ET4340 Electronics for Quantum Computing
Homework 5

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Problem 1: Warmup exercises

1. Consider an operator M and one of its eigenstates $|\psi\rangle$ (with eigenvalue λ). Consider another operator A that anticommutes with M (i.e., $\{M, A\} \equiv MA + AM = 0$). Show that the state $A|\psi\rangle$ is an eigenstate of M with eigenvalue $-\lambda$.

We can write $M|\psi\rangle = \lambda|\psi\rangle$.

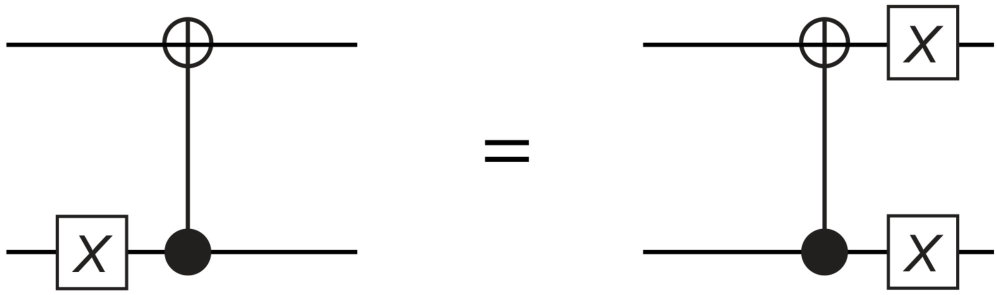
$$
\begin{aligned}
MA + AM &= 0 \\
MA|\psi\rangle + AM|\psi\rangle &= 0 \\
MA|\psi\rangle + A\lambda|\psi\rangle &= 0 \\
M(A|\psi\rangle) &= -\lambda(A|\psi\rangle)
\end{aligned}
$$

2. Now consider an operator B that commutes with M (i.e., $[M, B] \equiv MB - BM = 0$). Show that the state $B|\psi\rangle$ is an eigenstate of M with eigenvalue λ.

Assuming M again has an eigenstate $|\psi\rangle$ with eigenvalue λ.

$$
\begin{aligned}
MB - BM &= 0 \\
MB|\psi\rangle - BM|\psi\rangle &= 0 \\
M(B|\psi\rangle) &= \lambda(B|\psi\rangle)
\end{aligned}
$$

3. Prove the identity:



Feel free to do this either by multiplying matrices or by manipulating circuit diagrams. From this we see that a single-qubit bit-flip error prior to `CNOT` proliferates into a double bit-flip error.

It is an intuitive identity in my opinion. Flipping the control bit flips the output. So to 'simulate' the flip before a `CNOT` you can flip both the output and the control after the `CNOT`.

By matrice multiplication (note that the order of operations is reversed with respect of the diagram because we compute the combined matrix $M = M_n \dots M_2 M_1$ in $|out\rangle = M\,|in\rangle$ where M_n is operation n):

$$
\begin{aligned}
M_a = \mathtt{CNOT}_{01}(I \otimes X) \quad &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\
M_b = (X \otimes X)\mathtt{CNOT}_{01} \quad &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\
M_a = M_b
\end{aligned}
$$