From the simple Pareto process

$$\chi(q) = Pr(F_{W_1}(W_1) > q | F_{W_2}(W_2) > q) \tag{1}$$

$$= E\left(\frac{V_1}{E(V_1)} \wedge \frac{V_2}{E(V_2)}\right). \tag{2}$$

So $\chi(q)$ is the same value for all $0 \leq q \leq 1$ and this is what is reported as the measure of asymptotic dependence. Converting from the Pareto process W back to the original data X using

$$W_i = YV_i = TX_i = \left(1 + \xi_i \frac{X_i - u_i}{\sigma_i}\right)_+^{1/\xi_i} \tag{3}$$

we can compute a correspondence between two variables X_1 and X_2 (which could represent, say, observations and climate simulations) by

$$\chi(q) = P\left(W_1 > \frac{E(V_1)}{1-u}\middle| W_2 > \frac{E(V_2)}{1-u}\right) \tag{4}$$

$$= P\left(\left(1 + \xi_1 \frac{X_1 - u_1}{\sigma_1}\right)_+^{1/\xi_1} > \frac{E(V_1)}{1-q}\middle| \left(1 + \xi_2 \frac{X_2 - u_2}{\sigma_2}\right)_+^{1/\xi_2} > \frac{E(V_2)}{1-q}\right) \tag{5}$$

$$= P\left(X_1 > u_1 + \frac{\sigma_1}{\xi_1}\left[\left(\frac{E(V_1)}{1-q}\right)^{\xi_1} - 1\right]\middle| X_2 > u_2 + \frac{\sigma_2}{\xi_2}\left[\left(\frac{E(V_2)}{1-q}\right)^{\xi_2} - 1\right]\right) \tag{6}$$

$$= P\left(X_1 > x_1 | X_2 > x_2\right) \tag{7}$$

From (2), it must be the case that (7) is the same for all q. I think this is the reason for the issue I'm having. Choosing any x_1 or x_2 in which we might be interested doesn't yield itself to a straightfoward calculation. There is a single probability $\chi(q)$ that corresponds to many pairs of (x_1, x_2), which are both functions of q. And it would seem possible to find these pairs, but they are all share the same probability.

Ideally, we'd like to choose x_1 and x_2 to be very high, something like the thresholds. But even in this case, it doesn't appear that we can use the simple Pareto process calculation of (2). For x_1 and x_2 to be the thresholds only, we must have the additional terms in (6) be zero, but this cannot happen unless $E(V_1) = E(V_2)$, which cannot be true except in degenerate cases (if it were true, we would pick $1 - q = E(V_1) = E(V_2)$).

With what we've done with the simple Pareto process, it is not clear to me how to come up with a distribution for (7), whether to get probabilities or return levels.