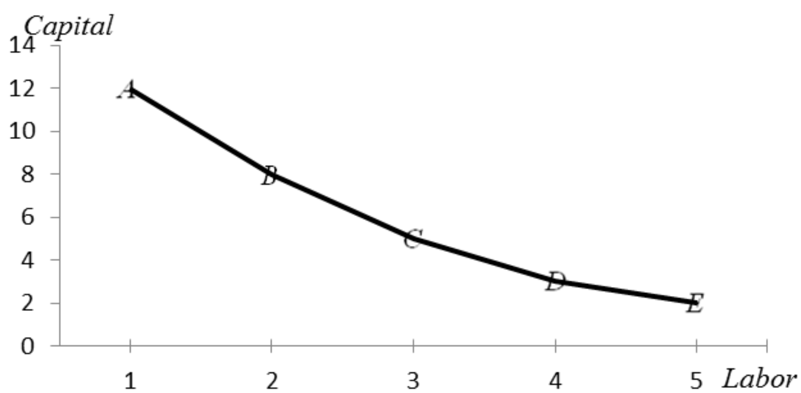
The following table presents 5 combinations of labor and capital. Each combination of labor and capital produce the same level of output, say 50 tons of wheat.

Table 4.2 Isoquant Table (Schedule)

Input Combination	*Labor (L)*	*Capital (K)*	*Maximum Output*
A	*1*	*12*	*50*
B	*2*	*8*	*50*
C	*3*	*5*	*50*
D	*4*	*3*	*50*
E	*5*	*2*	*50*

I hope that plotting the above combination between L and K is a very simple task for you. So, your plot of the data on the XY plane gives you the following iso-product or isoquant curve.

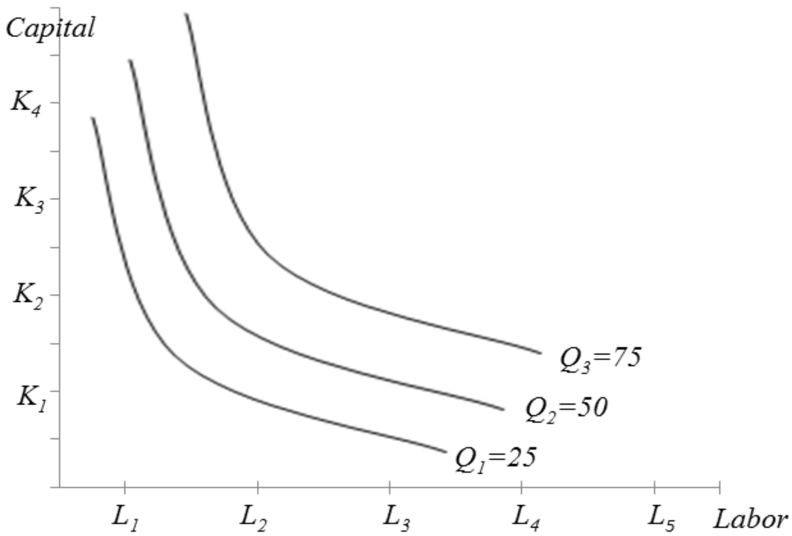
Figure 4.2. Isoquant Curve



Here also, whether the firm produces at point A or B or E, the same amount of output is realized. If the firm produces less or more of that amount, it produces at another point and in another isoquant.

Collection of isoquants is known as an isoquant map, as shown in *Figure 4.3*. The further an isoquant is from the origin, the larger the output level it represents as, for instance, 75 quintals of *teff* is greater than 25 quintals.

Figure 4.3 Isoquant Map



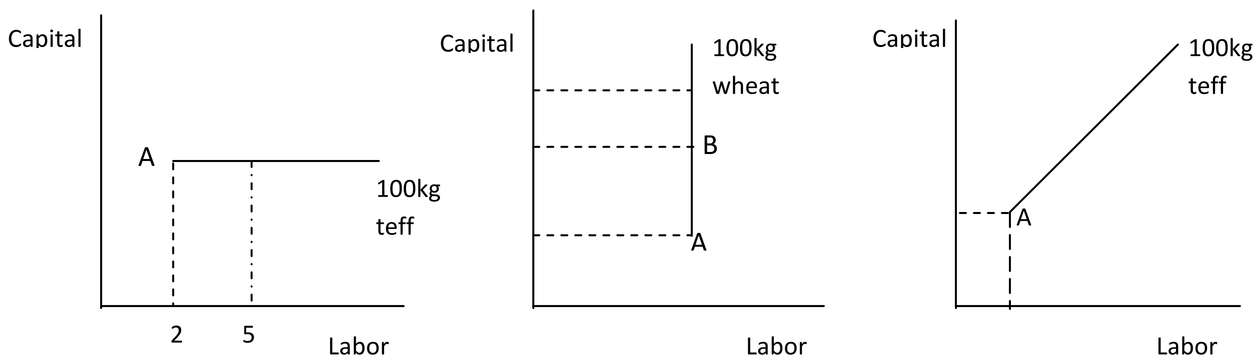
Characteristics (properties) of isoquants

Isoquants have most of the same properties as indifference curves. The biggest difference between them is that output is constant along an isoquant where as indifference curves hold utility constant. Most of the properties of isoquants, results from the word '**efficient**' in its definition.

1. Isoquants slope down ward.

Because isoquants denote efficient combination of inputs that yield the same output, isoquants always have negative slope. Isoquants can never be horizontal, vertical or upward sloping. If for example, isoquants have to assume zero slopes (horizontal line) only one point on the isoquant is efficient. See the following figures.

Fig 4.4



A

An isoquant can never be horizontal. In this figure, the firm can produce 100kg of teff by using either of the following alternatives: 4 capital and 2 labor,4 capital and 5 labor or any other combination of labor and capital along the curve. Obviously, only the first alternative is efficient as it uses the least possible combination of inputs. Thus, all points, except A, are inefficient and not part of isoquant.

B

In this figure, a firm can produce 100kg of wheat by using any combination of labor and capital along the isoquant. But only point A is efficient. For example, point B shows the same number of labor as point A, but higher capital. Thus point B is in efficient because it shows higher combination of inputs. Thus, isoquants can never be vertical

C

In this figure, all points above point A utilize higher combination of both inputs to produce the same output (100 kg coffee). Point A shows the least combination of inputs that can yield 100 kg coffees. Thus all other points are inefficient and not part of the isoquants.

Thus, efficiently requires that isoquants must be negatively sloped. As employment of one factor increases, the employment of the other factor must decrease to produce the same quantity efficiently.

2. The further an isoquant lays away from the origin, the greater the level of output it denotes.

Higher isoquants (isoquants further from the origin) denote higher combination of inputs. The more inputs used, more outputs should be obtained if the firm is producing efficiently. Thus efficiency requires that higher isoquants must denote higher level of output.

3. Isoquants do not cross each other.

This is because such intersections are inconsistent with the definition of isoquants. Consider the following figure.

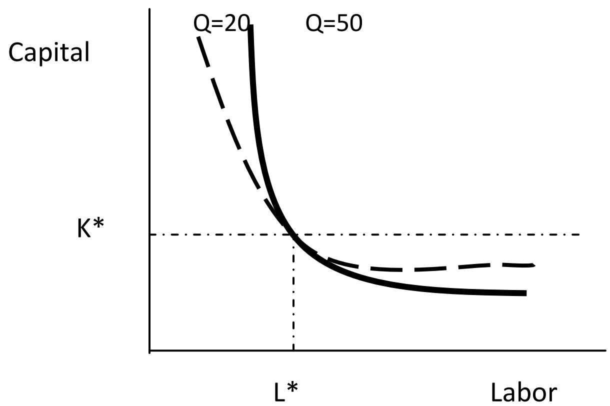


Fig 4.5 *Efficiency requires that isoquants do not cross each other, because the point of their intersection implies that there is inefficiency at this point*.

This figure shows that the firm can produce at either output level (20 or 50) with the same combination of labor and capital (L* and K*). The firm must be producing inefficiently if it produces q = 20, because it could produce q = 50 by the same combination of labor and capital (L* and K*). Thus, efficiency requires that isoquants do not cross each other.

4. Isoquants must be thin.

If isoquants are thick, some points on the isoquant will become inefficient. Consider the following isoquant.

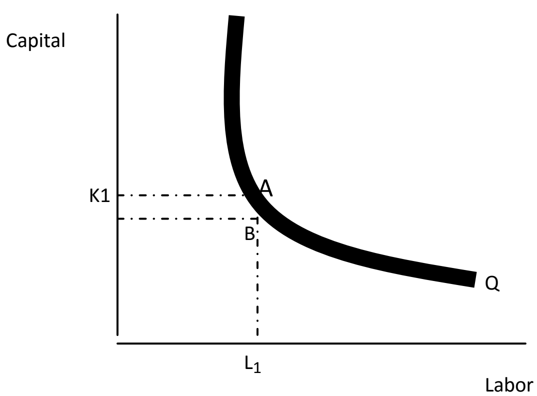


Fig.4.6: *Isoquants can never be thick. Points A and B are on the same isoquant. But point A denotes higher amount of capital and the same amount of labor as point B. Hence point A*

denotes inefficient combination of inputs and thus it lies out of the isoquant. The isoquant should be thin if point A is to be excluded from the isoquant.

Marginal Rate of Technical Substitution (MRTS)

It is the rate at which one input can be substituted for another given constant output. It is the absolute value of the slope of the isoquant at the given point, *i.e.,* marginal rate of technical substitution is the slope of an isoquant.

$$Slope\ of\ an\ isoquant = MRTS_{L\ for\ K} = \frac{amount\ of\ K\ given\ up}{amount\ of\ L\ employed} = \frac{\Delta K}{\Delta L} = \frac{dK}{dL} = \frac{\partial K}{\partial L} \qquad (3.9)$$

The *d* before a variable indicates a change or total differential in the variable.

MRTS and marginal products are highly related. Recall the two-factor long-run production function in *equation (3.8*).

$$Q = f(L, K)$$

The total differential of *Q* is given by:

$$\begin{aligned} dQ &= \frac{\partial Q}{\partial L} dL + \frac{\partial Q}{\partial K} dK \\ &= MP_L(dL) + MP_k(dK) \end{aligned}$$

However, along an isoquant the change in output is zero so that

$$\begin{aligned} &dQ = MP_L(dL) + MP_k(dK) = 0 \\ &\Rightarrow \frac{dK}{dL} = -\frac{MP_L}{MP_K} \end{aligned}$$

This means that the ratio of marginal products is also similar to MRTS, *i.e*,

$$MRTS_{L\ for\ K} = -\frac{MP_L}{MP_K} \qquad (3.10)$$

Note that the following two things:

(i) It is also possible to arrive at the same finding using the following concept. When a producer employs additional labor, he will get additional output given by $MP_L(\Delta L)$. When the producer reduces the use of capital input, he will face a decrease in output given by

$MP_K(\Delta K)$. Since along an isoquant the change in output is zero, the increase and the decrease in output should be equal in magnitude, *i.e*,

$$MP_L(\Delta L) = -MP_K(\Delta K)$$
$$\Rightarrow \frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K} \quad \Rightarrow \quad MRTS_{L\ for\ K} = -\frac{MP_L}{MP_K}$$

(ii) Marginal rate of technical substitution of labor for capital ($MRTS_{L\ for\ K}$ or $MRTS_{L,K}$) & marginal rate of technical substitution of capital for labor ($MRTS_{K\ for\ L}$ or $MRTS_{K,L}$) are not identical. Hence,

$$MRTS_{K,L} = \frac{\Delta L}{\Delta K} = \frac{dL}{dK} = -\frac{MP_K}{MP_L} \qquad (3.11)$$

<u>The Law of Diminishing Marginal Rate of Technical Substitution (LDMRTS)</u>

Most of the time inputs are imperfect substitutes. The LDMRTS states that as the amount of a given input increases, given other inputs, its marginal product decreases. On the other hand when the amount of a given input decreases, its marginal product increases. The more labor the firm has, the harder it is to replace the remaining capital with labor. So, $MRTS_{L,K}$ falls as the isoquant becomes flatter.

Table 4.3 Marginal Rate of Technical Substitution

Input Combination	*Labor (L)*	*Capital (K)*	$\boldsymbol{MRTS_{L,K}}$	$\boldsymbol{MRTS_{K,L}}$
A	*1*	*12*	*-*	*0.25*
B	*2*	*8*	*4*	*0.33*
C	*3*	*5*	*3*	*0.5*
D	*4*	*3*	*2*	*1*
E	*5*	*2*	*1*	*-*

$MRTS_{L,K}$ increases as more and more labor is employed – it falls to 1 from zero. It is that we called the law of diminishing marginal rate of technical substitution. On the other hand, $MRTS_{K,L}$ increases as the employment of capital increases. This is easily observable from the above table.

Properties of Isoquants

From the isoquant tables, curves and discussions we had so far, we can observe general features or properties about isoquants.

All isoquants share the following common properties.

(i) They are downward slopping (negatively slopped).
(ii) They are convex to the origin. This happens because of the law of diminishing marginal rate of technical substitution.
(iii) Two isoquants never cross each other. If two isoquants cross each other, a single combination of the two inputs (K and L) will represent two different levels of output, which is false.
(iv) Isoquants far from the origin represent higher levels of output.

(B) Isocost

As we will discuss in the upcoming chapter, production requires money at least for the acquisition of factors of production. This money is a cost to the producer. A very interesting and simple fact is that a firm can spend an identical total cost for various combinations of inputs. Here comes the concept of an isocost.

An isocost is simply a line that shows the various combinations of two inputs (in our case, labor and capital) that can be purchased for the same amount of outlay (total cost).

Let us assume that the total cost incurred by a firm is only *TC.* And if the price of labor (*L*) and price of capital (*K*) are symbolized by *w* and *r* respectively*,* then

$$TC = wL + rK \qquad (3.12)$$

If we express *K* in terms of others, we get

$$K = \frac{TC}{r} - \frac{w}{r}L \qquad (3.13)$$

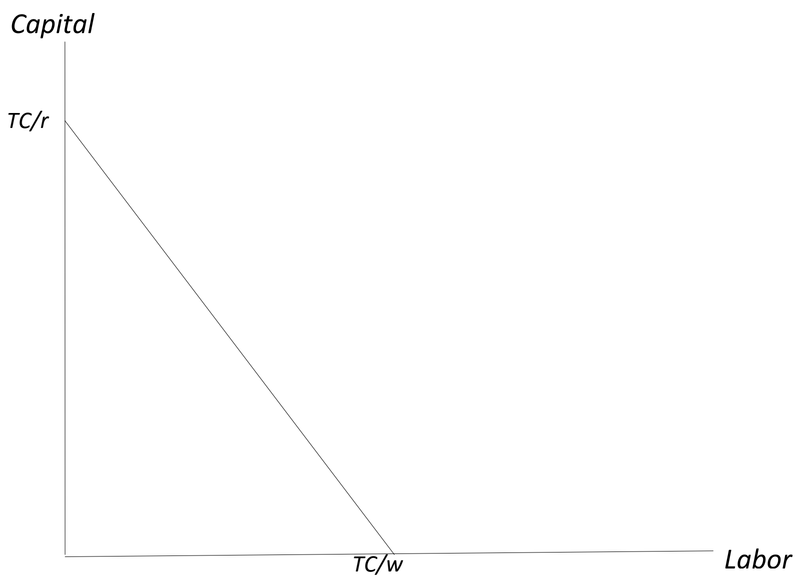
This equation is known as *isocost equation*. It has the following implications:

- If the firm devotes its entire fund to purchase *K*, then it can employ *TC/r* amount of *K*, leaving no money to hire labor (*L*=0).
- If the firm devotes all the funds to buy labor, then it can hire *TC/w* amount of *L*, leaving no money to purchase *K* (*K*=0).

- All the intermediate positions on these two extreme points show any other combinations of *L* and K the firm can hire at a cost of exactly *TC*.

Graphically, this could easily be put as follows.

Figure 4.7 Isocost Line



Slope of an isocost line is given by the ratio of price of labor (*w*) and the price of capital (*r*), i.e., it is just the derivative of the isocost equation given by *equation (3.13)* with respect to labor (*L*).

$$Slope\ of\ an\ isocost = \frac{dK}{dL} = -\frac{w}{r} \tag{3.14}$$

Note that the slope an isoquant and the slope of an isocost are both negative, implying the use or purchase of one input disregards the other. (We have omitted it deliberately above.)

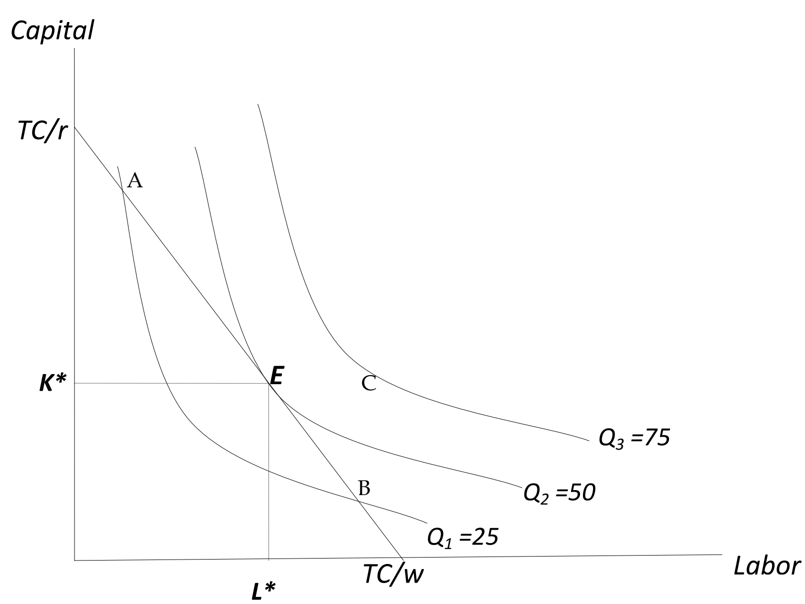
4.4. Optimum of the Producer in the Long-Run

The objective of any firm is maximizing profit which could be achieved by either output maximization or cost minimization or by both. In general, the optimum of the producer refers to the least cost combination of inputs providing the maximum achievable output level. The optimum levels of the two inputs (*K* and *L*) can be found by the use of isoquant curves and isocost lines.

A rational firm wants to produce more output *i.e*. to have isoquants far from the origin (like *Q_3=75* in *Figure 3.6*). However, that desire may be constrained by shortage of outlay for the purchase of factors of production. The money may not buy enough inputs which help produce no more than 50 units of the output *(Q_2=50*). So, there must be a certain level of employment of inputs which maximizes output at the given cost of production.

The optimum of the producer is achieved at the point where an isocost line is tangent to an isoquant curve. This means the point at which the slope of an isoquant is equal to the slope of an isocost line.

Figure 4.8. Equilibrium of the Producer



- At point A, the firm uses all its money to buy L and K. But, it produces only 25 units of a product; it is very low. The firm's outlay is able to buy more inputs to produce further output. Point B is similar to point A in terms of level of production.
- At point C, the firm can produce 75 units of a product. However, it cannot purchase the combination of L and K that produces this much quantity.
- At point E, the firm produces 50 units of a product; and this is the maximum possible amount given the cost. Therefore, the optimum combination of K and L is found at point E where the isocost line is tangent to the second isoquant curve. At this equilibrium point, the slope of the isoquant equals with the slope of the isocost, *i.e*. $MRTS_{L,K} = \frac{w}{r}$.

- Accordingly, the firm has to purchase *L** units of labor and *K** units of capital to realize the maximum possible level of output (=50 units) using the limited outlay of *TC*.

Numerical Illustration

Suppose a certain small enterprise allocates only *20,000 birr* for the production of furniture (school armchairs). The enterprise wants to employ workers (L) whose wage is *w=1000 birr* and purchase implements (K) at a price of *r=4000 birr.* Suppose further that the production function for furniture is given by $Q = 10L^{0.5}K^{0.5}$.

(a) Determine the marginal product functions of workers and implements.

(b) Find $MRTS_{L,K}$ and $MRTS_{K,L}$.

(c) How many workers (*L*) and implements (*K*) must be acquired for the small enterprise to produce the maximum possible number of armchairs.

(d) How many armchairs will be produced at the equilibrium of the enterprise?

Solution:

We are given the production function as $Q = 10L^{0.5}K^{0.5}$ *and the total cost (TC) function as* $TC = wL + rK \;\Rightarrow\; 1000L + 4000K = 20{,}000$.

(a) *Marginal product of a worker (MP_L):*

$$MP_L = \frac{\partial Q}{\partial L} = 0.5(10)L^{(0.5-1)}K^{0.5} = \underline{\underline{5L^{-0.5}K^{0.5}}}$$

Marginal product of an implement (MP_K):

$$MP_K = \frac{\partial Q}{\partial K} = 0.5(10)L^{0.5}K^{(0.5-1)} = \underline{\underline{5L^{0.5}K^{-0.5}}}$$

(b) $$MRTS_{L,K} = -\frac{MP_L}{MP_K} = -\frac{5L^{-0.5}K^{0.5}}{5L^{0.5}K^{-0.5}} = \underline{\underline{-\frac{K}{L}}}$$ *and*

$$MRTS_{K,L} = -\frac{MP_K}{MP_L} = -\frac{5L^{0.5}K^{-0.5}}{5L^{-0.5}K^{0.5}} = \underline{\underline{-\frac{L}{K}}}$$

(c) *The optimum of the small enterprise is found at the following point:*

$$MRTS_{L,K} = -\frac{w}{r} \quad \text{or} \quad MRTS_{K,L} = -\frac{r}{w}.$$

If we use the first equality, we have:

$$MRTS_{L,K} = -\frac{w}{r} \quad \Leftrightarrow \quad -\frac{K}{L} = -\frac{1000}{4000} = -\frac{1}{4} \quad \Rightarrow \quad \underline{\underline{L = 4K}}$$

Substituting this last equation into the TC equation, gives:

$$1000L + 4000K = 20{,}000$$
$$\Rightarrow 1000(4K) + 4000K = 20{,}000$$
$$\Rightarrow 5000K = 20{,}000$$
$$\Rightarrow \underline{\underline{K^* = 4}}$$

Hence, $L^* = 4K = 4(4) = \underline{\underline{16}}$

If the enterprise purchases 4 implements and employs 16 workers, it will have the least cost optimal level of armchair production.

(d) *At those levels of employments, the least cost amount of armchairs produced will be:*

$$Q_{max} = 10(L^*)^{0.5}(K^*)^{0.5}$$
$$= 10(16)^{0.5}(4)^{0.5}$$
$$= 10(\sqrt{64})$$
$$= \underline{\underline{80}}$$

4.5. Returns to Scale

In the long-run, supply of both labor and capital becomes elastic. Firms can therefore employ more of both labor and capital to increase their production. We now turn to determine the amount by which output changes if a firm increases all its inputs proportionately (by the same percentage). This is what we mean by returns to scale. It shows the percentage change in output as the firm changes all its inputs *by the same proportion.*

We have three types of returns to scale.

I. ***Increasing Returns to Scale (IRS):***

If output rises more than in proportion to an equal percentage increase in all inputs the production function is said to exhibit increasing returns to scale (IRS).

Example: A given production function exhibits IRS if doubling all inputs more than doubles the output: *f(aL, aK) > af(L, K) , a>1.*

Reason for IRS:

Although a firm could duplicate a small factory and double its output, the firm might be able to more than double its output by building a single large plant, allowing for greater specialization of labor or capital. In the two smaller plants, workers have to perform many unrelated tasks such as operating, maintaining, and fixing the machines they use. In the large plant, some workers may specialize in maintaining and fixing machines, thereby increasing efficiency. Similarly, a firm may use specialized equipment in large plant but not in small plant.

II. ***Decreasing Returns to Scale (DRS):***

If output rises less than in proportion to an equal percentage increase in all inputs, the production function exhibits decreasing returns to scale (DRS).

Example: Doubling all inputs less than doubles output: $f(aL, aK) < af(L, K), a>1$.

Reasons for DRS:

- Difficulty of organizing, coordinating and integrating activities increase with firm size. An owner may be able to manage one plant well but may have trouble of running two plants.
- Large team of workers may not function as well as small teams, in which individuals take greater personal responsibility.

III. ***Constant Returns to Scale (CRS):***

If output rises by the same proportion to an equal percentage increase in all inputs, the production function exhibits constant returns to scale.

Example: When all inputs double, output also doubles: $f(aL, aK) = af(L, K), a>1.$

Review question

Part I. Multiple Choice Items

1. Which one of the following is true about marginal product?

A. It is the measure of how outputs affect inputs.

B. It lets producers know how much a product is going to cost.

C. It describes the relationship between average total cost and marginal cost.

D. It is the measure of impact of an added unit of input on output.

2. In the second stage of short-run production function,

A. There is negative return to a variable input.

B. TP is increasing at a decreasing rate.

C. AP is increasing throughout.

D. MP is greater than AP.

E. MP is increasing throughout.

3. When $AP_L > MP_L$,

A. AP_L is falling.

B. The firm is producing in the second stage of production.

C. The firm is producing in the third stage of production.

D. All are correct.

4. The very reason for the existence of negative marginal returns in production is:

A. Mere presence of too much variable inputs.

B. Presence of too much fixed inputs.

C. Presence of too many variable inputs compared to fixed inputs.

D. None of the above

5. The optimum of the firm in production in the long-run is attained at the point where:

A. An isocost line intersects an isoquant curve.

B. Marginal rate of technical rate of substitution is equal to the ratio of input prices.

C. An isocost line is tangent to isoquant curve.

D. All of the above.

E. Only B and C.

Part II. <u>Work Out Items</u>

6. Assume a firm with a production function given by $Q = 8LK$ and with a limited outlay of *15,000 birr*. If input prices are as *w=500 birr* and *r=1500 birr,* answer the questions that follow.

(a) Determine following functions: AP_L, AP_K, MP_L and MP_K.
(b) Find $MRTS_{L,K}$ and $MRTS_{K,L}$.
(c) How much *L* and *K* must be acquired at the optimum of the producer?

How much is the maximum possible output at that cost of production?

7. A firm has the production function $f(x,y) = 40x^{\frac{2}{5}}y^{\frac{3}{5}}$. The slope of the firm's isoquant at the point (x, y) = (70, 50) is ________________________.

8. suppose the production function for potato chips is a Cobb-Douglas production function that can be written:

$QPC = 200K^{.6}L^{.4}$

Where QPC = number of bags of potato chips producer per hour, K is the number of deep kettle frying machines employed per hour and L is the number of persons working per hour.

a) Derive expressions for the marginal product of capital and the marginal product of labor in this particular production process.

b) You, as plant manager, are currently leasing 100 frying machines and employing 3000 employees. Distributors pay you $.65 per bag for the chips. At the moment, no more deep kettle frying machines are available for leasing but you can choose to hire more if they will add to your profits. If labor is currently receiving a wage of $12.00 per hour, would you consider hiring one more person? More than one? Explain your reasoning. (To frame the question, remember that an extra person would generate additional revenue via the extra product they produce, and additional cost via the wage they are paid.)

c) Several months have passed and it is once again possible to release more deep kettle frying machines if you choose to do so. Briefly describe the information you will need to make that decision.

CHAPTER FIVE : THEORY OF COST

In the previous chapter you have discussed the nature of production. This chapter will discuss about the nature of cost in order to understand the supply side of the market in a more rigorous way.

5.1 Cost Function and Some Types of Costs

Social cost versus private cost

The cost of producing a good can be seen from two perspectives. These perspectives are the society and the individual producer. The social cost of producing a good measures the cost incurred by all society. Social cost takes in to account the cost incurred by those who are directly involved and those who are indirectly affected by the production of the good in question. In other word, social cost takes into account all negative externalities due to the product. Therefore, social cost will be given by the sum of producer's cost and external cost.

This means; **Social cost= Producer's cost + external cost**

Producer's cost includes all expenses the producer incurs in order to produce the good. However, if the cost has to be seen from the society's perspective external agents can incur some cost. This cost can come, for example in the form of pollution, which may damage the interest of the society by causing health problems or by decreasing the productivity of inputs. Therefore, if one wants to measure the full cost of a commodity to the society all negative externalities have to be taken in to account.

Social costs are usually considered in implementing for non – profit projects aiming at improving the welfare of the society. However, some private projects may also consider social costs.

While social costs attempts to see how much the society as a whole pay for a commodity, private costs looks at the costs from the individual's perspective.

In measuring private cost, no attention is given to external costs. Therefore, private cost measures the cost incurred by the individual producer to produce goods and services. So we can say private cost is the producer's cost.

This chapter concerns about the cost incurred by the firm. Therefore, we will limit ourselves to the private cost.

Though, all producers consider the cost incurred by them, ignoring the external cost, there is no uniformity in measuring private cost. This will lead us to the discussion on accounting and economic cost.

Accounting versus economic cost

Accounting cost refers to the money expenses incurred by a firm in producing a commodity. In other words, accounting cost is the monetary value of all purchased inputs used in production.

Accounting cost ignores the cost of non – purchased (self – owned) inputs. It takes in to account the cost of purchased input only. In the real world economy, entrepreneurs may use some resources, which may not have direct monetary expense, since the entrepreneur can own these inputs himself or herself.

Accounting cost takes only those expenses which are direct such as wage and salaries of hired laborers, cost of raw materials, depreciation allowances, interest on borrowed funds, utility expenses such as electricity, water supply, telephone, etc. These costs are said to be explicit cost.

Explicitly costs are out of pocket expenses for the purchased inputs.

Therefore, we can say accounting cost considers only the explicit costs by ignoring the cost of non –purchased inputs.

If a producer calculates her cost by considering only the costs incurred for purchased inputs, then her profit will be accounting profit.

Therefore; Accounting profit = Total Revenue – Accounting cost=Total Revenue – Explicit cost

On the other hand, the economic cost of producing a commodity considers the monetary value of all inputs – purchased and non – purchased. Calculating economic costs will be difficult, since there are no direct monetary expenses for non – purchased inputs.

The monetary value of non – purchased inputs is obtained by estimating the monetary cost by considering the opportunity cost of the non – purchased input. This means, we try to calculate the sacrifice we have made by using the resources in the firm and not in the second best alternative employments.

For example, if Mr. X quit a job which pays him Birr 1500.00 per month in order to run a firm he has established, then the opportunity cost of his labor's will be taken to bee Birr 1500.00 per month (the salary he has forgone in order to run his own business).

Another example, if Mrs. Y, withdrew Birr 50,000 from her saving account which she used it to purchase raw materials for her factory. Then, though nobody will ask her to pay interest since the money is her own, this does not mean, the money she has used is a free resource. It has a cost and the cost can be the forgone interest income she has sacrificed. Since, the bank will no more pay her interest on the money she has withdrawn. If for example, the saving interest rate is 4% per year, then Mrs. Y would sacrifice an interest income of Birr 2000 per year. This will be the opportunity cost of using the money in her business.

So, in calculating economic cost by making such estimations, it is possible to get the monetary value of all inputs. The estimated opportunity cost is not direct expenses like explicitly cost but they are indirect or implicit

The estimated monetary cost for non –purchased inputs is known as implicit cost.

Economic cost, therefore, is given by the sum of implicit cost and explicit cost. Firms measuring their cost in economic way will have economic profit which is given us.

Economic profit =Total Revenue – Economic cost= Total Revenue – Explicit cost – Implicit cost

Economic profit will give the real profit of the firm since all costs are taken in to account. Accounting profit of a firm will be greater than economic profit by the amount of implicit cost. If all inputs are purchased from the market accounting and economic profit will be the same. But, if implicit costs exist, then accounting profit will be larger than economic profit.

In the theory of cost we assume costs to be measured in economic cost way in which the cost of all inputs will be taken in to account.

5.2. Short – Run Cost Function

Cost function

A cost function shows the total cost of producing a given level of output. A cost function can be described using equations tables or curves. A cost function can be represented using an equation as follows.

C = f (Q) ………………… Where C = total cost of production

Q = the level of output.

Short – run total costs

In the short – run total cost (TC) can be broken down in to two – total fixed cost (TFC) and total variable cost (TVC).

Total Fixed cost (TFC): Total fixed cost refers to that part of the total cost which does not change as output changes. It is the cost of fixed inputs used in production. The most commonly cited examples include rent for land and building, interest on borrower funds, insurance expenses, salaries of permanent staffs, etc.

Total variable cost (TVC): Total variable cost (TVC) is that part of the total cost that varies with the level of output. This means TVC depends on the level of output. TVC is the cost incurred for the variable inputs used in the production. The most commonly cited examples include cost of raw materials, cost of fuel, expenses for utilities such as power consumption, water, telephone, wages of temporary workers, etc.

The sum of TFC and TVC will always be equal to TC in the short –run. In short this can be written as:

TC = TFC + TVC

Average costs in the short –run

From the total costs, it is possible to derive their respective average costs.

Average total cost (average cost): Average total cost is the total cost per unit of output. It is calculated by dividing the total cost for the level of output symbolically;

AC = TC/Q ……………… Where AC = Average cost

TC = Total cost

Q = Level of output

Average Fixed cost (AFC): Average fixed cost is total fixed cost per unit of output. It is calculatedly by dividing TFC for the level of output.

Symbolically; AFC = TFC/Q …………… Where AFC = Average fixed cost

TFC = Total fixed cost

Q = Level of output

Average variable cost (AVC): Average variable cost is total variable cost per unit of output. It is calculated by dividing total variable cost for the level of output.

Symbolically; AVC = TVC /Q …………………. Where AVC = Average variable cost

TVC = Total variable cost

The short –run average costs have a relationship among them. We can derive this relationship by starting with the total cost.

TC = TFC + TVC

Dividing both sides by output (Q) will give

TC/Q = TFC/Q + TVC/Q or:

AC = AFC + AVC

Therefore, AC is given by the sum of AFC and AVC

Marginal cost (MC)

Marginal cost refers to the additional cost incurred in order to produce one additional output. It can be also be defined as change in total cost per unit change in output. Symbolically,

$MC = \frac{\Delta TC}{\Delta Q}$ .... Where MC = marginal cost

ΔTC = Change in total cost

ΔQ = Change in output

From a total cost function, C=f(Q), the marginal cost will be given by the first derivative of total cost function with respect to output. Symbolic,

Given C = f(Q)

MC = dC/dQ

It is also possible to get MC from total variable cost (TVC). This can be seen as follows. We defined MC as:

$MC = \frac{\Delta TC}{\Delta Q}$, But TC= TFC + TVC, so

$$MC = \frac{\Delta(TFC + TVC)}{\Delta Q} \quad = \frac{\Delta TFC + \Delta TVC}{\Delta Q}$$

$MC = \frac{\Delta TFC}{\Delta Q} + \frac{\Delta TVC}{\Delta Q}$, but $\frac{\Delta TFC}{\Delta Q} = 0,$ since TFC is constant. This implies that;

$$MC = \frac{\Delta TVC}{\Delta Q}$$

Therefore, $MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TVC}{\Delta Q}$.

This implies MC is the change in either TC or TVC per unit change in output. This means TFC does not affect the marginal cost.

To examine the behavior of the short – run costs and understand their relationship, let us take the following typical short – run cost function.

Output 1	TFC 2	TVC 3	TC 4	AFC 5	AVC 6	AC 7	MC 8
0	60	0	60	-	-	-	-
1	60	15	75	60	15	75	15
2	60	25	85	30	12.5	42.5	10
3	60	30	90	20	10	30	5
4	60	50	110	15	12.5	27.5	20
5	60	80	140	12	16	28	30
6	60	125	180	10	25.3	35.3	45

Table 5.1 Short-run cost Schedule

As you can see from the table, the TFC is 60, which is a constant and remains constant at all levels of output. However, TVC and TC continuously rise as output increase

The difference between TC and TVC at all levels of output is constant 60 which is the fixed cost. At output level zero, TVC is zero but TC has the value of the TFC.

AFC continuously declines as output increases. AVC, AC and MC have the same change of variation all first decrease; reach their respective minimum and then increase.

We can sketch, the short – run cost curves as follows.

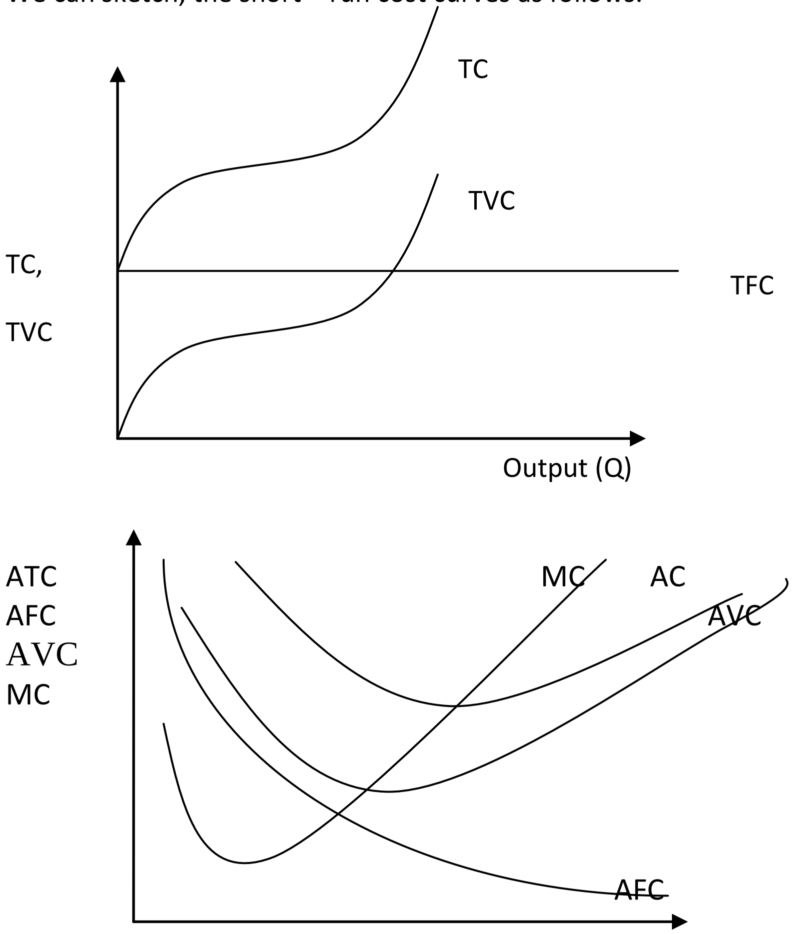


Fig 5.1 Short – run cost curves.

TFC is represented by a horizontal line, since its value is constant at all levels of output.

TC and TVC are represented by continuously rising curves they are parallel since they have the same slope given by MC. They both increase continuously first by a decreasing then by an increasing rate which is the result of the behaviors of the MC. However, there is a constant gap between TC curve and TVC curve and this gap is the fixed cost.

When Q = 0
TVC = 0 and TC = TFC

AFC declines continuously and approaches both axes asymptotically. AFC is rectangular hyperbola. AVC, AC and MC curves have ‘U’ shape. They first decline reaches their respective minimum and then declines. MC curve cuts the AVC curve and the AC curve at their respective minimum point. This is because as long as MC is below AC, it pulls the averages down. When MC is above AC or AVC, it pulls the averages up. When MC equals AC it is neither falling nor rising (i.e. AC or AVC are at their minimums). The gap between AC curve and AVC curve diminishes as output increases because the difference between them is AFC and it declines as output increase. So long as AFC and AVC fall, AC will also fall because AC = AFC + AVC

Example

Suppose the short – run cost function of a firm is given by: $C=2Q^3 - 2Q^2 + Q + 10$. Find

A. The expressions for TFC & TVC
B. The expressions for AFC, AVC & AC and MC
C. The minimum values of MC and AVC.

Solution

Given $C=2Q^3 - 2Q^2 + Q + 10$

A. TFC = TC at Q=0 Therefore ,
   TFC = 10

   $TVC = TC - TFC \qquad = 2Q^3 - 2Q^2 + Q + 10 - 10$

   $TVC = 2Q^3 - 2Q^2 + Q$

B. $AFC = TFC/Q = 10/Q$
   $AVC = TVC/Q = (2Q^3 - 2Q^2 + Q)/Q = 2Q^2 - 2Q + 1$

   $AC = TC/Q = (2Q^3 - 2Q^2 + Q + 10)/Q = 2Q^2 - 2Q + 1 + 10/Q$

   $MC = dC/dQ = 6Q^2 - 4Q + 1$

C. Minimum value of MC is obtained at output level, which will make the first derivative of MC zero. Therefore, to find the minimum value of MC:

$$dMC/dQ = 12Q - 4 = 0$$

$$Q = 1/3$$

To find the minimum value of $MC = 6(1/3)^2 - 4(1/3) + 1 = 0.37$

We follow the same step to get the minimum value of AVC

$$AVC = 2Q^2 - 2Q + 1$$

$d\,AVC/dQ = 4Q - 2 = 0$, $Q=0.5$. To find the minimum value of AVC

$AVC = 2(0.5)^2 - 2(0.5) + 1$

$= 0.5 - 1 + 1$

$\underline{= 0.5}$

5.2.1. The relation between production and cost.

As we know production involves cost. Therefore, we have to analyze how production and cost are related

In this section we will see how marginal product of labor and marginal cost are related, and also how average product of labor and average variable cost are related.

Suppose a firm in the short – run uses labor as a variable input and capital as a fixed input. We can put the production function as follows.

Q = f(L,K) …………………….Where Q = level of output

L = variable input (labor)

K = fixed input (capital)

Let the price of the variable input labor be given by w, which is constant. Using the production function it is possible to show the relations between production cost variables.

a) How MC and MPL are related?

Let as start by defining the MC of this firm as:

$MC = \frac{\Delta TVC}{\Delta Q}$, But TVC=w.L

$MC = \frac{\Delta(w.L)}{\Delta Q} \quad = w.\frac{\Delta L}{\Delta Q}$, but $\frac{\Delta L}{\Delta Q} = \frac{1}{MP_L}$ Therefore, $MC = \frac{w}{MP_L}$

From the above relation we can clearly see the relationship between MC and MP_L.

The above expression shows that MC and MP_L are inversely related, which means when initially MP_L increase, MC decreases and when MP_L is at maximum MC must be at a minimum and when finally MP_L declines MC increases.

B. How average variable cost is related with average product of labor?

$AVC = \frac{TVC}{Q}$, but TVC=w.L, so

$AVC = \frac{w.L}{Q} = w.\frac{L}{Q}$, but $\frac{L}{Q} = \frac{1}{AP_L}$

Therefore, $AVC = \dfrac{w}{AP_L}$

This relation also shows inverse relation between AVC and APL which means when AP_L increase AVC decreases & when AP_L is at a maximum AVC is at a minimum and when finally AP_L declines, AVC increases.

From the above relations use can conclude that the reason for the 'U' shape of the MC of AVC is the law of variable proportion which causes the MP_L to decline.

We can also see the relationship between these production and cost variables using graphs. This is given as follows.

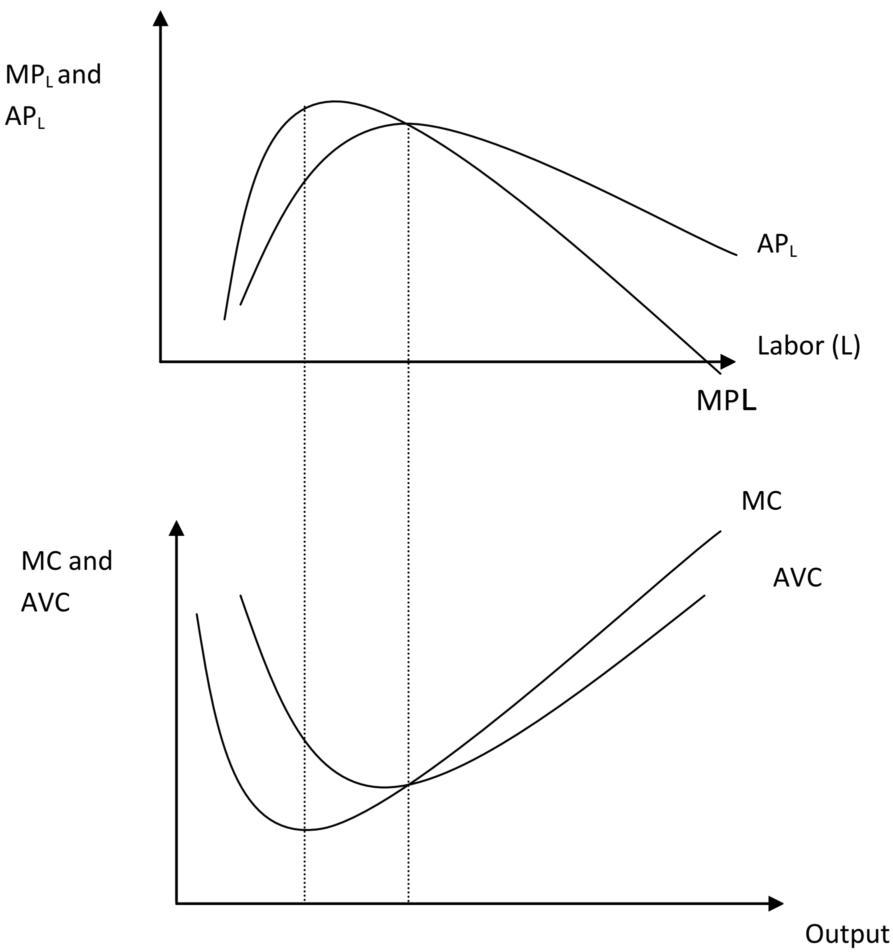


Fig 5.2 Production and cost curves

From the above diagram we can conclude that the MC curve is the mirror image of MP_L curve and AVC curve is the mirror image of AP_L curve

Review Questions

PART I: Multiple choice questions

--------------- 1. Which one of the following statement is true?

a. Accounting cost can never be greater than economic cost
b. Explicit costs are the estimated cost of non – purchased inputs
c. Accounting costs ignore the cost of purchased inputs
d. Accounting profit can be smaller than economic profit

---------------- 2. Implicit costs are costs______.

a. having direct expenses
b. which are not incurred by the firm
c. of non – purchased inputs
d. which are not part of private cost

---------------- 3. In the short – run period of production, when output increases, the AFC

a. Increase
b. Decreases
c. Remain the same
d. None of the above

--------------- 4. MC is given by all of the following except______

a. The slope of total product curve
b. The slope of TVC curve
c. The slope of TC curve
d. None of the above

--------------- 5. All of the following curves are U – shaped except

a. LAC b. AFC c. AVC d. MC

--------------- 6. Suppose a firm pays Br.60 to each unit of the variable input used and if the marginal product of labor is 3 units, then the marginal cost will be ------

a. Birr.20 b. Birr.120 c. Birr.40 d. None of the above

--------------- 7. In the short – run, if the firm does not produce any output (Q = 0), then------.

a. Total cost is zero

b. Variable cost is zero

c. Total cost and total fixed cost will be equal

d. All except A

--------------- 8. Which one of the following cost is more likely to be a variable cost of the firm?

a. Insurance payment
b. Rent for land
c. Costs of raw materials
d. Interest on borrowed funds.

--------------- 9. The reason for the eventual rise of LAC is --------------.

a. Economies of scale
b. Diseconomies of scale
c. Over – specialization
d. All of the above

--------------- 10. Which one of the following is true at the minimum point of LAC?

a. LAC = LMC > SAC = SMC
b. LAC = SAC < SMC = LMC
c. LAC = LMC = SAC = SMC
d. LAC = LMC = SAC > SMC

PART II: Discussion questions

1. Compare and contrast the following cost concepts.
   a. Social cost and private cost
   b. Explicit cost and implicit cost
   c. Economic and accounting cost
2. Distinguish between short – run and long – run cost functions
3. Why does long – run average cost curves fall, and then rise? Why do short – run average cost curves first fall, and then rise?
4. What happens to average total cost as:
   a. Marginal product increases?
   b. Average product increases?
5. When does average cost increase?
   Answer this in terms of:

   a. The relation of average cost to marginal cost
   b. The relation between the increase in AVC and the decrease in AFC.

PART III: Workout Questions

1. Fill in the blanks in the following table.

Output (Q)	TC	TFC	TVC	AFC	AVC	AC	MC
0							
1					30		
2			40				
3		10			15		
4	65						
5							15

2. Given a short run cost function as.

$C = \frac{1}{3}Q^3 - 2Q^2 + 60Q + 100$, Find the minimum value of AVC and MC