

Lecture Notes Thermal Physics

Mick Veldhuis

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1 Introduction: a short overview of important notions

The mole

A **mole** of a certain atom is equivalent to an **Avogadro number** N_a of atoms. Where $N_A = 6.022 \cdot 10^{23}$.

The ideal gas law

Suppose we have N particles in a gas, then we can relate the pressure, volume, temperature, and amount of particles as follows:

$$pV = Nk_B T \quad (1)$$

where k_B is Boltzmann's constant. In order to apply this formula one assumes that (i) there are no intermolecular forces and (ii) that the particles are point-like and have zero size.

Combinatorics

Suppose we have n atoms, and r of those atoms are for instance in an excited state. If we want to calculate the number of possible configurations Ω we apply the following formula,

$$\Omega = \frac{n!}{(n-r)!r!} \equiv {}^nC_r \quad (2)$$

Since factorials grow incredibly quick, we shall often use $\ln \Omega$ instead of Ω .

$$\ln \Omega = \ln(n!) - \ln((n-r)!) - \ln(r!) \quad (3)$$

We also have **Sterling's formula**:

$$\ln(n!) \approx n \ln n - n \quad (4)$$

2 Heat

Heat (Q , measured in Joules) is thermal energy in transit. Note that in contrast to for instance fuel in a car, one cannot say that something has a certain quantity of heat. One can only add heat to something, like adding fuel to a car.

2.1 Heat capacity

We can ask the question, how much heat needs to be supplied to an object to raise its temperature by a small amount dT . The answer is $dQ = C dT$, such that we define the **heat capacity** C of an object using

$$C = \frac{dQ}{dT} \quad (5)$$

There is also c , the **specific heat capacity** in $\text{J K}^{-1} \text{kg}^{-1}$. And the **molar heat capacity**, the heat capacity of one mole of a substance.

If we were to add heat to a system, we'd have to constrain, either the volume of the gas or the pressure of the gas to be fixed. We should therefore change the definition of heat capacity, we need new quantities: C_V , the heat capacity at constant volume, and C_p , the heat capacity at constant pressure.

$$C_V = \left(\frac{\partial Q}{\partial T} \right)_V \quad (6)$$

$$C_p = \left(\frac{\partial Q}{\partial T} \right)_p \quad (7)$$

In practice $C_p > C_V$, due to more heat being added whilst heating at constant pressure. While heating at constant volume will expend additional energy on doing work on the atmosphere as the gas expands.

3 Probability

3.1 Discrete probability distributions

Let x be a discrete random variable which takes values x_i with probability P_i . The sum of P_i 's must always be $\sum P_i = 1$. And we define the **mean** of x to be

$$\langle x \rangle = \sum_i x_i P_i \quad (8)$$

and in general

$$\langle f(x) \rangle = \sum_i f(x_i) P_i \quad (9)$$

3.2 Continuous probability distributions

Let x be a continuous random variable which has a probability $P(x) dx$ of having a value between x and $x + dx$. We require that

$$\int P(x) dx = 1 \quad (10)$$

With the mean defined as

$$\langle x \rangle = \int x P(x) dx \quad (11)$$

and in general

$$\langle f(x) \rangle = \int f(x) P(x) dx \quad (12)$$

3.3 Linear transformation

If one want to relate say a random variable x to the random variable y by e.g. $y = ax + b$ then

$$\langle y \rangle = \langle ax + b \rangle = a\langle x \rangle + b \quad (13)$$

where a and b are just constants.

3.4 Variance

One measurement of spread in a distribution is the variance (the square root of it is the standard deviation), defined as

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \quad (14)$$

3.5 Independent variables

If u and v are independent random variables the probability that they are in there respective range is given by

$$P_u(u) du P_v(v) dv \quad (15)$$

and by taking the double integral of that product we obtain

$$\langle uv \rangle = \langle u \rangle \langle v \rangle \quad (16)$$

3.6 Binomial distribution

It is a probability distribution based on **Bernoulli trials**, which are experiments with two outcomes, success and failure. Where the success happens with a probability p , and failure with probability $1 - p$. The **binomial distribution** is the discrete probability distribution of getting k successes from n independent Bernoulli trials. It is defined as

$$P(n, k) = p^k (1 - p)^{n-k} \binom{n}{k} \quad \text{where} \quad \binom{n}{k} = \frac{n!}{(n-k)!k!} \quad (17)$$

and the mean of the distribution is $\langle k \rangle = np$ and the variance is $\sigma_k^2 = np(1 - p)$.