ECE421 Problem Set 2

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1 Problem 1

Given data:

$x^{(i))}$	$t^{(i)})$
1	6
2	4
3	2
4	1
5	3
6	6
7	10

Table 1: Given data

1.1

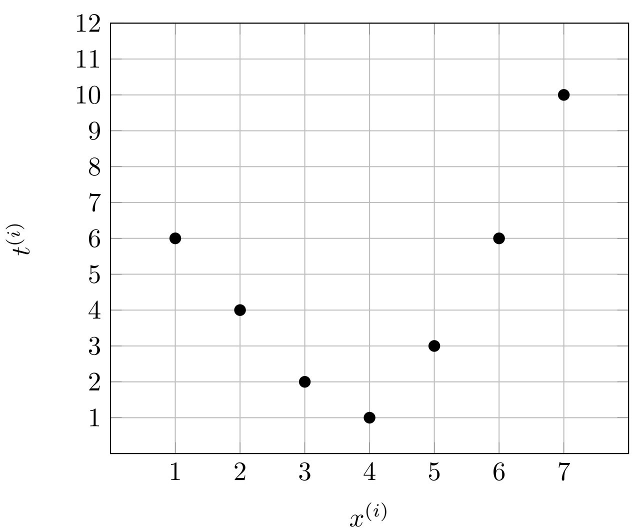

Figure 1: Scatter plot of $x^{(i)}$ vs. $t^{(i)}$

1.2

$$
\begin{aligned}
\mathcal{E}(w,b) &= \frac{1}{2N}\sum_{i=1}^{N}(y^{(i)}-t^{(i)})^2 \\
&= \frac{1}{2N}\sum_{i=1}^{N}(wx^{(i)}-t^{(i)})^2 \\
&= \frac{1}{2N}\sum_{i=1}^{N}\left[(wx^{(i)})^2 + wx^{(i)}b - wx^{(i)}t^{(i)} + wx^{(i)}b + b^2 - bt^{(i)} - wx^{(i)}t^{(i)} - t^{(i)}b + (t^{(i)})^2\right] \\
&= \frac{1}{2N}\sum_{i=1}^{N}\left[(w^2x^{(i)})^2 + 2wx^{(i)}b - 2wx^{(i)}t^{(i)} - 2bt^{(i)} + b^2 + (t^{(i)})^2\right] \\
&= \frac{1}{2N}\sum_{i=1}^{N}\left[(x^{(i)})^2w^2 + 1b^2 + 2x^{(i)}wb - 2t^{(i)}x^{(i)}w - 2t^{(i)}b + (t^{(i)})^2\right] \\
\implies & A_i = (x^{(i)})^2, B_i = 1, C_i = 2x^{(i)}, D_i = -2t^{(i)}x^{(i)}, E_i = -2t^{(i)}, F_i = (t^{(i)})^2 \\
& \text{in } \frac{1}{2N}\sum_{i=1}^{N} A_iw^2 + B_ib^2 + C_iwb + D_iw + E_ib + F_i
\end{aligned}
$$

1.3

The loss function is minimized when $\frac{\partial\mathcal{E}}{\partial w} = 0$ and $\frac{\partial\mathcal{E}}{\partial b} = 0$. Where $A = \sum_i A_i$, etc....

$$
\begin{aligned}
\frac{\partial\mathcal{E}}{\partial w} &= \frac{1}{2N}\sum_{i=1}^{N} 2wA_i + C_ib + D_i && (1)\\
&= 2wA + Cb + D = 0 && (2)\\
\implies w &= \frac{-Cb - D}{2A} && (3)\\
\frac{\partial\mathcal{E}}{\partial b} &= \frac{1}{2N}\sum_{i=1}^{N} 2B_ib + C_iw + E_i && (4)\\
&= 2Bb + Cw + E = 0 && (5)\\
\implies b &= \frac{-Cw - E}{2B} && (6)\\
\implies w &= && (7)
\end{aligned}
$$

1.4

By plugging in numerical values from the dataset D (Table 1), the values are found to be approximately:

$$w = b \qquad\qquad = \qquad (8)$$

1.5

Using Excel's linear regression tool, it is found that $w = 2.1429$ and $b = 0.6071$.

2 Problem 2

2.1

2.2

2.3

2.4

2.5

3 Problem 3

3.1

3.2

3.3

3.4

3.5

3.6