PROBLEMS

5.1 Determine the real roots of $f(x) = -0.5x^2 + 2.5x + 4.5$:
(a) Graphically.
(b) Using the quadratic formula.
(c) Using three iterations of the bisection method to determine the highest root. Employ initial guesses of $x_l = 5$ and $x_u = 10$. Compute the estimated error ε_a and the true error ε_t after each iteration.

5.2 Determine the real root of $f(x) = 5x^3 - 5x^2 + 6x - 2$:
(a) Graphically.
(b) Using bisection to locate the root. Employ initial guesses of $x_l = 0$ and $x_u = 1$ and iterate until the estimated error ε_a falls below a level of $\varepsilon_s = 10\%$.

5.3 Determine the real root of $f(x) = -25 + 82x - 90x^2 + 44x^3 - 8x^4 + 0.7x^5$:
(a) Graphically.
(b) Using bisection to determine the root to $\varepsilon_s = 10\%$. Employ initial guesses of $x_l = 0.5$ and $x_u = 1.0$.
(c) Perform the same computation as in (b) but use the false-position method and $\varepsilon_s = 0.2\%$.

5.4 (a) Determine the roots of $f(x) = -12 - 21x + 18x^2 - 2.75x^3$ graphically. In addition, determine the first root of the function with (b) bisection, and (c) false position. For (b) and (c) use initial guesses of $x_l = -1$ and $x_u = 0$, and a stopping criterion of 1%.

5.10 Find the positive real root of $f(x) = x^4 - 8x^3 - 35x^2 + 450x - 1001$ using the false-position method. Use initial guesses of $x_l = 4.5$ and $x_u = 6$ and perform five iterations. Compute both the true and approximate errors based on the fact that the root is 5.60979. Use a plot to explain your results and perform the computation to within $\varepsilon_s = 1.0\%$.

5.11 Determine the real root of $x^{3.5} = 80$: (a) analytically and (b) with the false-position method to within $\varepsilon_s = 2.5\%$. Use initial guesses of 2.0 and 5.0.

5.12 Given

$$f(x) = -2x^6 - 1.5x^4 + 10x + 2$$

Use bisection to determine the *maximum* of this function. Employ initial guesses of $x_l = 0$ and $x_u = 1$, and perform iterations until the approximate relative error falls below 5%.

5.13 The velocity v of a falling parachutist is given by

$$v = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$$

where $g = 9.81\,\mathrm{m/s^2}$. For a parachutist with a drag coefficient $c = 15$ kg/s, compute the mass m so that the velocity is $v = 36$ m/s at $t = 10$ s. Use the false-position method to determine m to a level of $\varepsilon_s = 0.1\%$.

5.5 Locate the first nontrivial root of $\sin x = x^2$ where x is in radians. Use a graphical technique and bisection with the initial interval from 0.5 to 1. Perform the computation until ε_a is less than $\varepsilon_s = 2\%$. Also perform an error check by substituting your final answer into the original equation.

5.6 Determine the positive real root of $\ln(x^2) = 0.7$ (**a**) graphically, (**b**) using three iterations of the bisection method, with initial guesses of $x_l = 0.5$ and $x_u = 2$, and (**c**) using three iterations of the false-position method, with the same initial guesses as in (**b**).

5.7 Determine the real root of $f(x) = (0.8 - 0.3x)/x$:

(**a**) Analytically.

(**b**) Graphically.

(**c**) Using three iterations of the false-position method and initial guesses of 1 and 3. Compute the approximate error ε_a and the true error ε_t after each iteration. Is there a problem with the result?

5.8 Find the positive square root of 18 using the false-position method to within $\varepsilon_s = 0.5\%$. Employ initial guesses of $x_l = 4$ and $x_u = 5$.

5.9 Find the smallest positive root of the function (x is in radians) $x^2|\cos\sqrt{x}| = 5$ using the false-position method. To locate the region in which the root lies, first plot this function for values of x between 0 and 5. Perform the computation until ε_a falls below $\varepsilon_s = 1\%$. Check your final answer by substituting it into the original function.

5.14 Use bisection to determine the drag coefficient needed so that an 82-kg parachutist has a velocity of 36 m/s after 4 s of free fall. Note: The acceleration of gravity is 9.81 m/s^2. Start with initial guesses of $x_l = 3$ and $x_u = 5$ and iterate until the approximate relative error falls below 2%. Also perform an error check by substituting your final answer into the original equation.

5.15 As depicted in Fig. P5.15, the velocity of water, υ (m/s), discharged from a cylindrical tank through a long pipe can be computed as

$$\upsilon = \sqrt{2gH}\tanh\left(\frac{\sqrt{2gH}}{2L}t\right)$$

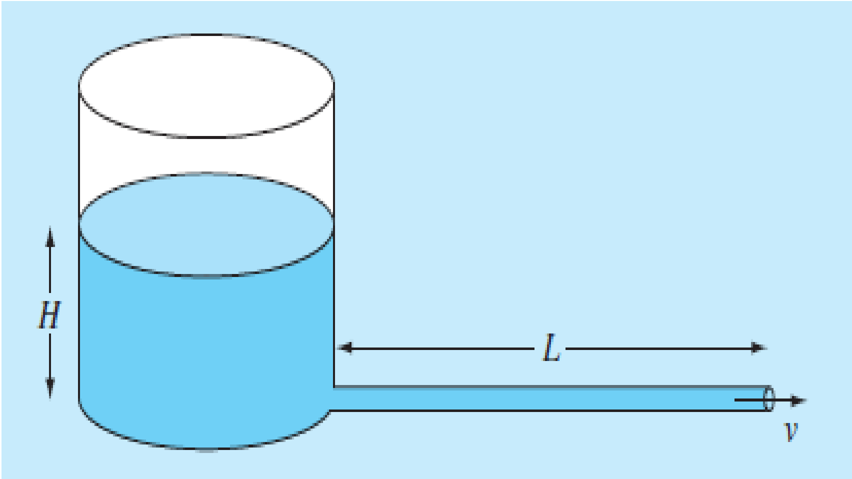

FIGURE P5.15

where $g = 9.81$ m/s^2, H = initial head (m), L = pipe length (m), and t = elapsed time (s). Determine the head needed to achieve $v = 5$ m/s in 2.5 s for a 4-m-long pipe (**a**) graphically, (**b**) by bisection, and (**c**) with false position. Employ initial guesses of $x_l = 0$ and $x_u = 2$ m with a stopping criterion of $\varepsilon_s = 1\%$. Check you results.

5.16 Water is flowing in a trapezoidal channel at a rate of $Q = 20$ m^3/s. The critical depth y for such a channel must satisfy the equation

$$0 = 1 - \frac{Q^2}{gA_c^3}B$$

where $g = 9.81$ m/s^2, A_c = the cross-sectional area (m^2), and B = the width of the channel at the surface (m). For this case, the width and the cross-sectional area can be related to depth y by

$$B = 3 + y \qquad \text{and} \qquad A_c = 3y + \frac{y^2}{2}$$

Solve for the critical depth using (**a**) the graphical method, (**b**) bisection, and (**c**) false position. For (**b**) and (**c**) use initial guesses of $x_l = 0.5$ and $x_u = 2.5$, and iterate until the approximate error falls below 1% or the number of iterations exceeds 10. Discuss your results.

your answer. Determine the approximate relative error after each iteration. Employ initial guesses of 0 and R.

5.18 The saturation concentration of dissolved oxygen in freshwater can be calculated with the equation (APHA, 1992)

$$\ln o_{sf} = -139.34411 + \frac{1.575701 \times 10^5}{T_a} - \frac{6.642308 \times 10^7}{T_a^2} + \frac{1.243800 \times 10^{10}}{T_a^3} - \frac{8.621949 \times 10^{11}}{T_a^4}$$

where o_{sf} = the saturation concentration of dissolved oxygen in freshwater at 1 atm (mg/L) and T_a = absolute temperature (K). Remember that $T_a = T + 273.15$, where T = temperature (°C). According to this equation, saturation decreases with increasing temperature. For typical natural waters in temperate climates, the equation can be used to determine that oxygen concentration ranges from 14.621 mg/L at 0°C to 6.413 mg/L at 40°C. Given a value of oxygen concentration, this formula and the bisection method can be used to solve for temperature in °C.

(**a**) If the initial guesses are set as 0 and 40°C, how many bisection iterations would be required to determine temperature to an absolute error of 0.05°C?

5.17 You are designing a spherical tank (Fig. P5.17) to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

$$V = \pi h^2 \frac{[3R - h]}{3}$$

where V = volume (m^3), h = depth of water in tank (m), and R = the tank radius (m).

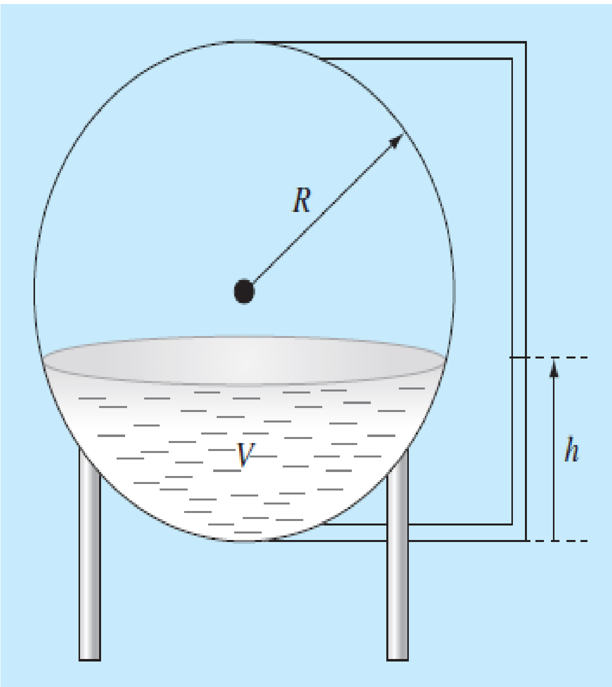

FIGURE P5.17

If R = 3 m, to what depth must the tank be filled so that it holds 30 m^3? Use three iterations of the false-position method to determine

(b) Develop and test a bisection program to determine T as a function of a given oxygen concentration to a prespecified absolute error as in (a). Given initial guesses of 0 and 40°C, test your program for an absolute error = 0.05°C and the following cases: o_{sf} = 8, 10, and 12 mg/L. Check your results.

5.19 According to *Archimedes principle*, the *buoyancy* force is equal to the weight of fluid displaced by the submerged portion of an object. For the sphere depicted in Fig. P5.19, use bisection to determine the height h of the portion that is above water. Employ the following values for your computation: r = 1 m, ρ_s = density of sphere = 200 kg/m^3, and ρ_w = density of water = 1000 kg/m^3. Note that the volume of the above-water portion of the sphere can be computed with

$$V = \frac{\pi h^2}{3}(3r - h)$$

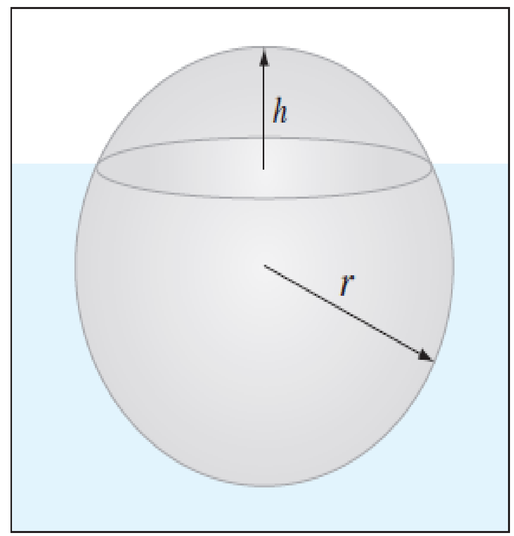

FIGURE P5.19