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(1) $\exists x\exists y(S(x,y)\vee S(y,x))\vdash\exists x\exists yS(x,y)$

$$\dfrac{\exists x\exists y\big(S(x,y)\vee S(y,x)\big)\quad \boxed{\begin{array}{l} x_0,\exists y\big(S(x_0,y)\vee S(y,x_0)\big)\ \textit{assumption} \\ \dfrac{\exists y\big(S(x_0,y)\vee S(y,x_0)\big)\quad \boxed{\begin{array}{l} y_0, S(x_0,y_0)\vee S(y_0,x_0)\ \textit{assumption} \\ \dfrac{S(x_0,y_0)\vee S(y_0,x_0)\quad \boxed{\begin{array}{l} S(x_0,y_0)\ \text{assume} \\ \dfrac{\dfrac{S(x_0,y_0)}{\exists yS(x_0,y)}\exists yi}{\exists x\exists yS(x,y)}\exists xi \end{array}}\boxed{\begin{array}{l} S(y_0,x_0)\ \text{assume} \\ \dfrac{\dfrac{S(y_0,x_0)}{\exists yS(y_0,y)}\exists yi}{\exists x\exists yS(x,y)}\exists xi \end{array}}}{\exists x\exists yS(x,y)}\vee e \end{array}}}{\exists x\exists yS(x,y)}\exists ye \end{array}}}{\exists x\exists yS(x,y)}\exists xe$$

(2) $\forall x\forall y\forall z\big(S(x,y)\wedge S(y,z)\Longrightarrow S(x,z)\big),\forall x\neg S(x,x)\vdash\forall x\forall y(S(x,y)\Longrightarrow\neg S(y,x))$

$$\dfrac{\boxed{\begin{array}{l} x_0 \\ \dfrac{\boxed{\begin{array}{l} y_0 \\ \dfrac{\boxed{\begin{array}{l} S(x_0,y_0)\ \textit{assumption} \\ \dfrac{\boxed{\begin{array}{l} S(y_0,x_0)\ \textit{assumption} \\ \dfrac{\dfrac{\dfrac{S(x_0,y_0)\ S(y_0,x_0)}{S(x_0,y_0)\wedge S(y_0,x_0)}\wedge i\quad \dfrac{\dfrac{\dfrac{\forall x\forall y\forall z\big(S(x,y)\wedge S(y,z)\Longrightarrow S(x,z)\big)}{\forall y\forall z\big(S(x_0,y)\wedge S(y,z)\Longrightarrow S(x_0,z)\big)}\forall xe}{\forall z\big(S(x_0,y_0)\wedge S(y_0,z)\Longrightarrow S(x_0,z)\big)}\forall ye}{S(x_0,y_0)\wedge S(y_0,x_0)\Longrightarrow S(x_0,x_0)}\forall ze}{S(x_0,x_0)}\Longrightarrow e\quad \dfrac{\forall x\neg S(x,x)}{\neg S(x_0,x_0)}\forall xe}{\bot}\neg e \end{array}}}{\neg S(y_0,x_0)}\neg i \end{array}}}{S(x_0,y_0)\Longrightarrow\neg S(y_0,x_0)}\Longrightarrow i \end{array}}}{\forall y(S(x_0,y)\Longrightarrow\neg S(y,x_0))}\forall yi \end{array}}}{\forall x\forall y(S(x,y)\Longrightarrow\neg S(y,x))}\forall xi$$

(3) $\exists x \exists y \big(S(x,y) \vee S(y,x)\big), \neg \exists x S(x,x) \vdash \exists x \exists y \neg (x=y)$

$x_0, \exists y\big(S(x_0,y) \vee S(y,x_0)\big)$ *assumption*

$y_0, S(x_0,y_0) \vee S(y_0,x_0)$ *assumption*

$x_0 = y_0$ *assumption*

$S(x_0,y_0)$ *assumption*

$$\cfrac{\cfrac{\cfrac{x_0 = y_0 \quad S(x_0,y_0)}{S(x_0,x_0)} = e}{\exists x S(x,x)} \exists x i \quad \neg\exists x S(x,x)}{\bot} \neg e$$

$S(y_0,x_0)$ *assumption*

$$\cfrac{\cfrac{\cfrac{x_0 = y_0 \quad S(y_0,x_0)}{S(y_0,y_0)} = e}{\exists x S(x,x)} \exists x i \quad \neg\exists x S(x,x)}{\bot} \neg e$$

$$\cfrac{S(x_0,y_0) \vee S(y_0,x_0) \quad \bot \quad \bot}{\bot} \vee e$$

$$\cfrac{\cfrac{\cfrac{\cfrac{\bot}{\neg(x_0 = y_0)} \neg i}{\exists y \neg(x_0 = y)} \exists y i}{\exists x \exists y \neg(x = y)} \exists x i}{}$$

$$\cfrac{\exists y\big(S(x_0,y) \vee S(y,x_0)\big) \quad \exists x \exists y \neg(x = y)}{\exists x \exists y \neg(x = y)} \exists y e$$

$$\cfrac{\exists x \exists y\big(S(x,y) \vee S(y,x)\big) \quad \exists x \exists y \neg(x = y)}{\exists x \exists y \neg(x = y)} \exists x e$$

(4) No natural deduction proof for $\forall x(P(x) \vee Q(x)) \vdash \forall x P(x) \vee \forall x Q(x)$

考慮一個丟硬幣問題: x 表示丟一次硬幣；P(x)表示這次結果為數字面；Q(x)表示這次結果為頭像面

$\forall x\big(P(x) \vee Q(x)\big)$: 每次丟的結果是數字面或頭像面其中一個 => True

$\forall x P(x) \vee \forall x Q(x)$: 每次丟的結果都是數字面或每次都是頭像面 => 不一定

兩者結果不同 => no natural deduction proof

(5) $\forall x \neg \phi \vDash \neg \exists x \phi$ (Semantically)

Let $\mathcal{M}$ be a model that $\mathcal{M} \vDash \forall x \neg\phi$ => $\forall t \in \mathcal{M}, \neg\phi(t/x)$ holds.

Suppose that $\mathcal{M} \nvDash \neg\exists x\phi$ => $\mathcal{M} \vDash \exists x \phi$ => there exists some $t \in \mathcal{M}, \phi(t/x)$ holds.

=> 矛盾! $\therefore \mathcal{M} \vDash \neg\exists x\phi$

=> $\forall x \neg\phi \vDash \neg\exists x\phi$