HW3: Exercises with module 7

Exercise 3

Background: The dataset "marriage.csv" contains (simulated) data on the age of first marriage for a number of women in Kenya. Information on the ethnic group is provided as well. For the questions below, let y_i denote the age of first marriage for the i-th woman, and $j[i]$ her ethnic group. The goal is to learn more about the mean age of marriage within ethnic groups, using a Bayesian hierarchical model.

To answer the questions below, fit the following Bayesian hierarchical model to the marriage data:

$$\begin{aligned} y_i|\alpha_{j[i]}, \sigma_y &\overset{i.i.d}{\sim} N(\alpha_{j[i]}, \sigma_y^2), \\ \alpha_j|\mu_\alpha, \sigma_\alpha &\overset{i.i.d}{\sim} N(\mu_\alpha, \sigma_\alpha^2), \end{aligned}$$

using default priors for μ_α, σ_α, σ_y as set in the brm function. In the questions below, if you present default brm-output, please indicate what brm-parameter name refers to what greek letter in the equations above.

(a) State the point estimates and 95% CIs of each of the following parameters and interpret (explain what information is given by) these estimates: μ_α, σ_α, σ_y.

(b) State the point estimate and 95% CIs for α_1 and interpret (explain what information is given by) the estimate.

(c) Construct two plots: (a) plot of $\hat{\alpha}_j - \bar{y}_j$ against the within-ethnic-group sample size n_j and (b) plot $\hat{\alpha}_j$ against $\bar{y}_j$, with the identity line added. Explain what information these plots provide regarding the comparison of $\bar{y}_j$ and $\hat{\alpha}_j$ for estimating the mean age of marriage within ethnic groups.

Optional/just for fun exercise: As stated in the slides, for the multilevel model under consideration here, the full conditional distribution for the j-th state mean is given by:

$$\begin{aligned} \alpha_j|\boldsymbol{y}, \mu_\alpha, \sigma_y, \sigma_\alpha &\sim N(m, v), \\ v &= \left(n_j/\sigma_y^2 + 1/\sigma_\alpha^2\right)^{-1}, \\ m &= v \cdot \left(\frac{n_j}{\sigma_y^2}\bar{y}_j + \frac{1}{\sigma_\alpha^2}\mu_\alpha\right) = \frac{\frac{n_j}{\sigma_y^2}\bar{y}_j + \frac{1}{\sigma_\alpha^2}\mu_\alpha}{\frac{n_j}{\sigma_y^2} + \frac{1}{\sigma_\alpha^2}}. \end{aligned}$$

Give the derivation of the full conditional.
Hint: Check slide in part 1 for the derivation of the normal distribution for μ in the normal-normal setting.