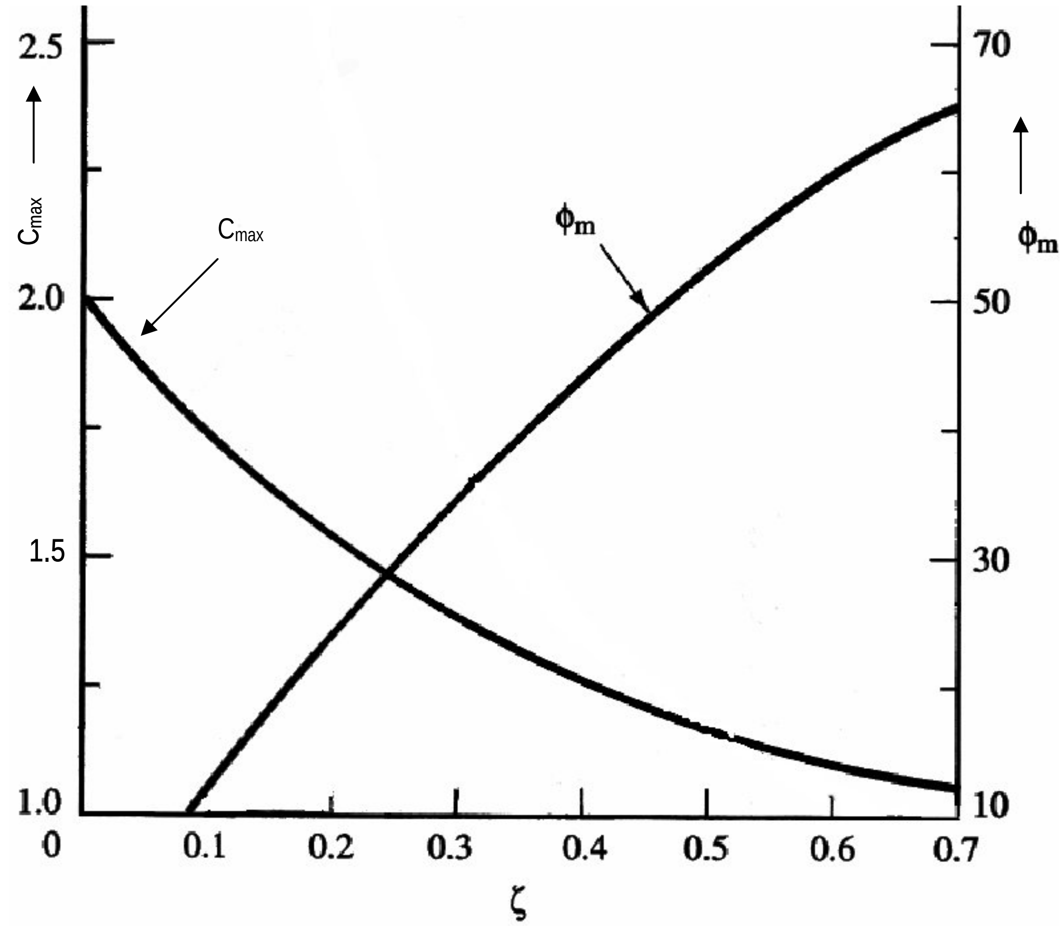
System stability and dynamics from open-loop Bode plots.
Phase margin and gain margin.

The closed loop system is stable if the open loop magnitude is less than unity where the phase is -180^{o} or less.
Phase margin φ_m is the change in open-loop phase shift required at unity gain, to make the closed loop system unstable.
Gain margin G_m is the change in open loop gain required at -180^{o} of phase shift to make the closed loop system unstable.

Assuming the system has 2 dominant poles, the phase margin φ_m is a function of ζ and related to the system overshoot, %OS:

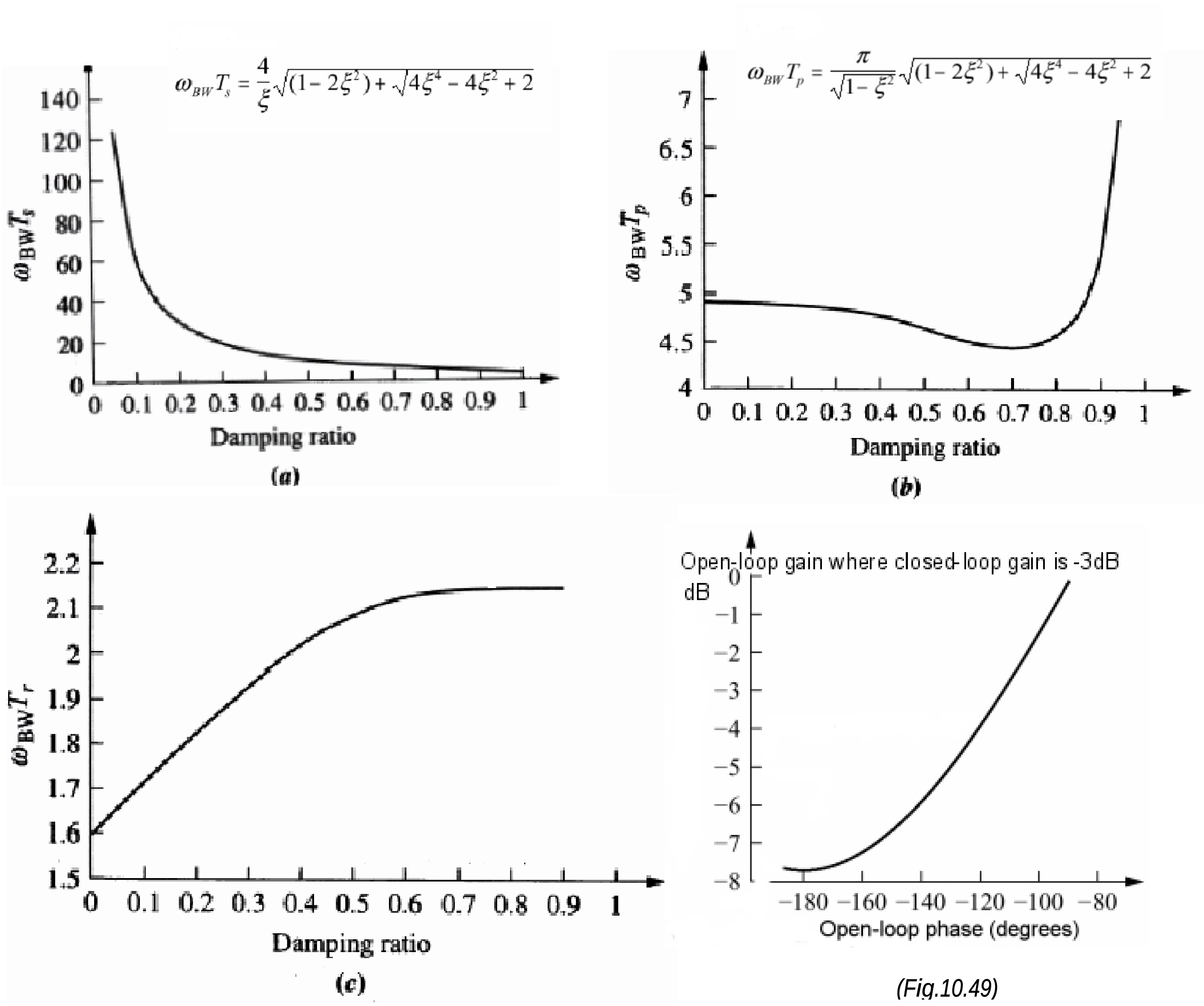
$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \varphi_m = \tan^{-1}\left(\frac{2\zeta}{(-2\xi^2 + (1+4\xi^4)^{1/2})^{1/2}} \right) \cong 100\varsigma$$

$$C_{\max} = 1 + e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \qquad \zeta = \frac{-\ln(\%OS/100}{(\pi^2 + \ln^2(\%OS/100))^{1/2}}$$



Characteristics of a second-order system

The bandwidth ω_{BW} of a system is defined as the frequency where the pass band gain has decreased with 3db from its maximum value (here the DC gain).
The bandwidth ω_{BW} for the closed loop is approximated as equal to the phase-margin frequency, $\omega_{\varphi m}$, for the open loop. Bandwidth and ζ decides the settling time, peak time and rise time for the closed loop. The relations is shown graphically below as a function of the damping ratio, ζ :



(Fig.10.49)

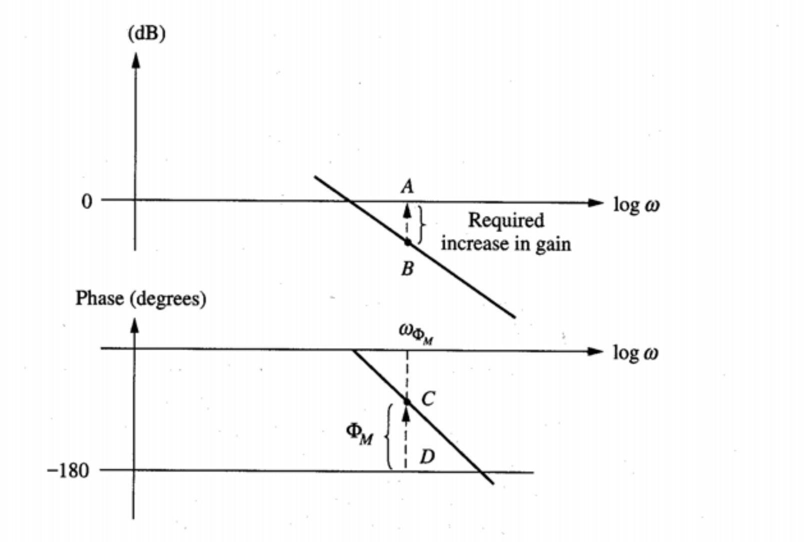
Analysis:
$\varphi_{m.}$ decides ζ and by that the overshoot
$\omega_{\varphi m}$ decides the bandwidth, and by that settling-, peak- and rise time.

DC-gain decides the steady state error $e(\infty)$.

Dynamic design criteria, demands:
Overshoot decides ζ and by that $\varphi_{m.}$
Settling-, peak- or rise time decides the bandwidth, and by that $\omega_{\varphi m}$
Realised by a P- and/or Lead controller

Static design criteria, demand:
$e(\infty)$ decides the DC-gain.
Realised by a Lag controller.

Design procedure for gain compensators. $G_c(s) = K_c$



1. Draw the Bode magnitude and phase plots for the open-loop system at a convenient value of gain.

2. Determine the required phase margin φ_m from the knowledge of percent overshoot. If you expect to add a lag compensation, choose φ_m 5-10^0 larger, to compensate the negative phase contribution at $\omega_{\varphi m}$.

3. Find the frequency, $\omega_{\varphi m}$, on the the Bode phase diagram that yields the desired phase margin φ_m, found in item 2). CD on the figure.

4. Change the gain by the amount AB on the figure, to force the magnitude curve to go through 0 dB at $\omega_{\varphi m.}$ The amount of gain adjustment is the additional gain needed to produce the required phase margin.

Design Procedure for lag compensator, $G_c(s)$.

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

1. Decide the gain value α that satisfies the steady-state error specifications. The steady-state characteristic can be improved with $1 < \alpha < \infty$. If $\alpha = \infty$ the compensator is called an integral compensator.

2. The zero of the lag compensator should be placed 10 – 20 times lower than the phase margin frequency, $\omega_{\varphi m}$. Then the lag compensator will contribute with from -5 to -10^o of phase at $\omega_{\varphi m}$. This should be taken in consideration when choosing φ_m. Decide T from:

$$\frac{1}{T} = \frac{\omega_{\varphi_m}}{10}$$

Design Procedure for lead compensator, $G_c(s)$.

$$G_c(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}K_c$$

1. Draw the frequency response for the open-loop system, without compensation.

2. Decide φ_m to meet the damping ratio and percent overshoot requirement. If you expect to add a lag compensator afterwards, choose φ_m 5-10^0 higher than required (see the lag procedure).

3. Settling-, peak- or rise time decides the bandwidth (taken as $\omega_{\varphi m}$) of the closed loop, and by that $\omega_{\varphi m}$. The bandwidth can be increased to a 10- 20 times higher value with the lead compensator.

4. Decide the positive phase contribution, φ_{m+}, required to meet the phase-margin and the bandwidth. Then find β from fig.b or the equation: $\beta = \frac{1-\sin\varphi_{m+}}{1+\sin\varphi_{m+}}$ $0 < \beta < 1$

5. Decide T on the basis of β and $\omega_{\varphi m}$ while $\omega_{\varphi m} = \omega_{max} = \frac{1}{T\sqrt{\beta}}$ or use the fig.a below, with $\omega_{\varphi m} = \omega_{.}$ $|G_c(j\omega_{max})| = K_c / \beta^{½}$

6. Decide K_c to force the total open-loop magnitude to go through 0 dB at $\omega_{\varphi m}$.

$$|G_c(j\omega_{\varphi_m})| \cdot |G(j\omega_{\varphi_m})| = 1$$

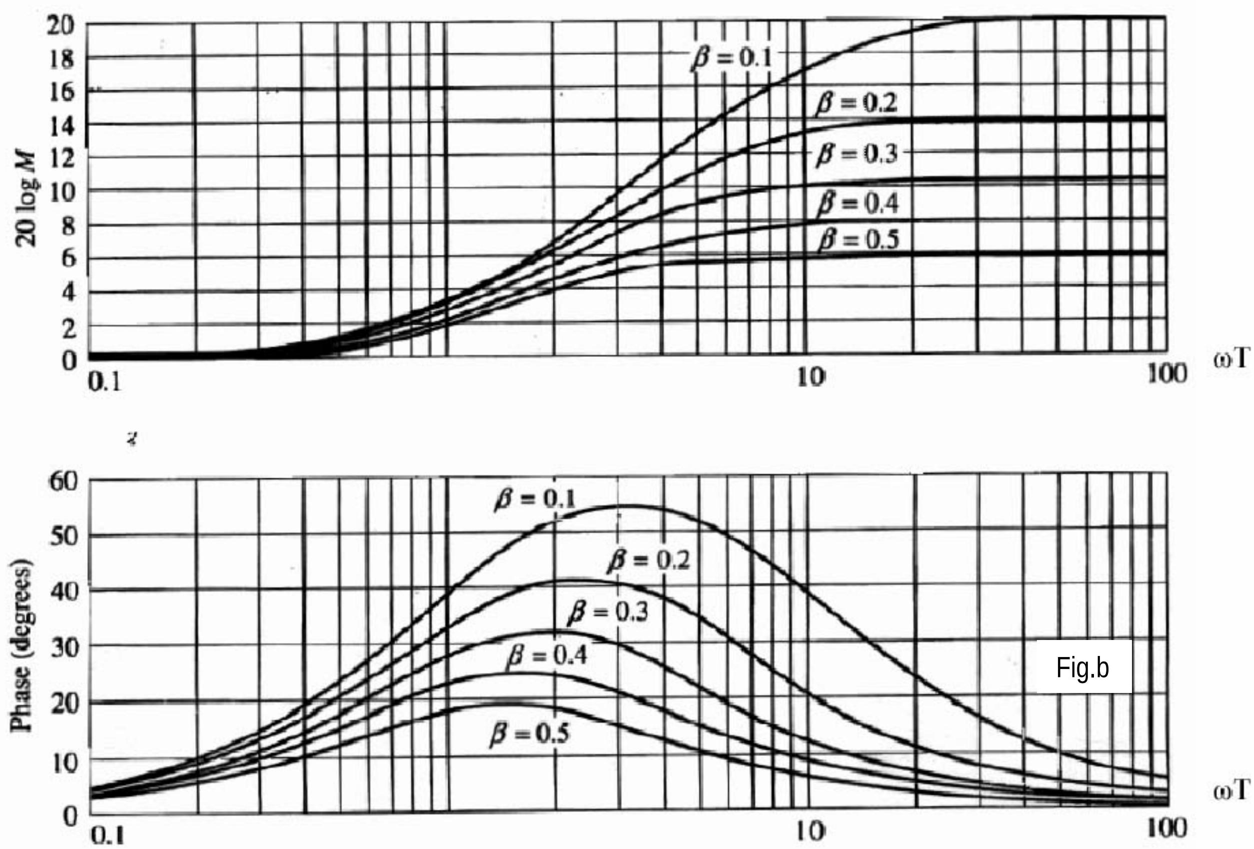


Fig.b

PID-Controller Design

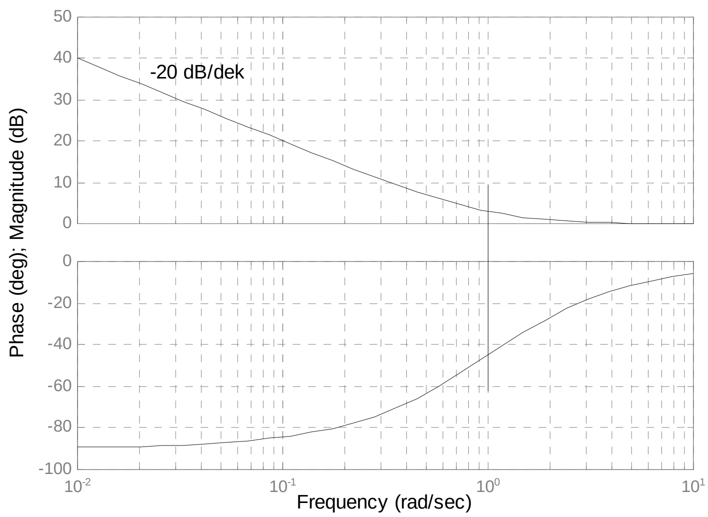
Ready build controllers are a common commercial product found in numerous applications. Often this product is called a PID-Controller even though it's a lead-lag compensator.
Theoretical the PID-controller has the transfer function:

$$G_c(s) = K_p (1 + 1/\tau_i s + \tau_D s)$$

This controller has two zero's and one pole. That's why it can't be realised. As a substitute is used a PI-lead controller, but still called PID-controller.
PID-control is a subset of P-, Lag- and Lead control.

Normally you should design the PD-part first and determine the characteristics at high frequencies. $G_c(s) = K_p (1 + \tau_D s)$ has a magnitude becoming infinity at high frequencies and in this way amplifies noise. That's the reason why a pole is always added, but it might be hidden for the user, so that only K_p and τ_D can be adjusted. We will always substitute the PD-part with a lead compensator and design it like this.

Next the PI-part is added $G_c(s) = K_p (1 + 1/\tau_i s) = K_p [(\tau_i s + 1) / \tau_i s]$. This is a lag compensator with it's pole placed at zero.
Bode plot for $G_c(s) = [(\tau_i s + 1) / \tau_i s]$ with $\tau_i = 1$ and $K_p = 1$ is shown below:



The PI-part will increase the system type with one because of the integration. The open-loop phase is increased with -90^o at lower frequencies which could cause problems in some systems. On the other hand not only the static error but also the sensitivity to noise and parameter changes is improved in this area.

The design procedure for lead- and lag compensators in general should be followed.

In practice the lead-part would often be placed in the feedback line, where the dynamic change is limited within the bandwidth. This will cause less saturation problems.