SIT718 Real world Analytics
Assessment Task 3

1. A food factory is making a beverage for a customer from mixing two different existing products A and B. The compositions of A and B and prices ($/L) are given as follows,

	Amount (L) in /100 L of A and B			
	Lime	Orange	Mango	Cost ($/L)
A	2	6	4	4
B	7	4	8	12

The customer requires that there must be at least 5 Litres (L) Orange per 100 Litres of the beverage and at least 5 Litres of Mango concentrate per 100 Litres of the beverage, but no more than 6 Litres of Lime concentrate per 100 Litres of beverage. The customer needs at least 140 Litres of the beverage per week.

(a) Explain why a linear programming model would be suitable for this case study.

[5 marks]

(b) Formulate a Linear Programming (LP) model for the factory that minimises the total cost of producing the beverage while satisfying all constraints.

[5 marks]

(c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution on the graph. Annotate all lines on your graph. [The graph can be drawn by hand or using any graphical solvers, but make sure that your graph is clear, all variables involved are clearly represented and annotated, and each line is clearly marked and related to the corresponding equation.]

[5 marks]

(d) What is the range for the cost ($) of A that can be changed without affecting the optimum solution obtained above?

[5 marks]

2. A factory makes three products called Spring, Autumn, and Winter, from three materials containing Cotton, Wool and Silk. The following table provides details on the sales price, production cost and purchase cost per ton of products and materials respectively.

	Sales price	Production cost		Purchase price
Spring	\$60	\$5	Cotton	\$30
Autumn	\$55	\$3	Wool	\$45
Winter	\$65	\$8	Silk	\$50

The maximal demand (in tons) for each product, the minimum cotton and wool proportion in each product is as follows:

	Demand	min Cotton proportion	min Wool proportion	min Silk proportion
Spring	3200	55%	30%	1%
Autumn	3800	45%	40%	2%
Winter	4200	30%	50%	3%

(a) Formulate an LP model for the factory that maximises the profit, while satisfying the demand and the cotton and wool proportion constraints. There is no penalty for the shortage.

[20 Marks]

(b) Solve the model using R/R Studio. Find the optimal profit and optimal values of the decision variables. (You don't need not copy the code to the report as we will directly evaluate the code you submit.)

[20 Marks]

Hints:
You may refer to Week 8.7 Example - Blending Crude Oils into Gasolines. For example, let $x_{ij} \geq 0$ *be a decision variable that denotes the number of tons of products* j *for* $j \in \{1 = Spring, 2 = Autumn, 3 = Winter\}$ *to be produced from Materials* $i \in \{C{=}Cotton,\ W{=}Wool,\ S{=}Silk\}$.

3. Consider the following parlor game to be played between two players. Each player begins with three chips: one red, one white, and one blue. Each chip can be used only once. To begin, each player selects one of her chips and places it on the table, concealed. Both players then uncover the chips and determine the payoff to the winning player. In particular, if both players play the same kind of chip, it is a draw; otherwise, the following table indicates the winner and how much she receives from the other player. Next, each player selects one of her two remaining chips and repeats the procedure, resulting in another payoff according to the following table. Finally, each player plays her one remaining chip, resulting in the third and final payoff.

Winning Chip	**Payoff ($)**
Red beats white	50
White beats blue	20
Blue beats red	5
Matching colors	0

(a) Formulate the payoff matrix for the game and identify possible saddle points.

[10 Marks]

(b) Construct a linear programming model for each player in this game. In other words, one LP model for Player I and the other LP model for Player II.

[10 Marks]

(c) Produce an R code to solve the linear programming model for this game. (You don't need not copy the code to the report as we will directly evaluate the code you submit.)

[10 Marks]

(d) Solve the game for both players using the linear programming model. Identify optimal solutions and payoffs for both players. Interpret the corresponding strategies. Are these optimal solutions unique?

[10 Marks]

[Hint: Each player has the same strategy set. A strategy must specify the first chip chosen, the second and third chips chosen. Denote the white, red and blue chips by W, R and B respectively. For example, a strategy "WRB" indicates first choosing the white and then choosing the red, before choosing blue at the end.]