Definition. (Constructive approach) A stochastic process W in $C_+(S)$ with constant $\omega_0 > 0$ is a simple Pareto process if $W(s) = YV(s)$, for all $s \in S$, for some Y and $V = \{V(s)\}_{s\in S}$ satisfying:

a) $V \in C_+(S)$ is a stochastic process satisfying $\sup_{s\in S} V(s) = \omega_0$ almost surely, $E[V(s)] > 0$ for all $s \in S$,

b) Y is a standard Pareto random variable, $P(Y > y) = y^{-1}, y > 1$,

c) Y and V are independent.

See Ferreira and de Haan (2014) for other variants of the definition.

Coefficient of asymptotic dependence. For random variables X_1 and X_2 having common marginal distribution F, let

$$\chi_{12} = \lim_{z\to z_+} P(X_1 > z | X_2 > z)$$

where z_+ is the (possibly infinite) right end-point.

For $s_1, s_2 \in S$ and $x > \omega_0$, then for $i = 1, 2$,

$$\begin{aligned}
P(W(s_i) > x) &= P(YV(s_i) > x) \\
&= P\left(Y > \frac{x}{V(s_i)}\right) \\
&= E_{Y,V(s_i)}\left[\mathbb{1}\left(Y > \frac{x}{V(s_i)}\right)\right] \\
&= E_{V(s_i)}\left\{E_{Y|V(s_i)}\left[\mathbb{1}\left(Y > \frac{x}{V(s_i)}\right)\middle| V(s_i)\right]\right\} \\
&= E_{V(s_i)}\left\{P\left(Y > \frac{x}{V(s_i)}\middle| V(s_i)\right)\right\} \\
&= E_{V(s_i)}\left\{\frac{V(s_i)}{x}\right\} = \frac{E[V(s_i)]}{x}
\end{aligned}$$

and

$$\begin{aligned}
P(W(s_1) > x, W(s_2) > x) &= P(YV(s_1) > x, YV(s_2) > x) \\
&= P\left(Y > \frac{x}{V(s_1)}, Y > \frac{x}{V(s_2)}\right) \\
&= P\left(Y > \frac{x}{V(s_1)} \vee \frac{x}{V(s_2)}\right) \\
&= P\left(Y > x\left(\frac{1}{V(s_1)} \vee \frac{1}{V(s_2)}\right)\right) \\
&= P\left(Y > x\left(\frac{1}{V(s_1) \wedge V(s_2)}\right)\right) \\
&= \frac{E[V(s_1) \wedge V(s_2)]}{x}
\end{aligned}$$

by using arguments similar in the first set of equations. Then for a simple Pareto process at points $s_1, s_2 \in S$, we have

$$\begin{aligned}
\chi_{12}^W &= \lim_{x \to \infty} P(W(s_1) > x | W(s_2) > x) \\
&= \lim_{x \to \infty} \frac{xE[V(s_1) \wedge V(s_2)]}{xE[V(s_2)]} \\
&= \frac{E[V(s_1) \wedge V(s_2)]}{E[V(s_2)]}
\end{aligned}$$

This is from Ferreira and de Haan (2014), but with some clarity (for the dummies of the universe) as to how they got to the expectations.