Contents

```matlab
% Evaluate performance and scaling of Gaussian elimination, Jacobi
% iteration, and Thomas algorithm by solving systems of different
% size and timing the solves

clear
clc
close all

nvals=1:1:50;
testtimes=zeros(size(nvals));
lrep=10;      %how many times to repeat each test
```

Gaussian elimination

```matlab
disp('Start of tests of Gaussian-elimination scaling');
for in=1:numel(nvals)
    nlarge=nvals(in);
    Blarge=diag(-1*ones(nlarge-1,1),-1)+diag(-1*ones(nlarge-1,1),1)+diag(4*ones(nlarge,1),0);      %this must be diagonally dominant or else the method won't converge
    blarge=ones(nlarge,1);

    for irep=1:lrep      %benchmark will repeat the same solution several times to eliminate random variations from CPU load, etc.
        tstart=cputime;
        [Blargemod,ordlarge]=Gauss_elim(Blarge,blarge);
        xlarge=backsub(Blargemod(ordlarge,:));
        tend=cputime;
        testtimes(in)=testtimes(in)+(tend-tstart)/lrep;
    end %for
    disp([' GE solution for system of size ',num2str(nlarge),' takes average time ',num2str(testtimes(in)),' s']);
end %for

TT(:,1) = testtimes';

figure(1);
plot(nvals,testtimes,'-o','LineWidth',2,'MarkerSize',10,'MarkerEdgeColor','blue','MarkerFaceColor','blue')
xlabel('system size');
ylabel('time to solve (s)');
title('Empirically Determined Performance');
```

```
Start of tests of Gaussian-elimination scaling
 GE solution for system of size 1 takes average time 0.0015625 s
 GE solution for system of size 2 takes average time 0 s
 GE solution for system of size 3 takes average time 0 s
 GE solution for system of size 4 takes average time 0 s
 GE solution for system of size 5 takes average time 0 s
 GE solution for system of size 6 takes average time 0 s
 GE solution for system of size 7 takes average time 0 s
 GE solution for system of size 8 takes average time 0 s
 GE solution for system of size 9 takes average time 0 s
 GE solution for system of size 10 takes average time 0 s
 GE solution for system of size 11 takes average time 0.0015625 s
 GE solution for system of size 12 takes average time 0 s
 GE solution for system of size 13 takes average time 0 s
 GE solution for system of size 14 takes average time 0 s
 GE solution for system of size 15 takes average time 0 s
 GE solution for system of size 16 takes average time 0.0015625 s
 GE solution for system of size 17 takes average time 0 s
 GE solution for system of size 18 takes average time 0 s
 GE solution for system of size 19 takes average time 0.0015625 s
 GE solution for system of size 20 takes average time 0 s
 GE solution for system of size 21 takes average time 0.0015625 s
 GE solution for system of size 22 takes average time 0 s
 GE solution for system of size 23 takes average time 0.0015625 s
 GE solution for system of size 24 takes average time 0 s
 GE solution for system of size 25 takes average time 0 s
 GE solution for system of size 26 takes average time 0.0015625 s
 GE solution for system of size 27 takes average time 0.0015625 s
 GE solution for system of size 28 takes average time 0.0015625 s
 GE solution for system of size 29 takes average time 0 s
 GE solution for system of size 30 takes average time 0 s
 GE solution for system of size 31 takes average time 0.0015625 s
 GE solution for system of size 32 takes average time 0.0015625 s
 GE solution for system of size 33 takes average time 0.003125 s
 GE solution for system of size 34 takes average time 0.0015625 s
 GE solution for system of size 35 takes average time 0.0015625 s
 GE solution for system of size 36 takes average time 0.0015625 s
 GE solution for system of size 37 takes average time 0.0015625 s
 GE solution for system of size 38 takes average time 0.0015625 s
 GE solution for system of size 39 takes average time 0.003125 s
 GE solution for system of size 40 takes average time 0.010937 s
 GE solution for system of size 41 takes average time 0.00625 s
```

```
GE solution for system of size 42 takes average time 0.0046875 s
GE solution for system of size 43 takes average time 0.00625 s
GE solution for system of size 44 takes average time 0.0078125 s
GE solution for system of size 45 takes average time 0.003125 s
GE solution for system of size 46 takes average time 0.003125 s
GE solution for system of size 47 takes average time 0.0046875 s
GE solution for system of size 48 takes average time 0.0046875 s
GE solution for system of size 49 takes average time 0.0046875 s
GE solution for system of size 50 takes average time 0.0046875 s
```
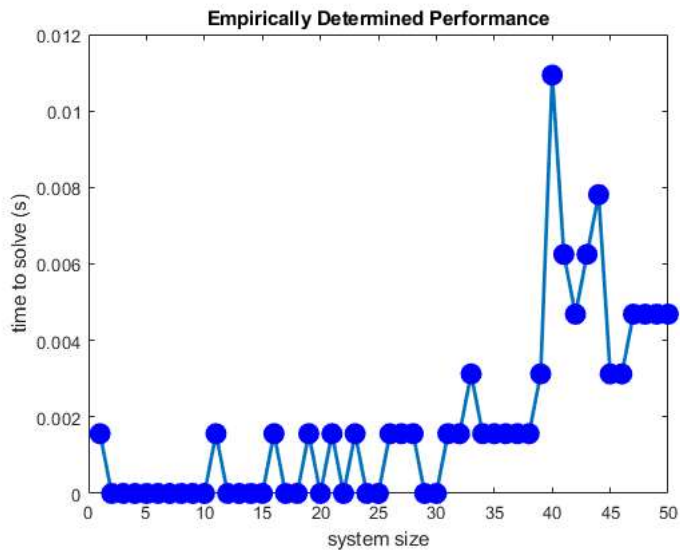

Jacobi iteration

```matlab
disp('Start of tests for Jacobi iteration');
tol=1e-9;
testtimes=zeros(size(nvals));
for in=1:numel(nvals)
    nlarge=nvals(in);
    Blarge=diag(-1*ones(nlarge-1,1),-1)+diag(-1*ones(nlarge-1,1),1)+diag(4*ones(nlarge,1),0);    %this must be diagonally dominant or else the method won't converge
    blarge=ones(nlarge,1);

    for irep=1:lrep      %benchmark will repeat the same solution several times to eliminate random variations from CPU load, etc.
        tstart=cputime;
        x0=randn(nlarge,1);
        [xit,iterations]=Jacobi(x0,Blarge,blarge,tol,false);
        tend=cputime;
        testtimes(in)=testtimes(in)+(tend-tstart)/lrep;
    end %for
    disp([' JI solution for system of size ',num2str(nlarge),' takes average time ',num2str(testtimes(in)),' s']);
end %for

TT(:,2) = testtimes';

figure(1);
hold on
plot(nvals,testtimes,'-^','LineWidth',2,'MarkerSize',10,'MarkerEdgeColor','green','MarkerFaceColor','green')
```
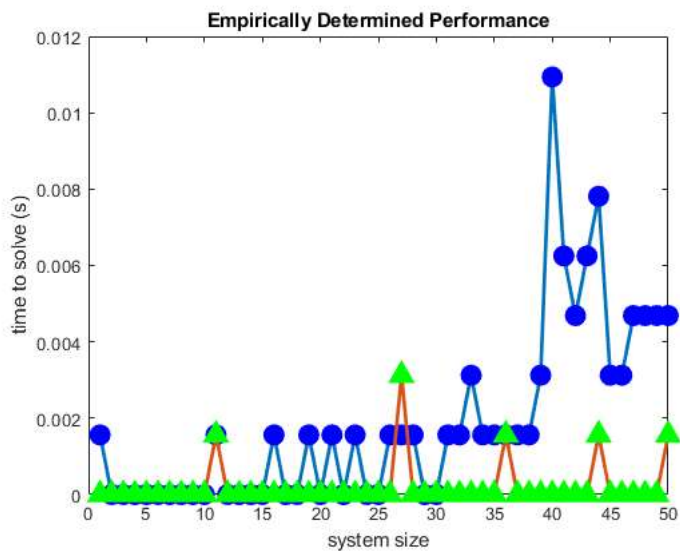
```
Start of tests for Jacobi iteration
 JI solution for system of size 1 takes average time 0 s
 JI solution for system of size 2 takes average time 0 s
 JI solution for system of size 3 takes average time 0 s
 JI solution for system of size 4 takes average time 0 s
 JI solution for system of size 5 takes average time 0 s
 JI solution for system of size 6 takes average time 0 s
 JI solution for system of size 7 takes average time 0 s
 JI solution for system of size 8 takes average time 0 s
 JI solution for system of size 9 takes average time 0 s
 JI solution for system of size 10 takes average time 0 s
 JI solution for system of size 11 takes average time 0.0015625 s
 JI solution for system of size 12 takes average time 0 s
 JI solution for system of size 13 takes average time 0 s
 JI solution for system of size 14 takes average time 0 s
 JI solution for system of size 15 takes average time 0 s
 JI solution for system of size 16 takes average time 0 s
 JI solution for system of size 17 takes average time 0 s
 JI solution for system of size 18 takes average time 0 s
 JI solution for system of size 19 takes average time 0 s
 JI solution for system of size 20 takes average time 0 s
 JI solution for system of size 21 takes average time 0 s
 JI solution for system of size 22 takes average time 0 s
 JI solution for system of size 23 takes average time 0 s
 JI solution for system of size 24 takes average time 0 s
```

```
JI solution for system of size 25 takes average time 0 s
JI solution for system of size 26 takes average time 0 s
JI solution for system of size 27 takes average time 0.003125 s
JI solution for system of size 28 takes average time 0 s
JI solution for system of size 29 takes average time 0 s
JI solution for system of size 30 takes average time 0 s
JI solution for system of size 31 takes average time 0 s
JI solution for system of size 32 takes average time 0 s
JI solution for system of size 33 takes average time 0 s
JI solution for system of size 34 takes average time 0 s
JI solution for system of size 35 takes average time 0 s
JI solution for system of size 36 takes average time 0.0015625 s
JI solution for system of size 37 takes average time 0 s
JI solution for system of size 38 takes average time 0 s
JI solution for system of size 39 takes average time 0 s
JI solution for system of size 40 takes average time 0 s
JI solution for system of size 41 takes average time 0 s
JI solution for system of size 42 takes average time 0 s
JI solution for system of size 43 takes average time 0 s
JI solution for system of size 44 takes average time 0.0015625 s
JI solution for system of size 45 takes average time 0 s
JI solution for system of size 46 takes average time 0 s
JI solution for system of size 47 takes average time 0 s
JI solution for system of size 48 takes average time 0 s
JI solution for system of size 49 takes average time 0 s
JI solution for system of size 50 takes average time 0.0015625 s
```


Thomas algorithm

```
disp('Start of tests for tridiag solver');
% tol=1e-9;
testtimes=zeros(size(nvals));
for in=1:numel(nvals)
    nlarge=nvals(in);
    Blarge=diag(-1*ones(nlarge-1,1),-1)+diag(-1*ones(nlarge-1,1),1)+diag(4*ones(nlarge,1),0);    %this must be diagonally dominant or else the method won't converge
    blarge=ones(nlarge,1);

    for irep=1:lrep    %benchmark will repeat the same solution several times to eliminate random variations from CPU load, etc.
        tstart=cputime;
        [xit,iterations]=tridiag(Blarge,blarge);
        tend=cputime;
        testtimes(in)=testtimes(in)+(tend-tstart)/lrep;
    end %for
    disp([' TA solution for system of size ',num2str(nlarge),' takes average time ',num2str(testtimes(in)),' s']);
end %for

TT(:,3) = testtimes';
disp('Time required for GE, JI, & TA (s)')
disp(TT)

figure(1);
hold on
plot(nvals,testtimes,'-s','LineWidth',2,'MarkerSize',10,'MarkerEdgeColor','red','MarkerFaceColor','red')
legend('Gauss elim.','Jacobi it.','Thomas algorithm')
```

```
Start of tests for tridiag solver
 TA solution for system of size 1 takes average time 0 s
 TA solution for system of size 2 takes average time 0 s
 TA solution for system of size 3 takes average time 0 s
 TA solution for system of size 4 takes average time 0 s
 TA solution for system of size 5 takes average time 0 s
```

```
TA solution for system of size 6 takes average time 0 s
TA solution for system of size 7 takes average time 0 s
TA solution for system of size 8 takes average time 0 s
TA solution for system of size 9 takes average time 0 s
TA solution for system of size 10 takes average time 0 s
TA solution for system of size 11 takes average time 0 s
TA solution for system of size 12 takes average time 0 s
TA solution for system of size 13 takes average time 0 s
TA solution for system of size 14 takes average time 0 s
TA solution for system of size 15 takes average time 0 s
TA solution for system of size 16 takes average time 0 s
TA solution for system of size 17 takes average time 0 s
TA solution for system of size 18 takes average time 0 s
TA solution for system of size 19 takes average time 0 s
TA solution for system of size 20 takes average time 0 s
TA solution for system of size 21 takes average time 0 s
TA solution for system of size 22 takes average time 0 s
TA solution for system of size 23 takes average time 0 s
TA solution for system of size 24 takes average time 0 s
TA solution for system of size 25 takes average time 0 s
TA solution for system of size 26 takes average time 0 s
TA solution for system of size 27 takes average time 0 s
TA solution for system of size 28 takes average time 0 s
TA solution for system of size 29 takes average time 0 s
TA solution for system of size 30 takes average time 0 s
TA solution for system of size 31 takes average time 0 s
TA solution for system of size 32 takes average time 0 s
TA solution for system of size 33 takes average time 0 s
TA solution for system of size 34 takes average time 0 s
TA solution for system of size 35 takes average time 0 s
TA solution for system of size 36 takes average time 0 s
TA solution for system of size 37 takes average time 0 s
TA solution for system of size 38 takes average time 0 s
TA solution for system of size 39 takes average time 0 s
TA solution for system of size 40 takes average time 0 s
TA solution for system of size 41 takes average time 0 s
TA solution for system of size 42 takes average time 0 s
TA solution for system of size 43 takes average time 0 s
TA solution for system of size 44 takes average time 0 s
TA solution for system of size 45 takes average time 0 s
TA solution for system of size 46 takes average time 0 s
TA solution for system of size 47 takes average time 0 s
TA solution for system of size 48 takes average time 0 s
TA solution for system of size 49 takes average time 0 s
TA solution for system of size 50 takes average time 0 s
Time required for GE, JI, & TA (s)
    0.0016        0         0
        0         0         0
        0         0         0
        0         0         0
        0         0         0
        0         0         0
        0         0         0
        0         0         0
        0         0         0
    0.0016    0.0016        0
        0         0         0
        0         0         0
        0         0         0
    0.0016        0         0
        0         0         0
        0         0         0
    0.0016        0         0
        0         0         0
    0.0016        0         0
        0         0         0
    0.0016        0         0
        0         0         0
        0         0         0
    0.0016        0         0
    0.0016    0.0031        0
    0.0016        0         0
        0         0         0
        0         0         0
    0.0016        0         0
    0.0016        0         0
    0.0031        0         0
    0.0016        0         0
    0.0016        0         0
    0.0016    0.0016        0
    0.0016        0         0
    0.0016        0         0
    0.0031        0         0
    0.0109        0         0
    0.0063        0         0
    0.0047        0         0
    0.0063        0         0
    0.0078    0.0016        0
    0.0031        0         0
```

```
0.0031        0        0
0.0047        0        0
0.0047        0        0
0.0047        0        0
0.0047   0.0016        0
```

Empirically Determined Performance