Comparative Analysis of surrogates of chaos based on Permutation Entropy
順列エントロピーに基づくカオスのサロゲートデータに対する比較分析

小林未佳 (1021148)
指導教員　ヴラジミール・リアボフ

November 26 2024

Background-1

Distinguishing noise and chaos is important problem in the field of signal processing. If chaotic case, we can forecast its evolution exactly.

Figure: Noise

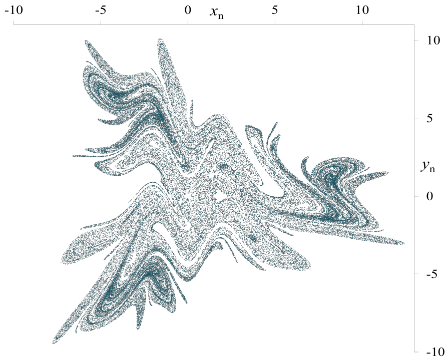


Figure: Chaos(Gumowski-Mira Map)

Reference：Wikipedia(グモウスキー・ミラー写像)

Background-2

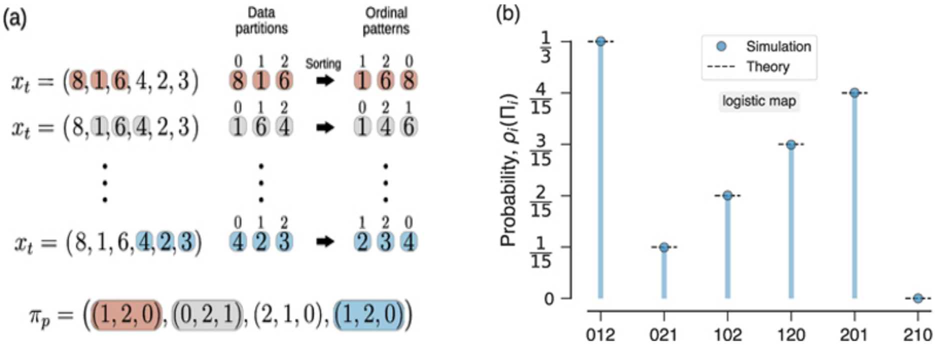
Lyapnov Exponent and Fractal dimension can handle with this problem,
but still difficult due to some reasons:

- data pollution
- the method to control parameters
- restricted data length

Permutation Entropy [1] and Complexity-Entropy Plane [3] was introduced for this problems and there is Ordpy python package [2] for effiecient calculation.

Prior Work - Ordinal Patterns

There are some prior work for this problem. Permutation Entropy had been proposed by Bandt and Pompes in 2002 [1].



Reference : [2]

Prior Work - Permutation Entropy

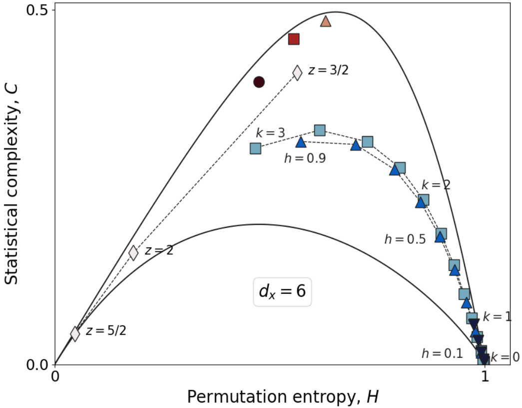
Count numbers of each patterns
and Calculate Shannon Entropy for probabilities.

Shannon entropy

$$S(P) = -\Sigma_{i=1}^{n_\pi} \rho(\pi_i) log \rho_i(\pi_i)$$

Prior Work - Complexity-Entropy Plane1

After Permutation Entropy, Complexity-Entropy Plane was introduced for distingusishing between chaos and stochastic time series by Rosso et al. in 2007 [3].



Reference : [2]

Prior Work - Complexity-Entropy2

Complexity-Entropy Plane is 2 dimensional plane for Statistical Complexity C vs Shannon Entropy H.

Statistical Complexity

$$C(P) = \frac{D(P, U)H(P)}{D^{max}}$$

where D(P, U) is Jensen-Shanon divergence.
Note U means uniform distribution.

Abstract

Background

Distinguishing noise from chaos is important, because chaos is deterministic.

Purpose

Further study for some chaotic maps using ordpy package [2].

Method

Analyze Complexity-Entropy Plane for some chaotic maps, such as

- Ikeda map
- Standard map

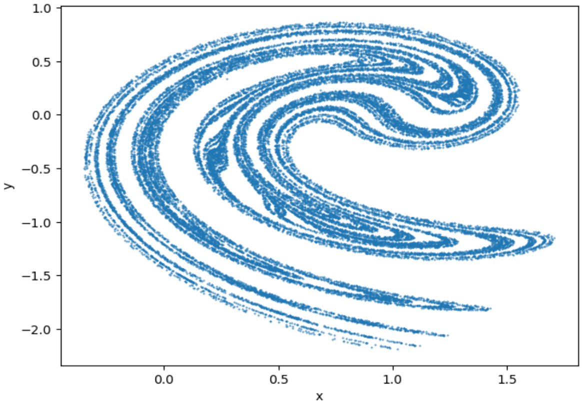
Discuss its propertis using surrogate method.

Ikeda Map

Ikeda map(left) is calculated as

$$\begin{aligned} x_{n+1} &= 1 - ux_n^2 + y_n \\ y_{n+1} &= ux_n \end{aligned}$$

Ikeda map(left) has spiral.

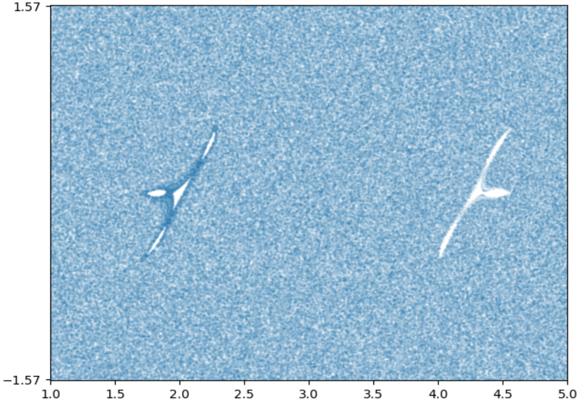


Standard Map

Stardard map is calculated as

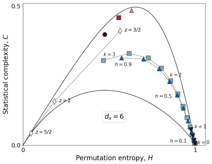
$$\begin{aligned} p_{n+1} &= p_n + \kappa sin x_n \\ x_{n+1} &= x_n + p_{n+1} \end{aligned}$$

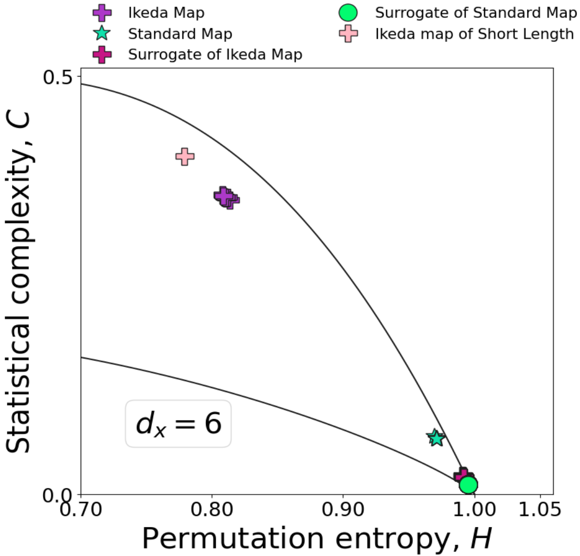
This map seems noise, but chaos.



Preliminary Calculation-1

The position of Ikeda map and standard map:





Preliminary Calculation-2

Result

Ikeda map with parameter u=0.9 and standard map are chaotic maps, but it's not clear

Next step

Using surrogate method, calculate how far timeseries generated from random and x-coordinate of chaotic maps are.

Surrogate

Algorithm to generate randomized time series from original data. [4]

In AAFT (Amplitude adjusted Fourier Transform) ,

- Fourier Transform without windowing
- Randomize phases
- Symmetrize phases and Calculate inverse Fourier Transform

Surrogate have the same amplitude distribution and Fourier Spectrum as the original (chaotic) data.

Results-1

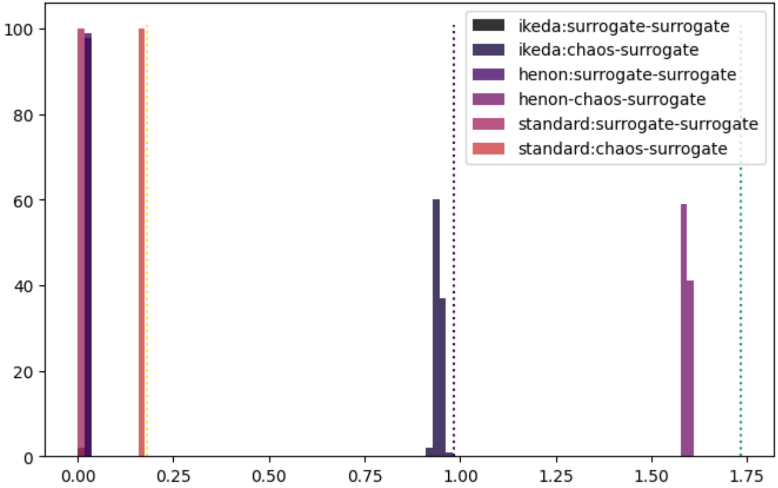


Figure: Chi-square statistic.The dot line means the value of chi-square statistic between uniform distribution and ordinal distribution of original data.

Result-2

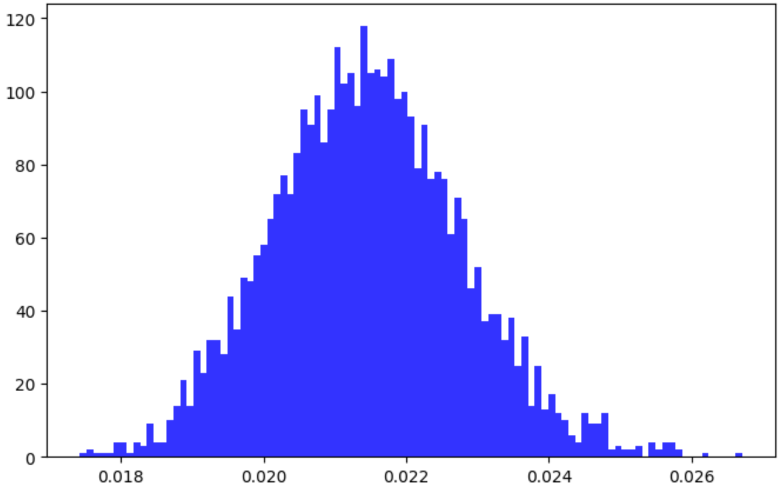


Figure: Histogram of chi-square statistic of surrogate data.

Results-3

Result

The differences between chi-square distances are quite greater than the prior expectation.

Consideration

The difference between chaos and noise got clearer in this study.

Next Step

Using surrogate method seems to be useful when distinguishing chaos from noise. Trying Ikeda map with another parameter will be interesting using this method to distinguish chaos, noise, and constant.

Questions?

References

C. Bandt and B. Pompe.
Permutation entropy: A natural complexity measure for time series.
Phys.Rev.Lett, 88(17):174102, 2002.

A. A. B. Pessa and H. V. Ribeiro.
ordpy: A python package for data analysis with permutation entropy and ordinal network methods.
Chaos, 31(6):063110, 2021.

O. A. Rosso, H. A. Larrondo, M. T. Martin, A. Plastino, and M. A. Fuentes.
Distinguishing noise from chaos.
Phys. Rev. Lett., 99:154102, Oct 2007.

James Theiler, Stephen Eubank, André Longtin, Bryan Galdrikian, and J. Doyne Farmer.
Testing for nonlinearity in time series: the method of surrogate data.
Physica D: Nonlinear Phenomena, 58(1):77–94, 1992.