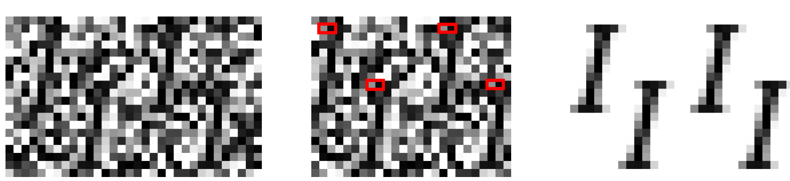
1 Introduction

Frequent pattern mining is the subfield of data mining that aims to find and extract recurring substructures as a means of data analytics. These patterns can be any kind of datastructure, for example item sets (transactions), graphs (networks) and sequences (in time). So far, little research has been done in applying pattern mining to raster-based, tabular data or matrices. Patterns in these types of data can be more complex as elements cannot only coincide, but their geometric relation is also important. As the first contribution of this paper, we introduce exactly this problem, that we will call **geometric pattern mining** and formally introduce notation and theoretical constructions for this problem. Potential applications include analysis of (satellite) imagery, texture recognition and clustering of matrices.

Geometric Pattern Mining Geometric pattern mining is the problem of finding recurring local structure or **patterns** in matrices. It is different from graph mining, as a matrix is more rigid and each element has a fixed degree of connectedness/adjacency. It is also unrelated to linear algebra, other then using the term 'matrix' and a comparable style of notation. Furthermore in this context, matrix elements can only be discrete, rows and columns have a fixed ordering and the semantics of a value is position-independent. Although the concept applies to any number of dimensions, we will limit the scope to two dimensional data from here on.

The problem of geometric pattern mining can roughly be divided into three classes. The first class consists of three subclasses: finding identical patterns in (1a) an otherwise empty (sparse) matrix, (1b) differently distributed noise and (1c) similarly distributed noise. The second class contains the same subclasses but adds that the patterns can also be overlaid with noise and are therefore not identical. The third class is also a continuation of the first class and requires that the patterns are identical after some optional transformation (such as mirror, inverse, rotate, etc.). These classes also represent an increasing difficulty level and serves as a rough benchmark for the performance of an algorithm.

Let us demonstrate the problem of geometric pattern mining with a brief example. Figure 1a shows a 32×24 grayscale 'matrix' filled with noise on the interval $[0; 255]$. If we look at all horizontal pairs of elements, we find that the



(a) 32×24 noise 'matrix' (b) Pair $(146, 11)$ (c) Pattern 'I' occurs four times.

Fig. 1: Brief example of geometric pattern mining

pair $(146, 11)$ is, among others, statistically more prevalent than 'random noise' suggests. If we would continue to try all combinations of elements that 'stand out' from the background noise, Figure 1c shows that we will eventually find four copies of the letter 'I' set in 16 point Garamond Italic.

The 35 elements that make up a single 'I' in the example form a **pattern**. We can use this pattern to describe the matrix in an other way than '768 unrelated values'. For example, we could describe it as 628 unrelated values plus pattern 'I' at locations $(5, 4), (11, 11), (20, 3), (25, 10)$, separating the structure from the accidental (noise) data. Since this requires less storage space than before, we have compressed the data. At the same time we have learned something about this data, namely that it contains four 'hidden I's.

Datamining by Compression In recent years, a class of algorithms utilizing the *Minimum Description Length (MDL) principle* [**?**,**?**] have become more and more common in the field of explanatory data analysis. Examples of such approaches include Krimp [**?**] or more recently Classy [**?**]. The MDL principle was first described by Rissanen in 1987 [**?**] as a practical implementation of Kolmogorov Complexity [**?**]. Central to MDL is the notion that 'learning' can be thought of as 'finding regularity' and that regularity itself is a property of data that is exploited by *compressing* said data. Therefore by compressing a dataset, we actually learn its structure — how regular it is, where this regularity occurs, what it looks like — at the same time.

The problem that MDL solves first and foremost, is that of *model selection*: given a multitude of explanations (models), select the one that fits the data best. In addition to this, MDL has also been demonstrated to be very effective in materialization of a specific model given the data. In this case, the model is predetermined and we want to find the parameters to fit the data. A similar problem class is solved in pattern mining: here the 'parameters' are the discrete building blocks that make up patterns in the data. We will specifically look at a variant of MDL called two-part MDL. As the second contribution of this paper, we present a geometric pattern mining algorithm that (1) is precise, (2) requires no parameters and (3) is tolerant to noise in the data, based on two-part MDL.