California precipitation extremes, 1950–1999

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AMS 263 — Stochastic Processes

22 March 2017

Introduction

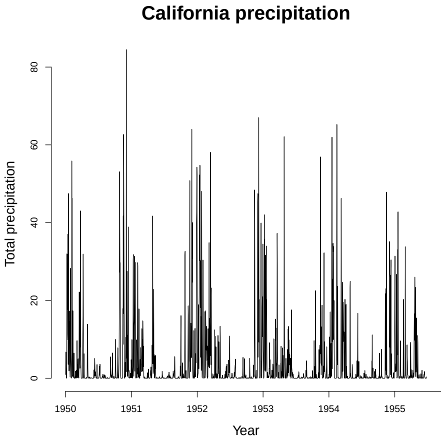
Data:

- Observation product spanning 50 years
- Daily measurements summed to produce daily total

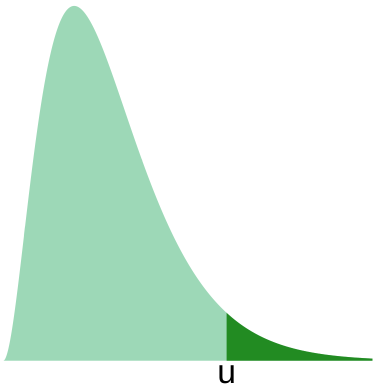
Goal:

- Characterize the tail of the distribution
- Make inferences based on extreme value theory

Introduction — Data



Introduction — Extremes



Poisson process characterization of extremes

Theorem: Let X_1,X_2,... be a series of independent and identically distributed random variables for which there are sequences of constants $\{a_n > 0\}$ and $\{b_n\}$ such that

$$\Pr\{(M_n - b_n)/a_n \leq z\} \to G(z),$$

where

$$G(z) = \exp\left\{ -\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\},$$

and let z_- and z_+ be the lower and upper end points of G, respectively. (Note, $M_n = \max(X_1, \ldots, X_n)$)

Poisson process characterization of extremes

Theorem continued: Then, the sequence of point processes

$$N_n = \{(i/(n+1), (X_i - b_n)/a_n) : i = 1, \dots, n\}$$

converges on regions of the form $(0,1) \times [u, \infty)$, for any $u > z_-$, to a Poisson process, with intensity measure on $A = [t_1, t_2] \times [z, z_+)$ given by

$$\Lambda(A) = (t_2 - t_1) \left[1 + \xi \left(\frac{z - \mu}{\sigma}\right)\right]^{-1/\xi}$$

$\square$

Poisson process characterization of extremes

Theorem (restated): Let $X_1, \ldots, X_n$ be a series of independent and identically distributed random variables, and let

$$N_n = \{(i/(n+1), X_i) : i = 1, \ldots, n\}.$$

Then, for sufficiently large u, on regions of the form $(0, 1) \times [u, \infty)$, N_n is approximately a Poisson process, with intensity measure on $A = [t_1, t_2] \times (z, \infty)$ given by

$$\Lambda(A) = (t_2 - t_1)\left[1 + \xi\left(\frac{z - \mu}{\sigma}\right)\right]^{-1/\xi}$$

$\square$

Likelihood

Select a high threshold u, and set $A = (0,1) \times [u, \infty)$. Denote the $N(A)$ observations in A by $\{(t_1, x_1), \ldots, (t_{N(A)}, x_{N(A)})\}$.

The (restated) theorem leads to the following likelihood, with $[t_1, t_2] = [0, 1]$:

$$L_A(\mu, \sigma, \xi; x_1, \ldots, x_n) = \exp\{-\Lambda(A)\} \prod_{i=1}^{N(A)} \lambda(t_i, x_i)$$

$$\propto \exp\left\{-n_y \left[1 + \xi \left(\frac{u-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \prod_{i=1}^{N(A)} \sigma^{-1} \left[1 + \xi \left(\frac{x_i - \mu}{\sigma}\right)\right]^{-1/\xi - 1}$$

Likelihood

By convention, n_y is chosen to be the number of years of observations. In our example, $n_y = 50$.

This results in parameters (μ, σ, ξ) that correspond to the annual maximum distribution.

The threshold u must be chosen to be large enough so the approximation holds, but not so large that we have too few data to work with.

Time-varying parameters

We let $\mu = \mu_t = \beta_0 + \beta_1 \cos(2\pi/365 \times t) + \beta_2 \sin(2\pi/365 \times t)$, corresponding to an annual cycle for the location parameter.

Similar adjustments are made for $\log \sigma$, and ξ.

This gives is model flexibility and the Poisson process handles the changes fine.

Return levels

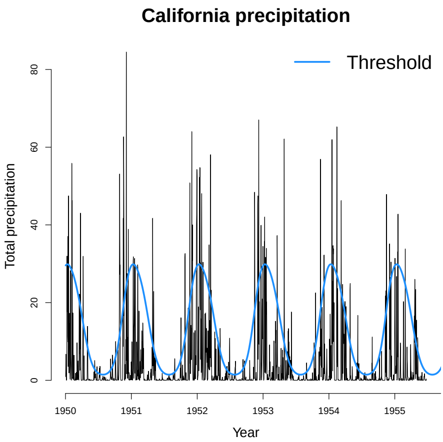
The return level z_m is the value that is exceeded on average once every m years, and z_m satisfies

$$1 - \frac{1}{m} = \Pr\{\max(X_1, \ldots, X_n) \leq z_m\} \approx \prod_{i=1}^{n} p_i$$

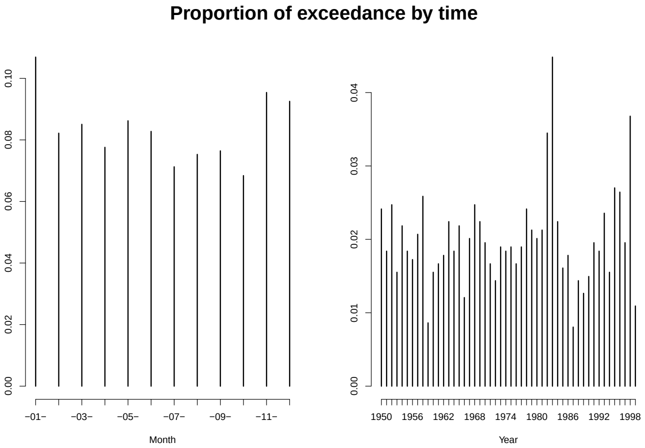
where $p_i = 1 - n^{-1}[1 + \xi_i(z_m - \mu_i)/\sigma_i]_+^{-1/xi_i}$ and $x_+ = \max(0, x)$.

If there is no time-varying components to the parameters, then this simplifies. Otherwise, we need to solve for z_m using numerical methods.

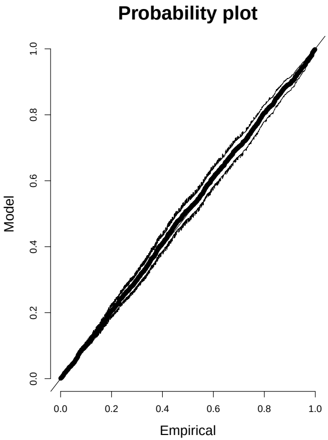
Results — time-varying threshold

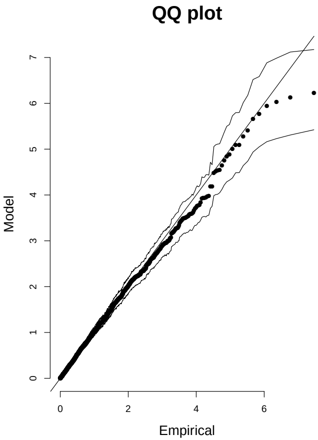


Results — rate of exceedance

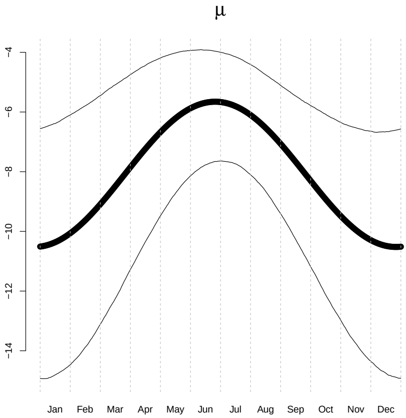


Results — diagnostics

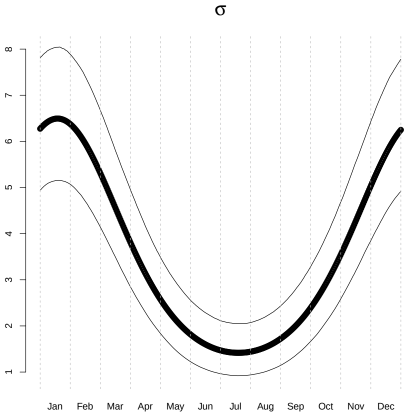




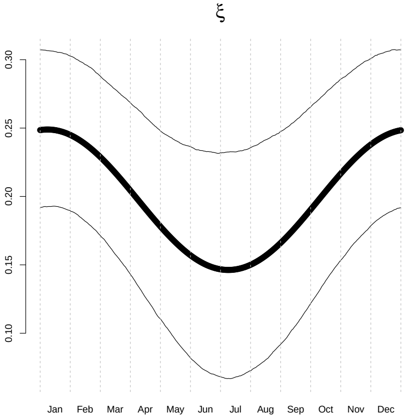
Results — posterior distributions



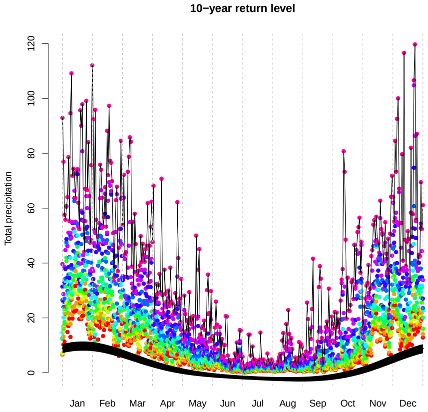
Results — posterior distributions



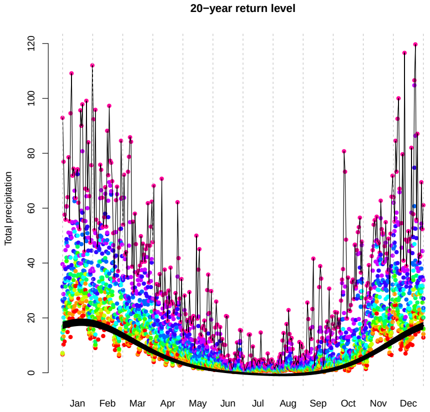
Results — posterior distributions



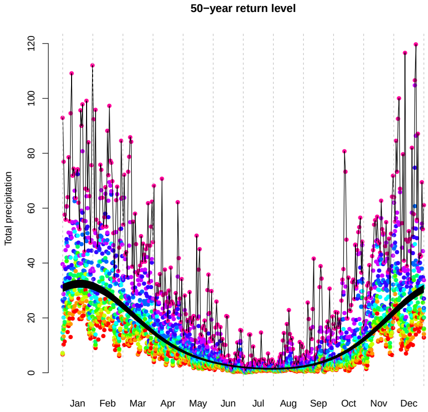
Results — 10-year return levels (by day)



Results — 20-year return levels (by day)



Results — 50-year return levels (by day)



Conclusion

The distributions for the three parameters were sensitive to my choice of time-varying threshold. More work needs to be considered in selecting an appropriate threshold.

There may be some evidence that does not vary over time.

The return levels are smaller than expected. Also, the bounds seem to get narrower as the return period increases. This the opposite of what is expected, especially when $\xi > 0$.

The return levels we are actually interested in are over a particular season or a year (not daily which is presented).