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Dynamics of an electron driven by relativistically intense laser radiation

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The dynamics of an electron driven by a relativistically intense laser pulse is analyzed on the basis of the equation of motion with the Lorentz force in the cases of linear and circular polarizations. Laser fields with nonplane phase fronts accelerate electrons in the longitudinal direction. An electron initially at rest is found not to move along figure-eight trajectories for the linear polarization, and not to move along circular trajectories for the circular polarization. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2839349]

I. INTRODUCTION

The dynamics of a charged particle (an electron, for example) in the electromagnetic field is determined by the Lorentz force. This is true in the case of any type of electromagnetic field, including that of a high-frequency electromagnetic field of a laser pulse. The character of the electron dynamics depends on the pulse parameters (the spatial-temporal intensity distribution and polarization). Focused laser beams with Gaussian transverse intensity distributions are considered in many papers. Such beams have a caustic waist in the focal areas. The typical radius of the caustic waist in the case of relativistic intensities is less than 10 wavelengths.

A number of studies were dedicated to the dynamics of electrons in electromagnetic fields studied on the basis of the Newton equation with the Lorentz force (for example, see Refs. 1–6). It was shown in Refs. 3 and 4 that an electron gets trapped by a laser pulse for a certain period of time and moves with it. The analysis of the electron motion in the case of a Gaussian transverse intensity distribution[7] demonstrated that an electron initially at rest on the beam axis gets accelerated up to a high velocity on the pulse front, but then decelerated on the pulse tail. An initially off-axis electron gets expelled at a certain angle to the propagation axis. The kinetic energy of such an electron can be quite high. In some works, the process is interpreted as the electron scattering. Note that at extremely high laser intensities, the amplitude of the electron oscillations becomes comparable to the wavelength of the laser beam.

The description of the electron dynamics in a high-frequency field is complicated because of the large number of oscillations. Because of this, the Lorentz force averaged over high-frequency oscillations, known as the ponderomotive force, is often used instead of the original Lorentz force.[8–11] Obviously, the concept of the ponderomotive force becomes inapplicable in the case of very short pulses involving small numbers of optical field oscillations, so that the exact expression for the Lorentz force has to be used to analyze the electron motion.

The comparison of the electron dynamics in the cases of the linear and circular polarizations of the optical fields did not receive enough attention in previous studies. Such a comparison is one of the aims of the present paper. The expulsion of an electron with a high kinetic energy from the interaction zone is another process that requires additional exploration. In addition, there was no clarity on the issue of the occurrence of figure-eight trajectories of electrons driven by linearly polarized electromagnetic fields.

II. EQUATIONS OF MOTION

Below, we consider the electron dynamics in an intense electromagnetic field. The electron motion in the field of a short relativistically intense laser pulse is analyzed using the Lorentz force expression for both linear and circular polarizations.

Assume that the laser radiation is a focused beam having a Gaussian transverse intensity distribution. The electron is driven by the high-frequency Lorentz force. The equation of motion is

$$\frac{d\mathbf{p}}{dt} = -e\mathbf{E} - \frac{e}{c}[\mathbf{vH}], \tag{1}$$

where $e>0$ is the electron charge magnitude. The initial conditions for Eq. (1) are

$$\mathbf{r}(0) = \mathbf{r}_0, \quad \mathbf{v}(0) = \mathbf{v}_0. \tag{2}$$

Let us use the reference frame in which the laser pulse propagates along the z axis. Its phase front is plane in the caustic waist and the phase surface being perpendicular to the z axis.

In terms of vector components, Eq. (1) is equivalent to

$$\frac{d}{dt}\frac{mv_x}{\sqrt{1-v^2/c^2}} = -eE_x - \frac{e}{c}(v_yH_z - v_zH_y), \tag{3}$$

$$\frac{d}{dt}\frac{mv_y}{\sqrt{1-v^2/c^2}} = -eE_y - \frac{e}{c}(v_zH_x - v_xH_z), \tag{4}$$

$$\frac{d}{dt}\frac{mv_z}{\sqrt{1-v^2/c^2}} = -eE_z - \frac{e}{c}(v_xH_y - v_yH_x). \tag{5}$$

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Electromagnetic beams with Gaussian transverse intensity distributions propagate developing caustic waists. Let ρ_0 be the minimal beam diameter at the waist and ρ the beam diameter

$$\rho(z) = \rho_0\sqrt{1 + z^2/z_R^2},$$

where $z_R = \pi\rho_0^2/\lambda$ is the Raleigh range and λ is the radiation wavelength.

Numerous studies of the structure of the electromagnetic fields in the cases of linearly polarized beams with Gaussian transverse intensity distributions are available.[10,12,13] Expressions for the elliptic polarization case (with the left rotation) are obtained below to the lowest order in the small parameter $\varepsilon = \lambda/(2\pi\rho_0)$. They are

$$\begin{aligned}
E_x &= E_0(x,y,\xi)\sqrt{\tfrac{1}{2}(1+\alpha)}\cos\varphi,\\
E_y &= -E_0(x,y,\xi)\sqrt{\tfrac{1}{2}(1-\alpha)}\sin\varphi,\\
E_z &= -2E_0(x,y,\xi)\frac{\varepsilon}{\rho}\Big(\sqrt{\tfrac{1}{2}(1+\alpha)}x\sin\tilde{\varphi}\\
&\quad + \sqrt{\tfrac{1}{2}(1-\alpha)}y\cos\tilde{\varphi}\Big),\\
H_x &= -E_y, \quad H_y = E_x,\\
H_z &= 2E_0(x,y,\xi)\frac{\varepsilon}{\rho}\Big(-\sqrt{\tfrac{1}{2}(1+\alpha)}y\sin\tilde{\varphi}\\
&\quad + \sqrt{\tfrac{1}{2}(1-\alpha)}x\cos\tilde{\varphi}\Big).
\end{aligned} \tag{6}$$

Equation (6) with $\alpha=1$ corresponds to the linear polarization along the x axis, $\alpha=0$ to the circular polarization, $\alpha=-1$ to the linear polarization along the y axis, and other values of α describe the elliptic polarization. Note that for the same intensity, the field amplitudes for the linear and circular polarizations differ by a factor of $\sqrt{2}$.

The function $E_0(x,y,\xi)$ is given by

$$E_0(x,y,\xi) = E_m\frac{\rho_0}{\rho}\exp\left\{-\left[\frac{\xi - z_d/c}{\tau}\right]^s - \left[\frac{\sqrt{x^2+y^2}}{\rho}\right]^2\right\}. \tag{7}$$

Here E_m is the maximal field strength, $\xi = t - z/c$, and z_d is the initial distance between the pulse and the electron (which guarantees a gradual field rise in the process of solving the equations numerically). Equation (7) assumes that the pulse temporal profile is described by a super-Gaussian shape with the parameter s, and τ is the pulse duration. The phase dependencies in Eq. (6) are given by $\varphi = 2\pi c\xi/\lambda + \arctan(z/z_R) - zr^2/z_R\rho^2 - \varphi_0$, $\tilde{\varphi} = \varphi + \arctan(z/z_R)$, φ_0 being the initial phase.

The intensity of the beam is

$$I(x,y,z,t) = \frac{c}{4\pi}\overline{[\mathbf{E}(x,y,z,t)\mathbf{H}(x,y,z,t)]}. \tag{8}$$

The maximal intensity I_m is reached at the pulse axis at the time moment corresponding to the laser field strength maximum. It makes

$$I_m = cE_m^2/8\pi.$$

In case the electron dynamics is localized in the proximity of the caustic waist, and its longitudinal displacement L over the interaction time is less than z_R, simplified expressions for the fields can be used in Eqs. (6) and (7) such that $E_z = H_z = 0$ and $\rho = \rho_0$. Under the circumstances, the phase front is assumed to be plane.

For $\rho_0 \to \infty$, the field becomes an infinite plane wave.

The simulations were performed for particles located on the beam axis at $t=0$, so that $(x_0 = y_0 = 0)$, but the impact of an initial displacement relative to the propagation axis was also considered.

III. DYNAMICS IN THE FIELD OF THE LINEARLY POLARIZED LASER RADIATION

The results of the present paper, obtained using Eq. (7), were generated for various values of ρ_0 and for $s=2$ and $s=4$.

Equations (3)–(5) for fields (6) can be cast in a dimensionless form in terms of the variables x/λ, z/λ, ct/λ, v/c. Dimensionless coordinates, velocity components, and accelerations $\lambda v_x'/c^2$ and $\lambda v_z'/c^2$, as well as the dimensionless particle kinetic energies W_k/mc^2 were calculated.

The dimensionless field amplitude is related to the dimensionless intensity I_m/I_r, where I_r is the relativistic intensity. Several expressions for the latter, differing by a factor, are used in various studies. From our standpoint, the most adequate criterion for defining the relativistic intensity is to compare the full energy of the electron oscillating in the short laser pulse field to the rest energy mc^2. In this case, we have

$$I_r = m^2c^3\omega^2/8\pi e^2 = 1.37\times10^{18}\cdot(1/\lambda[\mu\mathrm{m}])^2[\mathrm{W/cm^2}].$$

Figure 1 shows the temporal dependencies or x/λ [Fig. 1(a)], v_x/c [Fig. 1(b)], $\lambda v_x'/c^2$ [Fig. 1(c)], z/λ [Fig. 1(d)], v_z/c [Fig. 1(e)], $\lambda v_z'/c^2$ [Fig. 1(f)], and W_k/mc^2 [Fig. 1(g)]. They are calculated for the distribution given by Eq. (7) corresponding to a short pulse with the following parameters: $I_m/I_r=50$, $s=2$ (a Gaussian temporal profile), $c\tau/\lambda=4$, $\rho_0/\lambda=10$, a zero initial velocity, and a zero initial transverse displacement. The longitudinal and transverse accelerations are depicted in order to illustrate the corresponding forces acting on the electron. In this case $L<z_R$ ($L\approx60\lambda$, $z_R=314\lambda$) and the simulation results are identical to those in the framework of the simplified model assuming $E_z=H_z=0$ and $\rho=\rho_0$.

Figure 2 shows the temporal profiles for the electron velocity in the case in which $\rho_0/\lambda=3$, so that $L>z_R$. The electron is located on the beam axis, and its initial displacement relative to the waist changed, $z_0/\lambda=0$ in Fig. 2(a), $z_0/\lambda=-16$ in Fig. 2(b), $z_0/\lambda=-25$ in Fig. 2(c).

The following features of the electron dynamics are observed in the considered case of the linear polarization in the case $L<z_R$:

(i) The transverse coordinate and velocity oscillate at a variable frequency. On the pulse front and tail, where the intensity is low, the frequency of the oscillations equals that of the electromagnetic field. The frequency of the electron oscillations is substantially lower at higher intensities, where

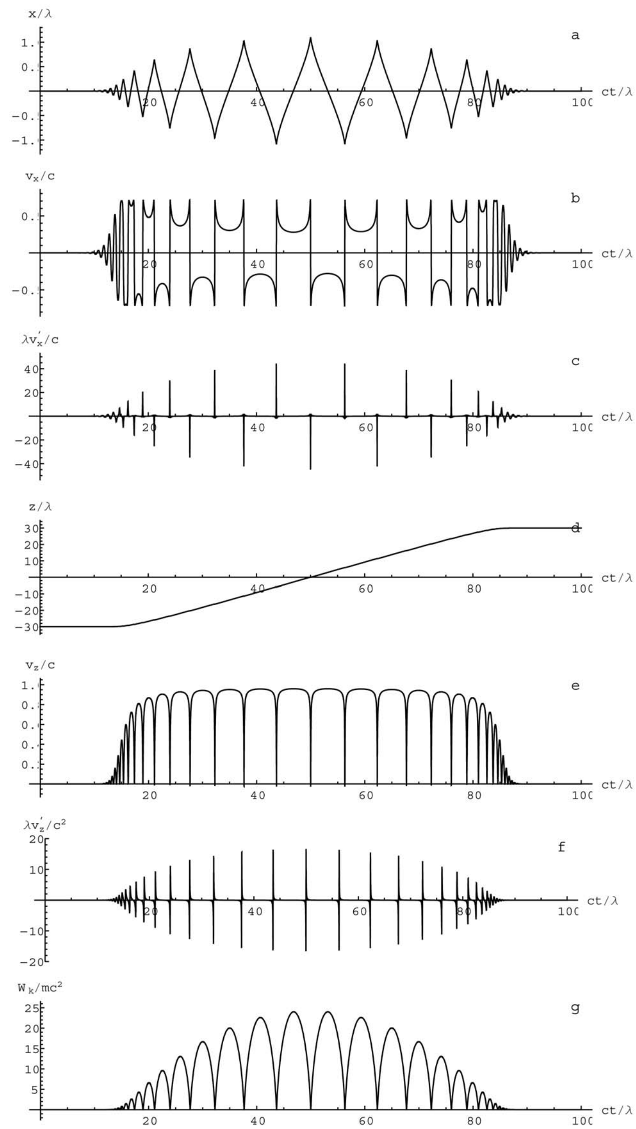

FIG. 1. The dependencies of x/λ (a), v_x/c (b), $\lambda v_x'/c^2$ (c), z/λ (d), v_z/c (e), $\lambda v_z'/c^2$ (f), and W_k/mc^2 (g) on time for the electron dynamics in the field of a relativistically intense $(I/I_r=50)$ linearly polarized short laser pulse. The laser pulse is described by Eq. (7) with $s=2$ (Gaussian temporal profile), $c\tau/\lambda=4$ and $\rho_0/\lambda=10$. The electron has a zero initial velocity and a zero initial displacement relative to the propagation axis.

the oscillations exhibit a complicated inharmonic temporal dependence. The inharmonic character of the oscillation was noted in Refs. 7 and 14.

(ii) The longitudinal velocity oscillates at twice the frequency of the oscillations of the transverse coordinate and velocity. The longitudinal velocity is non-negative at all times and substantially inharmonic (it has a nearly rectangular profile). When the pulse has gone, the longitudinal velocity equals zero.

(iii) The average longitudinal velocity can get close to the speed of light. This results in a sort of trapping of the particle by the laser pulse field and in a substantial increase in the duration of the field-particle interaction. The electron trapping by the field and its acceleration in this regime were discussed in Refs. 3 and 4.

(iv) The longitudinal acceleration oscillates around zero synchronously with the oscillations of the longitudinal velocity. The ranges of its positive and negative magnitudes are roughly the same, and the oscillations are narrow pikes centered at the times when the longitudinal velocity equals zero.

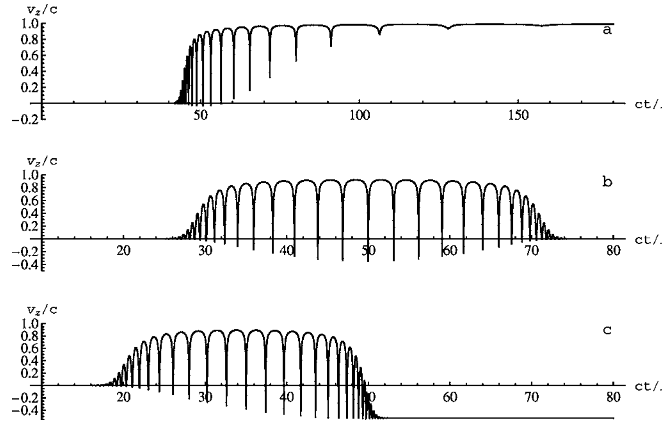

FIG. 2. The temporal dependencies of v_z/c for case $\rho_0/\lambda=3$ $(L>z_R)$. The electron has a zero initial velocity and is located on beam axes with different initial longitudinal displacement relative to the waist: $z_0/\lambda=0$ (a), $z_0/\lambda=-16$ (b), $z_0/\lambda=-25$ (c). The laser pulse parameters are the same as in Fig. 1.

(v) The longitudinal coordinate increases with time almost linearly with some fine step by step structure. The total displacement over the interaction time is finite and quite substantial (several dozen wavelengths in the case considered). Some analytical expression for the electron's longitudinal displacement in a strong field is proposed in Ref. 15.

When $L>z_R$, the electron dynamics becomes more complicated due to the presence of E_z and H_z. Even though the main features of the electron dynamics described above do remain, certain new details emerge. The most important one is that an electron initially at rest on the beam axis gains a fairly substantial longitudinal velocity component v_z, whose magnitude and sign depend on the initial longitudinal displacement. The presence of v_z results in the electron oscillation's frequency at the pulse tail being below that of the electromagnetic field. Within the pulse, v_z can be both positive and negative.

Note that in the case considered, the charged particle does not move along a figure-eight trajectory. Such trajectories were predicted in Ref. 16 for a plane linearly polarized monochromatic electromagnetic wave in a reference frame where the particle remains at rest on the average. In case the particle was at rest initially, it remains immobile only outside the laser pulse, and always has a nonzero average velocity (averaged over the fast oscillations) inside of its domain. The maximal value of this velocity v_m depends on the intensity and is reached at the maximum of the pulse. Therefore, the figure-eight trajectory can occur only on the plane top of a laser pulse, and initially the particle has to move in the direction opposite to that of the propagation with the velocity v_m to let this happen.

As an example of the figure-eight motion, Fig. 3 shows the results of simulating the dynamics of an electron initially moving in the direction opposite to that of the pulse propagation with the matched velocity v_m. Under the circum-

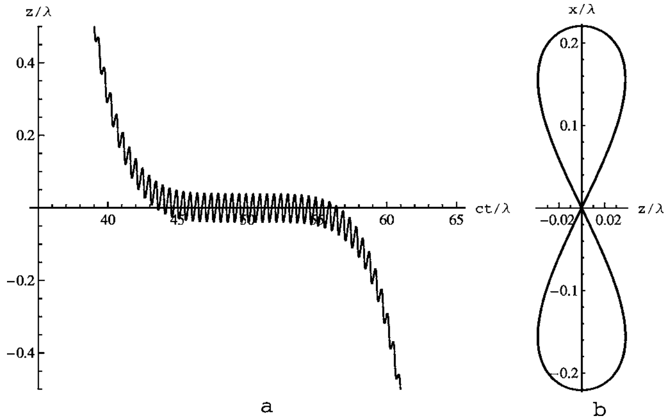

FIG. 3. The dynamics of an electron initially moving in the direction opposite to that of the laser pulse propagation with the velocity v_m. The case presented corresponds to a relatively long linearly polarized pulse with a plane top, the parameters being $s=4$, $c\tau/\lambda=16$, $\rho_0/\lambda=10$, and $I/I_r=50$. The matched value of the initial electron velocity is given by $v_m/c=-0.926$. No drift displacement in the longitudinal direction occurs on the plane top of the pulse (a), and the electron oscillates around a certain average location. The segment of the electron trajectory, shown in (b) in the (x,z) coordinates, exemplifies the figure-eight motion during five oscillations at the center of the plane top of the pulse.

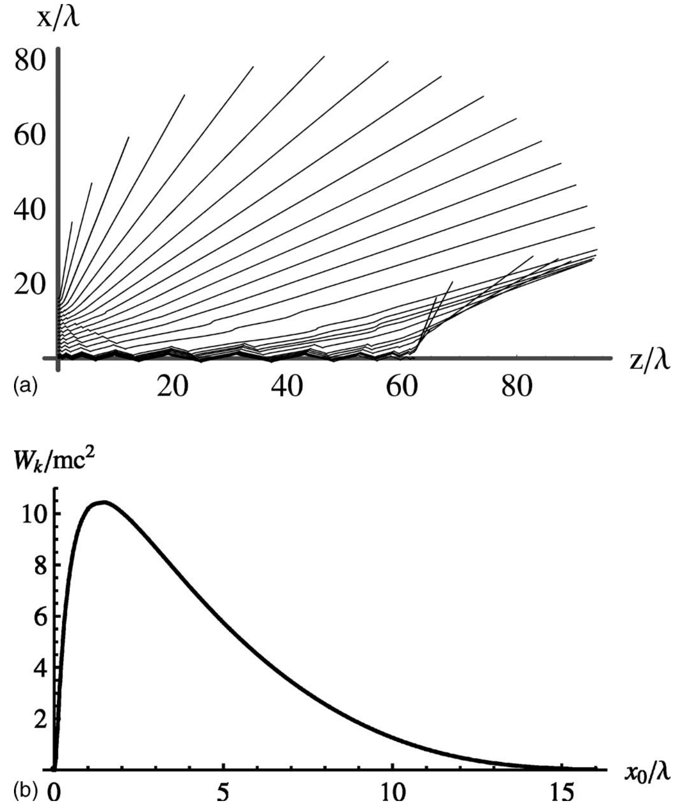

FIG. 4. The electron trajectories for various initial displacements relative to the laser pulse propagation axis (a) and the dependence of the electron kinetic energy after the interaction on the initial displacement (b). The laser pulse parameters are the same as in Fig. 1.

stances, the simulation was carried out for a longer pulse that had a plane top ($s=4$, $c\tau/\lambda=16$, $\rho_0/\lambda=10$, $I/I_r=50$). For these parameters, the value of the matched initial velocity of the electron is $v_m/c=-0.926$. As one can see in Fig. 3(a), there is no displacement of the electron due to the longitudinal drift on the plane top of the pulse, the dynamics being limited to oscillations relative to a certain average value. Figure 3(b) shows the segment of the trajectory in the (x,z) coordinates at the center of the plane part of the pulse. This segment is a figure-eight motion during five oscillations.

Another series of simulations were carried out to study the picture of the dynamics of an electron initially displaced in the x direction relative to the laser propagation axis. The basic result is that the particle gets expelled out of the laser pulse area in both radial and longitudinal directions. This phenomenon was studied, for example, in Ref. 5. In some publications, the process is interpreted as the scattering of an electron by the field. The particle continues to move after the laser pulse is gone. An important aspect of this dynamics is that the electron retains nonzero velocities in both the transverse and the longitudinal directions.

The trajectories of electrons for various initial displacements relative to the propagation axis are shown in Fig. 4(a). The laser pulse parameters are the same as in the case depicted in Fig. 1 (the condition $L<z_R$ is also valid). Figure 4(b) shows the dependence of the electron's post-interaction energy on the initial displacement.

The angle at which the electron gets expelled is sensitive to the magnitude of the initial displacement. The electrons accelerated by a laser pulse can generally be sorted into three groups. The first one consists of the electrons with the initial displacements x_0/λ from the 0.05–1 range. They exit the interaction zone close to the maximum of the laser intensity, thus getting maximal kinetic energies. In this group, the longitudinal motion prevails and they get expelled mostly in the forward direction. The second group comprises electrons whose initial displacement x_0/λ is greater than 1. They leave the interaction zone from the pulse front with low kinetic energy. The transverse motion prevails in this group of electrons and they get expelled nearly perpendicularly to the propagation axis. The third group consists of the electrons with the initial displacements x_0/λ below 0.05. They oscillate within the pulse for a relatively long time and leave the interaction zone from the pulse tail. Transverse motion prevails in this group of electrons as well, so that they also get expelled nearly perpendicularly to the laser pulse propagation axis.

For the linear polarization, an initial displacement in the y direction does not result in the expulsion of an electron.

IV. DYNAMICS IN THE FIELD OF A CIRCULARLY POLARIZED LASER PULSE

Figure 5 shows the trajectories in 3D for an electron initially located on the beam propagation axis [Fig. 5(a)]. It is clear that the electron's trajectory is a spiral. The transverse coordinates and velocities are described by harmonic functions with variable frequencies.

It should be noted that generally the trajectories of electrons driven by circularly polarized pulses are not circular. Similarly to the figure-eight motions in the linear polarization case, the electron has to move initially in the direction opposite to that of the laser propagation with a matched velocity for the circular orbit to occur on the top of the pulse.

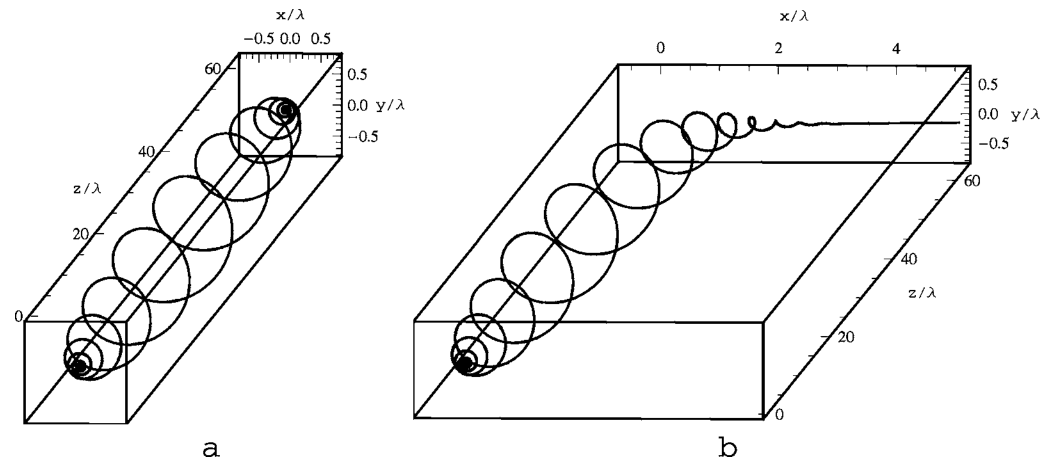

FIG. 5. The 3D trajectories of an electron initially located on the laser pulse propagation axis (a) and displaced relative to it (b), the displacement being $x_0/\lambda = 0.1$.

The case of an initial displacement in the transverse direction ($x_0/\lambda = 0.05$) is illustrated by Fig. 5(b). Here the electron is expelled from the interaction domain with a high kinetic energy at an angle to the propagation axis.

Figure 6 shows the temporal dependencies of r/λ [Fig. 6(a)], v_r/c [Fig. 6(b)], $\lambda v_r'/c^2$ [Fig. 6(c)], z/λ [Fig. 6(d)], v_z/c [Fig. 6(e)], $\lambda v_z'/c^2$ [Fig. 6(f)], and W_k/mc^2 [Fig. 6(g)] calculated for the distribution given by Eq. (7) in the case of a short laser pulse with $I/I_r = 50$ (a relativistic case), $s = 2$ (a Gaussian temporal profile), $c\tau/\lambda = 4$, $\rho_0/\lambda = 10$ ($L < z_R$), a zero initial velocity, and a zero initial transverse displacement.

The following is established in the case of the circular polarization:

(i) In contrast to what is seen in the case of the linear polarization, the coordinate oscillations are markedly sinusoidal.

(ii) The transverse coordinate and velocity oscillate at a variable frequency, and the frequency of the oscillations at the pulse front and tail, where the intensity is low, equals the frequency of the driving electromagnetic radiation. A substantial reduction of the frequency takes place at high intensities.

(iii) The longitudinal velocity does not oscillate, and the longitudinal displacement is not a steps-shaped function of time.

(iv) The longitudinal acceleration is described by a smooth function. The electron gets accelerated at the pulse front and decelerated at the pulse tail.

The overall longitudinal displacements in the cases of the linear and circular polarizations are the same for the same pulse shapes and maximal intensities. The smooth function describing the longitudinal velocity in the case of the circular polarization is the exact average of the longitudinal velocity in the linear polarization case.

Similarly to the linear polarization case, for $L > z_R$ the electron dynamics becomes more complicated due to the presence of E_z and H_z. An electron initially at rest gains a nonzero component v_z during the interaction. The electron motion is smooth, the trajectory being a spiral.

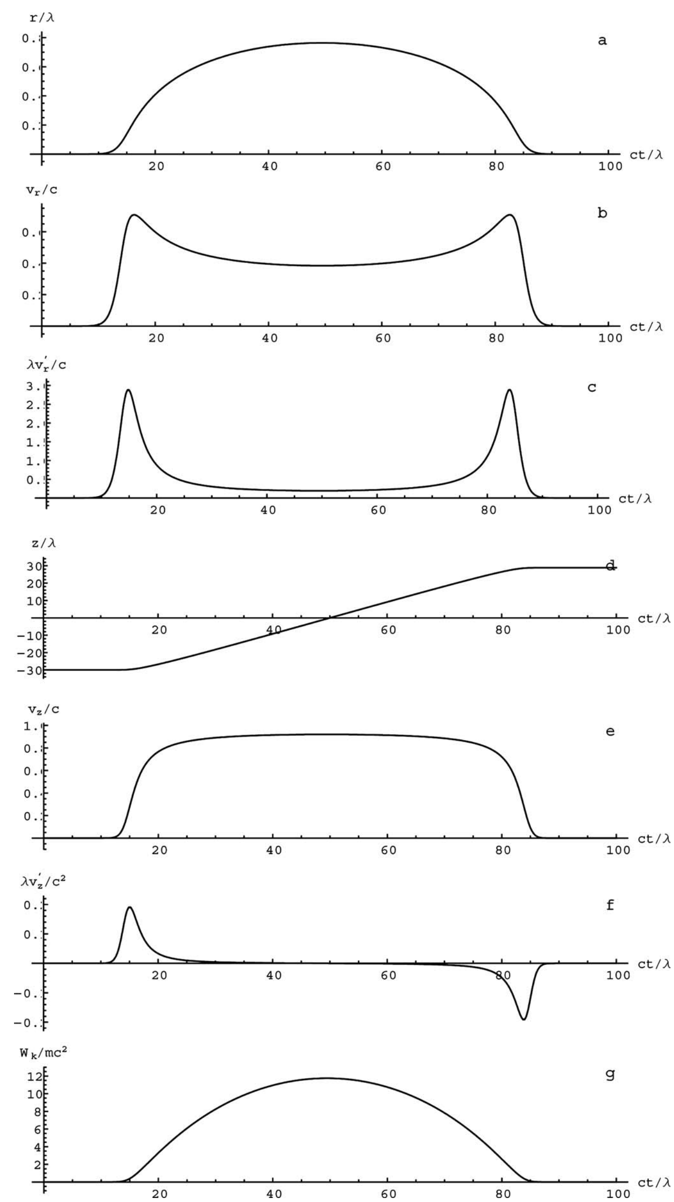

FIG. 6. The dependencies of r/λ (a), v_r/c (b), $\lambda v_r'/c^2$ (c), z/λ (d), v_z/c (e), $\lambda v_z'/c^2$ (f), and W_k/mc^2 (g) on time for the electron dynamics in the field of a relativistically intense ($I/I_r = 50$) short circularly polarized laser pulse. The laser pulse parameters and the initial conditions are the same as in Fig. 1.

V. INVARIANT OF ELECTRON MOTION IN THE LASER PULSE FIELD

It is well known[17] that an invariant of motion exists in the case of an electron driven by a high-frequency field with the plane phase distribution and homogenous intensity in radial direction. If the latter propagates along the z axis, the invariant is

$$(p^2+m^2c^2)^{1/2}-p_z=const. \tag{9}$$

It is of interest to verify the conditions under which the invariant holds.

Consider several specific examples of the electron interactions with a laser pulse. Assume that initially an electron is located at some distance from the propagation axis, has a certain initial velocity, and partially traverses the area where the field is strong. Consider the interactions for the following parameters: $x_0/\lambda=20$, $v_{x0}/c=-0.4$, $v_{z0}/c=-0.4$, and the values of z_0/λ equal to (a) 28.2, (b) 20.7, and (c) 20.2. The electron moved at 45° to the propagation axis prior to the interaction. Depending on the temporal delay (which depends on the specific value of z_0/λ), the electron can collide with the pulse front, its central part, or tail. In the case shown in Fig. 7(a), the electron's velocity longitudinal component changes its sign. The electron gains a substantial kinetic energy as a result of the interaction. In the case shown in Fig. 7(b), the electron's velocity transverse component changes its sign. The delay is chosen so that the electron kinetic energy remains unchanged after the interaction. In the case shown in Fig. 7(c), the electron moves along the z axis after the interaction. The conservation law given by Eq. (9) is found to be valid in the above three cases.

When we have $L > z_R$, the invariance breaks down.

Therefore, the invariant holds in the case of any transverse intensity distribution, but only in the area where the phase front can be assumed plane. For a Gaussian pulse, the invariant holds for $L < z_R$. The invariance can be regarded as the applicability condition for the simplified model.

The invariant defined by Eq. (9) can be used to derive a parametric solution describing the dynamics of a particle initially located on the beam axis.

To obtain such a parametric representation, v_z is expressed via v_x (assuming that $v_y=0$), and then substituted into Eq. (3). In the framework of the approximation, we have $E(x,y,\xi)=E(0,0,\xi)$, and Eq. (3) becomes an ordinary differential equation for $x(\xi)$. It can be solved using the multiple scale asymptotic technique similar to that used in Ref. 15. The solutions for x and p_x (for $v_{z0}=0$) can be presented in the form of the following series:

$$x=\frac{e}{m\omega^2}\sum_{n=0}^{\infty}(n+1)\omega^{-n}E_0^{(n)}(\xi)\cos\left(\omega\xi+\frac{n\pi}{2}\right), \tag{10}$$

$$p_x=-\frac{e}{\omega}\sum_{n=0}^{\infty}\omega^{-n}E_0^{(n)}(\xi)\sin\left(\omega\xi+\frac{n\pi}{2}\right). \tag{11}$$

The above solutions are applicable if the maximal intensity is such that the amplitude of the transverse oscillations is much less than the transverse scale of the intensity variation ρ_0.

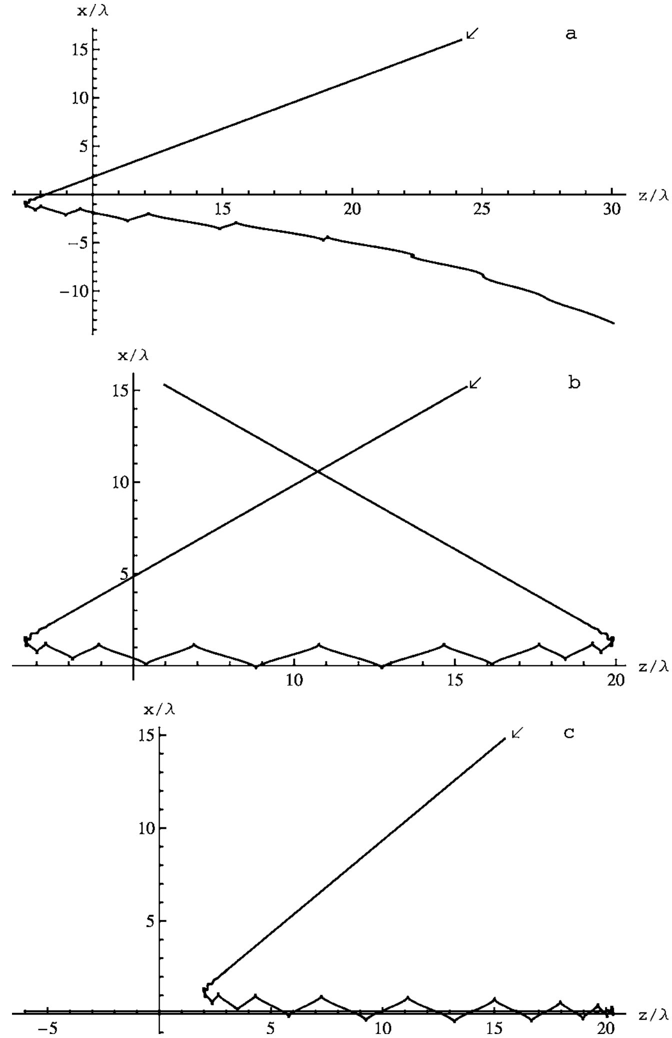

FIG. 7. The 2D trajectories of electrons for the following parameters: $x_0/\lambda=20$, $v_{x0}/c=-0.4$, $v_{z0}/c=-0.4$, and the values of z_0/λ equal to (a) 28.2, (b) 20.7, and (c) 20.2. The arrow shows the direction of the electron motion.

The above solutions can be regarded as a generalization of the solution describing the electron dynamics in a stationary plane-wave field, which is presented in Ref. 16. They can be used to estimate the ponderomotive force in a relativistically intense field.

Equations (10) and (11) were used to calculate x, z, v_x, and v_z on the pulse propagation axis. The result was compared to the numerical solutions based on the Newton equation with the Lorentz force, obtained for electrons initially located on the propagation axis. Satisfactory precision was provided for by the lowest two terms of the series given by Eqs. (10) and (11).

VI. CONCLUSIONS

(i) At relativistic intensities, the electron oscillations in a linearly polarized optical field are substantially inharmonic. The oscillations in a circularly polarized field are sinusoidal. The electron gets partially trapped by the field and the oscillation period depends on the local intensity.

(ii) In the case of the laser pulse having linear polarization, the electron motion does not follow the figure-eight trajectories. Such trajectories can materialize only on the plane top of a laser pulse for a particle initially moving in the direction opposite to that of the laser beam propagation with a carefully matched velocity. Similarly to the circularly polarized field, the electron trajectory is a spiral, and not circular.

(iii) When $L > z_R$, an electron initially at rest on the beam axis gains a considerable longitudinal velocity v_z as a result of the interaction with the field, its magnitude and sign depending on the electron's initial longitudinal displacement. This effect makes it possible to accelerate electrons to energies making a considerable fraction of its oscillation energy in the field.

(iv) For $L < z_R$, the longitudinal displacement of an electron that was at rest initially is the same for both polarizations provided that the pulse intensities are the same. Under this condition, the longitudinal velocity in the case of the circular polarization is the exact average of that in the case of the linear one.

(v) A parametric solution describing the motion of an electron in an intense linearly polarized electromagnetic wave is developed.

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