Homework 2 – AMS 276

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M1

The likelihood for the proportionl hazards model with a Weibull as the baseline hazard function is given by:

$$L(\mathbf{y}|\mathbf{x},\boldsymbol{\nu},\alpha,\gamma,\beta)=\prod_{\{i:\nu_i=1\}}\alpha\gamma y_i^{\alpha-1}\times\prod_{i=1}^{n}\exp\left[-\gamma y_i^{\alpha}e^{x_i^\top\beta}\right]$$

I place independent gamma priors on α, γ and a normal prior on β:

$$\begin{aligned}\alpha &\sim Gamma(1,0.1)\\ \gamma &\sim Gamma(1,0.1)\\ \beta &\sim Normal(0,10^2)\end{aligned}$$

The full conditional for the joint parameter vector (α,γ,β) is proportional to the likelihood times each of the priors:

$$\pi(\alpha,\gamma,\beta|\mathbf{y},\mathbf{x},\boldsymbol{\nu})=\prod_{\{i:\nu_i=1\}}\alpha\gamma y_i^{\alpha-1}\times\prod_{i=1}^{n}\exp\left[-\gamma y_i^{\alpha}e^{x_i^\top\beta}\right]\times e^{-0.1\alpha}e^{-0.1\gamma}\exp\left(-\frac{\beta^2}{2\cdot 10^2}\right)$$

The parameters are updated jointly using Metropolis-Hastings updates. Their marginal posterior distributions and an estimate of the survival functions for the two groups (Aneuploid in blue and Diploid in red) are given at the end.

M2

We define a partion $0=s_0<s_1<\cdots<s_J$ by the quantiles of the failure times. We let $J=20$, with s_j being the $5\times j$th quantile.

The likelihood for the piece-wise constant hazard model is given by

$$L(\mathbf{y}|\mathbf{x},\boldsymbol{\nu},\boldsymbol{\lambda},\boldsymbol{\beta})=\prod_{i=1}^{n}\prod_{j=1}^{J}(\lambda_j\exp^{\mathbf{x}_i^\top\boldsymbol{\beta}})^{\delta_{ij}\nu_i}\exp\left(-\delta_{ij}\left[\lambda_j(y_i-s_{j-1})+\sum_{g=1}^{j-1}\lambda_g(s_g-s_{g-1})\right]e^{\mathbf{x}_i^\top\boldsymbol{\beta}}\right)$$

This is the likelihood as given in ICS, which I noticed is different than the one you present on the slides. The difference is that you have the δ_{ij} term only over λ_j. I'm not sure what the consequences for this is, but I realized that it was necessary to include an intercept term, so $\boldsymbol{\beta}=(\beta_0,\beta_1)$. For whatever reason, excluding the intercept made my estimates for β_1 off.

For priors, we have

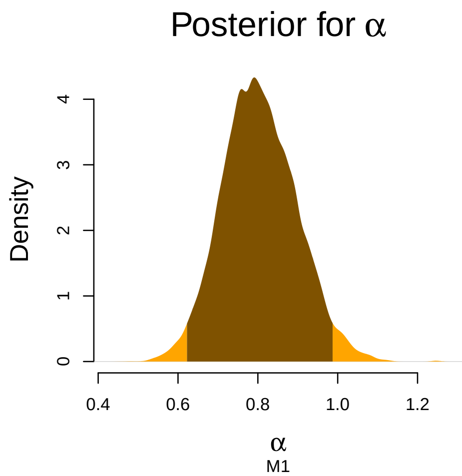
$$\begin{aligned}\lambda_j &\overset{iid}{\sim} Gamma(1,1/10), &\quad j=1,\ldots,J\\ \beta_j &\overset{iid}{\sim} Normal(0,10^2), &\quad j=0,1\end{aligned}$$

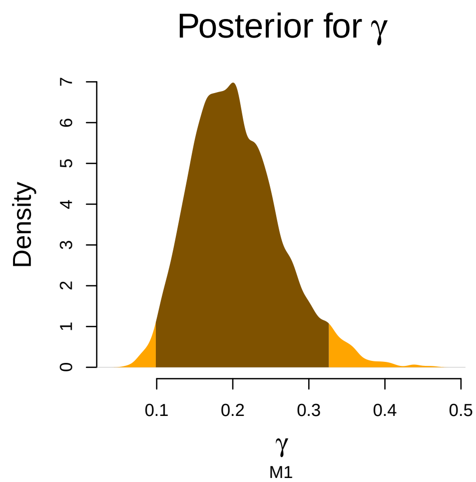
Again, I do a joint update of all the parameters. And yes, this is mostly so I don't have to derive individual full conditionals for each parameter.

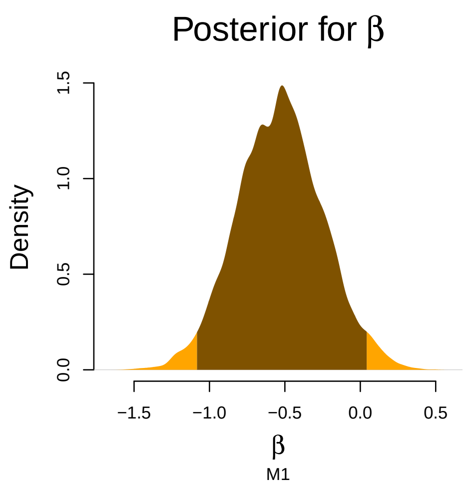
M3

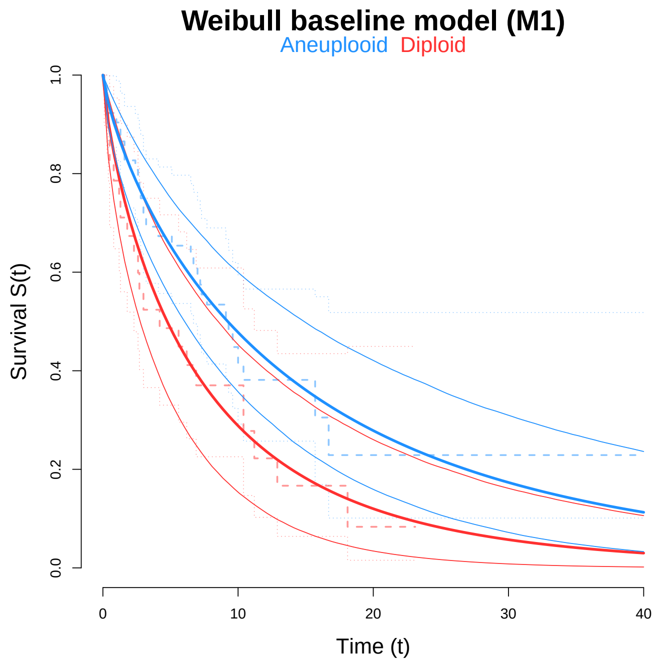
Remarks about the figures

Figures for M1

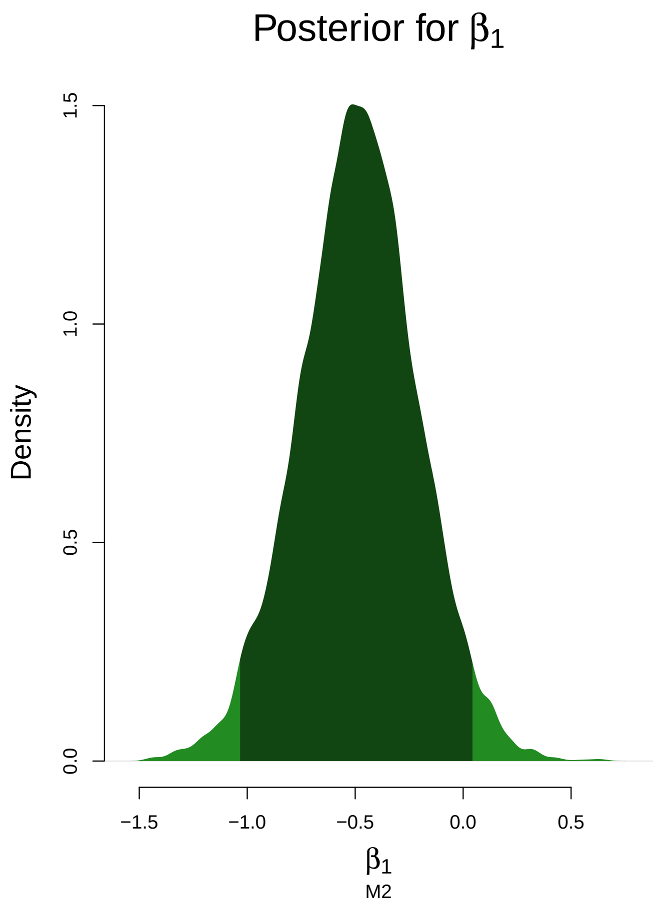


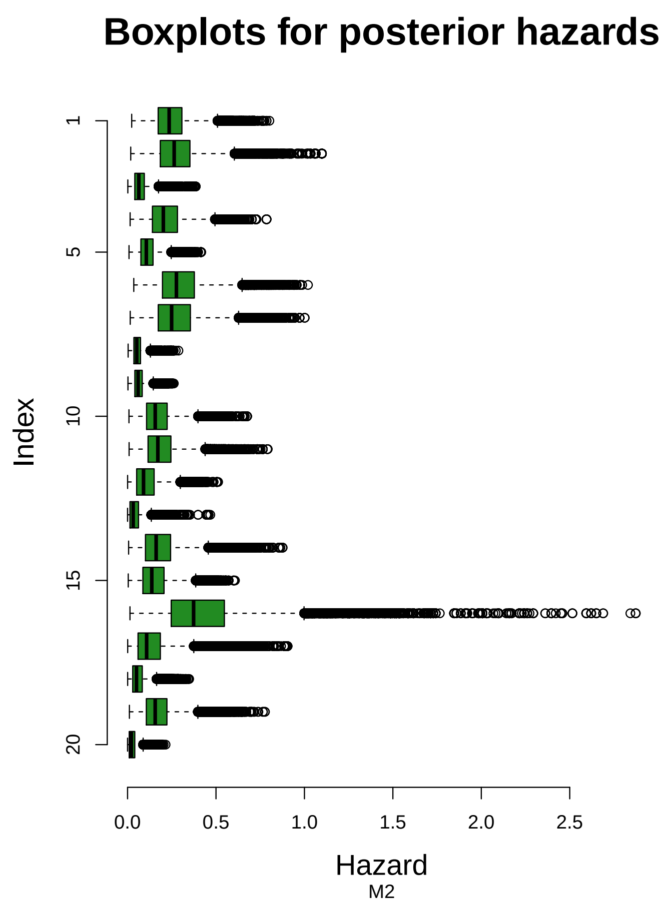


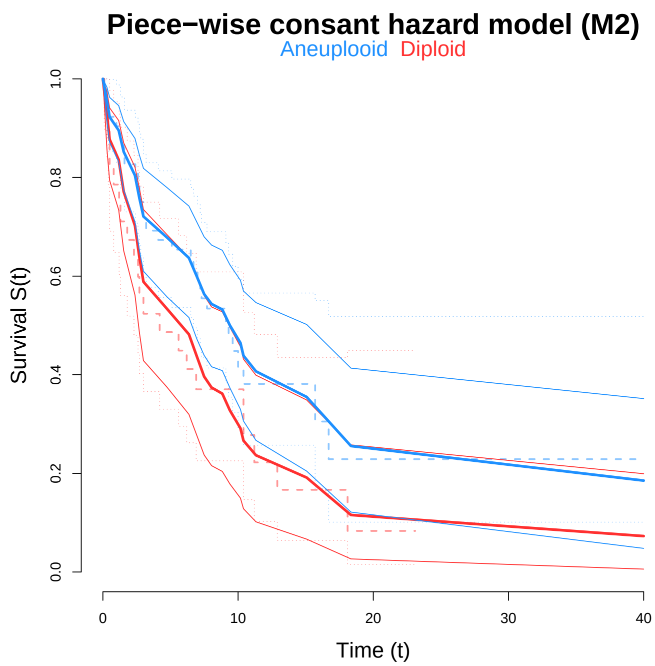




Figures for M2







Figures for M3