Unit 4: Direct Current Circuits

To define current more precisely, suppose that charges are moving perpendicular to a surface of area A, as shown in Figure 1. (This area could be the cross-sectional area of a wire, for example.)

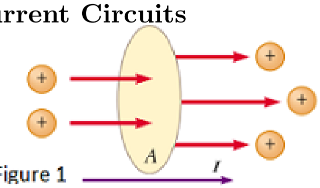

Figure 1

Current is the rate at which charge flows through this surface. If ΔQ is the amount of charge that passes through this area A in a time interval Δt, the **average current** I_{av} is

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous Current

If the flow is not steady, the instantaneous **electric current I** is defined as

$$I = \frac{dQ}{dt}$$

The SI unit of current is the ampere (A). Where $1A = 1\ C/s$.

Direction of current is:

⇒ **same** as direction of **positive** charges crossing a surface.

⇒ **opposite** to direction of **negative** charges crossing a surface.

Current Density

Look at a wire with cross-sectional area A shown in Figure 2. Assume the number of mobile charge carriers per unit volume in the wire is n. Let q and v_d be the charge and drift velocity of each particle in the wire.

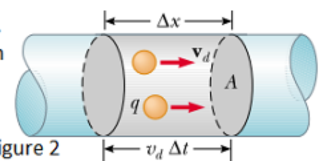

Figure 2

The total charge within a cylinder of length Δx and area A is $\Delta Q = n(A\Delta x)q$. This charge takes a time $\Delta t = \Delta x / v_d$ to pass the end of the cylinder. The current, given by $I = \Delta Q/\Delta t$, is

$$I = nAqv_d$$

The (average) **current density** is defined as the current per unit area:

$$J = \frac{I}{A} = \frac{nAqv_d}{A} = nqv_d$$

The SI unit of current density is A/m^2. The current density $\vec{J}$ is a vector quantity.

Resistance

In unit 2, we have seen that the **electric field inside a conductor in electrostatic equilibrium is zero**. However, **if the conductor is not in electrostatic equilibrium** charges in the conductor are not in equilibrium, in which case **there is an electric field in the conductor** which accelerate the charges (electrons) between collisions. However, their velocities do not increase indefinitely because they collide with the array of positive ions that form the crystal lattice. The **average extra velocity** that each electron in the conductor gains due to the electric field within it is called **drift velocity** $\vec{v}_d$. **The magnitude of drift velocity(** v_d **) due to field-induced acceleration between collisions is proportional to electric field in the conductor**, i.e., $v_d \propto E$ and thus **on potential difference per unit length** ($v_d \propto \Delta V/l$) in material.

J and **E** are established in a conductor whenever a $p.d.$ is maintained across the conductor. In some materials, the current density is proportional to the electric field:

$$\vec{J} = \sigma \vec{E}$$

where the constant of proportionality σ is called the **conductivity** of the conductor. Materials that obey this relationship between current density $\vec{J}$ and electric field $\vec{E}$ are said to follow **Ohm's law**, and are called **ohmic**. Materials that do not obey Ohm's law are called **nonohmic**.

We can obtain an equation useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length l, as shown in Figure 3.

Figure 3. A uniform conductor of length l and cross-sectional area A. A potentialdifference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field E, and this field produces a current I that is proportional to the potential difference.

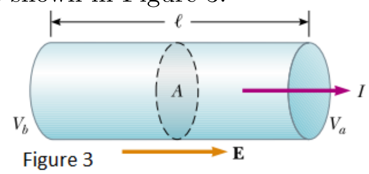

Figure 3

A $p.d.$ $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the $p.d.$ is related to the field as: $\Delta V = El$

Therefore, we can express the magnitude of the current density in the wire as: $J = \sigma E = \sigma \frac{\Delta V}{l}$

Because $J = I/A$, we can write the potential difference as

$$\Delta V = \frac{l}{\sigma} J = (\frac{l}{\sigma A}) I = RI$$

The quantity $R = l/\sigma A$ is called the **resistance** of the conductor. We can define the **resistance as the ratio of the potential difference across a conductor to the current in the conductor:**

$$R = \frac{\Delta V}{I}$$

Resistance has SI units of volts per ampere called **ohm**(Ω). Where $1\ \Omega = 1\ V/A$.

Resistivity ρ is inverse of conductivity:

$$\rho = \frac{1}{\sigma}$$

where ρ has the units ohm-meters $(\Omega \cdot m)$. Because $R = l/\sigma A$, we can express the resistance of a uniform block of material along the length l as

$$R = \frac{\rho l}{A}$$

• Every ohmic material has a characteristic **resistivity** and **conductivity** that depend on properties of the material and on temperature. Both do not depend on the shape of the conducting object.

Resistance and Temperature

Over a limited temperature range, the ***resistivity*** (ρ) of a conductor varies approximately ***linearly*** with temperature T (in degrees Celsius) according to the expression

$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

ρ_o is the resistivity at some reference temperature T_0 (usually taken to be 20^0C), and α is the **temperature coefficient of resistivity**. Note that the unit for ρ is $degrees\ Celsius^{-1}[(^0C)^{-1}]$.

Resistance of a material varies with temperature in the same way as resistivity.

$$R = R_o[1 + \alpha(T - T_o)]$$

Electrical Energy and Power

Power is defined as the time rate of doing work,

$$P = \frac{W}{\Delta t}$$

In typical electric circuits, energy is transferred from a source such as a battery, to some device, such as a lightbulb or a radio receiver. The amount of energy transferred is equal to the the work done by the source supplying the energy.

$$W = \Delta U = q\Delta V$$

So, power can also be defined as

$$P = q\frac{\Delta V}{\Delta t} = \frac{q}{\Delta t}\Delta V = I\Delta V$$

SI unit of power is **watt** where $1\ W = 1 A \cdot V$. Using Ohm's law, we can also express power as

$$P = I^2 R = \frac{(\Delta V)^2}{R}$$

Electro motive force ($e.m.f$)

A battery (Figure 4) is a source of energy for a circuit. Because the $p.d.$ at the battery terminals is constant in a particular circuit, the current in the circuit is constant and is called **direct current**(dc). A battery is called either a source of electromotive force or, more commonly, a source of emf.

•***emf* of a battery is the maximum voltage that battery can provide b/n its terminals**.

The $+ve$ terminal of the battery is at a higher potential than the $-ve$ terminal. Because a real battery is made of matter, there is resistance to the flow of charge within the battery. This resistance is called **internal resistance** r. For an **idealized battery with zero internal resistance, the *p.d.* across the battery (called its terminal voltage) equals its emf.** However, for a real battery (Fig.4), the **terminal voltage** between the ends of a battery is less than the emf of the battery when there is a flow of current through the battery. The terminal voltage is calculated as

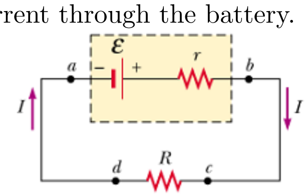

Figure 4. Circuit diagram of a source of emf (in this case, a battery), of internal resistance r, connected to an external resistor of resistance R.

$$\Delta V = V_b - V_a = \varepsilon - Ir$$

ε is equal to the terminal voltage when the current is zero.

From Ohm's law the potential difference between points c and d across the the external resistance R (Fig.4), called the **load resistance**, is $\Delta V = IR$,

$$IR = \varepsilon - Ir \qquad \Rightarrow \qquad \varepsilon = IR + Ir = I(R + r)$$

The current in the load resistor shown in Fig. 4 is

$$I = \frac{\varepsilon}{R+r}$$

Combination of Resistors

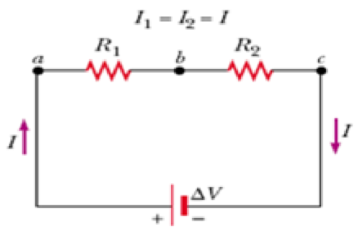

Figure 5. Series combination of two resistors

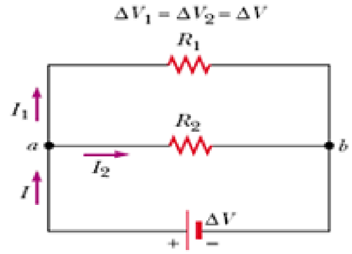

Figure 6. Parallel combination of two resistors

(i) Resistors in series

• The current through all resistors in a series circuit (Figure 5) is the same .

$$I = I_1 = I_2$$

• The potential difference applied across the series combination of resistors will divide between the resistors. For two resistors in series combination,

$$\Delta V = \Delta V_1 + \Delta V_2$$

Using $\Delta V_1 = IR_1$ and $\Delta V_2 = IR_2$,from Ohm's law,as potential drops across the two resistors in the above equation, we get

$$IR_{eq} = IR_1 + IR_2 = I(R_1 + R_2) \Rightarrow R_{eq} = R_1 + R_2$$

Generally for any number of resistors in series combination,

$$R_{eq} = R_1 + R_2 + R_3 + ... = \sum_i R_i..............(series\ combination\ of\ resistors)$$

The equivalent resistance of a series connection of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.

(ii) Resistors in parallel

• When resistors are parallel (Figure 6), the potential difference across them are the same.

$$\Delta V = \Delta V_1 = \Delta V_2$$

• The current entering a parallel circuit equals the sum of all currents traveling through each resistor:

$$I = I_1 + I_2$$

Using $I = \frac{\Delta V}{R_{eq}}$ for total current in the circuit, $I_1 = \frac{\Delta V_1}{R_1} = \frac{\Delta V}{R_1}$ and $I_2 = \frac{\Delta V_2}{R_2} = \frac{\Delta V}{R_2}$,from Ohm's law, for currents in the two resistors in the above equation, we get

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

An extension of this analysis to three or more resistors in parallel gives

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... = \sum_i \frac{1}{R_i}.............(Resistors\ in\ parallel)$$

The inverse of the equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

Kirchhoff's Rules

1. **Junction rule**: States that **the sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction**:

$$\sum I_{in} = \sum I_{out}$$

This is a statement of conservation of electric charge. All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point. If we apply this rule to the junction shown in Figure 7, we obtain

$$I_1 = I_2 + I_3$$

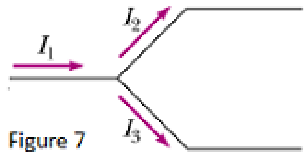

Figure 7

2. **Loop rule: The sum of the potential differences across all elements around any closed circuit loop must be zero**:

$$\sum \Delta V_{closed\ loop} = 0$$

When applying this rule in practice, we imagine **traveling** around the loop and **consider changes in electric potential**, rather than the changes in potential energy. You should note the following sign conventions when using the second rule:

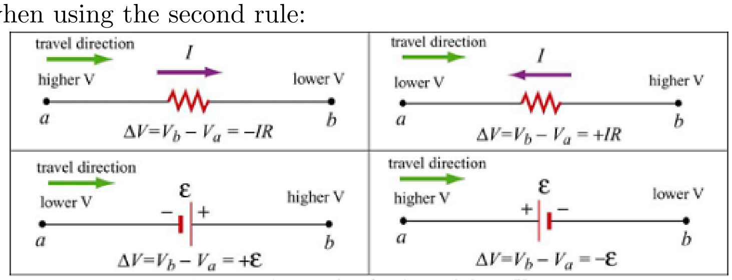

Figure 8 Convention for determining ΔV .

In order **to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents**.

RC circuits

In d.c circuits containing only resistors, current is constant. However, in d.c circuits containing capacitors, the current is always in the same direction but may vary in time. A circuit containing a series combination of a resistor and a capacitor is called an **RC circuit**.

a) **Charging a Capacitor**

Let us consider a simple RC circuit (Figure 9) and assume that the capacitor is initially uncharged. There is no current while switch S is open (Fig. 9(b)). If the switch is closed at $t = 0$, however, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge.

To analyze this circuit quantitatively, let us apply **Kirchhoffs loop** rule to the circuit after the switch is closed. Traversing the loop in Fig.9c clockwise gives

$$\varepsilon - \frac{q}{C} - IR = 0$$

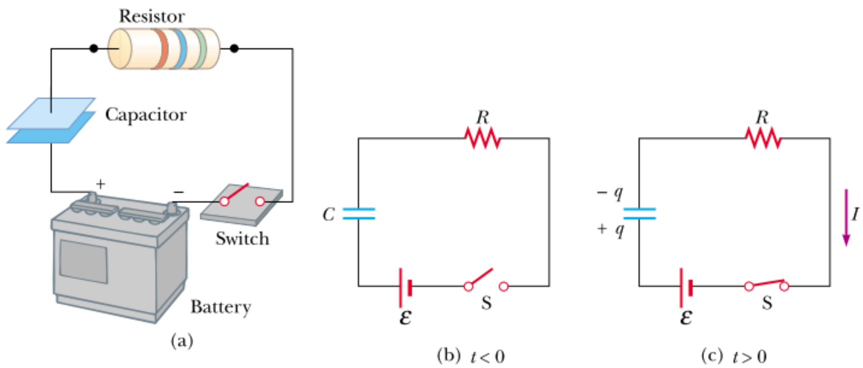

Figure 9 (a) A capacitor in series with a resistor, switch, and battery.
(b) Circuit diagram representing this system at time $t < 0$, before the switch is closed.
(c) Circuit diagram at time $t > 0$, after the switch has been closed.

Note that q *and* I are instantaneous values that depend on time (as opposed to steady-state values) as the capacitor is being charged. To determine analytical expressions for the time dependence of the charge and current, we must solve this Equation a single equation containing two variables, q and I. We substitute $I = dq/dt$ into the above differential Equation and rearranging the equation gives

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC}$$

To find an expression for q, we solve this separable differential equation. We first combine the terms on the right-hand side:

$$\frac{dq}{dt} = \frac{C\varepsilon}{RC} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC}$$

Now we multiply by dt and divide by $q - C\varepsilon$ to obtain

$$\frac{dq}{q - C\varepsilon} = -\frac{1}{RC}dt$$

Integrating this expression, using the fact that $q = 0$ at $t = 0$, we obtain

$$\int_0^q \frac{dq}{(q - C\varepsilon)} = -\frac{1}{RC}\int_0^t dt \quad \Rightarrow \quad \ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

From the definition of the natural logarithm, we can write this expression as

$$q(t) = C\varepsilon(1 - e^{\frac{-t}{RC}}) = Q(1 - e^{\frac{-t}{RC}})$$

where e is the base of the natural logarithm and $Q = C\varepsilon$ is the maximum charge on the plates of the capacitor. We can find an expression for the charging current by differentiating this Equation with respect to time.

$$I(t) = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$$

$I_o = \frac{\varepsilon}{R}$ is the maximum current in the circuit. Note that the charge is zero at $t = 0$ and approaches the maximum value $C\varepsilon$ as $t \to \infty$.The current has its maximum value $I_0 = \varepsilon/R$ at $t = 0$ and decays exponentially to zero as $t \to \infty$.

b) Discharging a Capacitor

Now consider the circuit shown in Figure 11, which consists of a capacitor carrying an initial charge Q, a resistor, and a switch. When the switch is open, a potential difference Q/C exists across the capacitor and there is zero potential difference across the resistor because $I = 0$.

Figure 11 (a) A charged capacitor connected to a resistor and a switch, which is open for t = 0. (b) After the switch is closed at t = 0, a current that decreases in magnitude with time is set up in the direction shown, and the charge on the capacitor decreases exponentially with time.

If the switch is closed at $t = 0$, the capacitor begins to discharge through the resistor. At some time t during the discharge, the current in the circuit is I and the charge on the capacitor is q (Fig.11(b)). The circuit in Figure 11 is the same as the circuit in Figure 9 except for the absence of the battery. Thus, we eliminate the emf to obtain the appropriate loop equation for the circuit in Figure 11.

$$-\frac{q}{C} - IR = 0$$

When we substitute $I = dq/dt$ into this expression, it becomes

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}dt$$

Integrating this expression, using the fact that $q = Q$ at $t = 0$ gives

$$\frac{dq}{q} = -\frac{1}{RC}dt$$

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$q(t) = Qe^{\frac{-t}{RC}}$$

Differentiating this expression with respect to time gives the instantaneous current as a function of time:

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}(Qe^{\frac{-t}{RC}}) = -\frac{Q}{RC}e^{\frac{-t}{RC}}$$

where $Q/RC = I_0$ is the initial current. The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged. We see that both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant $\tau = RC$.