Reversible jump Markov chain Monte Carlo

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16 November 2015
UC Santa Cruz – AMS 200

Markov chain Monte Carlo (MCMC)

A general set of methods that allows us to obtain random samples from a target distribution

In the Bayesian setting, the target distribution is the posterior distribution $p(\theta|y)$

MCMC is useful when directly sampling from $p(\theta|y)$ is difficult

Standard MCMC methods require the parameters θ to have fixed dimension

Reversible jump MCMC

What about when θ does not have fixed dimensions?

For example, consider the normal mixture model

$$y_i \overset{iid}{\sim} \sum_{j=1}^{\nu} \pi_j N(\phi_j, 1), \qquad i = 1, \ldots, n$$

$$\nu, \pi, \phi \sim p(\nu) p(\pi) p(\phi)$$

Here, $\theta = (\nu, \pi, \phi)$ has dimension $2\nu + 1$, with random $\nu \geq 1$, and $\pi = (\pi_1, \ldots, \pi_\nu)$, $\phi = (\phi_1, \ldots, \phi_\nu)$

Reversible jump MCMC

Generally, we are considering a collection of K models

$$\mathcal{M}_k = \{f(\cdot|\theta_k);\ \theta_k \in \Theta_k\}$$

We need a method that allows us to *jump* from one dimension, or model, to another (i.e. moving from $\mathcal{M}_i$ to $\mathcal{M}_j$)

Green's (1995) algorithm

Let $\pi(k, \theta_k)$ denote the posterior density for model $\mathcal{M}_k$

Define a $K \times K$ matrix $\{P\}_{ij} = p_{ij} \geq 0$ with row sums of 1

Define a deterministic transformation function T such that

$$(\theta_j, u_j) = T_{ij}(\theta_i, u_i)$$

where $\theta_k \in \Theta_k, u_k \sim g_k(u_k)$, for $k = \{i, j\}$ so (θ_i, u_i) has the same *number* of components as (θ_j, u_j)

Green's (1995) algorithm

At iteration t, if $x^{(t)} = (i, \theta_i^{(t)})$,

1. Select model $\mathcal{M}_j$ with probability p_{ij}
2. Generate $u_{ij} \sim g_{ij}(u)$
3. Set $(\theta_j, v_{ji}) = T_{ij}(\theta_i^{(t)}, u_{ij})$
4. Take $\theta_j^{(t)} = \theta_j$ with probability

$$\min\left(\frac{\pi(j,\theta_j)}{\pi(i,\theta_i^{(t)})}\frac{p_{ji}g_{ji}(v_{ji})}{p_{ij}g_{ij}(u_{ij})}\left|\frac{\partial T_{ij}(\theta_i^{(t)},u_{ij})}{\partial(\theta_j^{(t)},u_{ij})}\right|,1\right)$$

and take $\theta_i^{(t+1)} = \theta_i^{(t)}$ otherwise

References

Green, P. J. (1995), "Reversible jump Markov chain Monte Carlo computation and Bayesian model determination," *Biometrika*, 82, 711–732.

Richardson, S. and Green, P. J. (1997), "On Bayesian analysis of mixtures with an unknown number of components," *Journal of the Royal Statistical Society. Series B (Methodological)*, 731–792.

Robert, C. and Casella, G. (2013), *Monte Carlo statistical methods*, Springer Science & Business Media.