chapter_3_exercise_5

August 16, 2025

1 Example: Escanciano–Lobato Test with Volatility Modeling

This notebook demonstrates the **Escanciano–Lobato (EL) serial correlation test**, which is robust to heteroskedasticity and especially useful in situations where volatility clustering is expected. It shows how to apply the test alongside GARCH modeling in practice.

Workflow Overview: 1. Load and preprocess monthly simple returns of Intel stock from **January 1973 to December 2008** (from `m-intc7308.txt`). 2. Apply the EL test to assess serial correlation in the returns. 3. Fit a GARCH(1,1) model to the log-return series. 4. Re-apply the EL test on standardized residuals to verify mean and variance adequacy. 5. Generate 1- to 5-step-ahead volatility forecasts from the December 2008 origin.

Purpose: This notebook serves as an illustrative use case for the `escanciano_lobato` function implemented in the `rust_timeseries` library. It demonstrates how the test integrates into a typical volatility modeling workflow.

Exercise Context: This example follows **Exercise 3.5** from *Analysis of Financial Time Series* (3rd ed., Chapter 3) by **Ruey S. Tsay**, which asks for volatility forecasts of Intel stock log returns and to assess model adequacy.

Dataset Details: - **Source:** Faculty website of Ruey Tsay (m-intc7308.txt) - **Frequency:** Monthly - **Variable:** Monthly simple returns of Intel stock (to be transformed into log returns)

1.1 Data & Setup

Dataset - **Asset:** Intel Corporation (INTC) - **Variable:** Monthly *simple* returns (we'll transform to log returns) - **Span:** January 1973 – December 2008 (inclusive) - **Frequency:** Monthly - **Source file:** `m-intc7308.txt` (from Ruey S. Tsay's teaching materials)

Preprocessing summary 1) Parse the `date` column as calendar month-end timestamps.
2) Validate there are no missing values and no impossible returns (-100%).
3) Work with **log returns** $\log(1 + r_t)$ for modeling.

Why log returns?
Log returns are additively aggregable across periods and tame multiplicative effects; for small r_t, log and simple returns are numerically close, but logs are more convenient analytically.

Indexing
We analyze the series with a **DatetimeIndex** so plots, diagnostics, and forecasts align to calendar dates.

Reproducibility
Package versions and environment details are listed at the end so results are reproducible.

Load Intel monthly returns (1973–2008):

[288]:
```
import pandas as pd

url: str = 'https://faculty.chicagobooth.edu/-/media/faculty/ruey-s-tsay/
↪teaching/fts3/m-intc7308.txt'
data: pd.DataFrame = pd.read_table(url, sep=r'\s+')

# Preview of raw input (simple returns).
data.head()
```

[288]:
```
       date       rtn
0  19730131  0.010050
1  19730228 -0.139303
2  19730330  0.069364
3  19730430  0.086486
4  19730531 -0.104478
```

Verify lack of NaN/Null values in the data:

[289]:
```
data['date'].isnull().any()
```

[289]:
```
np.False_
```

Parse the date column as calendar month-end timestamps:

[290]:
```
data['date'] = pd.to_datetime(data['date'], format='%Y%m%d')
data.set_index(data['date'], inplace=True)
data.drop('date', inplace=True, axis=1)

# After parsing dates and setting them to be the index.
data.head()
```

[290]:
```
                 rtn
date
1973-01-31  0.010050
1973-02-28 -0.139303
1973-03-30  0.069364
1973-04-30  0.086486
1973-05-31 -0.104478
```

Confirm there are no missing values were introduced by the transformation, check numeric dtype, and verify no values $\leq -100\%$ (so $\log\ (1 + rt)$ is defined):

[291]:
```
returns: pd.Series = data['rtn']
nulls: bool = returns.isnull().any()
ret_type = returns.dtype
```

```python
less_than_one: bool = returns.loc[returns <= - 1].any()
print(f"Nulls: {nulls}, type: {ret_type}, less than 1: {less_than_one}")
```

```
Nulls: False, type: float64, less than 1: False
```

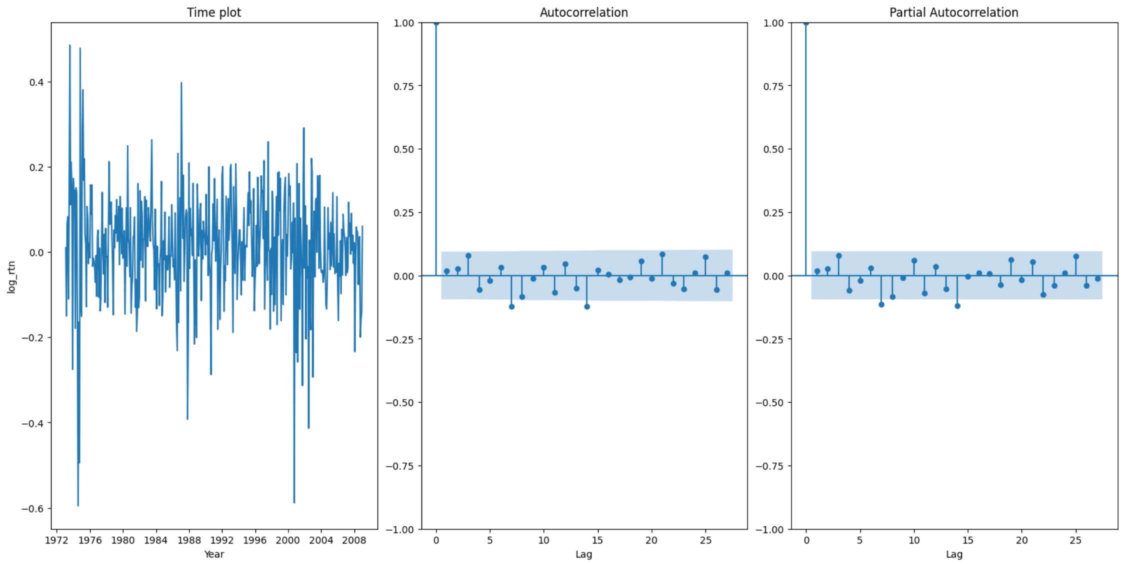
We compute $\log(1 + rt)$. Because we verified $rtn > -1$, $1 + rtn$ is positive for all observations, so the transform is well-defined:

[292]:
```python
import numpy as np

data['log_rtn'] = np.log(data['rtn'] + 1)

# After log-return transform.
data.head()
```

[292]:
```
                 rtn   log_rtn
date
1973-01-31  0.010050  0.010000
1973-02-28 -0.139303 -0.150013
1973-03-30  0.069364  0.067064
1973-04-30  0.086486  0.082949
1973-05-31 -0.104478 -0.110348
```

Initial diagnostics: time plot, ACF, and PACF:

[293]:
```python
import matplotlib.pyplot as plt
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

def plot_ts_pacf_acf(data: pd.Series):
    fig, ax = plt.subplots(1, 3, figsize=(16, 8), layout='constrained')
    ax[0].set_title('Time plot')
    ax[0].set_xlabel('Year')
    ax[0].set_ylabel(data.name)
    ax[0].plot(data.index, data);
    for i in range (1, 3):
        ax[i].set_xlabel('Lag')
    plot_acf(data, ax=ax[1]);
    plot_pacf(data, ax=ax[2]);
plot_ts_pacf_acf(data['log_rtn'])
```



We see clear volatility clustering in the time plot (e.g., 1973–1976 vs. 1980–1984). The ACF/PACF of returns show no strong ARMA signature. We proceed under a zero-mean specification and will test stationarity formally.

1.2 Stationarity Checks

We assess stationarity using **Phillips–Perron (PP)** and **KPSS** in tandem.

Why both?
The PP test is chosen over ADF because we suspect **heteroskedasticity**—visible as volatility clustering in the time plot—and PP is robust to such effects.
The KPSS test, also robust to heteroskedasticity, complements PP by reversing the null hypothesis:

- **PP test**: H_0 = unit root (nonstationary); reject H_0 → evidence of stationarity.
- **KPSS test**: H_0 = (trend-)stationary; fail to reject H_0 → evidence consistent with stationarity.

Using both, with complementary nulls, makes the conclusion more robust: agreement between PP and KPSS provides stronger evidence of stationarity. In our data, PP yields $p \approx 0.00$ and KPSS yields $p \approx 0.10$, consistent with stationarity of monthly log returns at the 5% level.

[294]:
```
import warnings
from arch.unitroot import PhillipsPerron
from statsmodels.tsa.stattools import kpss
from statsmodels.tools.sm_exceptions import InterpolationWarning

print(f"The p value for the PP test is: {PhillipsPerron(data['log_rtn'],␣
↪trend='n').pvalue}")
with warnings.catch_warnings():
    warnings.simplefilter("ignore", InterpolationWarning)
```

```
    print(f"The p value for the KPSS test is: {kpss(data['log_rtn'])[1]}")
```

```
The p value for the PP test is: 0.0
The p value for the KPSS test is: 0.1
```

Note. The KPSS statistic lies outside the tabulated range used by statsmodels, so the p-value is reported as a bound (e.g., $p < 0.01$ or $p > 0.10$). We rely on the statistic and the reported critical values to make the decision; the conclusion is unaffected by using a bound. Since the null for the Phillips-Perron test is the existence of a unit root and we got a very significant p value (and the opposite for KPSS), we can comfortably believe that our series is stationary at the 5% significance level.

1.2.1 Seasonal Differencing Attempt

Although the PP and KPSS tests suggest that the log return series is stationary, the time plot hinted at possible **seasonal structure**.
To check this, we difference the series at lag 7 (approximately half a year for monthly data) and re-examine the autocorrelation structure.

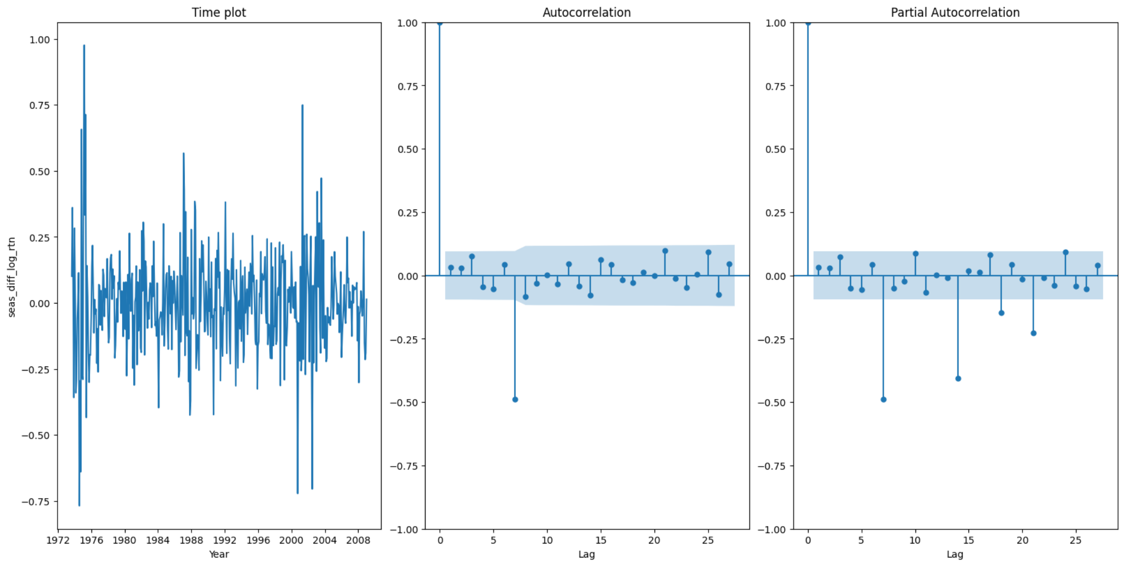
[295]:
```
data['seas_diff_log_rtn'] = data['log_rtn'].diff(7)

# After seasonal difference (lag 7).
data.head(15)
```

[295]:

date	rtn	log_rtn	seas_diff_log_rtn
1973-01-31	0.010050	0.010000	NaN
1973-02-28	-0.139303	-0.150013	NaN
1973-03-30	0.069364	0.067064	NaN
1973-04-30	0.086486	0.082949	NaN
1973-05-31	-0.104478	-0.110348	NaN
1973-06-29	0.133333	0.125163	NaN
1973-07-31	0.625000	0.485508	NaN
1973-08-31	0.117647	0.111226	0.101226
1973-09-28	0.234818	0.210924	0.360936
1973-10-31	0.144262	0.134760	0.067696
1973-11-30	-0.240688	-0.275343	-0.358291
1973-12-31	0.188679	0.172843	0.283191
1974-01-31	0.139683	0.130750	0.005587
1974-02-28	0.155989	0.144956	-0.340552
1974-03-29	-0.163855	-0.178953	-0.290179

Now we plot our ACF and PACF:

[296]:
```
plot_ts_pacf_acf(data['seas_diff_log_rtn'][7:])
```



As you can see, differencing did not help. There are massive negative lag spikes that imply over-differencing. We therefore turn to a heteroskedasticity-robust serial correlation test (instead of the usual Ljung-Box) developed by Escanciano and Lobato in their 2009 paper:

[297]:
```
from rust_timeseries.statistical_tests import EscancianoLobato
'%.6f'%EscancianoLobato(data['log_rtn']).pvalue
```

[297]: '0.770087'

As we can see, our p-value is not at all significant at the 5% level which means we can deduce no serial correlation.

1.3 Volatility Modeling

With no evidence of serial correlation in the mean of log returns, we now turn to modeling **time-varying volatility**.
The squared return plot and the ACF/PACF of squared residuals indicate strong **ARCH/GARCH effects**, which is typical in financial return series.

Our approach is:

1. **Demean the series** to focus on volatility dynamics rather than mean structure.

2. **Inspect squared residuals** to confirm the presence of volatility clustering.

3. **Apply the Escanciano–Lobato test** on squared residuals to formally check for dependence in second moments.

4. **Fit a GARCH(1,1) model** with standard Gaussian innovations, the standard benchmark for volatility modeling.

5. **Diagnose standardized residuals** to evaluate model adequacy.

6. **Generate volatility forecasts** (1- and 5-step ahead) to complete the exercise.

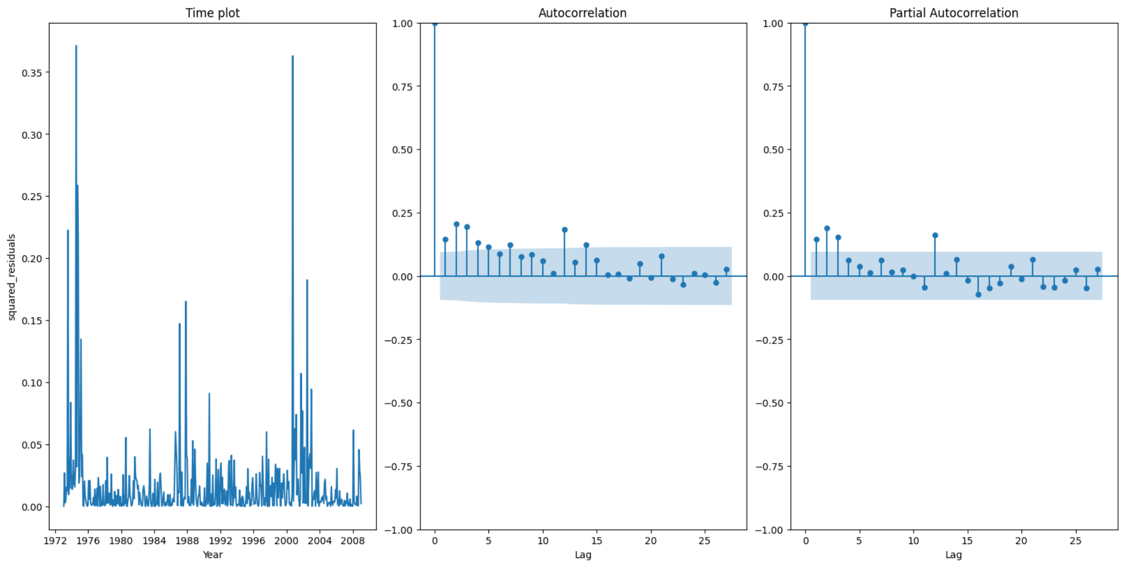
This workflow lets us verify that the GARCH(1,1) specification captures the volatility dynamics of Intel's monthly log returns. We start by calculating the residuals:

[298]:
```
data['residuals'] = data['log_rtn'] - data['log_rtn'].mean()
data['squared_residuals'] = data['residuals']**2

# Residuals and squared residuals (for ARCH checks).
data.head()
```

[298]:
```
                 rtn   log_rtn  seas_diff_log_rtn  residuals  \
date
1973-01-31  0.010050  0.010000                NaN  -0.003882
1973-02-28 -0.139303 -0.150013                NaN  -0.163895
1973-03-30  0.069364  0.067064                NaN   0.053182
1973-04-30  0.086486  0.082949                NaN   0.069067
1973-05-31 -0.104478 -0.110348                NaN  -0.124230

            squared_residuals
date
1973-01-31           0.000015
1973-02-28           0.026861
1973-03-30           0.002828
1973-04-30           0.004770
1973-05-31           0.015433
```

Now we can plot our ACF/PACFs:

[299]:
```
plot_ts_pacf_acf(data['squared_residuals'])
```



Seems like we have some ARCH and GARCH effects in the first 4 lags. Let's start by running the EL test on our squared series:

[300]:
```
el = EscancianoLobato(data['squared_residuals'])
print (f"The p-value is: {el.pvalue:.6f}")
print (f"The number of lags tested is: {el.p_tilde}")
```

```
The p-value is: 0.092370
The number of lags tested is: 1
```

Seems like we have 1 lag significance at the 10% level, let's try to fit a GARCH(1,1) model with standard Gaussian innovations:

[301]:
```
from arch.univariate import arch_model
from arch.univariate.base import ARCHModel, ARCHModelResult

# Rescaled the column to improve numerical stability.
data['scaled_residuals'] = data['residuals']*10
garch: ARCHModel = arch_model(data['scaled_residuals'], mean='Zero',␣
 ↪vol='GARCH', p=1, q=1, dist='normal')
fit_garch: ARCHModelResult = garch.fit(disp='off')
fit_garch.summary()
```

[301]:

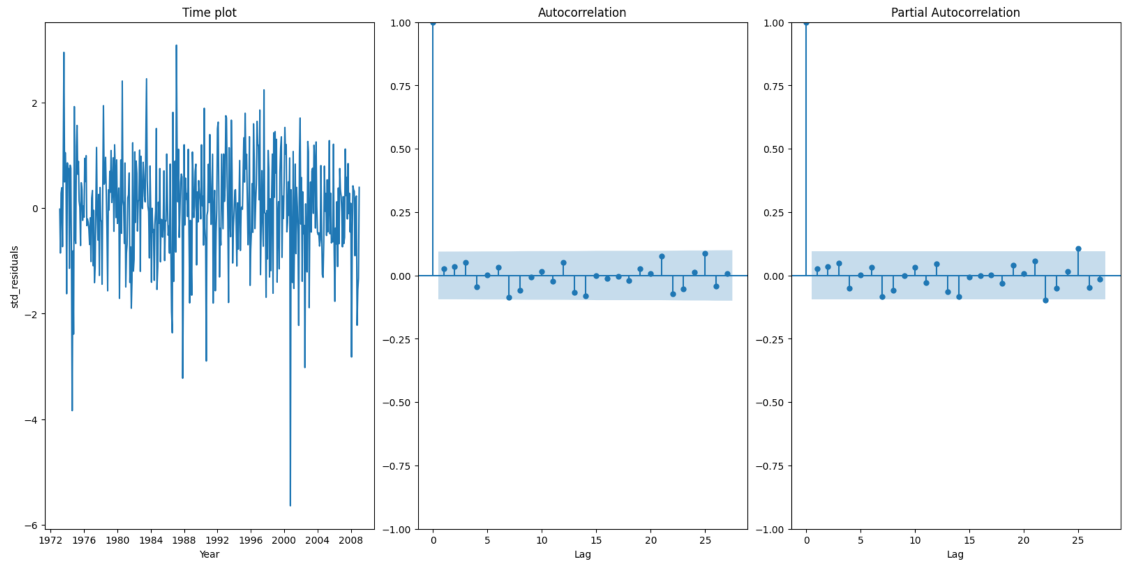
Dep. Variable:	scaled_residuals	**R-squared:**	0.000
Mean Model:	Zero Mean	**Adj. R-squared:**	0.002
Vol Model:	GARCH	**Log-Likelihood:**	-692.428
Distribution:	Normal	**AIC:**	1390.86
Method:	Maximum Likelihood	**BIC:**	1403.06
		No. Observations:	432
Date:	Sat, Aug 16 2025	**Df Residuals:**	432
Time:	15:15:12	**Df Model:**	0

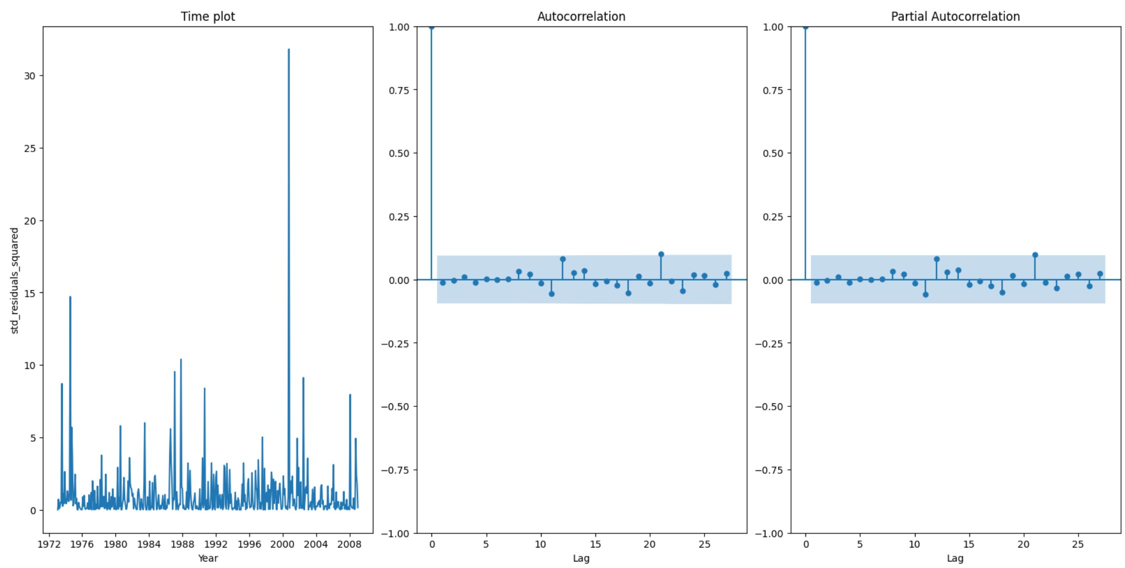
	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
omega	0.0777	3.223e-02	2.410	1.596e-02	[1.450e-02, 0.141]
alpha[1]	0.0686	2.311e-02	2.970	2.973e-03	[2.335e-02, 0.114]
beta[1]	0.8768	3.146e-02	27.867	6.713e-171	[0.815, 0.938]

Covariance estimator: robust

We can see that all parameter values are statistically significant at 5%. Now we check the quality of fit using the standardized residuals. First we look at the ACF/PACFs:

[302]:
```
data['std_residuals'] = data['scaled_residuals']/fit_garch.
 ↪conditional_volatility
data['std_residuals_squared'] = data['std_residuals']**2
plot_ts_pacf_acf(data['std_residuals'])
plot_ts_pacf_acf(data['std_residuals_squared'])
```





Visual diagnostics show no evidence of serial correlation. Now we test:

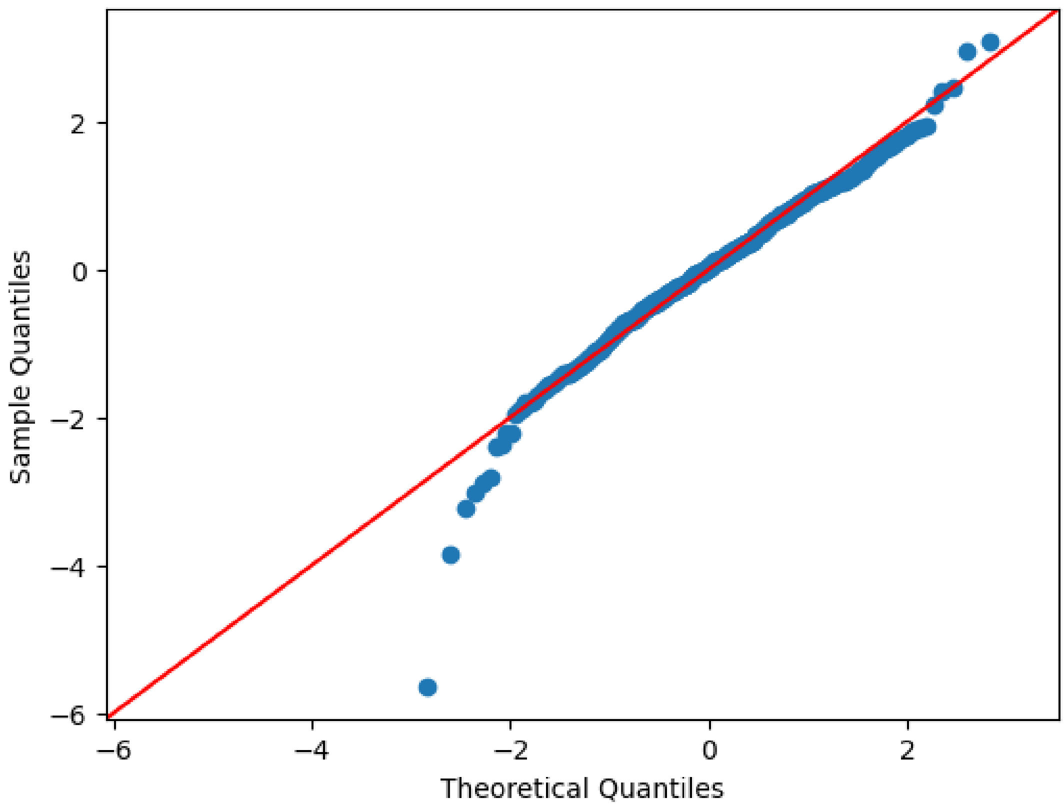
[303]:
```
el_std = EscancianoLobato(data['std_residuals'])
print (f"The p-value for the standardized residuals is: {el_std.pvalue:.6f}")
print (f"The number of lags tested for the standardized residuals is: {el_std.p_tilde}")
el_std_squared = EscancianoLobato(data['std_residuals_squared'])
print (f"The p-value for the squared standardized residuals is: {el_std_squared.pvalue:.6f}")
```

```
print (f"The number of lags tested for the squared standardized residuals is:␣
  ↪{el_std_squared.p_tilde}")
```

```
The p-value for the standardized residuals is: 0.551061
The number of lags tested for the standardized residuals is: 1
The p-value for the squared standardized residuals is: 0.610649
The number of lags tested for the squared standardized residuals is: 1
```

Both p-values are insignificant which support our visual reasoning. Now let us look at the QQ plot of the standardized residuals to check if tail behavior is accurate:

[304]:
```
import statsmodels.api as sm
qq_plt = sm.qqplot(data['std_residuals'], line='45')
```



The QQ plot suggests that our pick of the normal distribution for the innovations fits our data fairly well. There is some indication of slightly heavier tails by the S-shaped taper at the edges but it is fairly small so we will keep our model as is. Finally we forecast 1 and 5 step ahead forecasts from the origin of December 2008:

[305]:
```
from arch.univariate.base import ARCHModelForecast
from datetime import datetime
five_step_forecast: ARCHModelForecast = fit_garch.forecast(horizon=5)
```

```
h_1 = five_step_forecast.variance['h.1'].iloc[-1]/100.0
h_5 = five_step_forecast.variance['h.5'].iloc[-1]/100.0
print(f"The one step ahead forecast is: {h_1:.6f}")
print(f"The five step ahead forecast is: {h_5:.6f}")
```

```
The one step ahead forecast is: 0.013245
The five step ahead forecast is: 0.013445
```

1.4 Conclusion & Reproducibility

1.4.1 Key Takeaways

- **Stationarity**: The Phillips–Perron test rejected the unit root null, while the KPSS test did not reject stationarity. Together, these provide consistent evidence that Intel's monthly log returns (1973–2008) are stationary.

- **Serial correlation**: The Escanciano–Lobato (EL) test found no significant serial correlation in raw returns, supporting a zero-mean specification.

- **Volatility structure**: EL tests on squared returns indicated ARCH/GARCH effects. A GARCH(1,1) with Gaussian innovations was fit, with all parameters statistically significant.

- **Diagnostics**: Standardized residuals showed no remaining serial correlation or volatility clustering; QQ plots suggested mild heavy tails but overall adequacy under Gaussian innovations.

- **Forecasts**: From December 2008, one- and five-step-ahead volatility forecasts were produced, illustrating practical application of the fitted model.

Overall, this workflow demonstrates how the **Escanciano–Lobato test** integrates naturally into volatility modeling: confirming mean adequacy, guiding GARCH specification, and validating residuals.

1.4.2 Reproducibility

To reproduce results, ensure Python 3.10 with the following dependencies:

[306]:
```
import sys
import platform
import importlib

pkgs = [
    "pandas", "numpy", "matplotlib", "statsmodels", "arch"
]

print("Python:", sys.version)
print("Platform:", platform.platform())

for pkg in pkgs:
    try:
```

```python
        mod = importlib.import_module(pkg)
        print(f"{pkg}:", mod.__version__)
    except Exception:
        print(f"{pkg}: not installed")
```

```
Python: 3.13.5 (main, Jun 11 2025, 15:36:57) [Clang 17.0.0 (clang-1700.0.13.3)]
Platform: macOS-15.6-arm64-arm-64bit-Mach-O
pandas: 2.3.0
numpy: 2.3.2
matplotlib: 3.10.3
statsmodels: 0.14.5
arch: 7.2.0
```