Exercises 2 and 3

July 14, 2025

First, we download the data. By looking at it in advance I've noticed it lacked column names, we'll fix it here:

```
[1]: import pandas as pd
    import numpy as np
    url = 'https://www.chicagobooth.edu/-/media/faculty/ruey-s-tsay/teaching/fts2/
      ⇔m-ibm3dx7503.txt'
    col_names = ['date', 'ibm', 'crsp_vw', 'crsp_ew', 's&p']
    data = pd.read_table(url, header=None, names=col_names ,sep='\s+')
    <>:7: SyntaxWarning: invalid escape sequence '\s'
    <>:7: SyntaxWarning: invalid escape sequence '\s'
    /var/folders/f5/jdc2n89x0r11q017zdg17s9w0000gn/T/ipykernel_9124/908117592.py:7:
    SyntaxWarning: invalid escape sequence '\s'
      data = pd.read_table(url, header=None, names=col_names ,sep='\s+')
    Now we inspect it:
[2]: data.head()
[2]:
           date
                     ibm crsp_vw crsp_ew
                                                s&p
    0 19750131 0.12054 0.14150 0.29921
                                            0.12281
    1 19750228 0.15272 0.05842 0.05392 0.05989
    2 19750331 -0.04118 0.03019
                                   0.08150
                                            0.02169
    3 19750430 0.01573
                          0.04649
                                   0.03109
                                            0.04726
    4 19750530 0.03157 0.05514 0.07288
                                            0.04410
    We can immediately notice the same date issue as before, we'll fix it now:
[3]: data['date'] = pd.to_datetime(data['date'], format='%Y%m%d')
    data.head()
[3]:
            date
                      ibm crsp_vw crsp_ew
                                                 s&p
    0 1975-01-31 0.12054
                           0.14150 0.29921
                                             0.12281
    1 1975-02-28 0.15272
                           0.05842
                                    0.05392
                                             0.05989
    2 1975-03-31 -0.04118 0.03019
                                    0.08150
                                             0.02169
    3 1975-04-30 0.01573 0.04649
                                    0.03109
                                             0.04726
    4 1975-05-30 0.03157 0.05514 0.07288 0.04410
```

We now add \log returns and percentage columns for them and the simple net returns:

```
[4]: import utils as ut
     ut.add_log_returns(data, data.columns[1:])
     ut.add_percent(data, data.columns[1:])
     data.head()
[4]:
                                                          log_ibm
             date
                        ibm
                            crsp_vw
                                      crsp_ew
                                                    s&p
                                                                   log_crsp_vw
     0 1975-01-31
                  0.12054 0.14150
                                      0.29921
                                                         0.113811
                                                                      0.132343
                                               0.12281
     1 1975-02-28
                  0.15272 0.05842
                                      0.05392
                                               0.05989
                                                         0.142124
                                                                      0.056777
     2 1975-03-31 -0.04118 0.03019
                                      0.08150
                                               0.02169 -0.042052
                                                                      0.029743
     3 1975-04-30 0.01573
                            0.04649
                                      0.03109
                                               0.04726
                                                         0.015608
                                                                      0.045442
     4 1975-05-30 0.03157
                            0.05514
                                      0.07288
                                               0.04410
                                                         0.031082
                                                                      0.053673
                                                               crsp_ew_percent
        log_crsp_ew
                      log_s&p
                                ibm_percent
                                             crsp_vw_percent
     0
           0.261756
                    0.115834
                                     12.054
                                                       14.150
                                                                        29.921
     1
           0.052517
                    0.058165
                                     15.272
                                                        5.842
                                                                         5.392
     2
           0.078349 0.021458
                                     -4.118
                                                        3.019
                                                                         8.150
     3
           0.030616 0.046177
                                      1.573
                                                        4.649
                                                                         3.109
     4
           0.070347 0.043155
                                                        5.514
                                                                         7.288
                                      3.157
                                      log_crsp_vw_percent log_crsp_ew_percent
        s&p_percent
                     log_ibm_percent
     0
             12,281
                               11.381
                                                     13.234
                                                                           26.176
     1
              5.989
                               14.212
                                                      5.678
                                                                            5.252
     2
              2.169
                               -4.205
                                                      2.974
                                                                           7.835
     3
              4.726
                                                      4.544
                                                                            3.062
                                1.561
     4
                                                                           7.035
              4.410
                                3.108
                                                      5.367
        log_s&p_percent
     0
                 11.583
                  5.817
     1
     2
                  2.146
     3
                  4.618
     4
                  4.316
    Now to calculate the statistics:
[5]: num_data = data[data.columns[1:]]
     print(num_data.describe())
     print(num_data.skew())
```

s&p

348.000000

0.009032

0.044435

-0.217630

log_ibm

0.008722

0.077131

-0.303676

348.000000

crsp_ew

0.015908

0.056537

-0.272310

348.000000

num data.kurtosis()

count

mean

std

min

348.000000

0.011753

0.078139

-0.261900

ibm

crsp_vw

0.011909

0.045618

-0.225340

348.000000

```
25%
        -0.037757
                     -0.015807
                                  -0.015977
                                               -0.017552
                                                            -0.038489
50%
                      0.014945
         0.010110
                                   0.016995
                                                0.010130
                                                             0.010059
75%
         0.055410
                      0.043165
                                   0.047565
                                                0.038935
                                                             0.053929
         0.353800
                      0.141500
                                   0.299210
                                                0.131770
                                                             0.302915
max
       log_crsp_vw
                     log_crsp_ew
                                      log_s&p
                                                ibm_percent
                                                              crsp_vw_percent
count
        348.000000
                      348.000000
                                   348.000000
                                                 348.000000
                                                                   348.000000
mean
          0.010800
                        0.014210
                                     0.008006
                                                   1.175336
                                                                     1.190868
std
          0.045949
                        0.056522
                                     0.044695
                                                   7.813905
                                                                     4.561798
min
         -0.255331
                       -0.317880
                                    -0.245428
                                                 -26.190000
                                                                   -22.534000
25%
                                    -0.017708
         -0.015934
                       -0.016107
                                                  -3.775750
                                                                    -1.580750
50%
          0.014834
                        0.016852
                                     0.010079
                                                   1.011000
                                                                     1.494500
75%
          0.042259
                        0.046468
                                     0.038196
                                                   5.541000
                                                                     4.316500
          0.132343
                        0.261756
                                     0.123783
                                                  35.380000
                                                                    14.150000
max
                         s&p_percent
                                       log_ibm_percent
                                                         log_crsp_vw_percent
       crsp_ew_percent
count
            348.000000
                          348.000000
                                             348.000000
                                                                   348.000000
               1.590805
                            0.903213
                                               0.872198
                                                                     1.079974
mean
                             4.443510
                                                                     4.594929
std
               5.653741
                                               7.713083
min
            -27.231000
                          -21.763000
                                             -30.368000
                                                                   -25.533000
25%
              -1.597750
                           -1.755250
                                              -3.848750
                                                                    -1.593750
50%
               1.699500
                             1.013000
                                               1.006000
                                                                      1.483500
75%
               4.756500
                            3.893500
                                               5.393000
                                                                     4.226250
              29.921000
                           13.177000
                                              30.292000
                                                                    13.234000
max
                             log_s&p_percent
       log_crsp_ew_percent
                 348.000000
                                   348.000000
count
                   1.420997
mean
                                     0.800624
                   5.652252
std
                                     4.469428
min
                 -31.788000
                                   -24.543000
25%
                  -1.610750
                                    -1.771250
50%
                   1.685500
                                     1.008000
75%
                   4.647250
                                     3.819750
                                    12.378000
                  26.176000
max
ibm
                        0.332547
crsp_vw
                       -0.631580
crsp_ew
                       -0.182767
                       -0.476261
s&p
                       -0.071984
log_ibm
log_crsp_vw
                       -0.927199
                       -0.735505
log_crsp_ew
                       -0.749076
log_s&p
ibm_percent
                        0.332547
crsp_vw_percent
                       -0.631580
crsp_ew_percent
                       -0.182767
s&p_percent
                       -0.476261
log_ibm_percent
                       -0.071988
log_crsp_vw_percent
                       -0.927226
```

log_crsp_ew_percent -0.735485
log_s&p_percent -0.749123
dtype: float64

[5]:	ibm	1.693570
	crsp_vw	2.312882
	crsp_ew	4.423737
	s&p	1.947149
	log_ibm	1.559987
	log_crsp_vw	3.545620
	log_crsp_ew	5.403116
	log_s&p	2.997021
	ibm_percent	1.693570
	crsp_vw_percent	2.312882
	crsp_ew_percent	4.423737
	s&p_percent	1.947149
	log_ibm_percent	1.560051
	log_crsp_vw_percent	3.545596
	log_crsp_ew_percent	5.403043
	log_s&p_percent	2.997222
	dtype: float64	

Finally we run the asymptotic z-tests under the asymptotic normality assumption:

[6]: ut.t_test_for_mean(data, data.columns[1:5])

For ibm, the p_value is: 0.005299468414435815 For crsp_vw, the p_value is: 1.699901579383222e-06 For crsp_ew, the p_value is: 2.67082015055518e-07 For s&p, the p_value is: 0.0001763033264922363

Which means that we reject the null for all of them since they are all smaller then 0.05.

Now for exercise 3, we calculate the annual avarage log returns. Since the data is monthly, the mean we found for 'log_s&p' was $\frac{1}{m}\sum_{j=1}^m r_j$ where m is the number of months the data spans. Notice that, for any $t \in \mathbb{N}$ we have:

$$r_{t+12} = \ln(\frac{P_{t+12}}{P_t}) = \ln(\prod_{j=0}^1 1 \frac{P_{t+j+1}}{P_{t+j}}) = \sum_{j=0}^1 1 \ln(\frac{P_{t+j+1}}{P_{t+j}})$$

This means that, if $y = \frac{m}{12}$ is the amount of years the data spans (which is true in our case since our data spans from January of 75 to December of 03 so m%12 = 0) and t is the earliest date in our data, we get:

$$\frac{1}{y} \sum_{j=0}^{y} r_{t+12j} = 12 * \frac{1}{m} \sum_{j=0}^{m} r_{t+j}$$

Thus, it is enough to multiply the monthly mean by 12 to get:

[7]: np.float64(0.09607478867326014)

Now, if we were to invest 1\$ on the S&P composite index in January 75, we can calculate the investments value in December 2003 by summing the monthly log returns withing that span and exponentiating it. This is because the compounded simple returns turn from a product to a sum when the logarithm is applied. Thus, we get:

- [8]: np.exp(data['log_s&p'].sum())
- [8]: np.float64(16.218764453481448)

Which means it would be worth about 16\$ twenty nine years later.