ECE421 - Problem Set 1

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Problem 3

3.1. Computing the matrix product $\mathbf{AB}$ (where $\mathbf{A} \epsilon \mathbb{R}^{2\times 3}$ and $\mathbf{B} \epsilon \mathbb{R}^{2\times 3}$) is not possible since the inner dimensions are not equal. For $\mathbf{A} \epsilon \mathbb{R}^{a\times b}$ and $\mathbf{B} \epsilon \mathbb{R}^{c\times d}, b \neq c$.

3.2. Given:

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 0 & -2 & 4 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 2 & -0.5 \\ 1 & 0 \\ 1 & 0.5 \end{bmatrix},$$

$$\mathbf{AB} = \begin{bmatrix} 2 & 4 & 6 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & -0.5 \\ 1 & 0 \\ 1 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 2 \\ 2 & 2 \end{bmatrix}$$

3.3 Given:

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & -2 \\ 1 & 3 & -2 \\ 1 & 2 & -1 \end{bmatrix},$$

$$\mathbf{A} - \lambda\mathbf{I} = \begin{bmatrix} 4-\lambda & 0 & -2 \\ 1 & 3-\lambda & -2 \\ 1 & 2 & -1-\lambda \end{bmatrix}$$

$$det(\mathbf{A} - \lambda\mathbf{I}) = (4-\lambda)[(3-\lambda)(-1-\lambda)+4] - 2[2-(3-\lambda)]$$

$$0 = \boxed{-\lambda^3 + 6\lambda^2 - 11\lambda + 6} \text{ is the characteristic polynomial.}$$

$$= -(\lambda-3)(\lambda-2)(\lambda-1)$$

$$\implies \boxed{\lambda_0 = 1, \lambda_1 = 2, \lambda_2 = 3} \text{ are the eigenvalues of } \mathbf{A}.$$