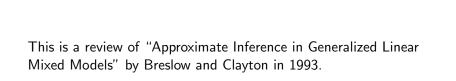
GLMM Parameter Estimation via Approximation Methods

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Hierarchical model

The *i*th of *n* observations has univariate response y_i with vectors \mathbf{x}_i and \mathbf{z}_i is covariates.

We have a q-vector **b** of random effects, where $\mathbf{b} \sim N(\mathbf{0}, \mathbf{D}(\boldsymbol{\theta}))$.

In vector notation, the conditional mean of $\mathbf{y} = (y_1, \dots, y_n)^{\top}$ given \mathbf{b} is assumed to satisfy

$$E(y|b) = \mu = h(X\alpha + Zb)$$

and $\mathrm{Var}(y_i|\mathbf{b}) = \phi a_i v(\mu_i)$ for $i=1,\ldots,n$, for known constant a_i and variance function $v(\cdot)$. Note, we do not have any distributional assumption on y_i .

Quasi-likelihood function

For independent observations y_i, \ldots, y_n , the quasi-likelihood function is defined as

$$\frac{\partial K(y_i, \mu_i)}{\partial \mu_i} = \frac{y_i - \mu_i}{a_i v(\mu_i)}$$

$$\Longrightarrow K(y_i, \mu_i) = \int_{y_i}^{\mu_i} \frac{y_i - u_i}{a_i v(u_i)} du_i$$

This has properties similar to the log-likelihood of a distribution from the exponential family.

The integrated quasi-likelihood function is defined by

$$e^{ql(\boldsymbol{\alpha},\boldsymbol{\theta})} \propto |\mathbf{D}|^{-1/2} \int \exp\left[\frac{1}{\phi} \sum_{i=1}^{n} K(y_i, \mu_i) - \frac{1}{2} \mathbf{b}^{\top} \mathbf{D}^{-1} \mathbf{b}\right] d\mathbf{b}$$

= $c|\mathbf{D}|^{-1/2} \int \exp\left[\kappa(\mathbf{b})\right] d\mathbf{b}$

where c is some multiplicative constant. Using Laplace's method for integral approximation (and ignoring the constants), we have

$$ql(\boldsymbol{\alpha}, \boldsymbol{\theta}) \approx -\frac{1}{2} \log |\mathbf{D}| - \frac{1}{2} \log |\kappa''(\tilde{\mathbf{b}})| - \kappa(\tilde{\mathbf{b}})$$

where $\ddot{\mathbf{b}}$ is the solution to $\kappa'(\mathbf{b}) = 0$.

Penalized quasi-likelihood (PQL)

Marginal quasi-likelihood (MQL)

Estimating parameters

Roughbutts and Whitesides