AI SCHOOL CHINA FY22

[AI + Game – Class No. 1]

From OR to RL

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What is Operations Research?

Past, now and future

“A discipline that deals with the **development** and **application** of advanced **analytical** methods to improve **decision-making**” ——Wikipedia

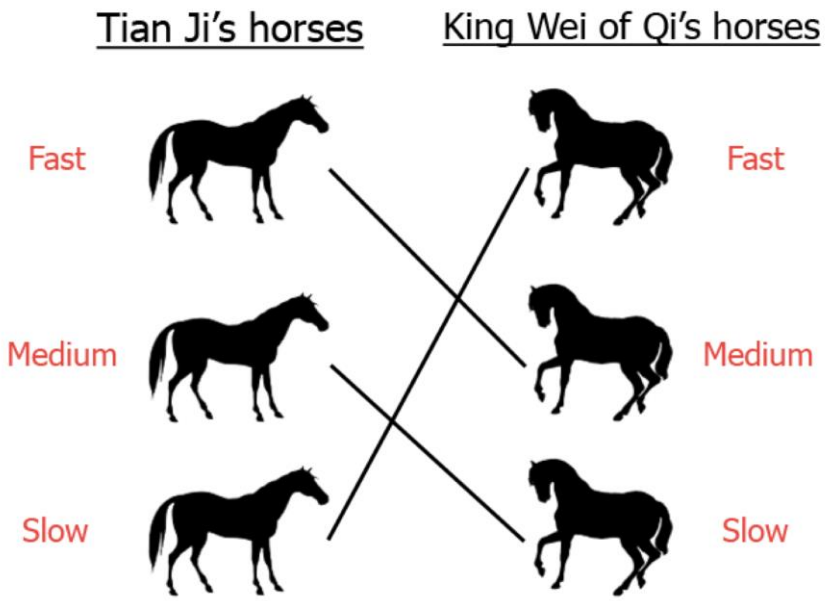
主要研究人类对各种资源的运用及筹划活动，以期通过了解和发展这种运用及筹划活动的基本规律，发挥有限资源的最大效益，达到总体最优的目标。

——运筹学发展的回顾与展望，《中国科学院院刊》

“运筹帷幄之中，决胜千里之外”

——《史记·高祖本纪》

田忌赛马



目标：赢的场次最多

资源：三匹不同等级的马

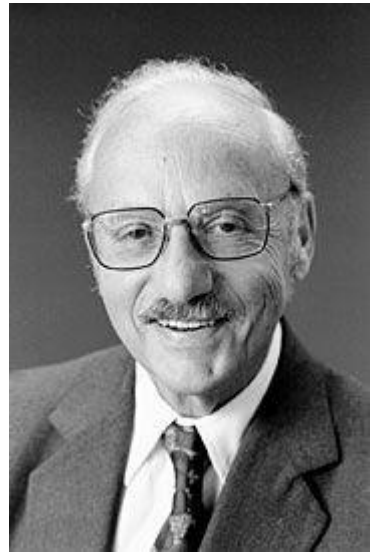
可行域：每匹马都要比赛一次

“Think in an optimization way”

Formal Origin of Operations Research

- British Army in World-War II
  - Objective: win the war
  - Resource: food, medicine, human power, etc.
  - Constraints: limited resource

- George Bernard Dantzig (1914-2005)
  - Invented “Simplex Method" to get **exact** solutions for linear programming problems



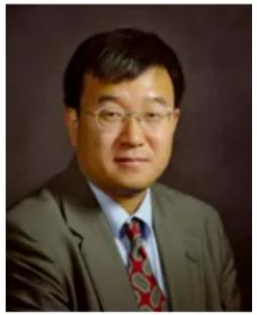
Operations Research (1945- 1975)

- Grow in many areas other than the military
- Can solve problems with hundreds of thousands of variables and constraints
- Application areas: city planning, football strategies, emergency planning, counter-terrorist attach planning

INFORMS

- Institute for Operations Research and the Management Sciences
- Only 21 Chinese out of 397 Fellows, none is located in China mainland

2008 INFORMS Fellows



Jianjun (Jan) Shi

For development of stream-of-variation theory and its contributions to quality improvements in manufacturing and other complex systems, and for leadership in education and the INFORMS community.

2009 INFORMS Fellows

C. F. Jeff Wu

For developing statistical methodologies and novel applications to engineering, and for leadership making statistical methods and thinking popular in engineering.

2018 INFORMS Fellows

Zuo-Jun (Max) Shen

For research contributions to supply chain management broadly and to inventory management and facility location specifically and to the education of exceptional graduate students.

INFORMS: Franz Edelman Award

- Nearly $250 billion in benefits have been tabulated among Franz Edelman Award finalists teams.

The finalists for the 2018 Edelman Award are:

China National Petroleum Corporation (CNPC). China's natural gas consumption has nearly doubled over the past five years. To better meet demand, CNPC, China's largest oil and natural gas producer and supplier, partnered with researchers from the University of California, Berkeley and Tsinghua University to develop and implement a new software that optimized the country's natural gas pipeline. Previously, all annual planning for CNPC, which controls 75 percent of the country's natural gas resources and pipeline network, was manually conducted using spreadsheets. However, the increasing complexity and size of China's natural gas pipeline meant the previous method had large measures of error, increasing costs and wasting resources. Since implementing its new systems at the end of 2014, CNPC has realized approximately $330 million in direct savings for CNPC, and the increased efficiency of the pipeline has enabled it to delay further pipeline expansions saving billions of additional dollars.

INFORMS: Franz Edelman Award

FINALISTS

Team: ***China National Petroleum Corporation***

Paper: "Natural Gas Pipeline Transmission Optimization for China National Petroleum Corporation"

Laureates: ***Tianhu Deng***, Tsinghua University

Jingkuan Han, China Petroleum Planning and Engineering

Dingzhi Liu, China Petroleum Planning and Engineering

Zuo-jun (Max) Shen, University of California, Berkeley/ Tsinghua University

Mengying Xue, Tsinghua University

Junchi Ye, Tsinghua University

Yanfang Zhao, China Petroleum Planning and Engineering Institute

Zhongde Zhao, China Petroleum Planning and Engineering Institute

Shuhui Zhou, China Petroleum Planning and Engineering Institute

Techniques in OR

- Linear Programming
- Integer Programming
- Nonlinear Programming
- Fuzzy Programming
- Graph and Network Theory
- Decision Theory
- Gaming Theory

Diet Problem -- Example of Linear Programming

Suppose you can only buy potatoes and steak in your community during a quarantine. You have limited budget, so you must decide how much of each food to buy, considering the nutrition objective and minimising the budget.

	Per unit of potatoes	Per unit of steak	Minimum requirements
Units of carbohydrates	3	1	8
Units of vitamins	4	3	19
Units of proteins	1	3	7
Unit cost	25	50	

Diet Problem -- Example of Linear Programming

Objective: minimize the total cost of buying food

Constraints: minimum requirement for carbohydrates, vitamins and proteins

	Per unit of potatoes	Per unit of steak	Minimum requirements
Units of carbohydrates	3	1	8
Units of vitamins	4	3	19
Units of proteins	1	3	7
Unit cost	25	50	

Diet Problem -- Example of Linear Programming

$$Minimize \quad 25X_1 + 50X_2$$

s.t.

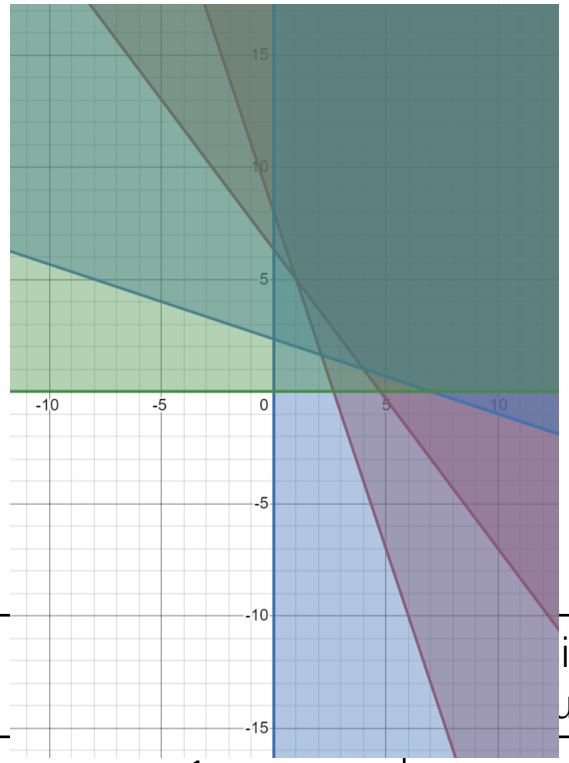
$$3X_1 + X_2 \geq 8$$

$$4X_1 + 3X_2 \geq 19$$

$$X_1 + 3X_2 \geq 7$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$



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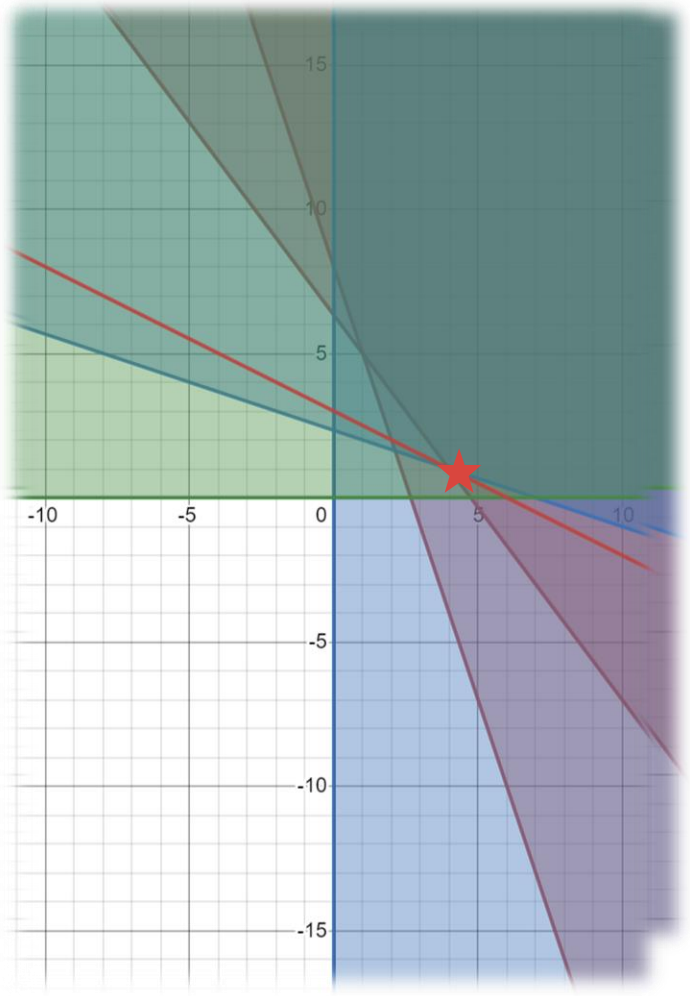
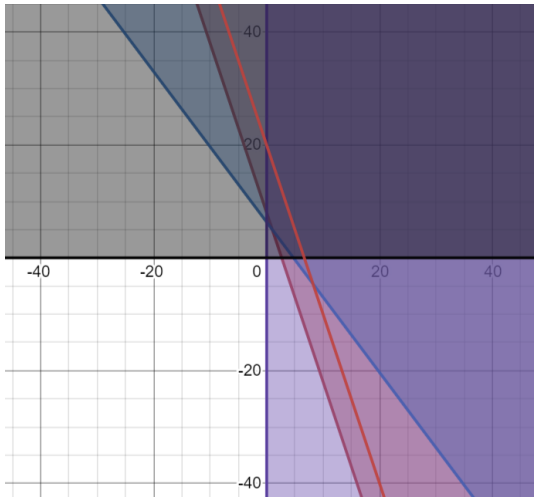


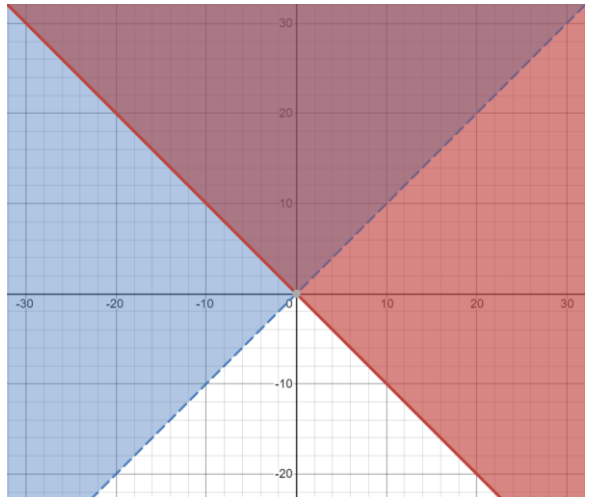
Solution:

$X_1 = 4, \quad X_2 = 1$

Solution of Linear Programming in a Graph

Solution Status	Scenario
One unique solution	One point of intersection
Infinite solutions	All the points on a segment are solutions
No feasible solution	Feasible domain is empty
Unbounded	Unbounded objective





Other Methods for Linear Programming

- Simplex Method by Dantzig
- Interior Point Method
- Ellipsoid Algorithm
- ……

Ultimate Method: Use Software!

- Cplex, LINDO, MATLAB, R, Excel ……

Other Programming Problems

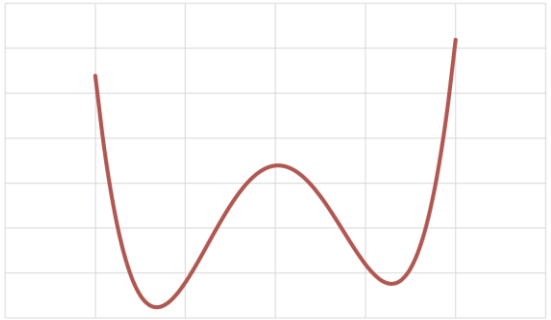
- Semidefinite Programming
- Conic Optimization
- Geometric Programming

Convex Optimization

Non-Convex Optimization

Combinatorial Optimization

- (Mixed) Integer Programming
- Most discrete optimization problems



Models in Practice

- Allocation models
- Inventory models
- Replacement models
- Competitive (Game Theory) models
- Waiting line or queueing models
- Network models
- Simulation Models

Back to the Diet Problem

Possible variants:

- Steak price changes to 40 **Sensitivity Analysis**
- Only integer number of steak can be purchased **Mixed Integer Programming**
- The unit price of potatoes decreases 0.5 for every extra unit purchased **Quadratic Programming**

	Per unit of potatoes	Per unit of steak	Minimum requirements
Units of carbohydrates	3	1	8
Units of vitamins	4	3	19
Units of proteins	1	3	7
Unit cost	25	~~50~~ 40	

New Challenges

If the price of steak is not fixed, it follows a Gaussian Distribution with mean 50 and standard deviation 10, how will you adjust your plan?

Optimization under Uncertainty
(Robust Optimization)

	Per unit of potatoes	Per unit of steak	Minimum requirements
Units of carbohydrates	3	1	8
Units of vitamins	4	3	19
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Unit cost	25	50	

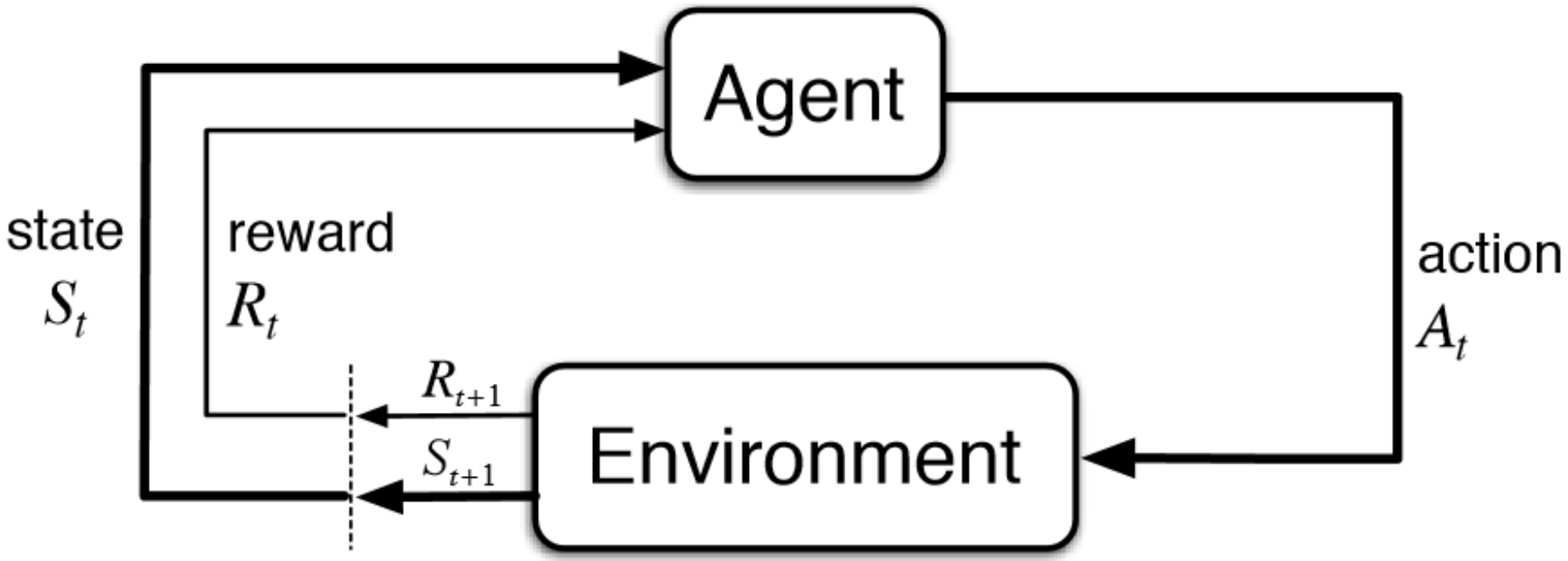
New Challenges

If the price of potatoes and steak would change by each hour, and you do not know the price unless you go to the store to check, how to modify your purchase plan?

Optimization under Dynamic Setting

	Per unit of potatoes	Per unit of steak	Minimum requirements
Units of carbohydrates	3	1	8
Units of vitamins	4	3	19
Units of proteins	1	3	7
Unit cost	25	50	

Reinforcement Learning



Similarities between OR and RL

- Three elements in OR
  - Constraints
  - Objective
  - Decision variables
- Correspondence in RL
  - State space
  - Outcome (reward) space
  - Action space

Difference between OR and RL

	OR	RL
Foundation	Math Theory	Application Scenarios
Setting	Static-based[1]	Dynamic-based
Solution	Exact	Not-guaranteed
Explanability	Easy	Hard
Data Requirement	Low	High
Computational Cost	Problem-specific	Problem-specific

[1]Dynamic Programming and Approximated Dynamic Programming are also investigated in OR, here we focus on classic solutions.

Opportunities brought by RL

- Incorporate different types of information into decision-making
- Interpretable RL, Safe RL and RL Ethics
- Theoretic guarantees of RL algorithms

[1]Dynamic Programming and Approximated Dynamic Programming are also investigated in OR, here we focus on classic solutions.

Thank you!