

$$1) a) P(A, B | X) = P(B | X) P(A | B, X)$$

$$\frac{P(A, B, X)}{P(X)} = \frac{P(A | B, X) P(B | X) P(X)}{P(X)} = P(A | B, X) P(B | X)$$

$$b) P(A, B, C, D) = P(A | B, C, D) P(B | C, D) P(C | D) P(D)$$

The general form of the chain rule is $P(A_1, A_2, \dots, A_n) = P(A_1 | A_2, \dots, A_n) P(A_2 | A_3, \dots, A_n) P(A_3 | A_4, \dots, A_n) P(A_n)$, therefore
 $P(A, B, C, D) = P(A | B, C, D) P(B | C, D) P(C | D) P(D)$.

c) Yes because the chain rule deals with multiplication, it doesn't matter what variables are put where.

$$2) a) P(A) P(B | A) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$b) P(C | A) P(C | \neg A) = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

c) 0 because you can never have B and C at the same time



	yellow 80%	green 20%
was green	$\frac{1}{4}$	$\frac{2}{3}$
not green	$\frac{3}{4}$	$\frac{1}{3}$

	yellow	green
was green	0.2	0.13333
not green	0.6	0.06666

$$0.2 + 0.13333 = \frac{1}{3}$$

$$\frac{0.13333}{\frac{1}{3}} = 0.4$$

4) a) It would be difficult because the variation would be too high. The odds of documents starting the same way and having words in the same positions is very slim