

# R Lab 1

Goals:

- Conceptual understanding of DAGs, SCMs, potential outcomes framework, MSMs
- Understand how R code relates to concepts
- ?other

Josh: Remember:

- Record this
- Use complete sentences on individual write-ups for Discussion Assignment 1
- Note that R HW 1 is very similar to this Lab
- Note that these slides will be posted to Google Drive after class
- Note that R Lab 1 Solutions will be posted to Google Drive as well and may help with R HW1
- Note that TEMPLATE exists for R HW 1 so that they won't forget any parts of any questions

Suppose we are interested in the causal effect of butterbeer consumption on happiness among wizards at Hogwarts. Specifically, we want to know if the average happiness would be higher if all wizards consumed butterbeer or if all wizards did not.

Let  $W1$  be a summary measure the student's pre-exposure covariates, including age, house, gender, friendship with Dumbledore, and enemy status with Snape.

Let  $W2$  be an additional baseline covariate, indicating whether the student had permission to travel to Hogsmeade, a location where butterbeer is sold.

We consider a binary exposure  $A$ , indicating consumption of butterbeer ( $A = 1$ ) or not ( $A = 0$ ).

Denote the outcome happiness with  $Y$ . Finally, suppose having a permission  $W2$  only affects the exposure  $A$ , but has no direct effect on the happiness  $Y$ .

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Endogenous Nodes:  $X = (W1, W2, A, Y)$

Exogenous (Unmeasured) Nodes:  $U = (U_{W1}, U_{W2}, U_A, U_Y) \sim \mathbb{P}_U$

Structural Equations  $F$ :

$$W1 = f_{W1}(U_{W1})$$

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**DRAW  
A  
DAG!**

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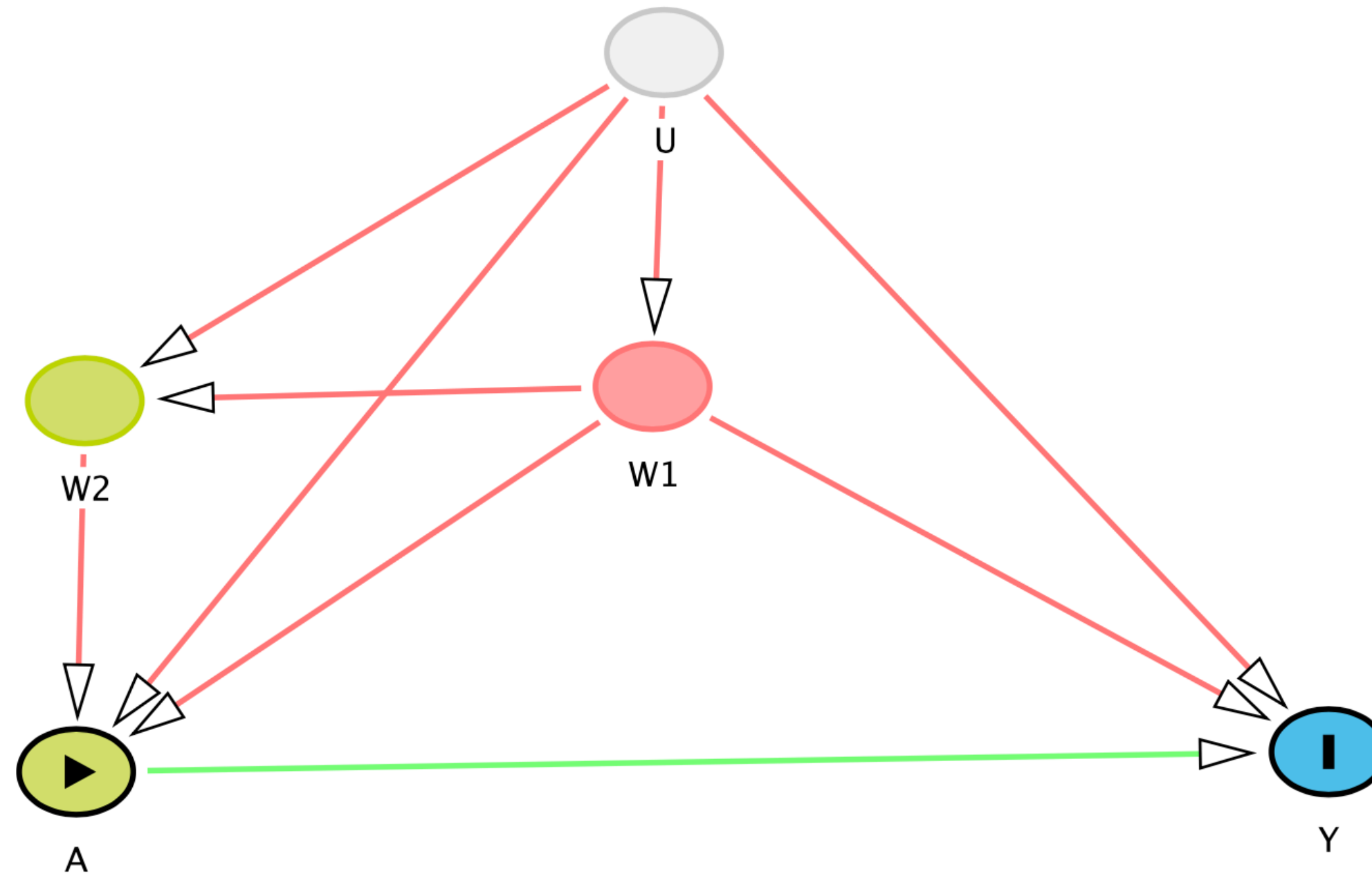
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Solution Fig. 1: Directed acyclic graph for the butterbeer consumption and happiness study. Here,  $U$  represents the background/unmeasured factors for all the endogenous nodes. Alternatively, we could have specified separate nodes  $(U_{W1}, U_{W2}, U_A, U_Y)$  and drawn double-headed arrows between them. We do not have any independence assumptions.



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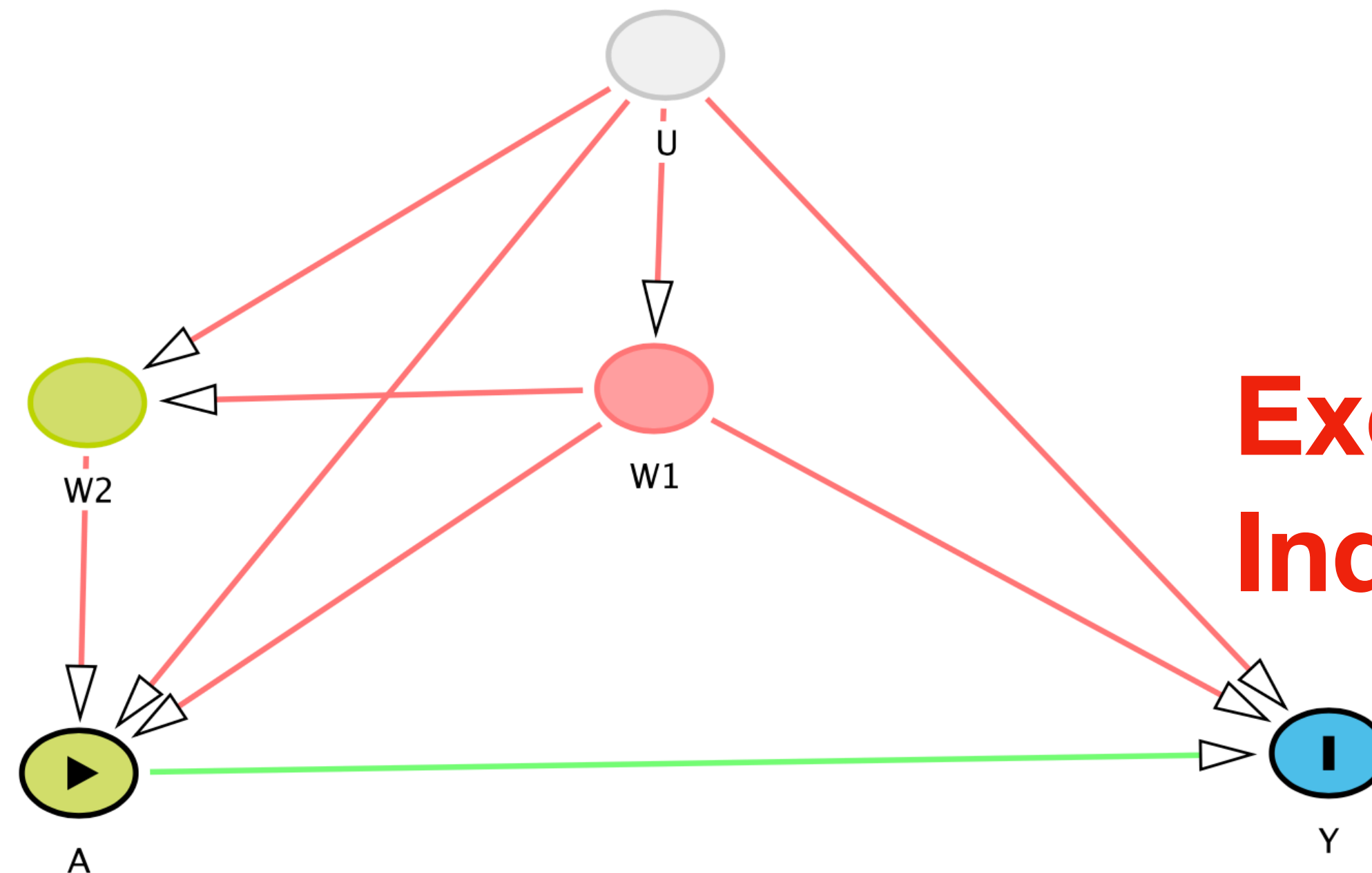
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**Exclusion restrictions?**  
**Independence assumptions?**

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1. Draw the corresponding directed acyclic graph (DAG).
2. Are there any exclusion restrictions? Are there any independence assumptions?
3. Define the counterfactual outcomes of interest with formal notation and in words.
4. In the structural causal model (SCM) framework, how are counterfactuals derived? What does the SCM tell us about all possible distributions for these counterfactuals?

**Solution:**

1. This study can be translated into the directed acyclic graph, given in Fig. 1.
2. We are making an exclusion restriction; the baseline covariate  $W_2$  does not directly affect the outcome  $Y$ . We have not made any independence assumptions. In other words, there are no restrictions on the joint distribution of the unmeasured factors  $\mathbb{P}_U$ .
3. The counterfactuals of interest are  $(Y_a : a \in \mathcal{A} = \{0, 1\})$ . Here,  $Y_1$  is the counterfactual happiness if, possibly contrary to fact, a wizard drank butterbeer, and  $Y_0$  is the counterfactual happiness if, possibly contrary to fact, a wizard did not drink butterbeer.
4. Counterfactuals are derived by intervening on SCM to set  $A = a$ . The distribution of counterfactuals is implied by the distribution of background factors  $\mathbb{P}_U$  and structural equations  $F$ . The SCM  $\mathcal{M}^*$  defines for the set of allowed counterfactual distributions. (This is why it is called a “model”.)



Suppose our target causal parameter is the average treatment effect:

$$\begin{aligned}\Psi^*(\mathbb{P}^*) &= \mathbb{E}^*(Y_1) - \mathbb{E}^*(Y_0) \\ &= \mathbb{E}^*[f_Y(W1, 1, U_Y)] - \mathbb{E}^*[f_Y(W1, 0, U_Y)]\end{aligned}$$

**Where is W2? A?**

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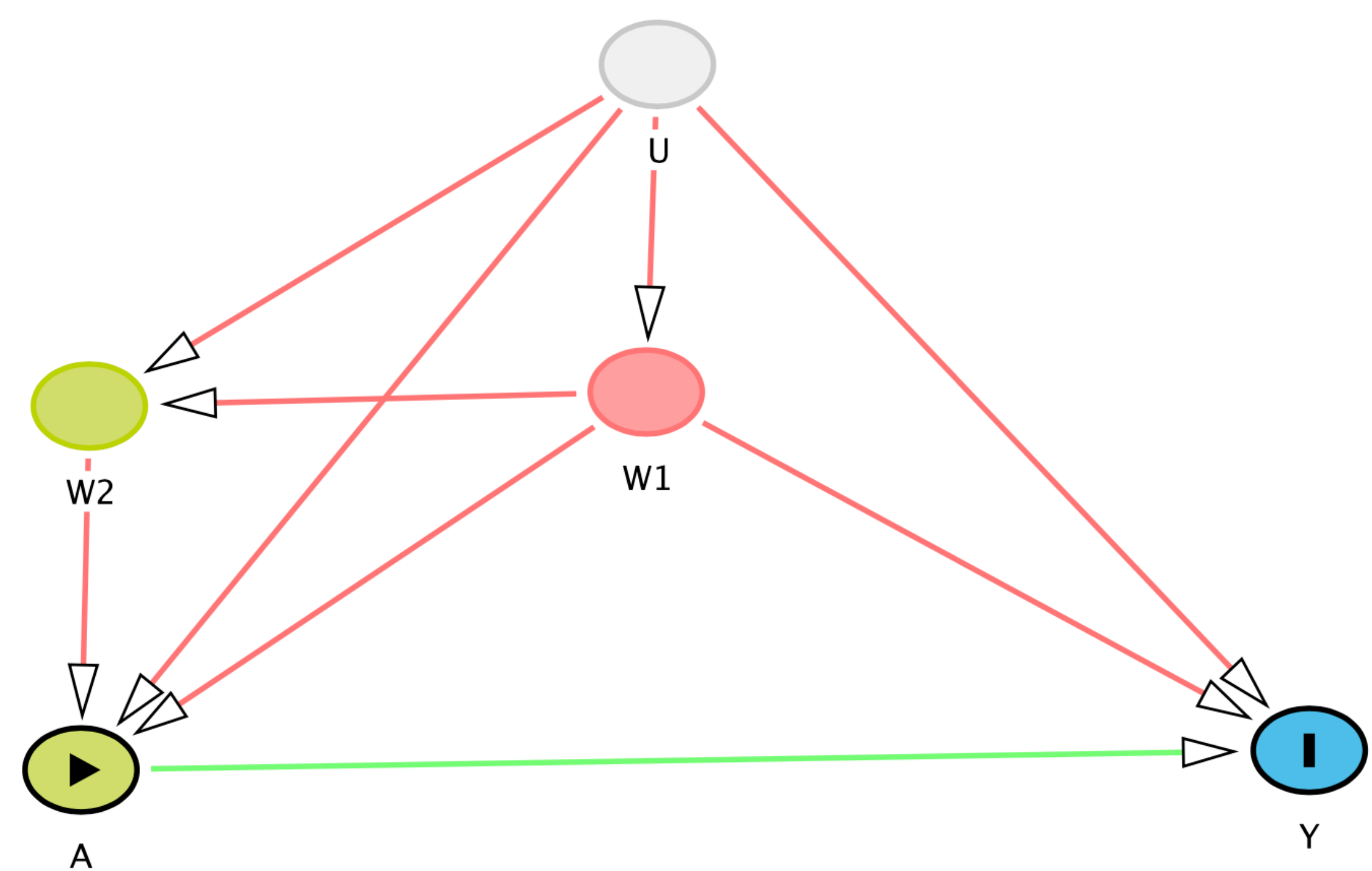
This is the difference in the expected counterfactual happiness if all wizards were to drink butterbeer and the expected counterfactual happiness if all wizards were not to drink butterbeer. In the second line, we have replaced the counterfactual outcome  $Y_a$  with the corresponding structural equation  $f_Y$

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Structural Equations  $F$ :

$$\begin{aligned}W1 &= f_{W1}(U_{W1}) \\ W2 &= f_{W2}(W1, U_{W2}) \\ A &= f_A(W1, W2, U_A) \\ Y &= f_Y(W1, A, U_Y)\end{aligned}$$

*In the previous section*, we specified a SCM  $\mathcal{M}^*$ , reflecting our limited knowledge of the data generating system. We did not place any assumptions on the joint distribution of the exogenous background factors ( $\mathbb{P}_U$ ). We made only an exclusion restriction of the impact of  $W2$  on the outcome  $Y$ . We did not make any assumptions about the functional form of the structural equations.

*Now*, we consider a particular data generating process  $\mathbb{P}^*$ , **one** of many compatible with the SCM  $\mathcal{M}^*$ .

- Each of the exogenous factors  $U$  is drawn independently from the following distributions:

$$U_{W1} \sim Uniform(min = 0, max = 1)$$

$$U_{W2} \sim Bernoulli(p = 0.5)$$

$$U_A \sim Normal(\mu = -3, \sigma^2 = 1^2)$$

$$U_Y \sim Normal(\mu = 0, \sigma^2 = 0.3^2)$$

- Let us also specify the structural equations  $F$ :

$$W1 = f_{W1}(U_{W1}) = \mathbb{I}[U_{W1} < 0.35]$$

$$W2 = f_{W2}(W1, U_{W2}) = W1 + 2 \times U_{W2}$$

$$A = f_A(W1, W2, U_A) = \mathbb{I}[(1 + W1 + 2 \times W2 + U_A) > 0]$$

$$Y = f_Y(W1, A, U_Y) = 1 + 2.5 \times A + 3 \times W1 - 0.25 \times A \times W1 + U_Y$$

where  $\mathbb{I}[\cdot]$  is the indicator function and equal to 1 if the statement in the brackets is true.



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- Each of the exogenous factors  $U$  is drawn independently from the following distributions:

$$U_{W1} \sim Uniform(min = 0, max = 1) \quad \dots \text{flat distribution between 0 and 1}$$

$$U_{W2} \sim Bernoulli(p = 0.5) \quad \dots \text{0 or 1 with probability .5}$$

$$U_A \sim Normal(\mu = -3, \sigma^2 = 1^2)$$

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- Let us also specify the structural equations  $F$ :

$$\text{W1 = general covariate} \quad W1 = f_{W1}(U_{W1}) = \mathbb{I}[U_{W1} < 0.35] \quad \dots \text{0 or 1 with probability .35}$$

$$\text{W2 = can they go to Hogsmeade?} \quad W2 = f_{W2}(W1, U_{W2}) = W1 + 2 \times U_{W2} \quad \dots \text{W2 is a function of W1 and unobserved variable } U_{W2}$$

$$\text{A = Did the have butterbeer?} \quad A = f_A(W1, W2, U_A) = \mathbb{I}[(1 + W1 + 2 \times W2 + U_A) > 0] \quad \dots \text{0 or 1 based on W1, W2, } U_A$$

$$\text{Y = happiness} \quad Y = f_Y(W1, A, U_Y) = 1 + 2.5 \times A + 3 \times W1 - 0.25 \times A \times W1 + U_Y$$

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1. In closed form, evaluate the average treatment effect  $\Psi^*(\mathbb{P}^*)$  for this data generating process.

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**Solution:**

1. In this particular data generating system (one of many compatible with the SCM), the expectation of the counterfactual outcome is a linear function of the treatment level  $a$ , covariate  $W1$  and random error  $U_Y$ :

$$\begin{aligned}\mathbb{E}^*[Y_a] &= \mathbb{E}^*[1 + 2.5 \times a + 3 \times W1 - 0.25 \times a \times W1 + U_Y] \\ &= 1 + 2.5a + 3\mathbb{E}^*[W1] - 0.25 \times a \times \mathbb{E}^*[W1] + \mathbb{E}^*[U_Y]\end{aligned}$$

We also know that  $W1$  is a Bernoulli random variable with probability 0.35:

$$\mathbb{P}^*(W1 = 1) = \mathbb{E}^*[W1] = \mathbb{E}^*(\mathbb{I}[U_{W1} < 0.35]) = 0.35$$

Finally, the mean value of  $U_Y$  is zero:

$$\mathbb{E}^*[U_Y] = 0$$

Therefore, the true value of the target causal parameter is

$$\begin{aligned}\Psi^*(\mathbb{P}^*) &= \mathbb{E}^*(Y_1 - Y_0) \\ &= 1 + 2.5 \times 1 + 3 \times 0.35 - 0.25 \times 1 \times 0.35 + 0 - (1 + 2.5 \times 0 + 3 \times 0.35 - 0.25 \times 0 \times 0.35 + 0) = 2.4125\end{aligned}$$

2. The counterfactual expected happiness would be 2.4125 units higher if all wizards consumed butterbeer than if none of the wizards consumed butterbeer.

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2. The counterfactual expected happiness would be 2.4125 units higher if all wizards consumed butterbeer than if none of the wizards consumed butterbeer.

**In section 2.1 of Lab 1 you will see how to estimate this true value (2.4125) using simulations.**

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$$= \mathbb{E}^*[f_Y(W1, 1, U_Y)] - \mathbb{E}^*[f_Y(W1, 0, U_Y)]$$

**Slight context change.**

**New situation: Ignore covariates W1 and W2.**

**Just interested in effect of butterbeer consumption on happiness Y.**

**Exposure variable A is now continuous: represents mugs of butterbeer consumed.**

**Brief overview here, see R Lab 1 solutions for more in-depth discussion of this example.**



We could use a marginal structural model (MSM) to summarize how the expected counterfactual outcome changes as function of the exposure. Suppose, however, that we do not know the exact shape of the butterbeer-happiness (dose-response) curve. Therefore, we use the following *working* MSM to define the target parameter  $\beta^*$  as the projection of the true causal curve  $\mathbb{E}^*(Y_a)$  onto a summary model  $m(a|\beta)$ :

$$\beta^* = \underset{\beta}{\operatorname{argmin}} \mathbb{E}^* \left[ \sum_{a \in \mathcal{A}} (Y_a - m(a|\beta))^2 \right] \quad \text{Linear regression targeting } \beta.$$

$$m(a|\beta) = \beta_0 + \beta_1 a$$

The causal parameter is then the value of the coefficients ( $\beta^*$ ) that minimize the sum of squared residuals between the counterfactual outcomes  $Y_a$  and the predicted  $m(a|\beta)$  for all possible exposure levels  $a \in \mathcal{A}$ . In this case, we are using a linear summary.



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# But...

Consider the following data generating process for  $\mathbb{P}^*$ :

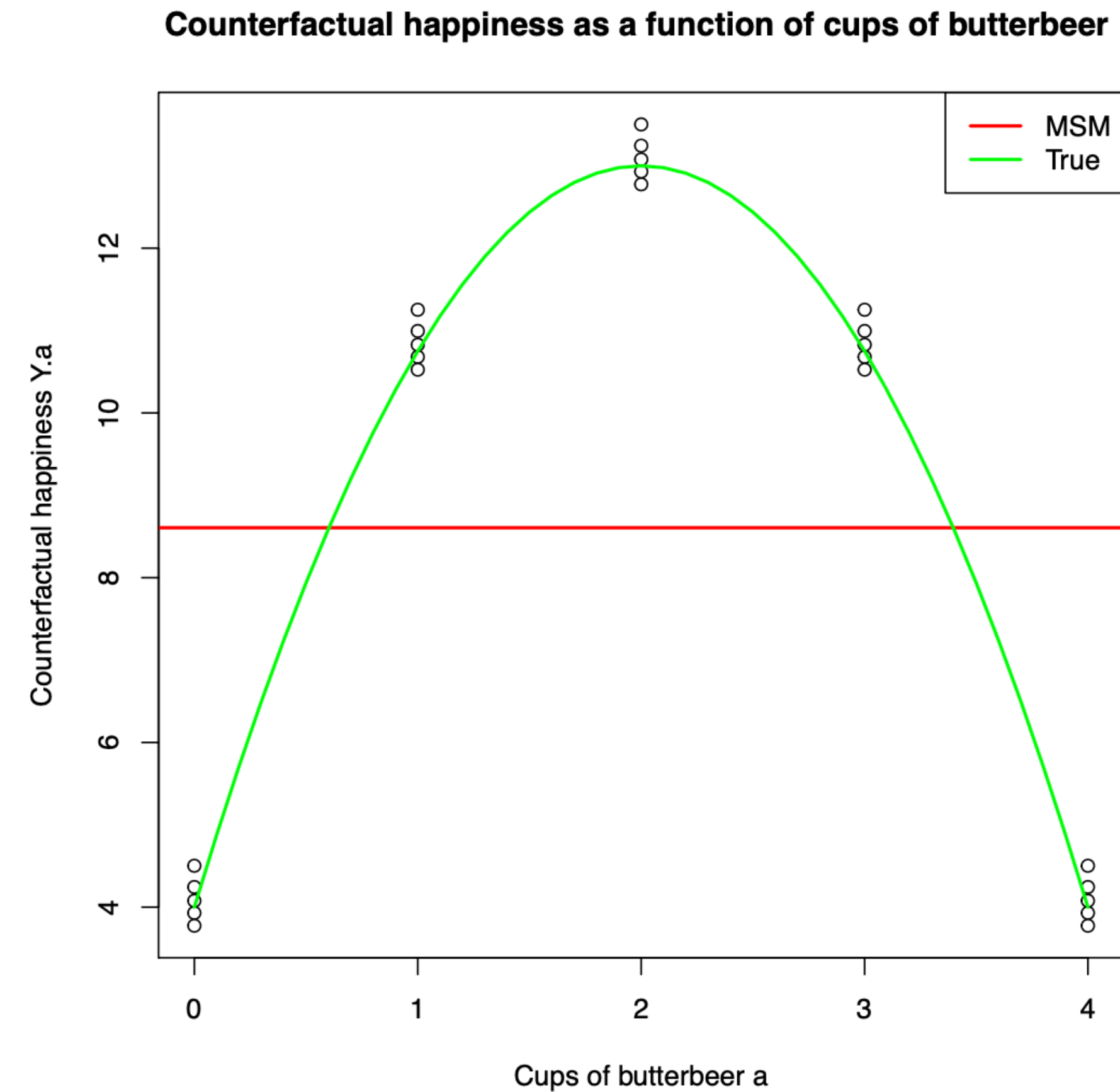
$$U_A \sim \text{Uniform}(\min = 0, \max = 2)$$

$$U_Y \sim \text{Normal}(\mu = 0, \sigma^2 = 0.3^2)$$

$$A = 2U_A$$

$$Y = 4 + 9A - 2.25A^2 + U_Y$$

**What do you notice?**



Solution Fig. 3: Plot the counterfactual outcomes  $Y_a$  as a function of  $a$ . The true causal curve is shown in green, while its projection onto the working MSM is shown in red.

**One more note that will help  
you on R HW 1...**

Now, consider a particular data generating process, one of many compatible with  $\mathcal{M}^*$ . Suppose that the each of the background factors is drawn independently from following distributions:

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Given the background  $U$ , the endogenous variables are deterministically generated as

$$W1 = \mathbb{I}[U_{W1} < 0.2]$$

$$W2 = \mathbb{I}[U_{W2} < \text{logit}^{-1}(0.5 \times W1)]$$

$$A = \mathbb{I}[U_A < \text{logit}^{-1}(W1 \times W2)]$$

$$Y = 4 \times A + 0.7 \times W1 - 2 \times A \times W2 + U_Y$$

Recall the  $\text{logit}^{-1}$  is the inverse-logit:

$$\text{logit}^{-1}(x) = \frac{\exp(x)}{1 + \exp(x)}$$

and given by the `plogis` function in R.

Now, consider a particular data generating process, one of many compatible with  $\mathcal{M}^*$ . Suppose that the each of the background factors is drawn independently from following distributions:

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Given the background  $U$ , the endogenous variables are deterministically generated as

$$W1 = \mathbb{I}[U_{W1} < 0.2] \text{ This will generate 1s 20\% of the time and 0s 80\% of the time.}$$

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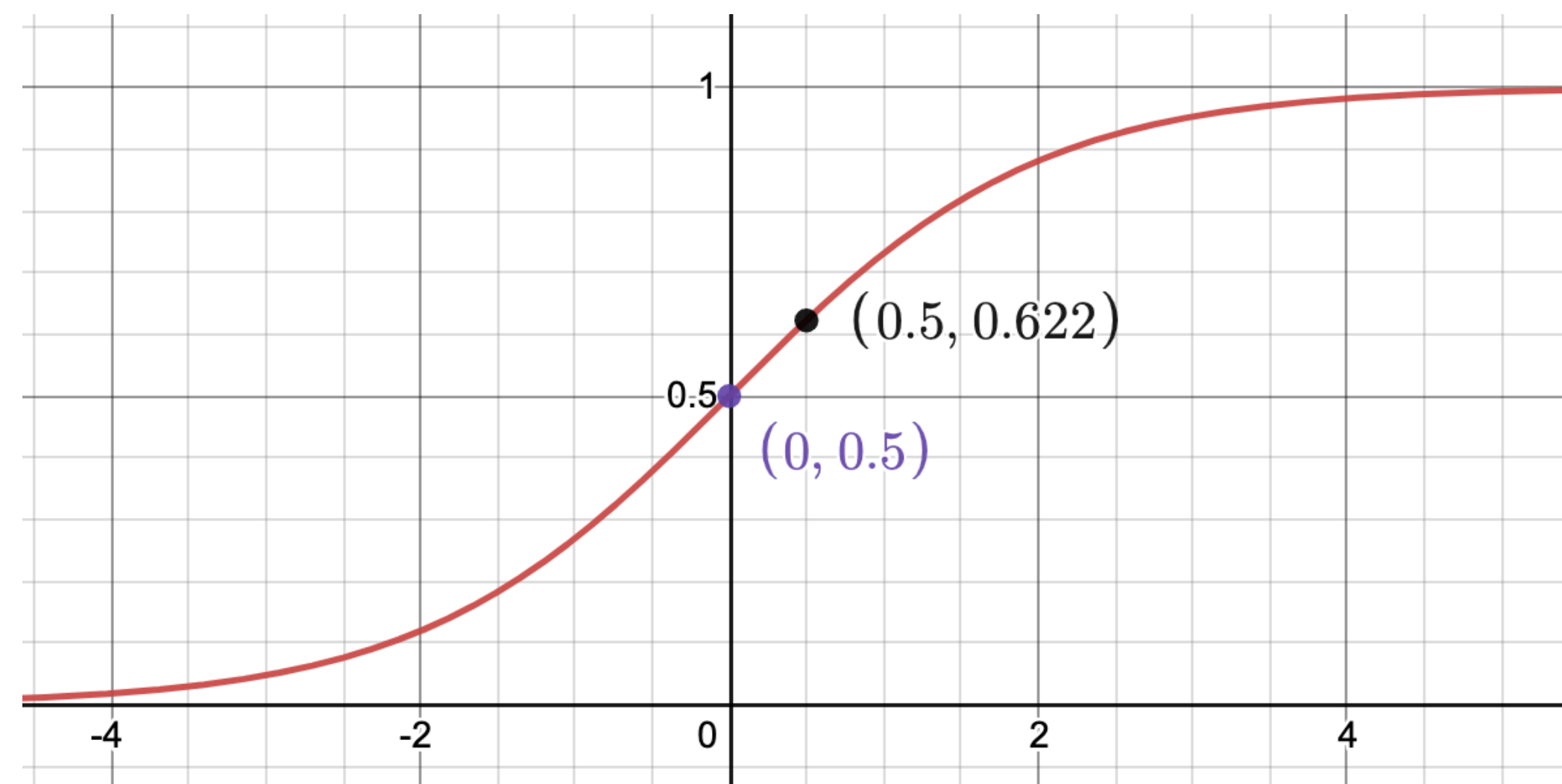
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$$W2 = \mathbb{I}[U_{W2} < \text{logit}^{-1}(0.5 \times W1)] \text{ This will generate 0s/1s depending on } W1 \text{ and } U_{W2}$$



**If  $W1 = 0$  (80% of the time), it'll make  $W2 = 1$  exactly 50% of the time.**

**If  $W1 = 1$  (20% of the time), it'll make  $W2 = 1$  exactly 62.2..% of the time.**

**From here you can work out the probability of  $W2 = 1$  or  $W2 = 0$ .**

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