

R Assignment 1: Causal Parameters & Simulations in R

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Biostat683 - Intro. to Causal Inference

Assigned: September 20, 2021

Write-ups due: Uploaded to your personal GoogleDrive folder by October 4, 2021 by 2:30pm. Please answer all questions and include relevant R code. You are encouraged to discuss the assignment in groups, but should not copy code or interpretations verbatim. Use of RMarkdown is strongly encouraged; please see the template on GoogleDrive.

1 Background Story

Suppose we are interested in the causal effect of ready-to-use therapeutic food (RUTF) on recovery from undernutrition in a resource-limited country. RUTF is peanut butter-type paste, fortified with milk proteins and essential nutrients, and does not require water for use (WHO, 2007). We propose a study to contrast the effect of RUTF with the standard supplement on weight gain over two months among school-aged children.

Suppose we only have two pre-exposure covariates. Specifically, $W1$ is an indicator equaling 1 if the child has access to potable water. Likewise, $W2$ is an indicator equaling 1 if the child suffered from an infectious disease within the two weeks prior to the study initiation. The intervention A is also an indicator equaling 1 if the child received RUTF and 0 if the child received the standard supplement. Finally, the outcome Y represents the child's weight gain in pounds at the study close.

The above study can be translated into the following structural causal model (SCM) \mathcal{M}^* :

- Endogenous nodes: $X = (W1, W2, A, Y)$
- Background variables: $U = (U_{W1}, U_{W2}, U_A, U_Y) \sim \mathbb{P}_U$
- Structural equations \mathcal{F} :

$$\begin{aligned}W1 &= f_{W1}(U_{W1}) \\W2 &= f_{W2}(W1, U_{W2}) \\A &= f_A(W1, W2, U_A) \\Y &= f_Y(W1, W2, A, U_Y)\end{aligned}$$

2 Steps 1-2 of the Roadmap

1. Step 1: Causal model representing real knowledge

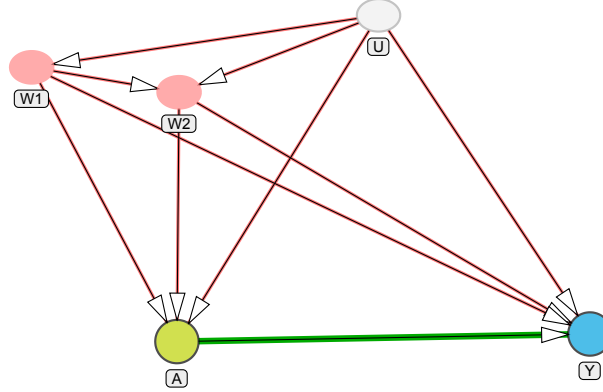
- (a) Draw the accompanying directed acyclic graph (DAG).
- (b) Are there any exclusion restrictions? Recall we are working with recursive (time-ordered) structural causal models.

(c) Are there any independence assumptions on the distribution of unmeasured factors \mathbb{P}_U ?

2. Step 2: Counterfactuals & causal parameter

- Define the counterfactual outcomes of interest with formal notation and in words.
- How are counterfactuals derived?
- Suppose we are interested in the average treatment effect. Specify the target causal parameter. Use formal notation as well as explain in words.

Solution:



Solution Fig. 1: Directed Acyclic Graph for the study of RUTF on weight gain. Recall $W1$ represents access to potable water, $W2$ represents recent history of an infectious disease, A is the exposure (nutritional supplementation) and Y is the outcome (weight gain in pounds).

1. Step1

- See Solution Fig. 1.
- There are no exclusion restrictions. All endogenous nodes are affected by the nodes preceding them temporally. We are assuming access to potable water $W1$ may affect presence of infectious disease within two weeks of study initiation $W2$.
- There are no independence assumptions explicitly specified in the background above. If you made any independence assumptions on \mathbb{P}_U , you should justify them.

2. Step 2

- The counterfactual outcomes of interest are $Y_a : a \in \mathcal{A} = \{0, 1\}$. Y_1 is a child's weight gain if, possibly contrary to fact, they received RUTF $A = 1$. Y_0 is a child's weight gain if, possibly contrary to fact, they received the standard supplement $A = 0$.
- Counterfactuals are derived by intervening on the SCM to set $A = a$. The distribution of counterfactuals, denoted \mathbb{P}^* , is implied by the distribution of background and endogenous factors, denoted \mathbb{P}_U , and set of structural equations, denoted \mathcal{F} . The SCM \mathcal{M}^* gives us a model for the set of possible counterfactual distributions.
- The target causal parameter is the difference in the expected counterfactual weight gain if all children were given RUTF and the expected counterfactual weight gain if all children were given the standard supplement:

$$\begin{aligned} \Psi^*(\mathbb{P}^*) &= \mathbb{E}^*(Y_1) - \mathbb{E}^*(Y_0) \\ &= \mathbb{E}^*[f_Y(W1, W2, 1, U_Y)] - \mathbb{E}^*[f_Y(W1, W2, 0, U_Y)] \end{aligned}$$

In the second line, we have replaced the counterfactual outcome Y_a with the corresponding (post-intervention) structural equation.

3 A specific data generating process

Now, consider a particular data generating process, one of many compatible with \mathcal{M}^* . Suppose that the each of the background factors is drawn independently from following distributions:

$$\begin{aligned} U_{W1} &\sim \text{Uniform}(0, 1) \\ U_{W2} &\sim \text{Uniform}(0, 1) \\ U_A &\sim \text{Uniform}(0, 1) \\ U_Y &\sim \text{Normal}(\mu = 0, \sigma^2 = 0.3^2) \end{aligned}$$

Given the background U , the endogenous variables are deterministically generated as

$$\begin{aligned} W1 &= \mathbb{I}[U_{W1} < 0.2] \\ W2 &= \mathbb{I}[U_{W2} < \text{logit}^{-1}(0.5 \times W1)] \\ A &= \mathbb{I}[U_A < \text{logit}^{-1}(W1 \times W2)] \\ Y &= 4 \times A + 0.7 \times W1 - 2 \times A \times W2 + U_Y \end{aligned}$$

Recall the logit^{-1} is the inverse-logit:

$$\text{logit}^{-1}(x) = \frac{\exp(x)}{1 + \exp(x)}$$

and given by the `plogis` function in R.

3.1 Closed form evaluation of the target causal parameter

Evaluate the target causal parameter $\Psi^*(\mathbb{P}^*)$ in closed form (i.e., by hand) for this data generating process.

Hints: In this particular data generating system (one of many compatible with the SCM), the expectation of the counterfactual outcome is a linear function of the treatment level a , the pre-exposure covariates $(W1, W2)$ and random error U_Y :

$$\mathbb{E}^*(Y_a) = \mathbb{E}^*[f_Y(W1, W2, a, U_Y)] = \mathbb{E}^*[4 \times a + 0.7 \times W1 - 2 \times a \times W2 + U_Y]$$

The marginal distribution of $W1$ (access to potable water) is Bernoulli with probability 0.20:

$$\mathbb{P}^*(W1 = 1) = \mathbb{E}^*(W1) = 0.20$$

The conditional expectation of $W2$ (presence or absence of an infectious disease), given $W1$, is given by

$$\mathbb{P}^*(W2 = 1 \mid W1) = \mathbb{E}^*(W2 \mid W1) = \text{logit}^{-1}(0.5 \times W1)$$

Solution: For this data generating process, the counterfactual mean outcome is

$$\begin{aligned} \mathbb{E}^*[Y_a] &= \mathbb{E}^*[4a + 0.7W1 - 2a \times W2 + U_Y] \\ &= 4a + 0.7\mathbb{E}^*[W1] - 2a \times \mathbb{E}^*[W2] + \mathbb{E}^*[U_Y] \end{aligned}$$

The marginal expectation of $W1$ is $\mathbb{E}^*[W1] = 0.20$. The marginal expectation of $W2$ is

$$\begin{aligned} \mathbb{E}^*[W2] &= \mathbb{E}^*(W2 \mid W1 = 1)\mathbb{P}^*(W1 = 1) + \mathbb{E}^*(W2 \mid W1 = 0)\mathbb{P}^*(W1 = 0) \\ &= \text{logit}^{-1}(0.5 \times 1) * 0.20 + \text{logit}^{-1}(0.5 \times 0) * (1 - 0.20) \end{aligned}$$

The expectation of the background factor U_Y is zero.

Plugging these values into the target causal parameter, we have

$$\begin{aligned}
 \Psi^*(\mathbb{P}^*) &= \mathbb{E}^*(Y_1) - \mathbb{E}^*(Y_0) \\
 &= 4 \times 1 + 0.7 \times \mathbb{E}^*[W1] - 2 \times 1 \times \mathbb{E}^*[W2] + \mathbb{E}^*[U_Y] \\
 &\quad - (4 \times 0 + 0.7 \times \mathbb{E}^*[W1] - 2 \times 0 \times \mathbb{E}^*[W2] + \mathbb{E}^*[U_Y]) \\
 &= 4 \times 1 + 0.7 \times 0.20 - 2 \times 1 \times [\text{logit}^{-1}(0.5 \times 1) * 0.20 + \text{logit}^{-1}(0.5 \times 0) * (1 - 0.20)] - (0.7 \times 0.20) \\
 &\approx 3.09 - 0.14
 \end{aligned}$$

Therefore, the average treatment effect is

$$\Psi^*(\mathbb{P}^*) = \mathbb{E}^*(Y_1) - \mathbb{E}^*(Y_0) \approx 3.09 - 0.14 = 2.95$$

```
> # corresponding R code for E*[Y_1]
> 4 + .7*.2 -2*(plogis(0.5)*.2 + plogis(0)*.8)
```

```
[1] 3.091016
```

Alternatively, we could have evaluated the target causal parameter as follows. The expected counterfactual outcome under the intervention to set $A = a$ is

$$\begin{aligned}
 \mathbb{E}^*(Y_a) &= \mathbb{E}^*[\mathbb{E}^*(Y_a|W1, W2)] \\
 &= \sum_{w1, w2} [\mathbb{E}^*(Y_a|W1 = w1, W2 = w2) \times \mathbb{P}^*(W1 = w1, W2 = w2)] \\
 &= \sum_{w1, w2} [\mathbb{E}^*(Y_a|W1 = w1, W2 = w2) \times \mathbb{P}^*(W2 = w2|W1 = w1) \times \mathbb{P}^*(W1 = w1)]
 \end{aligned}$$

We know conditional mean of the counterfactual Y_a given $W1$ and $W2$ is

$$\mathbb{E}^*(Y_a|W1, W2) = 4 \times a + 0.7 \times W1 - 2 \times a \times W2$$

We also know the conditional distribution of $W2$, given $W1$. Specifically, we know

$$\begin{aligned}
 \mathbb{P}^*(W2 = 1|W1 = w1) &= \text{logit}^{-1}(0.5w1) \\
 \mathbb{P}^*(W2 = 0|W1 = w1) &= 1 - \text{logit}^{-1}(0.5w1)
 \end{aligned}$$

Therefore, the expected value of the counterfactual outcome if all children received RUTF ($A = 1$) is

$$\begin{aligned}
 \mathbb{E}^*(Y_1) &= \mathbb{E}^*(Y_1|W1 = 1, W2 = 1)\mathbb{P}^*(W2 = 1|W1 = 1)\mathbb{P}^*(W1 = 1) \\
 &\quad + \mathbb{E}^*(Y_1|W1 = 0, W2 = 1)\mathbb{P}^*(W2 = 1|W1 = 0)\mathbb{P}^*(W1 = 0) \\
 &\quad + \mathbb{E}^*(Y_1|W1 = 1, W2 = 0)\mathbb{P}^*(W2 = 0|W1 = 1)\mathbb{P}^*(W1 = 1) \\
 &\quad + \mathbb{E}^*(Y_1|W1 = 0, W2 = 0)\mathbb{P}^*(W2 = 0|W1 = 0)\mathbb{P}^*(W1 = 0) \\
 &= (4 * 1 + 0.7 * 1 - 2 * 1 * 1)\text{logit}^{-1}(0.5 * 1)0.2 \\
 &\quad + (4 * 1 + 0.7 * 0 - 2 * 1 * 1)\text{logit}^{-1}(0.5 * 0)0.8 \\
 &\quad + (4 * 1 + 0.7 * 1 - 2 * 1 * 0)(1 - \text{logit}^{-1}(0.5 * 1))0.2 \\
 &\quad + (4 * 1 + 0.7 * 0 - 2 * 1 * 0)(1 - \text{logit}^{-1}(0.5 * 0))0.8 \\
 &\approx 3.09
 \end{aligned}$$

Likewise, the expected value of the counterfactual outcome if all children received that standard supplement ($A = 0$) is

$$\begin{aligned}\mathbb{E}^*(Y_0) &= (4 * 0 + 0.7 * 1 - 2 * 0 * 1) \text{logit}^{-1}(0.5 * 1) 0.2 \\ &\quad + (4 * 0 + 0.7 * 0 - 2 * 0 * 1) \text{logit}^{-1}(0.5 * 0) 0.8 \\ &\quad + (4 * 0 + 0.7 * 1 - 2 * 0 * 0) (1 - \text{logit}^{-1}(0.5 * 1)) 0.2 \\ &\quad + (4 * 0 + 0.7 * 0 - 2 * 0 * 0) (1 - \text{logit}^{-1}(0.5 * 0)) 0.8 \\ &= 0.14\end{aligned}$$

3.2 Translating this data generating process into simulations, generating counterfactual outcomes, and evaluating the target causal parameter.

1. First set the seed to 252.
2. Set $n=50,000$ as the number of i.i.d. draws from the data generating process.
3. Simulate the background factors U . Note the syntax for `rnorm`.
4. Evaluate the structural equations \mathcal{F} to deterministically generate the endogenous nodes X . Recall the logit^{-1} function is given by the `plogis` function in R.
5. Intervene to set the supplement to RUTF ($A = 1$) and generate counterfactual outcomes Y_1 for n units. Then intervene to set the supplement to the standard ($A = 0$) and generate counterfactual outcomes Y_0 for n units.
6. Create a data frame X to hold the values of the endogenous factors ($W1, W2, A, Y$) and the counterfactual outcomes Y_1 and Y_0 . The rows are the n children and the columns are their characteristics. Use the `head` and `summary` to examine the resulting data. Does the counterfactual value Y_a equal the observed Y when $A = a$?
7. Using these simulations, evaluate the causal parameter $\Psi^*(\mathbb{P}^*)$ for this population of 50,000 units.
8. Interpret $\Psi^*(\mathbb{P}^*)$.

Solution:

```
> # 1. set seed and number of draws
> set.seed(252)

> # 2. set the number of draws n=50,000
> n = 50000

> # 3. sample the background factors U
> U.W1<- runif(n, 0, 1)
> U.W2<- runif(n, 0, 1)
> U.A<- runif(n, 0, 1)
> U.Y<- rnorm(n, 0, 0.3)
```

```

> # 4. generate the endogenous factors X
> W1 <- as.numeric( U.W1 < 0.2)
> W2 <- as.numeric( U.W2 < plogis(0.5*W1) )
> A <- as.numeric( U.A < plogis(W1*W2))
> Y <- 4*A + 0.7*W1 - 2*A*W2 + U.Y

> # 5. generate the counterfactuals for large n
> Y.1<- 4*1 + 0.7*W1 - 2*1*W2 + U.Y
> Y.0<- 4*0 + 0.7*W1 - 2*0*W2 + U.Y

> # 6. create data frame X
> X<- data.frame(W1,W2,A,Y, Y.1, Y.0)
> head(X)

  W1 W2 A          Y          Y.1          Y.0
1  0  0  0 -0.188412377 3.811588 -0.188412377
2  0  0  0  0.007554149 4.007554  0.007554149
3  0  1  0  0.485863200 2.485863  0.485863200
4  0  0  0  0.207939977 4.207940  0.207939977
5  0  1  0 -0.189080671 1.810919 -0.189080671
6  0  1  0 -0.017563026 1.982437 -0.017563026

> summary(X)

      W1          W2          A          Y
Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   : -1.15726
1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:  0.09119
Median :0.0000   Median :1.0000   Median :1.0000   Median :  1.67785
Mean    :0.1976   Mean    :0.5254   Mean    :0.5292   Mean    :  1.67314
3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:  3.04173
Max.    :1.0000   Max.    :1.0000   Max.    :1.0000   Max.    :  5.66234

      Y.1          Y.0
Min.   :0.8427   Min.   : -1.15726
1st Qu.:2.0867   1st Qu.: -0.15171
Median :2.9530   Median :  0.08838
Mean    :3.0871   Mean    :  0.13794
3rd Qu.:4.0423   3rd Qu.:  0.38180
Max.    :5.6670   Max.    :  1.94849

> # 7. evaluate the causal parameter
> Psi.star<- mean(Y.1) - mean(Y.0)
> Psi.star

[1] 2.94912

```

The counterfactual mean weight gain would be 2.95 pounds higher if all children were given RUTF as opposed to the standard supplement.

4 Defining the target causal parameter with a working MSM

Now suppose we are interested in knowing if age (in years) V modifies the effect of RUTF A on weight gain Y . As before, $W1$ is an indicator of access to potable water and $W2$ is an indicator of having an infectious disease within two weeks of the study initiation.

Consider the following SCM \mathcal{M}^* :

- Endogenous nodes: $X = (V, W1, W2, A, Y)$
- Background nodes: $U = (U_V, U_{W1}, U_{W2}, U_A, U_Y) \sim \mathbb{P}_U$
- Structural equations \mathcal{F} :

$$\begin{aligned} V &= f_V(U_V) \\ W1 &= f_{W1}(U_{W1}) \\ W2 &= f_{W2}(V, W1, U_{W2}) \\ A &= f_A(V, W1, W2, U_A) \\ Y &= f_Y(V, W1, W2, A, U_Y) \end{aligned}$$

- We have made an exclusion restriction that age V does not effect access to potable water $W1$.

Let us summarize how the counterfactual outcome changes as a function of the intervention and age with the following *working* marginal structural model (MSM):

$$\begin{aligned} \beta^* &= \operatorname{argmin}_{\beta} \mathbb{E}^* \left[\sum_{a \in \mathcal{A}} (Y_a - m(a, V|\beta))^2 \right] \\ m(a, V|\beta) &= \beta_0 + \beta_1 a + \beta_2 V + \beta_3 a \times V \end{aligned}$$

Then the target parameter is defined as a projection of the true causal curve $\mathbb{E}^*(Y_a|V)$ onto a working model $m(a, V|\beta)$. In other words, the causal parameters are the values of the β coefficients that minimize the sum of squared residuals between the counterfactuals Y_a and the model predictions $m(a, V|\beta)$ for all possible exposure levels $a \in \mathcal{A}$.

Based on our knowledge of the data generating system, as represented in \mathcal{M}^* , this working MSM with an interaction term may or may not be a good summary of how the effect of RUTF on the counterfactual average weight gain is modified by age.

4.1 A specific data generating process:

Consider a new data generating process (one of many compatible with the SCM). Suppose that the each of the background factors is drawn independently from following distributions:

$$\begin{aligned} U_V &\sim \text{Uniform}(0, 3) \\ U_{W1} &\sim \text{Uniform}(0, 1) \\ U_{W2} &\sim \text{Uniform}(0, 1) \\ U_A &\sim \text{Uniform}(0, 1) \\ U_Y &\sim \text{Normal}(\mu = 0, \sigma^2 = 0.1^2) \end{aligned}$$

Given the background factors U , the endogenous variables X are deterministically generated as

$$\begin{aligned} V &= 2 + U_V \\ W1 &= \mathbb{I}[U_{W1} < 0.2] \\ W2 &= \mathbb{I}[U_{W2} < \text{logit}^{-1}(0.5 * W1)] \\ A &= \mathbb{I}[U_A < \text{logit}^{-1}(W1 * W2 + V/5)] \\ Y &= 4 * A + 0.7 * W1 - 2 * A * W2 + .3 * V - .3 * A * V + U_Y \end{aligned}$$

1. For $n = 5,000$ children, generate the background factors U and the pre-exposure covariates $(V, W1, W2)$. Then set $A = 1$ to generate the counterfactual weight gain under RUTF Y_1 . Likewise, set $A = 0$ to generate the counterfactual weight gain under the standard supplement Y_0 .
2. Create a data frame `X.msm` consisting of age V , the set treatment levels a , and the corresponding outcomes Y_a .

$$X_{MSM} = (V, a, Y_a) = \begin{pmatrix} V(1) & 1 & Y_1(1) \\ V(2) & 1 & Y_1(2) \\ \vdots & \vdots & \vdots \\ V(n) & 1 & Y_1(n) \\ V(1) & 0 & Y_0(1) \\ V(2) & 0 & Y_0(2) \\ \vdots & \vdots & \vdots \\ V(n) & 0 & Y_0(n) \end{pmatrix}$$

where $V(i)$ and $Y_a(i)$ denote the age and counterfactual outcome for the i^{th} participant. See R lab 1 for a similar example.

3. **Evaluate the target causal parameter.** We have defined the target parameter using the least square projection (i.e., with the L2 loss function). Use the `glm` function to fit the coefficients of the working MSM. Specifically, regress the counterfactual outcomes Y_a on a and V according to the working MSM. Be sure to specify the argument: `data=X.msm`.
4. **Interpret the results.**
5. **Bonus:** Plot of the counterfactual outcomes Y_a as a function of age (V) and treatment group (a).

Solution:

```
> # set the number of draws n=5000
> n = 5000
> # sample the background factors U
> U.V<- runif(n, 0, 3)
> U.W1<- runif(n, 0, 1)
> U.W2<- runif(n, 0, 1)
> U.A<- runif(n, 0, 1)
> U.Y<- rnorm(n, 0, 0.1)

> # generate the pre-exposure covariates and the counterfactuals
> V <- 2 + U.V
> W1 <- as.numeric( U.W1 < 0.2)
> W2 <- as.numeric( U.W2 < plogis(0.5*W1) )
> Y.1 <- 4*1 + 0.7*W1 - 2*1*W2 + .3*V - .3*1*V + U.Y
> Y.0 <- 4*0 + 0.7*W1 - 2*0*W2 + .3*V - .3*0*V + U.Y
> mean(Y.1)

[1] 3.091404

> mean(Y.0)
```



```
[1] 1.188447

> # 2. create data frame X.msm
> V<- rep(V,2)
> A<- c( rep(1,n), rep(0,n))
> Ya<- c(Y.1, Y.0)
> X.msm<- data.frame(V, A, Ya)
> head(X.msm)

      V A      Ya
1 2.431051 1 4.619084
2 2.898271 1 4.520995
3 2.087969 1 2.201596
4 3.034100 1 4.044636
5 2.060597 1 2.552383
6 3.503814 1 1.957732

> tail(X.msm)

      V A      Ya
9995 3.631921 0 1.0303005
9996 2.984406 0 1.0241824
9997 3.224092 0 0.8730339
9998 2.966538 0 1.5122934
9999 3.611784 0 1.0968881
10000 2.405089 0 0.6879200

> # 3. evaluate the causal parameter
> #We can use ordinary least squares regression to obtain betas.
> workMSM<- glm(Ya ~ A + V+ A*V, data=X.msm)
> #
> # the target parameter are the coefficient values that minimize the L2 risk function
> workMSM

Call:  glm(formula = Ya ~ A + V + A * V, data = X.msm)

Coefficients:
(Intercept)          A           V          A:V
      0.1442       2.9954       0.2987      -0.3125

Degrees of Freedom: 9999 Total (i.e. Null);  9996 Residual
Null Deviance:          15050
Residual Deviance: 5661      AIC: 22700

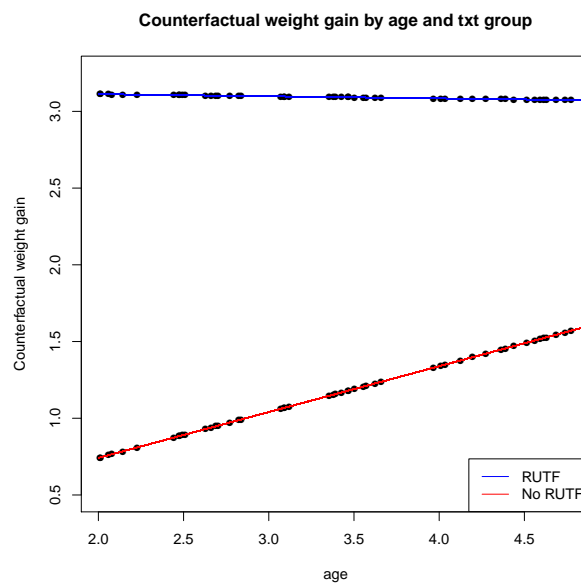
> # creating a plot
> X.msm1 <- X.msm[X.msm$A==1,]
> X.msm0 <- X.msm[X.msm$A==0,]
> # getting predicted outcomes for each unit given the txt & control
> E.Y1V <- predict(workMSM, newdata=X.msm1, type='response')
> E.Y0V <- predict(workMSM, newdata=X.msm0, type='response')
```

```

> pdf('Figures/HW1_MSM.pdf')
> #plotting for a random 50 units in the sample
> these<- sample(1:n, 50)
> plot(X.msm1[these, 'V'], E.Y1V[these], pch=16, xlab='age',
+       ylab='Counterfactual weight gain',
+       main='Counterfactual weight gain by age and txt group', ylim=c(0.5,3.25))
> points(X.msm0[these, 'V'], E.Y0V[these], pch=16)
> # adding the projections
> lines(X.msm1$V, E.Y1V, col='blue')
> lines(X.msm0$V, E.Y0V, col='red')
> legend('bottomright', c('RUTF', 'No RUTF'), col=c('blue', 'red'), lty=c(1,1))
> dev.off()

```

Projecting the true causal curve onto the working marginal structural model yields coefficients of $\beta_0^* = 0.14$, $\beta_1^* = 3$, $\beta_2^* = 0.3$ and $\beta_3^* = -0.31$. These coefficients summarize how the counterfactual mean outcome changes a function of treatment A and age V . Specifically, the coefficient on the interaction between the intervention and age is non-zero, indicating that the effect of RUTF on counterfactual mean weight gain is modified by age. See Figure 2.



Solution Fig. 2: Plot of the counterfactual outcomes Y_a as a function of age (V) and treatment group (a). The predicted counterfactual outcomes, according to the working model, are shown as lines: blue for RUTF and red for the standard supplement.

References

World Health Organization (WHO), World Food Programme (WFP), United Nations System Standing Committee on Nutrition (SCN), and United Nations Children's Fund (UNICEF). *Community-based management of severe acute malnutrition*. WHO/WFP/SCN/UNICEF, Geneva/Rome/Geneva/New York, 2007.