Comparison of extreme values of different climate model simulations and observations

Mickey Warner

30 Oct 2017

Introduction

CanCM4 simulation classes (with $R = 10$ replicates each):

1. Decadal
2. Historical
3. Control

Observations over U.S. interpolated from weather stations

Factors:

1. Variable — Total Precipitation (`pr`) or Average Maximum Temperature (`tasmax`)
2. Season — Winter or Summer
3. Decade — 1962–1971 or 1990–1999
4. Region — California or USA

Locations

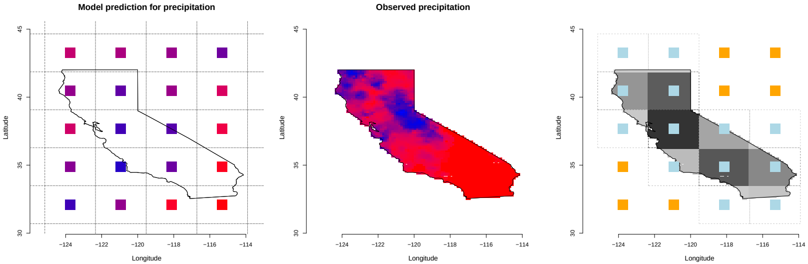
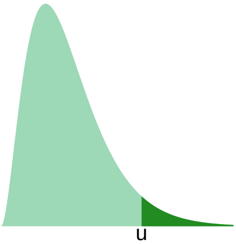


Figure: Left: CanCM4 simulation grid cells. Center: Observation locations. Right: method for computing weighted sum or average for CanCM4 to make values comparable with observations.

Extremes

For r.v. X and large threshold u, the exceedance $Y = X - u$, for $X > u$, approximately follows the generalized Pareto distribution (GPD), which has density

$$f_Y(y) = \frac{1}{\sigma}\left(1 + \xi\frac{y}{\sigma}\right)_+^{-1/\xi - 1}$$



Data processing

Two objectives before performing the analysis:

1. Make climate simulations comparable to observations
2. Get near-independent random variables for model fitting

These are accomplished by

1. Taking weighted sums (`pr`) or weighted averages (`tasmax`)
2. Computing anomalies based on DLMs, and
3. Declustering

Weighted sum or average

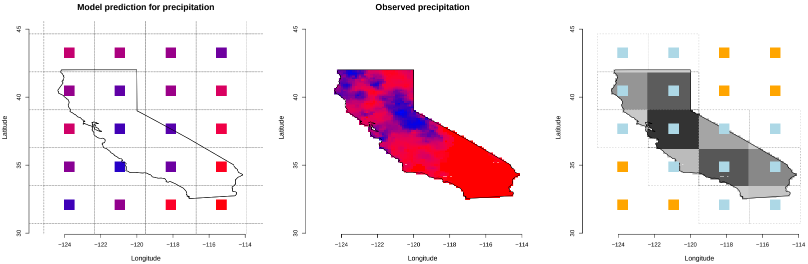
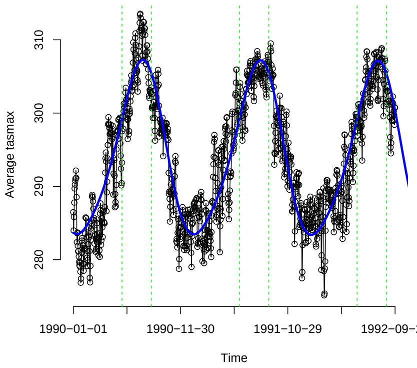
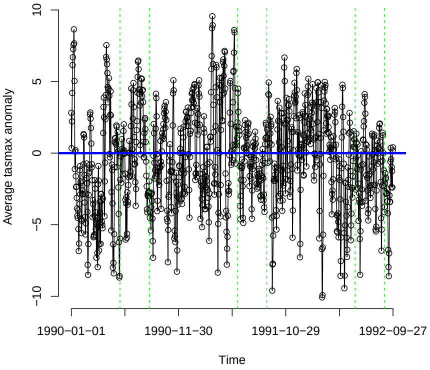


Figure: Left: CanCM4 simulation grid cells. Center: Observation locations. Right: method for computing weighted sum or average for CanCM4 to make values comparable with observations.

DLM-based anomaly





Extremal index (declustering)

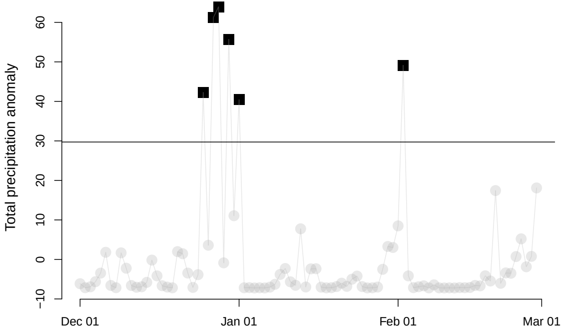
The extremal index θ is the inverse of the limiting mean cluster size

It can be estimated using interexceedance times, $T_i = S_{i+1} - S_i$, with a log-likelihood of

$$l(\theta, p; \mathbf{T}) = m_1 \log(1 - \theta p^{\theta}) + (N - 1 - m_1)\{\log(\theta) + \log(1 - p^{\theta})\} + \theta \log(p) \sum_{i=1}^{N-1} (T_i - 1)$$

p is the probability of not exceeding the threshold

Declustering



Likelihood

Replicate i, observation j, exceedances $Y_{ij} = X_{ij} - u$, and keep only those $Y_i > 0$. These have likelihood

$$L(\mathbf{y};\boldsymbol{\sigma},\boldsymbol{\xi},\boldsymbol{\zeta}) = \prod_{i=1}^{R}\left[(1-\zeta_i)^{n_i-k_i}\zeta_i^{k_i}\prod_{j=1}^{k_i}\frac{1}{\sigma_i}\left(1+\xi_i\frac{y_{ij}}{\sigma_i}\right)_+^{-1/\xi_i-1}\right]$$

n_i is the number of X_{ij}'s
k_i is the number of Y_{ij}'s
ζ_i is the probability of exceeding the threshold

Priors

These priors complete the hierarchical model formulation. Greek letters are random variables while English letters are fixed.

$$\begin{aligned}
\sigma_i|\alpha, \beta &\sim Gamma(\alpha, \beta) \\
\xi_i|\xi, \tau^2 &\sim Normal(\xi, \tau^2) \\
\zeta_i|\mu, \eta &\sim Beta(\mu\eta, (1-\mu)\eta)
\end{aligned}$$

$$\begin{aligned}
\alpha_\sigma \sim Gamma(a_\alpha, b_\alpha) &\qquad \beta_\sigma \sim Gamma(a_\beta, b_\beta) \\
\xi \sim Normal(m, s^2) &\qquad \tau^2 \sim Gamma(a_\tau, b_\tau) \\
\mu \sim Beta(a_\mu, b_\mu) &\qquad \eta \sim Gamma(a_\eta, b_\eta)
\end{aligned}$$

Return level

For a distribution G, the return level x_m is the solution to

$$G(x_m) = 1 - \frac{1}{m}.$$

The value x_m is exceeded on average once every m observations.

For the GPD, the return level is given by

$$x_m = u + \frac{\sigma}{\xi}\left[(m\zeta\theta)^{\xi} - 1\right]$$

Bhattacharyya distance

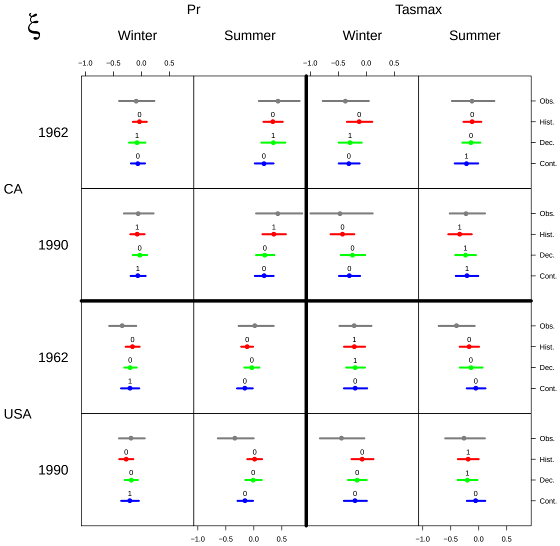
Bhattacharyya coefficient

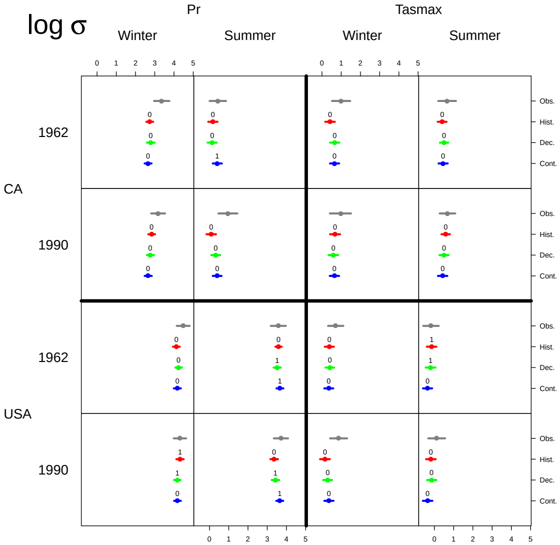
$$BC(p,q) = \int_{\mathcal{X}} \sqrt{p(x)q(x)}dx$$

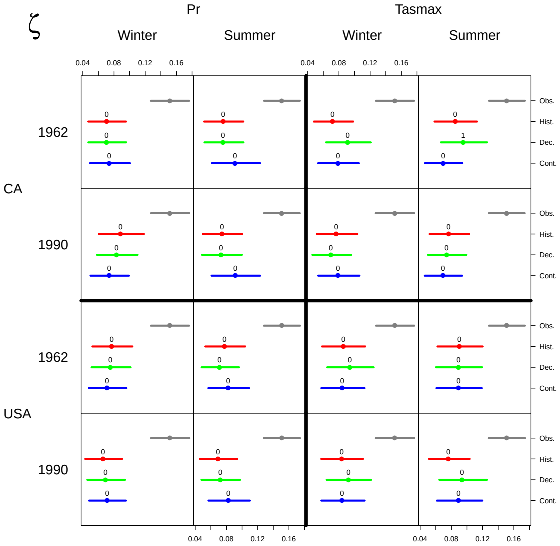
Bhattacharyya distance

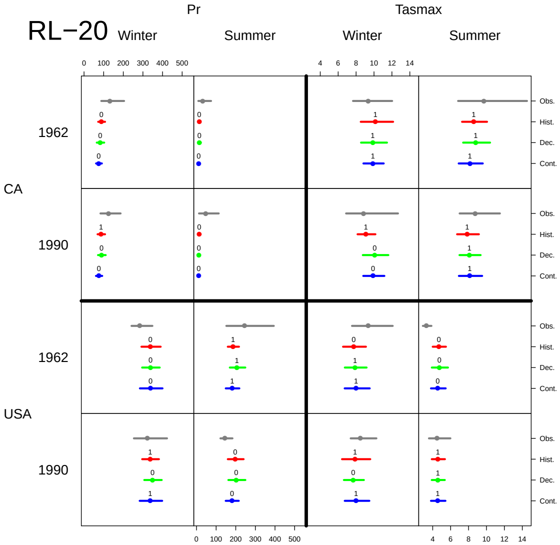
$$D_B(p,q) = -\log BC(p,q).$$

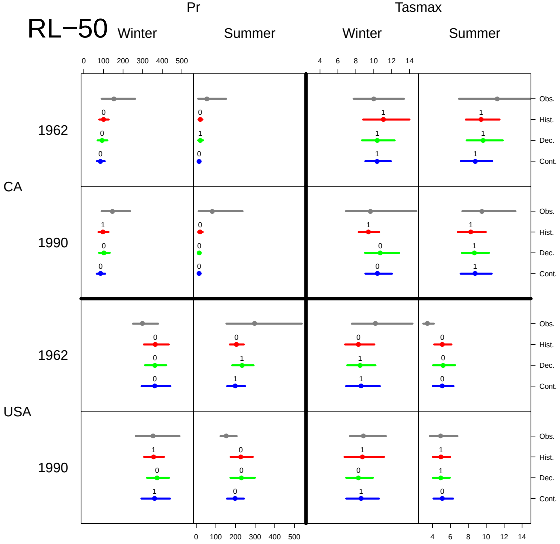
D_B is computed between parameters in the replicates (and observations) and parameters in the hierarchy.


ξ
Pr
Tasmax
Winter
Summer
Winter
Summer
CA
1962
1990
USA
1962
1990
Obs.
Hist.
Dec.
Cont.
−1.0 −0.5 0.0 0.5

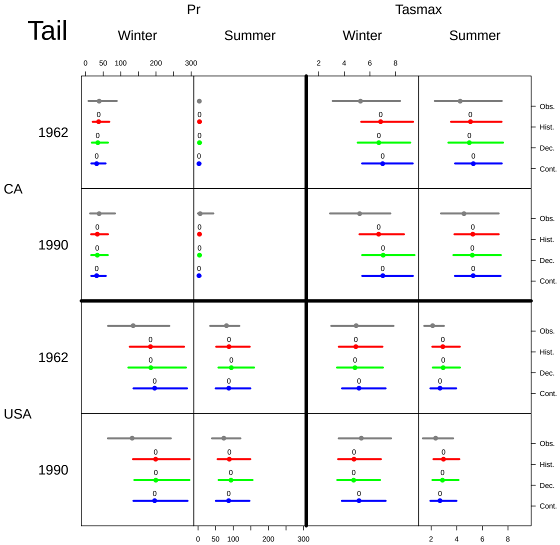

log σ
Pr
Tasmax
Winter
Summer
Winter
Summer
CA
USA
1962
1990
1962
1990
Obs.
Hist.
Dec.
Cont.
0 1 2 3 4 5

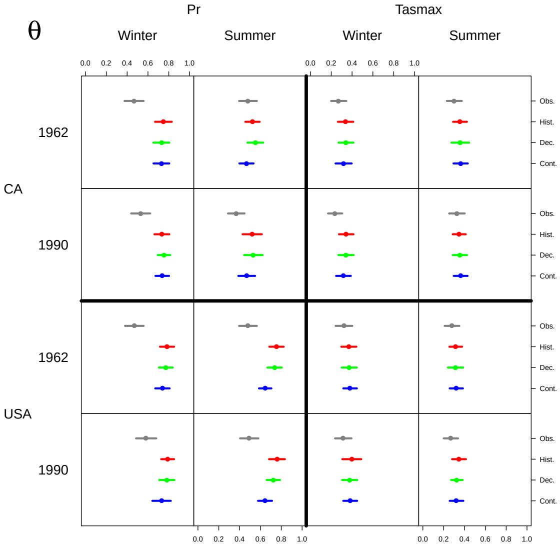

ζ
Pr
Tasmax
Winter
Summer
Winter
Summer
0.04
0.08
0.12
0.16
CA
USA
1962
1990
1962
1990
Obs.
Hist.
Dec.
Cont.
0
1


RL−20
Pr
Tasmax
Winter
Summer
Winter
Summer
CA
USA
1962
1990
1962
1990
Obs.
Hist.
Dec.
Cont.


RL−50
Pr
Tasmax
Winter
Summer
Winter
Summer
1962
1990
CA
1962
1990
USA
Obs.
Hist.
Dec.
Cont.
0 100 200 300 400 500
4 6 8 10 12 14

Tail


Pr
Tasmax
Winter
Summer
Winter
Summer
0 50 100 200 300
2 4 6 8
CA
USA
1962
1990
1962
1990
Obs.
Hist.
Dec.
Cont.
0 50 100 200 300
2 4 6 8

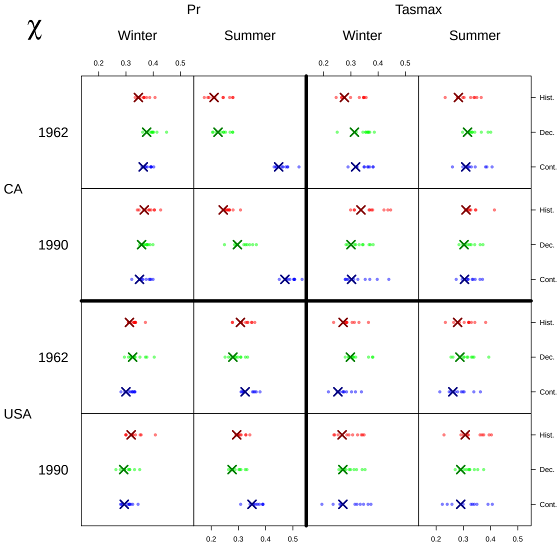

θ
Pr
Tasmax
Winter
Summer
Winter
Summer
CA
USA
1962
1990
1962
1990
Obs.
Hist.
Dec.
Cont.
0.0
0.2
0.4
0.6
0.8
1.0

Bivariate analysis

Bivariate analysis using simple Pareto processes

Simple pareto process

Limit
Transform
Figure of transformed (include supremum cone) and non-transformed data


χ
Pr
Tasmax
Winter
Summer
Winter
Summer
1962
1990
CA
1962
1990
USA
Hist.
Dec.
Cont.
0.2
0.3
0.4
0.5