

**INTRODUCTION TO COMPUTATIONAL LOGIC**  
**HOMEWORK 3**  
**DUE DATE: NOVEMBER 4, 2020**

Let  $S$  be a binary predicate symbol,  $P$  and  $Q$  unary predicate symbols.

- (1) Find a natural deduction proof to show

$$\exists x \exists y (S(x, y) \vee S(y, x)) \vdash \exists x \exists y S(x, y).$$

- (2) Find a natural deduction proof to show

$$\forall x \forall y \forall z (S(x, y) \wedge S(y, z) \implies S(x, z)), \forall x \neg S(x, x) \vdash \forall x \forall y (S(x, y) \implies \neg S(y, x)).$$

- (3) Find a natural deduction proof to show

$$\exists x \exists y (S(x, y) \vee S(y, x)), \neg \exists x S(x, x) \vdash \exists x \exists y \neg (x = y).$$

- (4) Show that there is no natural deduction proof for

$$\forall x (P(x) \vee Q(x)) \vdash \forall x P(x) \vee \forall x Q(x).$$

- (5) Semantically show

$$\forall x \neg \phi \models \neg \exists x \phi.$$