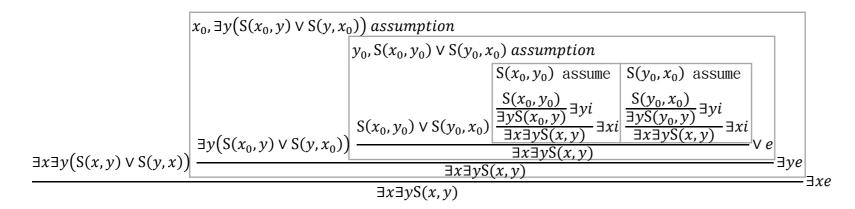
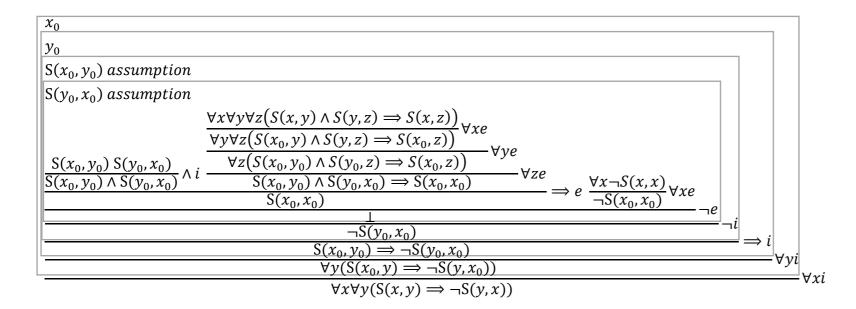
學號:B06902136 系級:資工四 姓名:賴冠毓

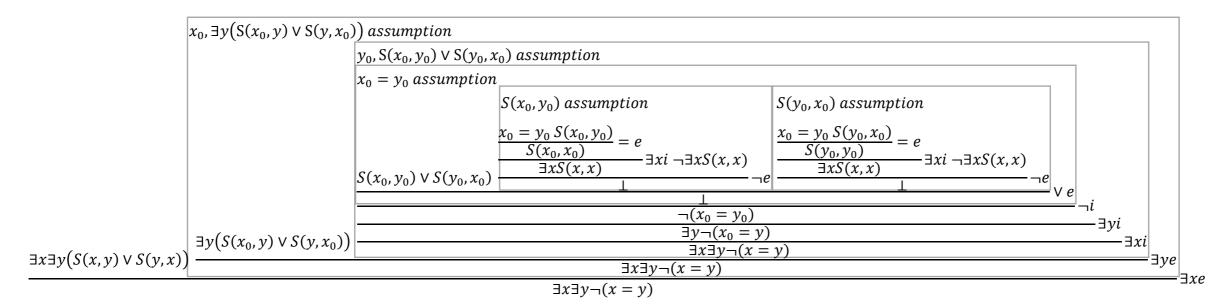
## $(1) \exists x \exists y (S(x,y) \lor S(y,x)) \vdash \exists x \exists y S(x,y)$



## $(2) \forall x \forall y \forall z \big( S(x,y) \land S(y,z) \Longrightarrow S(x,z) \big), \forall x \neg S(x,x) \vdash \forall x \forall y (S(x,y) \Longrightarrow \neg S(y,x))$



 $(3) \exists x \exists y (S(x,y) \lor S(y,x)), \neg \exists x S(x,x) \vdash \exists x \exists y \neg (x=y)$ 



(4) No natural deduction proof for  $\forall x (P(x) \lor Q(x)) \vdash \forall x P(x) \lor \forall x Q(x)$ 

考慮一個丟硬幣問題:X表示丟一次硬幣;P(X)表示這次結果為數字面;Q(X)表示這次結果為頭像面

 $\forall x (P(x) \lor Q(x))$ : 每次丢的結果是數字面或頭像面其中一個 => True

∀xP(x) V ∀xQ(x): 每次丢的結果都是數字面或每次都是頭像面 ⇒ 不一定

雨者結果不同 => no natural deduction proof

 $(5) \forall x \neg \varphi \models \neg \exists x \varphi \text{ (Semantically)}$ 

Let  $\mathcal{M}$  be a model that  $\mathcal{M} \models \forall x \neg \varphi => \forall t \in \mathcal{M}, \neg \varphi(t/x)$  holds.

Suppose that  $\mathcal{M} \not\models \neg \exists x \varphi => \mathcal{M} \models \exists x \varphi => \text{ there exists some } t \in \mathcal{M}, \varphi(t/x) \text{ holds.}$ 

=> 矛盾!:: *M* ⊨ ¬∃xф

 $\Rightarrow \forall x \neg \varphi \models \neg \exists x \varphi$