

BAYESIAN INFERENCE IN `geoR`

`geoR` uses the function `krige.bayes` to fit and provide prediction of a Gaussian process using Bayesian methods. The strategy of `krige.bayes` is that of discretizing the distribution of the range parameter as well as the *relative nugget* defined as $\gamma^2 = \tau^2 / \sigma^2$.

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Sampling of the posterior distribution is achieved by direct sampling of the parameters using the block conditioning

$$\pi(\beta, \psi, \tau^2, \gamma^2 | X) = \pi(\beta | \tau^2, \psi, \gamma^2, X) \pi(\tau^2 | \psi, \gamma^2, X) \pi(\psi, \gamma^2 | X)$$

CONTROLLING `geoR` INPUT/OUTPUT

The model to be fitted in `bayes.krige` can be specified with the function `model.control`. The model defaults to an exponential correlation with no nugget and no Box and Cox transformation.

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The output from `bayes.krige` can be specified using the function `output.control`. This includes the number of samples in the posterior and the predictive posterior distributions.

Set the priors:

```
prior.swiss=prior.control(phi.discrete=seq(7.5,150,len=20),  
phi.prior='reciprocal',  
tausq.rel.prior='uniform',tausq.rel.discrete=  
seq(0,0.5,len=11))
```

Set The model:

```
model.swiss=model.control(kappa=1,lambda=0.5)
```

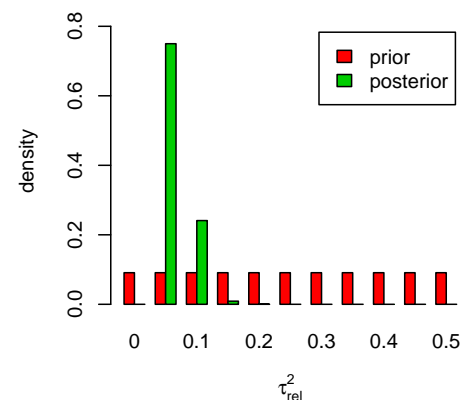
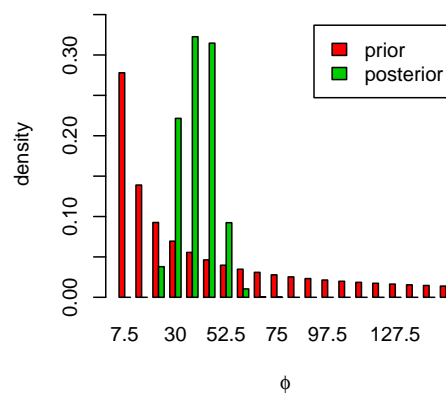
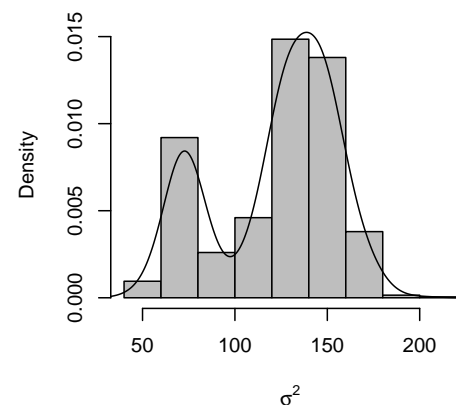
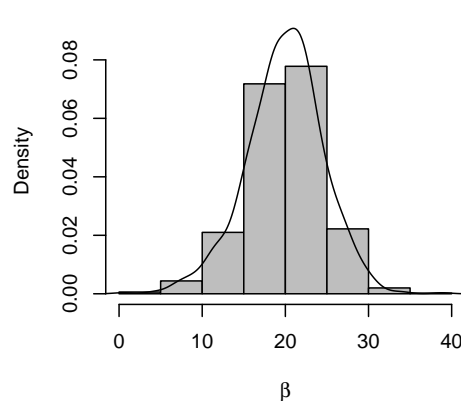
Fit the model:

```
bayes.fit=krige.bayes(sic.all,prior=prior.swiss,  
model=model.swiss)
```

POSTERIOR RESULTS

Summary of 1,000 samples from the posterior distribution for the Swiss data example obtained with

```
hist(bayes.fit,  
pars=c('beta',  
'sigma_sq')  
plot(bayes.fit,  
col=2:3)
```



POSTERIOR RESULTS

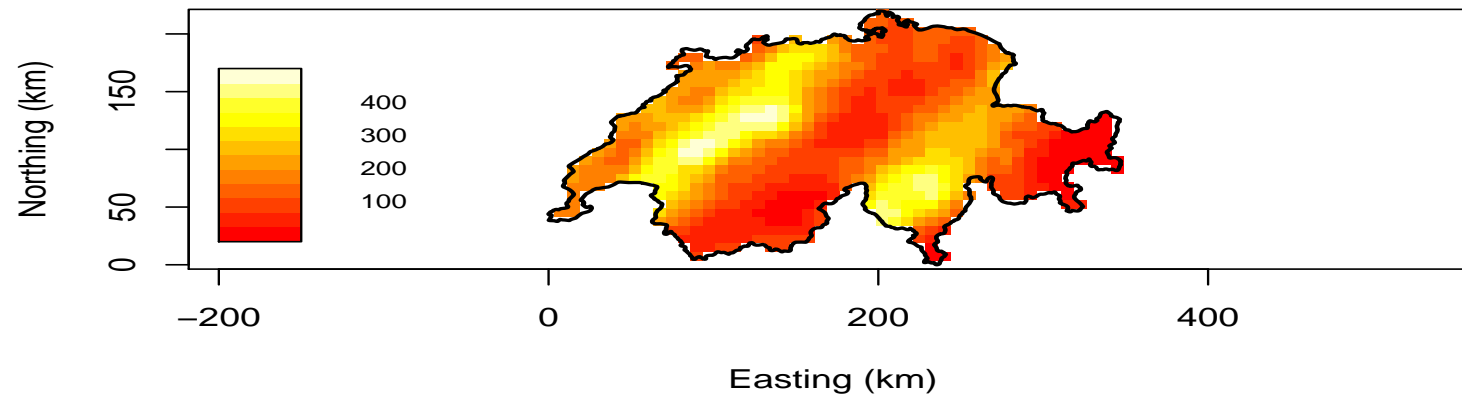
```
> summary(bayes.fit$posterior$sample$beta)
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.4443 17.0900 20.2000 19.9100 22.7900 38.7900
> summary(bayes.fit$posterior$sample$sigma2)
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
43.53   90.85  130.50  122.10  146.20  204.10
```


PREDICTIVE INFERENCE

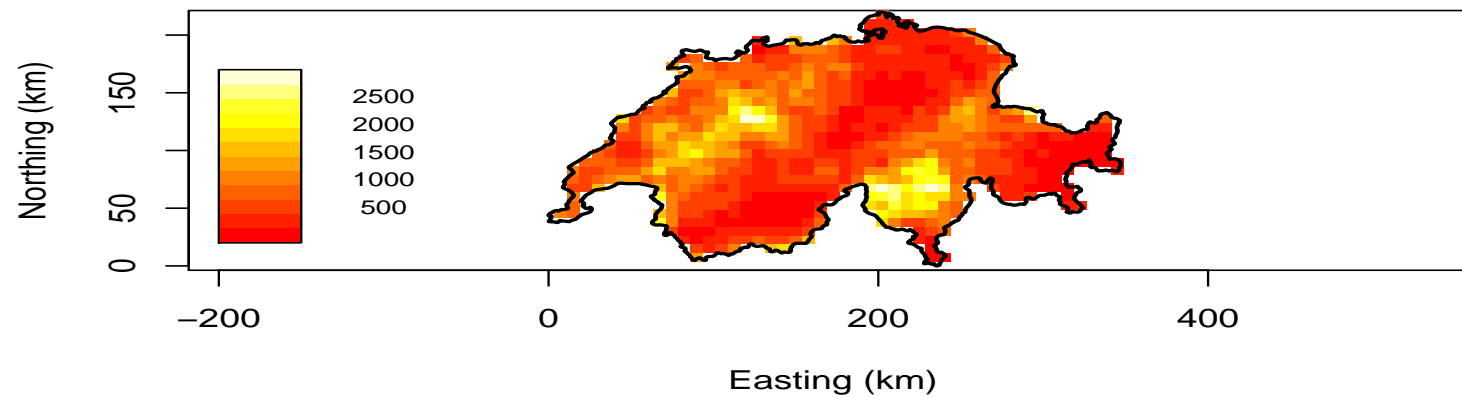
```
image(bayes.fit,borders=sic.borders,  
x.leg=c(-200,-150),y.leg=c(20,170),ylab='Northing (km)',  
xlab='Easting (km)',vertical=T)  
title('Posterior Predictive Mean Rainfall')  
#Variance  
image(bayes.fit,borders=sic.borders,  
val='variance',x.leg=c(-200,-150),  
y.leg=c(20,170),ylab='Northing (km)',  
xlab='Easting (km)',vertical=T)  
title('Posterior Predictive Rainfall Variance')
```

PREDICTIVE INFERENCE

Posterior Predictive Mean Rainfall



Posterior Predictive Rainfall Variance

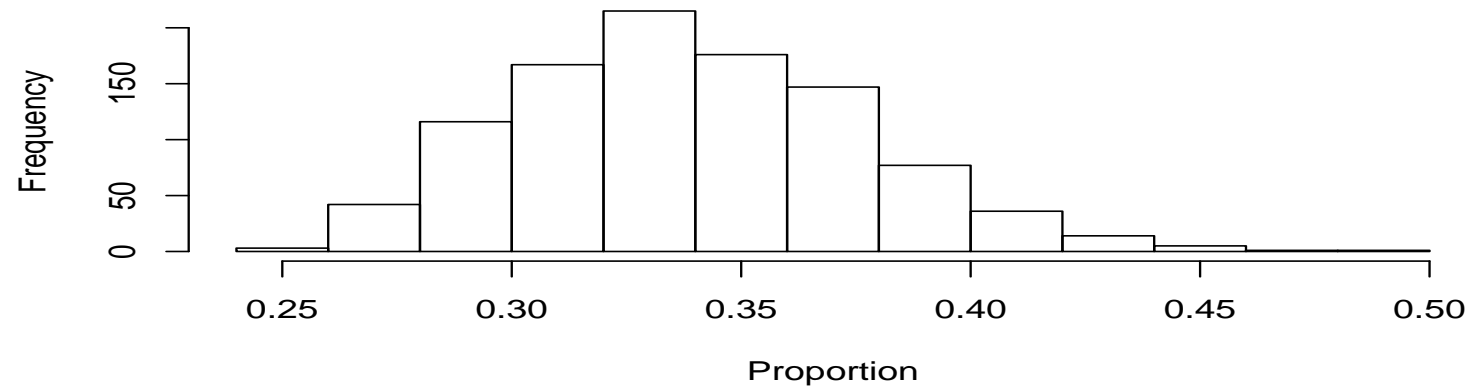


PREDICTIVE INFERENCE

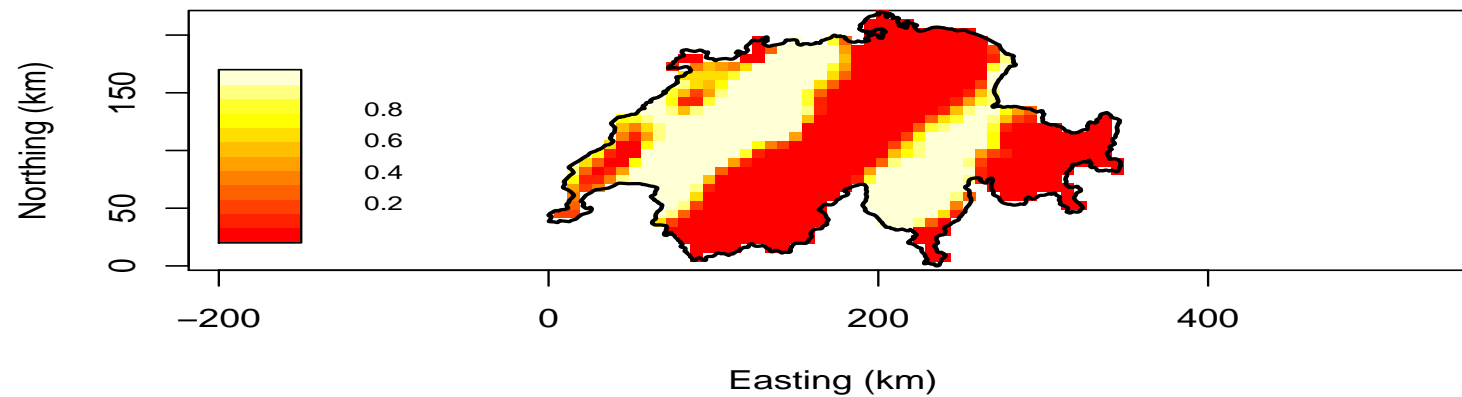
```
#Proportion of area with more than 200 mm of rainfall
A200=apply(bayes.fit$predictive$simulations,2,function(y)
sum(y>200)/length(y))
hist(A200,xlab='Proportion',
main='Predictive Proportion of Areas With Rainfall > 200 mm')
#Probabilities of exceeding 200 mm
prob=apply(bayes.fit$predictive$simulations,1,function(y)
sum(y>200)/length(y))
image(bayes.fit,,borders=sic.borders,
x.leg=c(-200,-150),y.leg=c(20,170),ylab='Northing (km)',
xlab='Easting (km)',vertical=T,val=prob)
title('Predictive Probability of Exceeding 200 mm of Rainfall')
```

PREDICTIVE INFERENCE

Predictive Proportion of Areas With Rainfall > 200 mm



Predictive Probability of Exceeding 200 mm of Rainfall



The core function in `spBayes` is `spGGT` and NOT `ggt.sp` as described in the tech. report!

Set the priors using the function `prior`:

```
phi.prior=prior(dist="UNIF",a=7.5,b=150) #range  
Psi.prior=prior(dist="IG",shape=50,scale=10) #nugget  
K.prior=prior(dist="IG",shape=1,scale=1) #sill  
nu.prior=prior(dist="FIXED") #smoothness
```

Set The model:

```
var.update.control=  
  list("K"=list(starting=12000,tuning=0.1,prior=K.prior),  
        "Psi"=list(starting=.3,tuning=0.5,prior=Psi.prior),  
        "phi"=list(starting=57,tuning=0.5,prior=phi.prior),  
        "nu"=list(starting=1,tuning=0.5,prior=nu.prior))  
beta.control=list(update="GIBBS",prior=prior(dist="FLAT"))
```

Run the model for the Swiss rainfall data

```
run.control=list("n.samples "=1000)
bayes.fit=spGGT(formula=sic.all$data~1,coords=sic.all$coords,
  run.control=run.control,var.update.control=var.update.control,
  beta.update.control=beta.control,cov.model="matern")
```

POSTERIOR RESULTS

```
summary(bayes.fit$p.samples[-seq(200),])
```

K	Psi	Phi	(Intercept)
Min. :11023	Min. :0.1337	Min. :53.55	Min. :169.4
1st Qu.:11984	1st Qu.:0.1842	1st Qu.:55.93	1st Qu.:180.4
Median :12510	Median :0.1981	Median :57.12	Median :183.9
Mean :12542	Mean :0.2019	Mean :57.72	Mean :184.2
3rd Qu.:13077	3rd Qu.:0.2192	3rd Qu.:58.94	3rd Qu.:187.8
Max. :14892	Max. :0.3224	Max. :63.47	Max. :200.9

PREDICTIVE INFERENCE

```
gr=pred_grid(sic.borders,by=7.5) #using geoR
bayes.predict=spPredict(bayes.fit,gr,start=200,
  pred.covars=matrix(rep(1,dim(gr)[1])))
#... forever and a day later:
# The names of the components of the resulting object
# do not coincide with those in the help file!
```

MODEL ASSESSMENT

The methods for assessing the goodness of fit a Bayesian spatial model follow along the lines of Bayesian predictive checking that is common for the Bayesian approach. The fundamental idea is to compare the posterior predictive distribution to observed data.

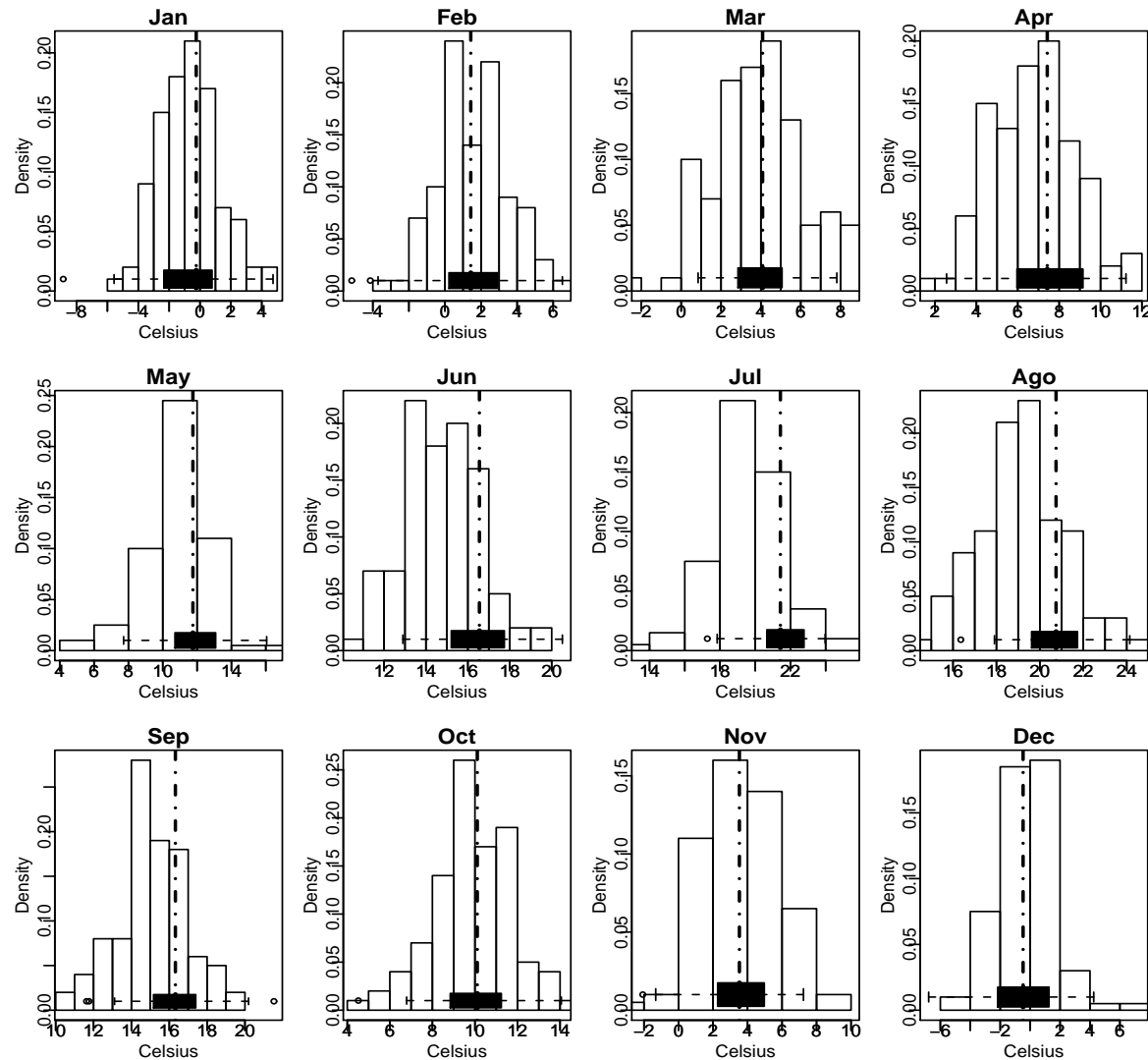
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The methods for assessing the goodness of fit a Bayesian spatial model follow along the lines of Bayesian predictive checking that is common for the Bayesian approach. The fundamental idea is to compare the posterior predictive distribution to observed data.

- Select some locations for validation
- Fit the model to the remaining data set
- Obtain the posterior predictive for the validation locations
- Compare to observed data

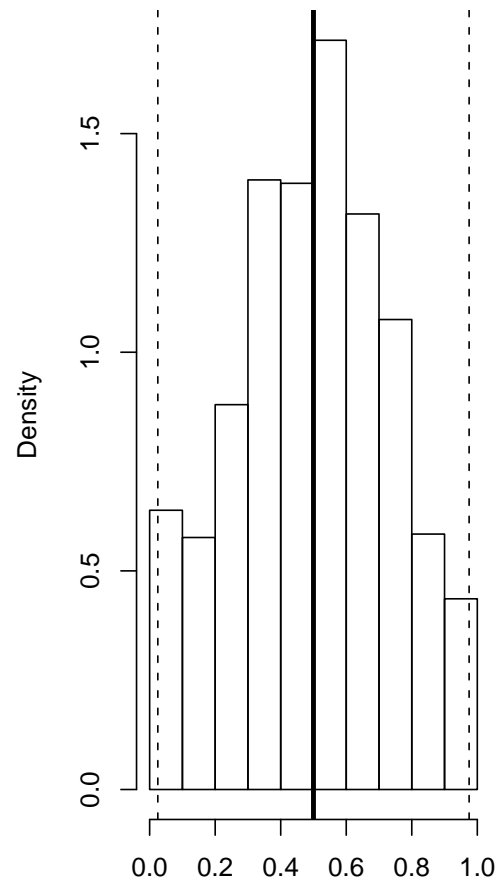
CALIFORNIA TEMPERATURE DATA

Posterior
predictive dis-
tribution for
temperature
at Bowman Dam.
Horizontal
boxplots
correspond to
50 years of
observations

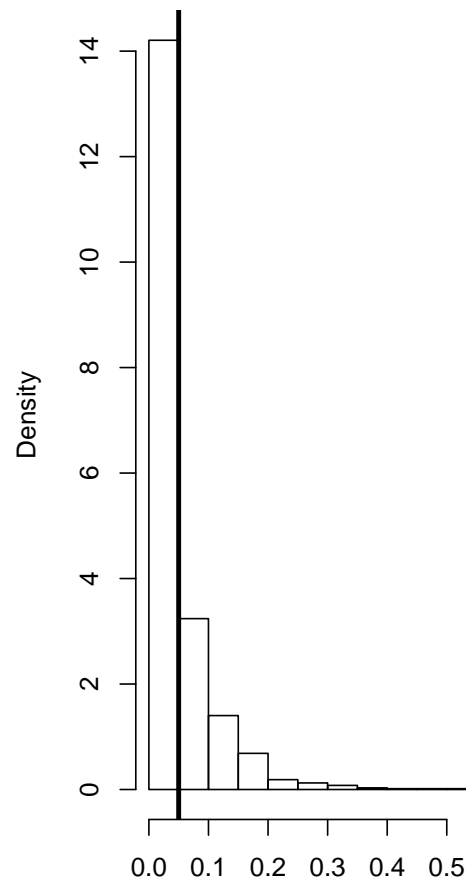


CALIFORNIA TEMPERATURE DATA

Proportion of predictive samples
above the observed median



Proportion of observations
outside prediction interval



Other summaries of the predictive distribution can also be explored. **Warning:** In these plots we pooled together samples from all other locations. So histograms are produced with dependent data

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For a single goodness of fit statistics:

- For validation locations s_1, \dots, s_k define $Z = (z(s_1) \dots z(s_k))'$ and z the corresponding observations
- Obtain samples $Z^{(1)} \dots Z^{(l)}$ of $p(Z|X)$.
- Let \bar{Z} and $\hat{\Sigma}$ be the sample mean and variance respectively
- $D^2 = (z - \bar{Z})' \hat{\Sigma}^{-1} (z - \bar{Z}) \sim \chi_k^2$