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## Lecture 1: Basic Concepts

Galaxies are gravitationally bound units composed of stars, gas, and dust, which live in dark matter halos.

### Basic concepts & formulae

- Hubble law:  $v_r = H_0 d$  where  $H_0 \approx 70$  km/s/Mpc, it is a good approximation for the nearby universes
- Redshift:  $z = 1 - \lambda_e/\lambda_o$ , note that  $v_r = cz$  if  $v_r \ll c$
- Luminosity:

$$L = 4\pi d_L^2 f = 4\pi R^2 \sigma T^3$$

where  $f$  is the integrated flux density

- Angular separation:  $\alpha = D/d$  where  $\alpha$  is in radians
- Telescope diffraction limit: For two point sources to be distinguishable we require that  $\alpha > \theta$ ,

$$\theta = \frac{1.22\lambda}{D}$$

- Magnitude systems:  $m = -2.5 \log(f_\nu/f_0) = -2.5 \log(f_\nu) + \text{ZP}$
- Flux through a filter:

$$f_\nu(\lambda) = \frac{\int f_\nu(\lambda) T(\lambda) d\lambda}{\int T(\lambda) d\lambda}$$

and

$$\lambda_{\text{eff}} = \frac{\int \lambda T(\lambda) d\lambda}{\int T(\lambda) d\lambda}$$

- Color:

$$m_A - m_B = -2.5 \log(f_\nu^A/f_\nu^B)$$

- Effects of dust extinction:

$$f_o = f_e e^{-\tau_\nu}$$

- Distance modulus:  $\mu = m - M = -2.5 \log(d/10 \text{ pc})$

## Lecture 2: Galaxy Morphological Classification

### Galaxy morphology

Different inclinations will influence the galaxy morphology, on top of that the morphology will also be different in different wavelength bands.

The morphology can be influenced by, for instance, mergers. Major mergers are mergers in which the two colliding galaxies have similar mass, whereas minor mergers consist of two galaxies with vastly different masses.

### The Hubble sequence

The Hubble sequence is valid up to redshifts  $z \approx 1$ .

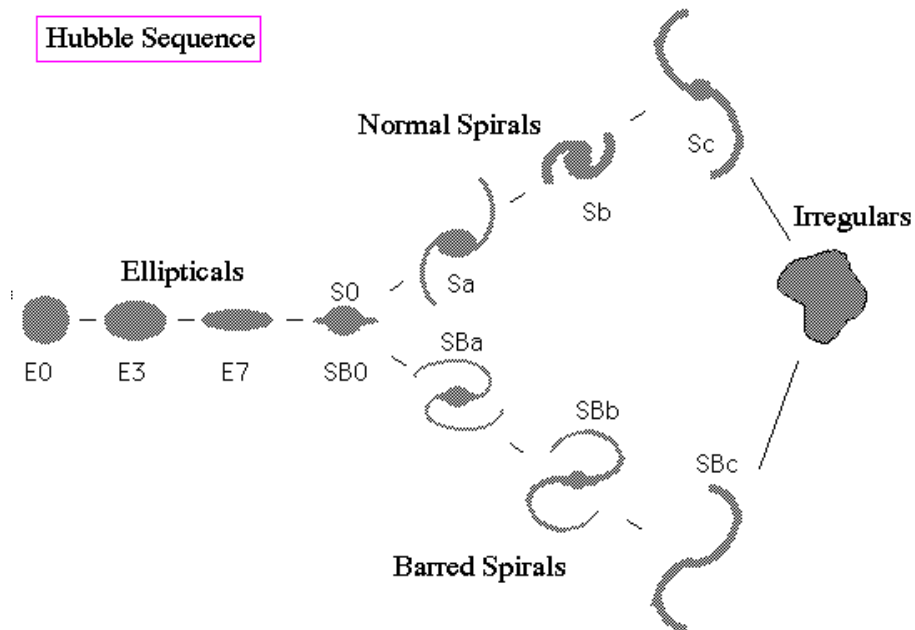


Figure 1: Hubble sequence

## Types of Galaxies

### Elliptical galaxies

Ellipticals seem featureless, typically red (and they therefore radiate mostly in the NIR) due to the abundance of old stars, and they are the most massive galaxies.

They can also show shells, which are probably due to mergers. Some contain a bit of dust, but not much.

Some also have an active black hole, i.e. BHs that accrete matter from the dark matter halo.

### Lenticular galaxies S0/SB0

They could be an intermediate stage between ellipticals and spirals.

### Spiral galaxies

They contain a disk of stars and gas in a spiral pattern. The bulge has old stars (population II stars), whereas the disk consists also of young stars (population I stars).

Basic characteristics: - A flat disk - A rotating disk - Orbits which are almost in a single plane (the random motion of stars is small, the disk)

From Sa to Sc on the Hubble sequence: - The bulge decreases - Fractional amount of gas increases - Contribution of young stars increases - More patchy disk - The spiral arms become more open

### Irregulars

Irregulars have a lot of stars formation and are therefore quite blue, they are also relatively small. Dwarf irregulars have a lot of HI gas, which might indicate that atomic gas can lead directly to stars formation.

### Ring galaxies

Ring galaxies might be the process of (nearly) head-on collisions with smaller galaxies.

## A Quantitative Analysis of Galaxies

### Light profiles of elliptical galaxies

The surface brightness of ellipticals can be characterized by

$$\mu(r) = \mu_e + 8.33 \left[ \left( \frac{r}{r_e} \right)^{1/4} - 1 \right]$$

Note that  $[\mu] = \text{mag/arcsec}^2$  and  $r_e$  is the effective radius, i.e. the projected radius within which one-half of the galaxies light is emitted.

### A general light profile

A more general model is given by the 3-parameter model, the **sersic profile** (see **Graham** for more detail):

$$I(r) = I_e \exp \left( -b_n [(r/r_e)^{1/n} - 1] \right)$$

where  $n$  is the Sersic index.  $n = 1$  indicates a perfect disk and  $n = 4$  is the Vaucouleurs profile.

### CAS classification

A non-parametric classification scheme used for  $z > 1$  galaxies, i.e. it does not assume a functional form.

- Asymmetry: Measures the rotation of the image,

$$A = \frac{|I - R|}{I}$$

- Smoothness: Measures how blurred the image is,

$$\frac{I - B}{I}$$

- Concentration: Measures to what extent the light is concentrated or spread out,

$$C = 5 \log \left( \frac{r_{80}}{r_{20}} \right)$$

where  $r_{80}$  and  $r_{20}$  denote respectively the radii where 80% and 20% of the light come from. High concentration implies high  $C$ .

## Lecture 3: Stellar Populations

### Stellar Properties

- Effective temperature  $T_{\text{eff}}$ :

$$L \equiv 4\pi R^2 \sigma T_{\text{eff}}^2$$

- Surface gravity  $\log g$ : causes, among others, pressure broadening,

$$g = \frac{GM}{R^2}$$



- Metallicity  $Z$  or  $[\text{Fe}/\text{H}]$ : Metals are everything heavier than helium.

$$[A/B] = \log \left\{ \frac{\# \text{atoms A} / \# \text{atoms B}}{(\# \text{atoms A} / \# \text{atoms B})_{\odot}} \right\}$$

Higher  $Z$  (and  $\log g$ ) will lead to more prominent absorption.

Note that the metallicity will change the MS turnoff and shift the isochrones. On top of that the effective temperature, luminosity, and opacity depend on  $Z$ . Thus higher  $Z$  more absorption, leading to cooler stars.

## Spectral Differences for Different Types of Galaxies

Early type, e.g. ellipticals and lenticulars, have very low flux at lower wavelengths ( $\lambda < 4000 \text{ \AA}$ ). Whereas late type galaxy, e.g. starburst galaxies, tend to have higher flux in the UV range (and below) than in the visible and IR part of the spectrum. On top of that late type galaxies have prominent emission lines.

## K-correction

The K-correction corrects for the fact that sources observed at different redshifts are, in general, compared with standards or each other at different rest-frame wavelengths. We can relate the wavelength of the observed versus emitted photon via  $\lambda_o = \lambda_e(1 + z)$ .

## Statistical Properties of Stars

### Stellar luminosity function

The stellar luminosity function is a measure of the number density of stars per unit of volume and per unit of luminosity (or absolute magnitude).

$$dN = \Phi(M, V) dM dV$$

If the distribution of stars of different luminosities is homogeneous in space:

$$dN = [\Phi(M) dM][\nu(V) dV]$$

Source surveys are apparent magnitude limited, i.e. the LF is affected by the Malmquist bias, meaning that the absolute magnitude of the observed sample is brighter than the mean of the full stellar population.

### The initial mass function (IMF)

The **IMF**, which we today know should be a broken power law, describes the distribution of stellar masses in a newly formed stellar population. A well known IMF is the **Salpeter IMF**,

$$\Psi(M) = \Psi_0 M^{-2.35}$$

Where  $M$  is the stellar mass. The IMF represents the number of stars with masses in a certain range, within a specified volume of space

## Star Clusters

### Globular clusters

- Very compact
- Around  $10^3$ - $10^4$  stars
- Contain some of the oldest stars (they constitute to a very tight sequence in the HR-diagram)
- Low metallicities (compared to the rest of the host galaxy)
- They set a minimum of the age of the host galaxy

### Open clusters

- Low mass
- Less than 1000 stars
- Young and they don't last very long
- Member stars have the same composition

## Lecture 4: The Cosmic Distance Scale

### Groups of distance measures

We can roughly distinguish between two groups of distance measures.

#### Absolute methods

- Distances can be determined quite directly.
- Typically involve geometric measurements.
- Typically used for nearby sources, i.e. sources within the Milky Way.

#### Relative methods

- Typically used for larger distances.
- Often refers to a standard candle.

### Primary Methods: Distances within the Milky-Way

#### Stellar Parallax

1 pc is the distance to some object (measured with respect to the earth being at a distance of 1 AU from the sun) such that the distance sun-earth subtends 1 arcsec.

$$d = \frac{1}{\varpi}$$

where  $\varpi$  is the observed parallax in arcsec.

$$(m - M)_0 \equiv 5 \log_{10} \left( \frac{0.1 \text{ [arcsec]}}{\varpi} \right)$$

### Secular Parallax

Instead of using the Earth's motion around the sun, one uses the motion of the sun around nearby stars. The mean proper motion of the stars  $d\theta/dt$ , which has a mean component proportional to  $\sin \theta$  and if the slope of this line is  $\mu$  then the distance  $D$  is given (in parsec) by  $D = 4.16/\mu$ , where  $\mu$  is given in arcsec per year.

### Gravitational Lensing

Typically one would use a cluster of galaxies ( $z \approx [0, 1]$ ) which would function as a lens. Due to the mass of the object, light passing it will bend. The distance can be worked out from the light delay  $t_{\text{lens}}^{(i)}$  due to the longer path (as a consequence of not reaching us directly).

### The Sunyaev-Zeldovich Method

Galaxy clusters are gravitationally bound, between those galaxies we have the intracluster medium (lots of gas). As CMB photons pass through the cluster they will increase in energy (the Sunyaev-Zeldovich effect), and you know the normal characteristic energy of the CMB, such that you can determine the distance (to the cluster). In the expression for  $D$ ,  $\sigma_T$  is the collision cross-section,  $F$  the flux,  $\tau_s$  the optical (scattering) depth, and  $T$  the electron temperature in the intracluster medium.

$$D = \frac{\tau_s}{\theta n_e \sigma_T} = \frac{1.78 \cdot 10^{-40} \tau_s^2 \theta T^{1/2}}{24 \sigma_T^2 F}$$

where

$$\tau_s = 2 \sigma_T R n_e$$

and if the cluster is spherical  $R = D\theta$ .

## Relative Methods (secondary/tertiary methods)

The general idea is to measure something that doesn't vary with the distance, e.g. a galaxy (i.e. flux specific phenomenon). Relate the velocity or period to the luminosity using local object, using  $F = L/4\pi d^2$  we can work out the distance or we use the distance modulus.

### Stellar main sequence fitting

One can calibrate the main sequence in terms of parallax in terms of magnitude and color. Then compare the apparent magnitude to other clusters and determine the distance.

$$m_{\text{cluster}} - M_{\text{ref}} = 5 \log(d) - 5 + A$$

where  $A$  is the extinction, i.e. dust will have the effect of yielding a lower flux than its true flux.

In general main sequence fitting compares the location of the main sequence for the cluster stars placed on the HR Diagram where apparent magnitude is used as the y-axis variable to the location of the main sequence for nearby stars whose distances are well-known from parallax where absolute magnitude is used on the y-axis. Any difference in position between the main sequences must be due to the distance of the cluster. The vertical position of the cluster main sequence is adjusted that it lines up with the main sequence of the nearby stars. The amount of vertical adjustment gives the distance modulus which in turn gives the distance.

### Variable stars

Variable stars vary periodically in luminosity (due to pulsations in their interiors). For instance Cepheids (young, quite luminous, with relatively long periods).

### Time-delay methods

Typically applied to supernovae. A supernova explosion at time  $t$  is seen at position  $A$ . At some time later,  $t_0$ , light from the supernova is seen at position  $B$ , and at an even later time,  $t_1$ , exactly the same signal is seen at position  $C$ . Using this information one can write

$$t_B - t_A = \frac{r(1 - \sin i)}{c} t_C - t_A = \frac{r(1 + \sin i)}{c}$$

and determine the distance.

### **Standard candles**

Standard candles are anything that has a predictable brightness. Using Cepheids is very useful for nearby galaxies, we can measure the distance out to  $\sim 30$  Mpc this way. We can determine the apparent magnitude by measuring the pulsation for several periods and using the period-luminosity relation determine the distance modulus.

### **Supernovae as distance estimators**

Type I supernovae involve a white dwarf that is part of a binary system. Type II supernovae involve very massive stars at the ends of their lives. Because supernovae are such energetic events, one can observe them at great distances. They are only useful as distance indicators if it is possible to calibrate them, i.e. to relate their observed brightness profile to absolute magnitudes. Type I supernovae are very uniform and easy to calibrate, more so than type II.

### **Globular cluster LF**

The number of globular clusters per unit magnitude in a galaxy has a Gaussian form, with a well defined peak (of the Gaussian) at turnover magnitude  $M_0 = -6.5$ . This turnover magnitude is a standard candle that can work out to  $\sim 50$  Mpc.

### **The Tully-Fisher relation**

The (linear) relation between absolute magnitude (or luminosity) and the rotational velocity in spiral galaxies. By comparing the intrinsic brightness with the apparent magnitude (which you measure), one can calculate its distance.

### **The Hubble Law**

For the vast majority of objects we use the Hubble Law  $v = H_0 d$  (where  $H_0 \approx 100h$  km/s/Mpc is the Hubble Constant, currently we use  $h = 0.69$  to parameterize the Hubble constant).

## **Lecture 5: Galaxy Dynamics**

### **Two components of the gravitational potential**

- Smooth potential: The average over a region containing many stars in a large volume.
- A deep potential well: Due to/depending on the local potential due to the stars.

## Stellar collisions

**Close encounters** are encounters that will change the orbits of stars significantly. Whereas **Weak encounters** are a rather small perturbation on the initial trajectory.

### Strong encounters

A strong encounter has happened if the change in potential energy is at least as large as the initial kinetic energy.

$$\frac{Gm^2}{r} \geq \frac{mV^2}{2}$$

which implies that

$$r \leq r_s = \frac{2Gm}{v^2}$$

where  $r_s$  is the strong encounter radius. The average time between collisions in a volume (in the galaxy) of a cylinder  $\pi r_s^2 vt_s$ . Define  $n\pi r_s^2 vt_s = 1$ , such that a star will on average have 1 close encounter (given  $n$  stars per unit volume). We find

$$t_s = 4 \cdot 10^{12} \text{ yr} \left( \frac{v}{10 \text{ km/s}} \right)^3 \left( \frac{m}{M_\odot} \right)^{-2} \left( \frac{n}{1 \text{ pc}^{-3}} \right)^{-1}$$

Note that strong encounters are only important in the dense cores of globular clusters.

### Distant weak encounters

The force of one star on another is so weak that the stars hardly deviate from their original paths. We consider the case of a star moving through a system of  $N$  identical stars of mass  $m$ . We assume that the change in velocity is very small  $\delta v/v \ll 1$ , the perturbing star is stationary (this is also called the *impulse approximation*). We assume that a projectile star (mass  $m$ ) is perturbed by a target star of mass  $M \gg m$ . The impact parameter  $b$  is the minimum distance between the two stars. The change in velocity in the smaller star is given by

$$\Delta v_\perp = \frac{2GM}{bv}$$

The faster the relative velocity, the smaller the perturbation. The (differential) number of weak encounters is equal to

$$dn_e = 2\pi n v t b dt$$

Each of these encounters will produce a small change in velocity  $dv$ . The accumulation of weak encounters can be measured by the sum of  $\langle \Delta v_{\perp}^2 \rangle$ .

$$\langle \Delta v_{\perp}^2 \rangle = \frac{8\pi G^2 M^2 n t}{v} \ln \left( \frac{b_{\max}}{b_{\min}} \right)$$

The relaxation time is the time necessary for that system to drastically change the system (i.e. lose its memory of the initial dynamics). Relaxation is the cumulative effect of individual encounters. In a relaxation time, the stellar velocity distribution randomizes.

$$t_{\text{relax}} = \frac{t_s}{2 \ln \Lambda}$$

where  $\Lambda = b_{\max}/b_{\min}$ . This expression can be approximated by

$$t_{\text{relax}} = \frac{2 \cdot 10^9 \text{ yr}}{\ln \Lambda} \left( \frac{v}{10 \text{ km/s}} \right)^3 \left( \frac{m}{M_{\odot}} \right)^{-2} \left( \frac{n}{10^3 \text{ pc}^{-3}} \right)^{-1}$$

Often the relaxation time is in the order of the Hubble time. This implies that when calculating the motions of stars like the Sun, we can ignore pulls of individual stars, and consider them to move in the smooth potential of the entire Galaxy.

## Motion under gravity

$$\frac{d}{dt}(m\mathbf{v}) = -m\nabla\Phi(\mathbf{x})$$

where

$$\Phi(\mathbf{x}) = - \sum_{\alpha} \frac{Gm_{\alpha}}{|\mathbf{x} - \mathbf{x}_{\alpha}|} \quad \text{for } \mathbf{x} \neq \mathbf{x}_{\alpha}$$

## Poission's equation

The potential at a point  $\mathbf{x}$  produced by a continuous mass distribution represented by density  $\rho(\mathbf{x})$  is given by

$$\Phi(\mathbf{x}) \equiv -G \int d\mathbf{x}' \frac{\rho(\mathbf{x}')}{|\mathbf{x}' - \mathbf{x}|}$$

and if the potential is known instead of the density we have

$$\nabla^2 \Phi = 4\pi G \rho$$

Note that not all  $\Phi$  are meaningful, only those where the mass is always positive.

### Theorem I

A body that is inside a spherical shell of matter experiences no net gravitational force from that shell.

### Theorem II

The gravitational force on a body that lies outside a closed spherical shell of matter is the same as it would be if all the shells' mass was concentrated in a point at its center. So the gravitational force within a spherical system that a particle feels at a radius  $R$  is only due to the mass inside that radius.

As such, if a star moves on a circular orbit, its acceleration is given by

$$\frac{v_c^2}{r} = \frac{GM(r)}{r^2}$$

where

$$v_c^2 = \frac{GM}{r}$$

### The orbit of stars on a plane

- In a time-independent gravitational potential energy is conserved
- In a spherical potential angular momentum is conserved

### The orbits of stars in a disk galaxy

We define the position using  $(R, \phi, z)$ , for axisymmetric systems (i.e. systems independent of  $\phi$ ) we write  $\Phi = \Phi(R, z)$ .

### Equations of motion

$$\frac{d^2 \mathbf{r}}{dt^2} = -\nabla \Phi$$

where  $\mathbf{r} = R\hat{R} + z\hat{z}$ .

$$\begin{aligned} \frac{d^2 R}{dt^2} &= -\frac{\partial \Phi}{\partial R} \\ \frac{d^2 z}{dt^2} &= -\frac{\partial \Phi}{\partial z} \end{aligned}$$

Note that  $L_z = R^2 \frac{d\phi}{dt} = \text{constant}$ . We define the effective potential



$$\Phi_{\text{eff}} = \Phi(R, z) + \frac{L_z^2}{2R^2}$$

The effective potential behaves like a potential energy for the star's motion in  $R$  and  $z$ . Note that the effective potential is minimum when the radius (let's call it  $R = R_g$ ) of the orbit is that of a circular orbit and when  $z = 0$ .

## Epicycles

The equation of motion for stars on nearly circular orbits in the symmetry plane are given by

$$x = x_0 \cos(\kappa t + \psi)$$

and

$$z = z_0 \cos(\nu t + \theta)$$

where  $\nu$  is the vertical frequency  $\nu^2 = \partial^2 \Phi_{\text{eff}} / \partial z^2|_{(R_g, 0)}$ ,  $\kappa$  the epicycle frequency  $\kappa^2 = \partial^2 \Phi_{\text{eff}} / \partial R^2|_{(R_g, 0)}$  and  $x = R - R_g$ .

## Lecture 6: Milky Way I

### The multi-wavelength view

- NIR pictures are dominated by the older stars, such that the bulge is best observed in the NIR.
- Molecular hydrogen (i.e. the fuel for stellar birth) is an indicator for the formation of new stars.
- Atomic hydrogen is observed by for instance the 21cm line.
- High intensity gamma rays in the galactic center indicate star formation.

### Galactic coordinates

The position of a star can be given by the angles  $l$  and  $b$ , respectively the angle with respect to the sun-galactic center line in the galactic plane and the angle with respect to the galactic plane.

### Galactic cylindrical coordinates

We also use  $(R, \phi, z)$  coordinates to define the position of a star in the MW.

## Galactic Cartesian coordinates

In case of using Cartesian coordinates  $x$  is radially outward (from the galactic center),  $y$  in the direction of the sun's rotation around the MW, and  $z$  out of the galactic plane, positive towards the NGP.

## Stellar density of disk galaxies

We can approximate the stellar density in the disc as double exponential

$$n(R, z, S) = n(0, 0, S) \exp \left[ -R/h_R(S) \right] \exp \left[ -|z|/h_z(S) \right]$$

$h_R$  and  $h_z$  are characteristic scale lengths, representing the the lengths over which the density falls by a factor of  $e$  for some (stellar) population  $S$ .

## The MW disk

The thick disk is mainly composed of dwarf stars, and contains no O, B, A type stars. Those are mainly found in the thin disk. In a similar fashion, most older stars are found farther out from the galactic plane, whereas younger stars are predominantly found close to the galactic plane.

## Analysis of MW components

If we plot  $V$  (the rotation velocity) versus  $T$  then the location of stars in the MW are very clearly denoted in groups. Note that  $T = \sqrt{U^2 + W^2}$ , where  $U$  is the velocity component towards the galactic center and  $W$  is the component towards the north galactic pole.

Similarly plotting the iron abundance  $[\text{Fe}/\text{H}]$  versus  $V$  shows a similar trend.

## The MW disks (stars)

The thin disk stars make up 90% of the stars near the sun. And integrated over the  $z$ -direction the thick disk has about a third of the intensity of the thin disk.

The vertical distribution of stars in the Milky cannot be fit by single exponential curve. There is only a good fit with two exponentials. So either (1) the functional form (exponential) is incorrect or (2) there are two physically distinct components of the disk of the Galaxy: a thin and a thick disk. For the second possibility to be correct, need to conclusively demonstrate that they have different and distinct properties!

## The MW disks (gas)

The neutral hydrogen gas is rather concentrated around the galactic plane, which is even more so for molecular gas.

## Stars in the MW halo

Looking even farther out from the plane of the Galaxy, it appears that the disk turns into a much more tenuous stellar halo. Whose density profile is given by

$$n(r) \approx n_0 \left( \frac{r}{r_0} \right)^{-3}$$

Note this relation is only valid until about a radius of  $R \approx 20$  kpc. They might tell something about the minor merger history. This stellar halo extends to about 80 kpc and it is about 1% of the luminosity of the MW.

The stellar halo is very important for understanding the formation of the Milky Way, as the stars are generally very old and metal-poor. The stellar halo preserves the early formation history of the Galaxy.

## Lecture 7: Milky Way II

### Differential rotation

Differential rotation implies that objects closer to the center take less time to complete an orbit with respect to object farther out.

This differential rotation maintains the spiral pattern in spiral galaxies and shows that stars exhibit both radial, tangential, and vertical motions.

### Galactic rotation

$$v_r = v \cos \alpha - v_0 \sin \ell$$

where  $v$  is the velocity of the star wrt the galactic center,  $\alpha$  is the angle ... In the end we find

$$v_r = R_0 \left( \frac{v}{R} - \frac{v_0}{R_0} \right) \sin \ell$$

### Oort's constants

#### The first constant: $A$

The first constant  $A$  is a measure of the local shear, i.e. the deviation from the rigid rotation.  $A \approx 14.8$  km/s/kpc.

### **The second constant: $B$**

The tangential velocity is given as

$$v_t = v \sin \alpha - v_0 \sin \ell = R_0 \left( \frac{v}{R} - \frac{v_0}{R_0} \right) \cos \ell - v \frac{d}{R}$$

$B$  measures the local vorticity (the tendency of something to rotate), or the angular momentum gradient in the disk.  $B \approx -12.4 \text{ km/s/kpc}$ .

### **Gas motion pattern**

We divide the milky way into four different quadrants. In Q1  $v_r > 0$  if within the sun's orbit, otherwise  $v_r < 0$ . In QII  $v_r < 0$  always. In QIII and QIV we have the opposite (in terms of sign) of respectively QII and QI.

Note that any point between the galactic center and the sun will move faster than us and outside that range will be faster than us.

### **Stellar streams**

The distribution of stars in velocity is not smooth. Projections in  $U$  and  $V$  space show a lot of structure including moving groups/streams.

Those streams are groups of stars born together and/or dynamical perturbations (which could be due to a bar/spiral structure).

### **MW kinematics: bulge versus halo**

Stars in the halo show a nearly Schwarzschild distribution in velocities. The halo also does not appear to rotate. Stars in the halo move on very eccentric orbits, and having very large speeds implies that the stars travel very far away from the galactic center.

Kinematically, the bulge is not an extension of the halo. On top of that, stars in the bulge also have higher metallicities (closer to the disk).

### **The interstellar medium**

#### **Gas recycling**

#### **Dust**

Dust is responsible for **reddening** and **extinction**.

Dust grains in the interstellar medium have a typical size that is comparable to the wavelength of blue light. The result is that the blue light coming from distant objects is strongly absorbed and scattered by the dust, essentially removing it from the light reaching us and making the objects appear redder than they really are.

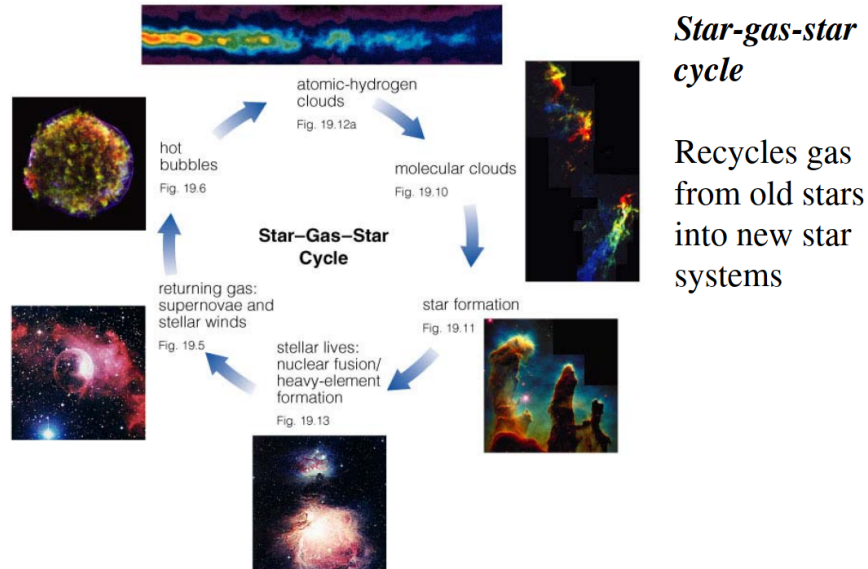


Figure 2: Gas recycling

Essentially due to the dust, longer wavelengths make it through the dust clouds but *redder*, short wavelength light is scattered away from its original direction.

### The local group

- Andromeda (M31): Halo with metal-rich stars,  $3/2$  times the luminosity and HI as the Milky Way.
- The Large Magellanic Cloud: About 10% as luminous as the Milky Way, it's a barred galaxy, rich in HI.
- The Small Magellanic Cloud: An irregular galaxy, rich in HI.

### Dwarf spheroidal galaxies

They are often about hundred times fainter than the SMC and LMC, rich in very old stars, and they are free of HI.

## Lecture 8: The Integrated Galaxy View

We generally study the integrated view of the galaxy, as we cannot resolve individual stellar populations for  $z > 0.3$ . Note  $z \approx 2$  is around 3 billion years ago.

## Galaxies in a blank field

If we look at high redshift ( $z > 0.3$ ) galaxies we tend to study multiple galaxies together. If we take these images containing thousands of galaxies, we estimate the redshift via a statistical treatment based on galaxy evolution.

## Galaxy physics from photometric measurements

### Galaxy photometry

The more point-like objects in your aperture are the more distant galaxies. We want to measure the light from each individual galaxy. To do so we measure the light encircled in an aperture (and subtract the background), this is done at multiple wavelengths (to measure light from different components).

### Dealing with different resolutions

Even if the resolution is the same, for instance the **PSF** could be different. To properly analyse these type of images one would need a theoretical model of the PSF.

### Galaxy color-magnitude diagrams

The fast majority of galaxies will lie in the so called *red sequence* (ellipticals) or *blue cloud* (mostly spirals). The green valley is a region in between the red and blue, which contains mostly stars that recently ... This is valid for  $z \lesssim 1$ , as at higher  $z$  dust extinction will start to play a bigger role.

### SED: Spectral energy distribution

We measure the galaxy at (as many) different wavelengths (as possible). We for instance determine the AB magnitude. We can then construct various models, using among others the IMF. We construct the model assuming  $z = 0$ , then fitting the model to the measured data we have to shift the model, thus obtaining the redshift.

Recall that the observed and emitted wavelengths are related via

$$\lambda_o = (z + 1)\lambda_e$$

### SED models

Synthetic models take into account the spectra of the individual stars, isochrones (the evolution on the MS), and the IMF. We generally assume a IMF, as it is not known for the fast majority of galaxies. Then you can determine the spectrum of a galaxy assuming a single stellar population (assuming all stars were born at the same time, at very high redshift). To form models of galaxies with more complicated star formation histories we combine models of single populations

born at different times. We also need to know the chemical composition and the presence of dust. Synthetic models are generally from UV to NIR. Combining all this information one would obtain the CSP (combined stellar population).

Emperical models are ...

### **Expected photometry from templates**

If you want to obtain the magnitudes from the observed flux, you'd need to solve

$$f_{\nu}^{\text{band}} = \frac{\int_0^{\infty} f_{\nu}(\lambda) T(\lambda) d\lambda}{\int_0^{\infty} T(\lambda) d\lambda}$$

Note  $T(\lambda)$  is the transmission profile/curve of the filter.

### **Dust in galaxies**

Dust was more important in the cosmic past than it is nowadays. As the star formation rates are lower now, than in the past.

### **Radio**

Radio emission is due to synchrotron acceleration as a result of SNs, also an indicator of (new) star formation.

### **Galaxy physics from spectroscopy**

The spectra of star forming galaxies are characterized by the presence of emission lines (due to the presence of gas).

The spectra of passive galaxies (i.e. no/negligible new star formation takes place) only have absorption lines.

AGNs can be classified as type I or type II. Type I spectra contain very pronounced broadening of specific lines. But note that not all AGNs show these broadened lines, which might be due to the orientation of the galaxy wrt us.

### **Multi-object spectroscopy**

This is traditionally obtained with slits or fibres. This method could (1) be used for instance to calibrate your redshifts obtained via the photometric route (as photometry is *less expensive* than spectroscopy). (2) Provide a backbone for large scale structure studies. (3) Use it for studies of metallicities, which can only be done via spectroscopy.

### **Spectral classification: BPT diagram**

These graphs plot the line ratios of various metals wrt  $H\alpha$  and  $H\beta$ .

### Integral field spectroscopy

Various slices of the galaxy are passed through a spectrograph, constructing a data cube containing two spatial coordinates and a spectral dimension.

## Lecture 9 Reionization & First Galaxies

### The evolution of the universe

- (1) After the big bang and before the recombination the particles were in thermal equilibrium. (2) Recombination, i.e. the moment at which charged electrons and protons first became bound to form hydrogen atoms. (3) The cosmic dawn, the moment at which the first stars and galaxies came into existence (4) Nowadays, basically all H and He is fully ionized (outside of the galaxies).

### Hydrogen reionization

- Pre overlap phase: each galaxy builds a region of ionized hydrogen around itself, the Stromgren sphere.
- Overlap phase: ionized spheres grow and start merging.
- Post overlap phase: really large ionized regions, merging with the left-over neutral islands.

### Thermal evolution of the intergalactic medium

At recombination the universe was at thermal equilibrium, then the universe started to cool adiabatically, i.e.

$$pV^\gamma = \text{constant}$$

Using the ideal gas law  $pV = NkT$ , with  $\gamma = 5/3$  for a monoatomic gas, we find

$$T \propto V^{-2/3} \propto a^{-2}$$

where  $a = 1/(1+z)$  is the scale factor, such that

$$T \propto (1+z)^2$$

Note that at the beginning  $a = 0$  and  $z = \infty$ , while today we have  $a = 1$  with  $z = 0$ .



## Sources of reionization

Main point: more and more galaxies form and galaxies become more and more massive over time.

Gravity causes the initial fluctuations in the density field to grow over time, i.e. slightly over-dense regions increase their density and become more confined, while slightly under-dense regions become more under-dense. Galaxies form in the density peaks. The most massive galaxies have formed in the very first density peaks, while less massive galaxies have formed later. At each time there are far more low massive galaxies than very massive galaxies.

Note: there are more fainter than brighter galaxies in general, because of the concentration of over-densities/under-densities. And the number of galaxies increase as the universe evolves, as the under-densities become more dense, such that galaxies can be formed there as well.

## Physical processes that affect the contribution of galaxies to reionization

1. Only a fraction of the photons escapes due to HI in the ISM.
2. Supernovae winds and heating lead to gas being ejected from the galaxy.
3. Reionization heats gas in the IGM & reduces cool gas for star formation.

Note 2 and 3 mainly have effect in smaller mass/fainter galaxies, whereas 1 dominates in brighter galaxies. This dictates how many stars can be formed in a galaxy.

## Reionization feedback on galaxies

The virial temperature of a halo with virial mass  $M$  at redshift  $z$  is given as

$$T = 1.69 \cdot 10^4 \left( \frac{\mu}{0.6} \right) \left( \frac{\Omega_M}{0.3} \right)^{1/3} \left( \frac{1+z}{10} \right) \left( \frac{hM_{\text{vir}}}{10^8 M_\odot} \right)$$

where  $\mu$  is the molecular mass and  $\Omega_M$  the matter density parameter.

## The Stromgren sphere

What is the size of the ionized sphere around a galaxy? We assume that the rate of ionizations equals that of recombinations. The ionization rate  $N_{\text{ion}} = \alpha n_e n_p V$ , where the volume  $V = 4\pi R_s^3/3$ . Such that the radius of the sphere is given by

$$R_s = \left( \frac{3N_{\text{ion}}}{4\pi\alpha n_e n_p} \right)^{1/3}$$

## Key physics of reionization

An expression for the ionization fraction of hydrogen is given by,

$$\frac{dQ_{\text{HII}}}{dt} = \frac{\dot{n}_{\text{ion}}}{\bar{n}_{\text{H}}} - \frac{Q_{\text{HII}}}{\bar{t}_{\text{rec}}}$$

the first term on the RHS denotes the ionization rate and the second the recombination rate. Note that

$$\dot{n}_{\text{ion}} = f_{\text{esc}} \dot{n}_{\text{ion}}^{\text{ISM}}$$

and

$$\bar{t}_{\text{rec}} = (\chi_e \bar{n}_{\text{H}} \alpha_B C)^{-1}$$

where  $\chi_e$  accounts of excess in free electrons due to ionized helium,  $\alpha_B$  is the case-B recombination coefficient, and  $C = \langle n^2 \rangle / \langle n \rangle^2$ . Note that for a homogeneous system the clumping factor is one. And if the gas only consists of hydrogen:  $\chi_e = 1$ .

## Observational evidence for reionization

### Electron scattering

The electron scattering optical depth is given by

$$\tau_e = \int^{z_{\text{reion}}} \sigma_T n_e \frac{dl}{dz} dz$$

From the CMB optical depth we can infer an instantaneous reionization redshift.  $\tau_e = 0.055 \pm 0.009$ ,  $\sigma_T$  the Thomson scattering cross section.

### Lyman- $\alpha$ photons

Photon wavelengths will be shifted by a factor  $1 + z$ . Using a Gunn-Peterson trough we can detect this shift. For instance

- Quasar emits radiation
- Radiation travels as Universe expands & is redshifted
- Neutral hydrogen in the IGM absorbs Lyman- $\alpha$  always at 1206 Å
- Quasar spectra shows absorption at wavelengths shorter than 1206 Å

If the photons of the quasar spectra passes the neutral gas in the IGM during reionization, they are nearly completely absorbed at wavelengths shorter than 1206 Å (in the rest frame of the quasar!)

### Lyman- $\alpha$ emitters

- Stars in the galaxy emit HI ionizing photons
- Ionizing photons are absorbed by HI in the interstellar medium (ISM)
- Ionized hydrogen in dense clumps recombines quickly
- Lyman- $\alpha$  is emitted as part of recombination radiation

Ly $\alpha$  line has been redshifted out of HI absorption. It is transmitted through the IGM.

The larger the ionized regions around a galaxy, the higher is its fraction of Ly $\alpha$  radiation that is transmitted through the IGM.

The decrease of galaxies with a detected Ly $\alpha$  line could be due to a higher HI fraction.

## Lecture 10: Spirals & Ellipticals I

### Plots of number density versus stellar mass/luminosity

One sees that at higher stellar masses, the shape will decline exponentially. Whereas at low mass the graph looks like a power-law. Most of the mass is around the turning point of the stellar mass function, this is in between where the bulk of spirals and ellipticals.

### Sersic profiles

$$\mu(r) = \mu_e + 8.3268 \left[ \left( \frac{r}{r_e} \right)^{1/n} - 1 \right]$$

This formalism can be used to discriminate between disk-dominated and bulge-dominated galaxies.

### Measuring the surface brightness

The surface brightness  $I(x)$  is the amount of light contained in an area at a particular point  $x$  in an image. The surface brightness is given by

$$I(x) = \frac{F}{\alpha^2}$$

where  $\alpha = D/d$ ,  $D$  width of galaxy,  $d$  distance to galaxy. Note that the surface brightness does not change with distance.

$$\mu_\lambda(x) = -2.5 \log I_\lambda(x) + \text{constant}_\lambda$$

## Cosmological dimming (Tolman test)

Surface brightness  $B = f/d\omega$ ,  $\omega$  is a solid angle.

$$B = \frac{L}{d_L^2} \frac{d_A^2}{dl^2}$$

In non-Euclidean space  $d_L = d(1+z)$  and  $d_A = d/(1+z)$ , such that

$$B = \frac{L}{dl^2(1+z)^{-4}}$$

## Sources of error in surface brightness

- Sky background subtraction: i.e. if the sky is over or under subtracted.
- Seeing effects: unresolved points are spread out due to effects of our atmosphere, quantified by a PSF. If you don't take seeing into account the central part will be flatter and the isophotes rounder.

## Surface brightness ellipticals

Ellipticals have very smooth profiles over 2 orders of magnitude in radius, which usually falling off as  $R^{1/4}$ .

## Surface brightness (bulge + disc)

The disk has an exponential profile  $I(R) = I_0 \exp(-R/h_R)$ ,

$$\mu(R) = \mu_0 + 1.09 \left( \frac{R}{h_R} \right)$$

. And the bulge follows an  $r^{1/4}$  law.

Stars in the bulge and disk probably have different origins.

## Colors of ellipticals and spirals

Galaxies get bluer as the disks become more prominent, on top of that they'll become fainter.

## Rotational curves of disk galaxies

$$v_r = \frac{\Delta\lambda}{\lambda_{\text{HI}}} c$$

When viewed at an inclination the radial velocity is given by

$$v_r(R, i) = v_{\text{sys}} + v(r) \cos \phi \sin i$$

### Three main classes of rotation curves

1. Nearly constant velocity at all radii
2. The typical increase in the beginning (rigid body regime) and then a constant velocity due to differential rotation.
3. A slow rise in the beginning (typical for dwarf irregulars)

### HI rotation curves and disk galaxies

The rotation curve depends on mass, not luminosity, so we need to transform the surface brightness profile to a mass profile using the mass-luminosity relation.

$$v^2(R) = v_{\text{disk}}^2(R) + v_{\text{bulge}}^2(R)$$

A flat rotation curve is due to additional matter, i.e. dark matter.

### The Tully-Fisher relation

In spiral galaxies a relation exists between its luminosity and maximum rotational velocity,  $L \propto v_{\text{max}}^4$ . Note: the exponent changes with bandpass.

So there is a link between the stellar mass (luminosity) of a disk and the mass of the dark matter halo.

Note that the mass-to-light ratio  $M/L$  depends on the age, star formation history, dust extinction, etc.

### Rotational velocities nellipticals from stellar absorption lines

The absorption lines are a composite spectrum of all the stars in the galaxy. The broadening can tell you about the gas kinematics.

### Scaling relations for ellipticals

A relation between the luminosity and the (central) velocity dispersion (The Faber-Jackson relation),

$$L \propto \sigma^4$$

### The fundamental plane for ellipticals

The fundamental plane is a correlation in a 3D space,

$$R \propto \sigma^{1.24} \bar{I}_e^{-0.82}$$

The mass-to-light ratios of ellipticals increases as they become more luminous, like due to larger galaxies being older.

$$\frac{M}{L} \propto L^{1/4}$$

## Lecture 11: Gravitational Lensing

Background reading:

- [Meneghetti Notes](#)
- [Notes on Gravitational Lensing in astronomy](#)
- [Differences between types of gravitational lensing](#)

### History

Using general relativity we could in 1919 calculate the (angle of) deflection of the light by a star we have

$$\alpha = \frac{4GM}{c^2} \frac{1}{\xi}$$

### Two regimes of gravitational lensing

Note: magnifications preserves surface brightness:

$$\mu = \frac{A_{\text{obs}}}{A_{\text{int}}}$$

#### Strong regime

You have the creation of multiple images, gravitational arcs are visible, and you have high magnifications (10-100 times the intrinsic luminosity). Due to very *good* alignment.

#### Weak regime

Small distortion of the images of (the) galaxies.

### Lens equation

The lens equation is given by:

$$\beta D_{\text{OS}} = \alpha(\theta) D_{\text{LS}} = \theta D_{\text{OS}}$$

where  $\beta$  is the real position on the source plane,  $\alpha$  and  $\theta$  are related to the observed position on the lens plane. Note  $\alpha$ , the deflection angle, depends on the *lens* mass distribution and on the observed position. We can write

$$\beta = \theta - \alpha(\theta) \frac{D_{LS}}{D_{OS}}$$

### The deflection angle

For a point mass, assuming a weak gravitational field, the deflection angle of a point mass is given by:

$$\vec{\alpha} = \frac{4GM}{c^2} \frac{\vec{\xi}}{\xi^2}$$

For an extended lens we have

$$\alpha(\vec{\xi}) = \frac{4G}{c^2} \int d^2\xi' \int dz \rho(\vec{\xi}', z) \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

define the projected mass density profile (considering no perturbers along the LOS)

$$\Sigma(\xi') = \int dz \rho(\vec{\xi}', z)$$

Such that

$$\vec{\alpha}(\vec{\theta}) = \vec{\alpha}(D_{OL}\vec{\theta}) \frac{D_{LS}}{D_{OS}}$$

### Magnification

In gravitational lensing the surface brightness is conserved. The magnification is given by the ratio between the observed and intrinsic fluxes,

$$\mu = \frac{d\Omega_{\text{obs}}}{d\Omega_{\text{int}}}$$

We decompose the magnification in an isotropic and traceless part, the isotropic part just magnifies the object whereas the traceless part (which is defined by the shear  $\gamma$ ) distorts the image. We find

$$\mu = \frac{1}{(1 - \kappa)^2 - \gamma^2}$$

note that  $\kappa$  is the dimensionless projected mass distribution, named convergence. It is defined as the projected mass density profile at  $D_{OL}\vec{\theta}$  divided by the critical density. We have a tangential and radial magnification

$$\mu_t = \frac{1}{1 - \kappa - \gamma}$$

and

$$\mu_r = \frac{1}{1 - \kappa + \gamma}$$

Every time you cross a caustic from *outside* an extra two degrees of multiplicity will be visible. So inside the first but outside the second caustic we have three (multiple) images.

## Time delay

The lens equation can be rewritten as

$$\nabla(\delta t) = 0, \quad \delta t = t - t_0$$

The time delay is the difference between the actual arrival time and the arrival time with no deflector. It defines a plane and the multiple images are formed in the stationary points. It can also be used to probe the Hubble constant

$$t_1 - t_2 \propto F(\psi)/H_0$$

where  $F$  depends on the lens potential, which is a projection of the 3D gravitational potential.

## Point mass model #1

This is an antisymmetric case, all vectors have the same direction, so we can reduce to one dimension. The magnification is given by:

$$\mu = \left[ 1 - \left( \frac{\theta_E}{\theta} \right)^4 \right]^{-1}$$

where

$$\theta_E^2 \equiv \frac{4GM}{c^2} \frac{D_{LS}}{D_{OL}D_{OS}}$$



is called the Einstein radius. The Einstein radius defines the Einstein ring, which is a special case of gravitational lensing, caused by the exact alignment of the source, lens, and observer. This results in symmetry around the lens, causing a ring-like structure.

## Axisymmetric mass distribution model #2

The deflection angle depends on the mass enclosed inside the image position. This way we can compute the mass of for instance a galaxy within an Einstein ring of a certain size  $\theta_E$ :

$$M(< \theta_E) = 1.1 \cdot 10^{14} M_{\odot} \left( \right)$$

## Lens model #3

The singular isothermal sphere, assuming that the matter behaves as an ideal gas and thermal and hydrostatic equilibrium.

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

where  $\sigma_v$  is the central velocity dispersion.

## Microensing

Point mass lenses are good approximations for stars. Note microlensing implies very small deflection angles  $\alpha < 0.01$  arcsec that we cannot resolve. So we mainly see a boost in luminosity and not really any change in the observed image.

## Lenses with elliptical symmetry

We go from radial coordinates to elliptical coordinates.

$$\kappa(x) \rightarrow \kappa(x_e)$$

where

$$x_e = \sqrt{\frac{x^2}{1-e} + y^2(1-e)}, \quad e = 1 - \frac{b}{a}$$

If we have multiple components, i.e. a cluster of galaxies:

$$\kappa(\vec{x}) = \sum \kappa_i(\vec{x})$$

Where each galaxy member is described as a singular isothermal sphere and the dark matter extended component by an elliptical mass distribution NFW or a cored IS. Lenses with elliptical symmetry can produce most of the observed configurations of multiple images. But still very simplistic for real systems.

## **Weak lensing regime**

At larger distances from the core, lensing weakly distorts the background galaxies. You can measure the mass of galaxy clusters at large radii by stacking the signal of several single galaxies to obtain an average profile. One can for instance measure the shear, which had some orientation w.r.t. to the center of the cluster.

## **Real world lenses**

Systems with Einstein rings are rare, we use multiple images or distortion to measure the mass distribution of galaxies. The mass distribution is never axisymmetric, so we at least have to consider elliptical symmetry and substructures. So we study the mass distribution of clusters and galaxies by using multiple images.

# **Lecture 12: Spiral & Elliptical Galaxies II**

## **Spirals**

### **Spiral structure in disk galaxies**

- 10% are grand design, very big galaxies with two well defined spiral arms
- 60% have multiple spiral arms, with slightly less structured spiral arms
- 30% are flocculent spirals, barely visible spiral arms, the arms are hard to distinguish

Leading spiral arms are pointing in the direction of rotation, if they are opposite to the rotation we call them trailing spiral arms.

### **Density waves**

The local gravitational field is not constant, as the density of matter at different times will be different. Which causes density waves, these waves resist the spiral's tendency to wind up and cause a rigidly rotating spiral pattern.

### **Self-propagating star formation**

Star formation produces supernovae, which will shock the gas, and triggers more star formation in the process. Then differential rotation stretches out the regions of star formation into trailing fragmentary arms with no global symmetry.

This process could explain the flocculant spiral structure.

## Ellipticals

### The centres of elliptical galaxies

Ellipticals either have a cusp, mainly in intermediate ellipticals (non-flat light curve at the centre), or core, giant ellipticals (flat light curve at the centre).

Cores could be due to mergers, such that the central nucleus is more diffuse. They might also be caused by *dry mergers*, without new gas, and thus barely any new star formation.

The cusp might be the result of *wet mergers*, i.e. mergers bringing in new gas.

Additionally, there are also dwarf ellipticals. They are are really faint.

### Shapes of ellipticals

We can, in the most general case, express the luminosity density  $\rho(x)$  as  $\rho(m^2)$ ,

$$m^2 = \frac{x^2}{\alpha^2} + \frac{y^2}{\gamma^2} + \frac{z^2}{\beta^2}$$

If all parameters  $\alpha, \beta, \gamma$  are different, the galaxy is triaxial. If  $\alpha = \gamma < \beta$ , the galaxy is prolate. And lastly if  $\alpha = \gamma > \beta$ , the galaxy is oblate.

### Deviation from elliptical isophotes

The diskiness/boxiness of an isophote (curves of equal brightness) can be classified by the difference between the real isophote and the best-fit ellipse.

### Boxy/disky & isophotal twisting

Note because the ellipticity changes with radius they will appear as if they will appear as if they were rotated in the projected image. This is called isophote twisting.

- Boxy galaxies: more luminous, more likely to show isophote twists, probably triaxial.
- Disky galaxies: intermediate ellipticals, often oblate, faster rotators.

### Slow and fast rotators

In minor mergers there is a higher transfer of angular momentum. The energy in major mergers is distributed differently.

### Fine structure in ellipticals

About 10-20% of ellipticals show distinct shells, which is probably the result of accretion (or mergers) of a small galaxy that was on a nearly radial orbit.

Density waves can produce ripples in the galaxy.

## Spectral properties

### Absorption lines

Mainly caused by atoms, molecules in a star's atmosphere (stellar lines). They can also be due to cold gas in the ISM, which extracts energy from the passing radiation (interstellar lines).

In ellipticals absorption lines are very important, as good as no emission lines, due to very little new star formation.

### Emission lines

Caused by gas being ionized and heated and then re-radiating at specific wavelengths. Young stars produce this ionized gas, due to UV photons being emitted. So emission lines tell you about the presence of young stellar populations.

For instance the spectra of spiral galaxies are characterized by the presence of emission lines. A prominent indicator of (a lot of) star formation is the H $\alpha$  line.

## Black holes in the centre of ellipticals

If a super-massive black hole lives at the centre of a galaxy, we should be able to detect this by looking at the speeds of stars that pass near to the black hole. We look for rotational speeds where

$$V^2(r) \approx \frac{GM_{\text{BH}}}{r} \gtrsim \sigma_c^2$$

where  $\sigma_c$  is the velocity dispersion of the stars at the centre. Which is within a radius

$$r_{\text{BH}} = 45 \text{ pc} \left( \frac{M_{\text{BH}}}{10^8 M_{\odot}} \right) \left( \frac{\sigma_c}{100 \text{ km/s}} \right)$$

### BH mass/bulge-velocity relation

It is likely that every elliptical galaxy, even every spheroidal system, for instance bulges, have a super massive black hole.

$$\log(M_{\text{BH}}/M_{\odot}) = 4.24 \log(\sigma/200 \text{ km/s}) + 8.12$$

And because  $M_{\text{bulge}} \propto \sigma^2 r$ ,

$$\frac{M_{\text{BH}}}{M_{\text{bulge}}} = 2.2 \cdot 10^{-3}$$

## Lecture 13: Chemical Enrichment & Galaxy Growth

### Chemical Enrichment

#### Closed-box model

$$Z(t) = -p \ln \left[ \frac{M_g(t)}{M_g(0)} \right]$$

where  $p = \text{cst}$  is the metal yield and  $M_g$  is the (interstellar) gas mass. So as the gas mass decreases the metallicity increases.

#### Star formation tracers

- Emission lines, particularly Balmer lines
- Mid-IR peak(s), due to PAHs
- far-IR/sub-mm emission
- Rest UV light

#### Star formation quenching

##### Early-type star formation quenching

1. Pre-quenching: Two galaxies on main sequence are about to merge, both bring in gas reservoir and are connected to cosmic gas flow.
2. Quenching: Merger may drive SFR above main sequence or not. Morphology becomes spheroid. Lots of outflowing gas, causes depletion of the cold gas reservoir. Further cosmological inflows/cooling stopped.
3. Post-quenching: SFR drops rapidly to zero and the galaxy moves to the green valley. Gas reservoir is destroyed. AGN becomes active.
4. (Far) Post-quenching: Galaxy becomes a passive red sequence galaxy.

##### Late-type star formation quenching

1. Pre-quenching: Galaxy is on main sequence, and  $M_\star \propto \text{SFR}^\beta$ . Inflows balance outflows, the system in quasi-equilibrium.
2. Quenching: Gas reservoir is no longer replenished, thus the galaxy leaves the main sequence. Note:  $\text{SFR} = \varepsilon M_{\text{gas}} / \tau_{\text{dyn}}$ .
3. Post-quenching: SFR goes into exponential decline, galaxy enters the green valley. Gas reservoir is slowly used up.
4. (Far) Post-quenching: Galaxy has become a passive red spiral galaxy (so morphological transformation). Very low SFR.

### High Redshift Galaxies

#### The cosmic SFR density

Galaxy formation and growth was much more efficient in the past.

## Selection of High Redshift Galaxies

The Lyman-break selection technique. If we change the set of filters, the technique can be extended to select higher redshift galaxies. But the technique is biased against dusty galaxies.

### Lyman-break galaxies versus Lyman-alpha emitters

Note: Lyman-break galaxies are not necessarily  $\text{Ly}\alpha$  emitters. The  $\text{Ly}\alpha$  line profile depends on the ability of  $\text{Ly}\alpha$  photons to escape dusty ISM. However there's one problem, namely resonant scattering of photons with neutral hydrogen.

### Optical versus NIR Surveys

Previously the search of  $z > 1.5$  galaxies was done primarily on optical images. However using NIR, it has been shown that optical images miss a lot of high redshift galaxies.

## Physics of High Redshift Galaxy Evolution

### Observational constraints - the $\text{SFR}-M_\star$ plane

Starbursts may be more important than was thought a few years ago. But one needs to look for them among galaxies with lower stellar masses  $M_\star$ .

### Searching for galaxy outflows

Evidence of outflows in spectra:

- Velocity offsets of emission and absorption lines.
  - Redshifted  $\text{Ly}\alpha$
  - Blueshifted interstellar absorption
- Stellar photospheric features are too weak to detect, so we need to guess the systemic redshift.

## Dusty Galaxies at High Redshifts

Dust was very common in galaxies at  $z \approx 2 - 4$ , at which time the universe was 1.5-3.5 Gyr old. Dust is rare in the first billion years, but evidence of dust has been found in galaxies up to  $z \approx 7 - 8$

## Lecture 14: Basics on AGN & General Review

### AGN Components

#### QSO & AGN

QSOs (quasi-stellar objects) are the most luminous AGNs, they look point-like. Some facts about AGNs:

- x-ray emission due to accretion
- Broadline in type I AGNs (due to a fast rotation disk around the accretion disk), type II AGNs have only narrow lines (especially in the optical)
- AGNs are either radio quiet or radio loud, radio loud implies the presence of a jet due to a strong magnetic field
- An excess of blue light
- Some AGNs contain light variability
- Optical light polarization, which might make the broad line regions visible even for type II AGNs

### The central engine

The central engine of an AGN is a supermassive black hole accreting gas, this BH has an event horizon the size of the solar system. With gas being accreted at a rate of  $1M_{\odot}/\text{yr}$ . This gas being accreted forms a disk which is heated by friction. Which results in UV, optical, and x-ray radiation.

The energy that is released by accretion is approximately

$$\Delta E = \frac{GMm}{R}$$

where  $M$  is the mass of the central BH,  $R$  the radius of the accretion disk, and  $m$  the accreted mass. And the accretion luminosity, i.e. the energy radiated due to accretion. is given by

$$L = \frac{GM\dot{M}}{R} \approx 1.3 \cdot 10^{21} \left( \frac{M/M_{\odot}}{R/\text{km}} \right) \left( \frac{\dot{M}}{\text{g/s}} \right) \text{ erg/s}$$

where we assume a steady accretion rate  $\dot{M}$ .

### The broad-line region

Extends to around 0.01-0.1 pc around the central engine, but is not directly visible. Here the gas is extremely hot (producing velocities around  $v \approx 10^3$  to  $v \approx 10^4$  km/s), this can only be the result of the presence of supermassive BHs.

### The dusty torus

To observe it we need to look in the IR, mainly in the mid-IR. Current evidence shows that the torus is rather clumpy and not homogeneous.

### The narrow-line region

Extends to about 100-1000 pc, outside of the central engine, essentially the host galaxy itself. This region contains gas clouds with velocities ranging from 100-500 km/s.

### **AGN Classification**

The AGN type I/II classification depends only on the orientation. So if the broad-line region is exposed to the observer we call the AGN, type I.

In polarized light both types of AGNs look the same.

### **Spectral properties of AGNs**

#### **SED contribution of different regions**

- UV/optical: accretion disk, following a power law
- Mid-IR: hot dust and torus
- Far-IR: cold dust and the host galaxy, due to star formation

#### **The x-ray spectrum**

For x-ray observations we need both soft/hard band photometry, to then confirm the presence of an AGN a color should be present.

#### **The optical spectrum**

We can construct a BPT diagram, then galaxies containing AGNs can be found in the upper right corner.