Trajectory Generation

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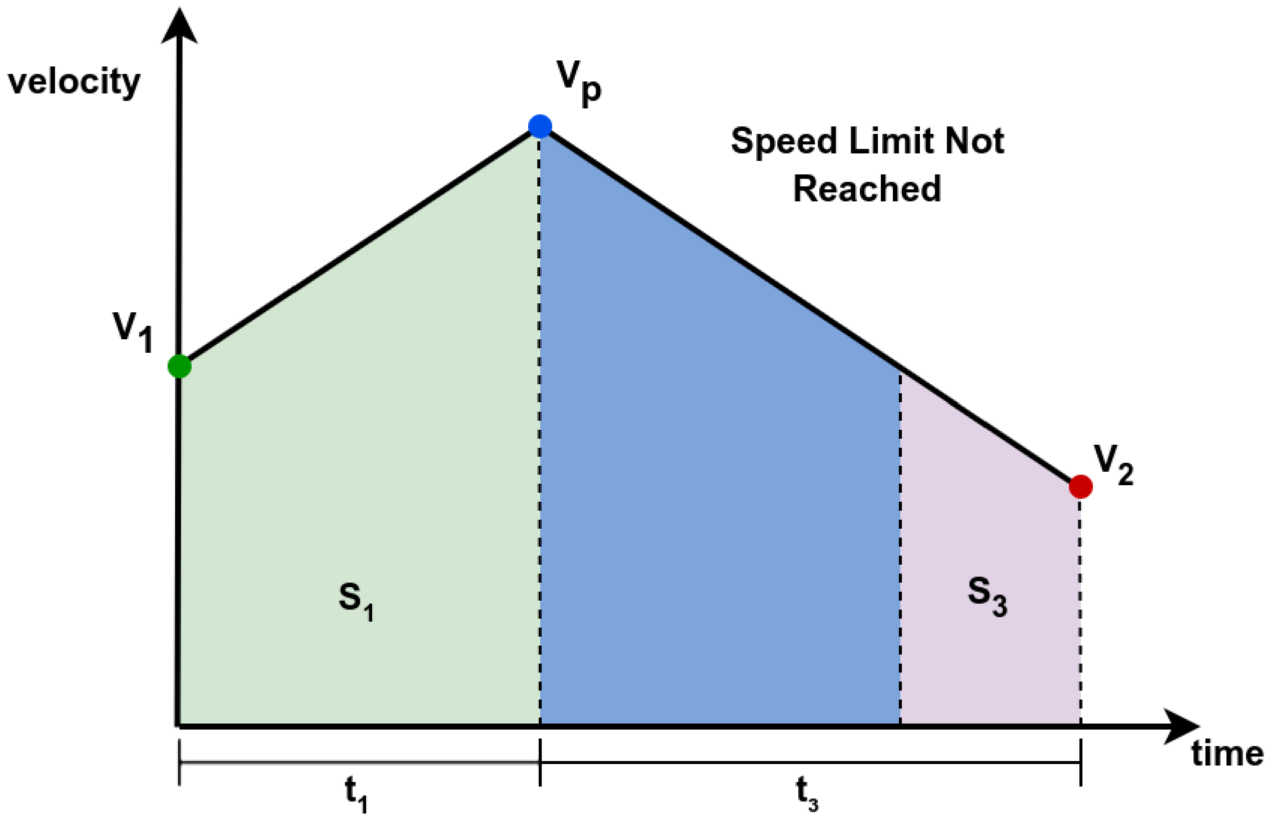
Trapezoidal

Ignoring edge cases for now, a typical straight line motion trajectory has a profile that looks like this.



We have a starting velocity,v_1, and an end velocity, v_2.

Assuming the speed does not reach a limit, it will peak at v_p after a time, t_1.

In the illustration the end speed is lower than the start speed. If that is not the case, the figure and the working below can simply be reversed.

The profile velocity, after reaching its peak will pass back through v_1 again before reaching v_2. During that period it is easy to calculate the distance travelled, S_3.

$$S_3 = \frac{(v_1^2 - v_2^2)}{2a} \tag{1}$$

For the profile generation, we need to calculate the time at which acceleration stops and deceleration begins. This is t_1

During that acceleration period, the distance travelled is S_1 (shaded in green) which can be calculated as:

$$S_1 = v_1 t_1 + \frac{1}{2} a t_1^2 \tag{2}$$

The period during which the speed reduces once more to v_1 is symmetrical about t_1 and so covers the same distance. Thus, the distance moved during the green and blue sections together is just $2S_1$

$$2S_1 = 2v_1 t_1 + a t_1^2 \tag{3}$$

Since we know the total distance, S, to be covered by the profile, we can write (3) in terms of things we know

$$2S_1 = S - S_3 = 2v_1 t_1 + a t_1^2 \tag{4}$$

Rearrange to get the quadratic equation in t_1:

$$a t_1^2 + 2v_1 t_1 + S_3 - S = 0 \tag{5}$$

Substitute for S_3 from (1):

$$a t_1^2 + 2v_1 t_1 + \frac{(v_1^2 - v_2^2)}{2a} - S = 0 \tag{6}$$

This can be solved in the normal way for quadratics by equating terms. Thus, if

$$Ax^2 + Bx + C = 0$$

has solutions:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

then we have for t_1:

$$t_1 = \frac{-2v_1 \pm \sqrt{4v_1^2 - 4a\left(\frac{(v_1^2 - v_2^2)}{2a} - S\right)}}{2a}$$

$$t_1 = \frac{-2v_1 \pm \sqrt{4v_1^2 - 2v_1^2 + 2v_2^2 + 4aS}}{2a}$$

$$t_1 = \frac{-2v_1 \pm \sqrt{2v_1^2 + 2v_2^2 + 4aS}}{2a} \tag{7}$$

Since the time taken to move the distance S_3 at the end it eay to calculate, $t_3 = (v_1 - v_2)/a$, we can easily deduce that t_3 is

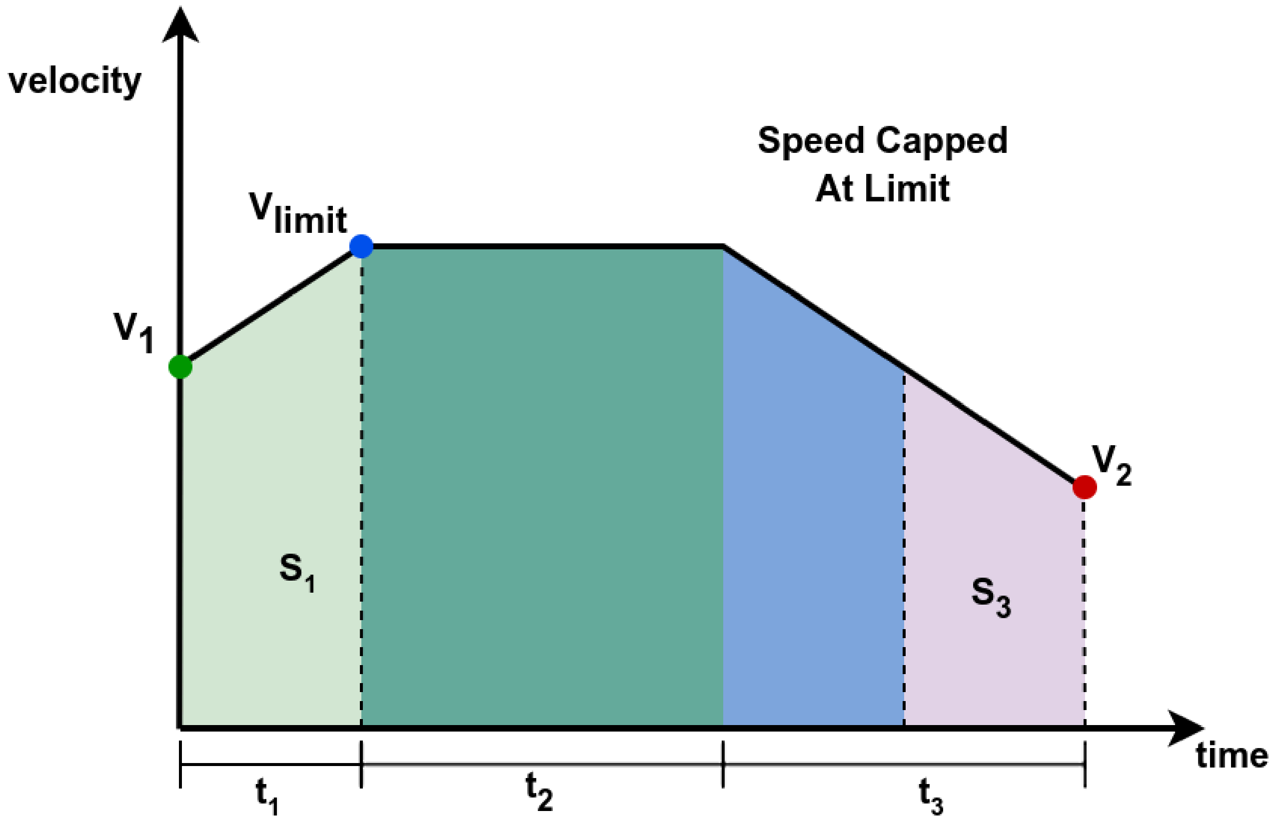
$$t_3 = t_1 + \frac{v_1 - v_2}{a} \tag{8}$$

Ensuring accuracy

In the code we can only perform calculations to update the velocity at fixed time intervals of dt so once the etimate of t_1 and t_3 have been made, the value used for the acceleration, a, is recalculated to quarantee no rounding errors.

Speed Limiting

If you were wondering what happened to t_2, consider the case where a limit is placed on the maximum achievable speed.



Now we must re-calculate the times to account for the coasting period where velocity is constant. This is done in the code and the derivation i left a an exercise for the reader.