

**CSci 4270 and 6270**  
**Computational Vision,**  
**Spring Semester 2021**  
**Homework 3**

**Due: Friday, March 5 at 11:59:59 pm EST**

Please refer back to HW 1 for guidelines.

This homework is divided into two parts. The first part consists of three programming problems. The second consists of a single analytical problem. **You must upload your solutions to the second part as a single pdf - absolutely no hand-written solutions.** The preferred method is L<sup>A</sup>T<sub>E</sub>X typesetting, though this is not strictly required. There is no difference between the graduate and undergraduate versions of this assignment.

### Programming Problems

1. (50 points) Ordinarily, image resize functions, like the one in OpenCV, treat each pixel equally — everything gets reduced or increased by the same amount. In 2007, Avidan and Shamir published a paper called “Seam Carving for Content-Aware Image Resizing” in SIGGRAPH that does the resizing along contours in an image — a “seam” — where there is not a lot of image content. The technique they described is now a standard feature in image manipulation software such as Adobe Photoshop.

Here is an example of an image with a vertical seam drawn on it in red.



A vertical seam in an image contains one pixel per row, and the pixels on the seam are 8-connected between rows, meaning that pixel locations in adjacent rows in a seam differ by at most one column. Formally a vertical seam in an image with  $M$  rows and  $N$  columns can be described as a set of pixels

$$\mathbf{s}_r = \{(i, c(i))\}_{i=0}^{M-1} \text{ s.t. } \forall i, |c(i) - c(i-1)| \leq 1. \quad (1)$$

In reading this, think of  $i$  as the row, and  $c(i)$  is the chosen column in each row. Similarly, a horizontal seam in an image contains one row per column and is defined as a set of pixels

$$\mathbf{s}_c = \{(r(j), j)\}_{j=0}^{N-1} \text{ s.t. } \forall j, |r(j) - r(j-1)| \leq 1. \quad (2)$$

Here, think of  $j$  as the column and  $r(j)$  as the row for that column.

Once a seam is selected — suppose for now that it is a vertical seam — the pixels on the seam are removed from the image, and the pixels that are to the right of the seam are shifted to the left by one. This will create a new image that has  $M$  rows and  $N - 1$  columns. (There are also ways to use this to add pixels to images, but we will not consider this here!) Here is an example after enough vertical seams have been removed to make the image square.



The major question is how to select a seam to remove from an image. This should be the seam that has the least “energy”. Energy is defined in our case as the sum of the derivative magnitudes at each pixel:

$$e[i, j] = \left| \frac{\partial I}{\partial x}(i, j) \right| + \left| \frac{\partial I}{\partial y}(i, j) \right|.$$

for  $i \in 1 \dots M - 2$ ,  $j \in 1 \dots N - 2$ . (Use the OpenCV Sobel function and no Gaussian smoothing to compute the partial derivatives.) The minimum vertical seam is defined as the one that minimizes

$$\sum_{i=0}^{M-1} e[i, c(i)]/M$$

over all possible seams  $c(\cdot)$ . Finding this seam appears to be a hard task because there is an exponential number of potential seams.

Fortunately, our old friend (from CSCI 2300) dynamic programming comes to the rescue, allowing us to find the best seam in linear time in the number of pixels. To realize this, we need to recursively compute a seam cost function  $W[i, j]$  at each pixel that represents the minimum cost seam that runs through that pixel. Recursively this is defined as

$$\begin{aligned} W[0, j] &= e[0, j] \quad \forall j \\ W[i, j] &= e[i, j] + \min(W[i - 1, j - 1], W[i - 1, j], W[i - 1, j + 1]) \quad \forall i > 0, \forall j \end{aligned}$$

Even if you don’t know dynamic programming, computing  $W[i, j]$  is pretty straightforward (except for a few NumPy tricks — see below).

Once you have matrix  $W$ , you must trace back through it to find the actual seam. This is also defined recursively. The seam pixels, as defined by the function  $c(\cdot)$  from above, are

$$\begin{aligned} c(N - 1) &= \operatorname{argmin}_{1 \leq j \leq N - 1} W[N - 1, j] \\ c(i) &= \operatorname{argmin}_{j \in \{c(i+1)-1, c(i+1), c(i+1)+1\}} W[i, j] \quad \text{for } i \in N - 2 \text{ downto } 0 \end{aligned}$$

In other words, in the last row, the column with the minimum weight (cost) is the end point of the seam. From this end point we trace back up the image, one row at a time, and at each row we choose from the three possible columns that are offset by -1, 0 or +1 from the just-established seam column in the next row.

A few quick notes on this.

- You need to be careful not to allow the seam to reach the leftmost or rightmost column. The easiest way to do this is to introduce special case handling of columns 0 and  $N - 1$  in each row, assigning an absurdly large weight.
- The trickiest part of this from the NumPy perspective is handling the computation of the minimum during the calculation of  $W$ . While you clearly must explicitly iterate over the rows (when finding the vertical seam), **I don't want you iterating over the columns**. Instead, use slicing in each row to create a view of the row that is shifted by +1, -1 or 0 and then take the minimum. For example, here is code that determines at each location in an array, if the value at that is greater than the value at either its left or right neighbors.

```
import numpy as np

a = np.random.randint( 0,100, 20 )
print(a)
is_max = np.zeros_like(a, dtype=np.bool)
left = a[ :-2]
right = a[ 2: ]
center = a[ 1:-1 ]
is_max[ 1:-1 ] = (center > right) * (center > left)
is_max[0] = a[0] > a[1]
is_max[-1] = a[-1] > a[-2]
print("Indices of local maxima in a:", np.where(is_max)[0])

,,,
Example output:
```

```
[93 61 57 56 49 40 51 85  5 13 28 89 31 56 11 10 60 93 26 86]
Indices of local maxima in a: [ 0  7 11 13 17 19]
,,,
```

- Recompute the energy matrix  $e$  after each seam is removed. Don't worry about the fact that the energy of most pixels will not change.
- The seam should be removed from the color image, but the energy is computed on a gray scale image. This means you will have to convert from color to gray scale before each iteration energy matrix computation and seam removal.
- Convert the image to float immediately after reading it (and before any derivative computation). This will ensure the greatest consistency with our results. In particular, if `fname` stores the name of the image file, use

```
img = cv2.imread(fname).astype(np.float32)
```

**Command-line and output:** Your program should take an image as input and remove enough rows or columns to make the image square. The command-line should be

```
python p1_seam_carve.py img
```

For 0th, 1st and last seam, please print the index of the seam (0, 1, ...), whether the seam is vertical or horizontal, the starting, middle and end pixel locations on the seam (e.g. if there are 11 pixels, output pixels 0, 5 and 10), and the average energy of the seam (accurate to two decimal places). Finally, output the original image with the first seam drawn on top of image in red and output the final, resized image. If `foo.png` is the input image, the output images should be `foo_seam.png` and `foo_final.png`.

**Finally, you may not be able to reproduce the exact output of my code.** Do not worry about this too much as long as your energies and seams are close. Especially important is that the final square image looks close.

2. **(40 points)** In class we started to implement an edge detector in the Jupyter notebook `edge-demo.ipynb`, including Gaussian smoothing and the derivative and gradient computations. The code is posted on Submittit.

In this problem, you will implement the non-maximum suppression step and then a thresholding step, one that is simpler than the thresholding method we discussed in class. Here are the details:

- For non-maximum suppression, a pixel should be marked as a maximum if its gradient magnitude is greater than or equal to those of its two neighbors along the gradient direction, one “ahead” of it, and one “behind” it. (Note that by saying “greater than or equal”, edges that have ties will be two (or more) pixels wide — not the right solution in general, but good enough for now.) As examples, if the gradient direction at pixel location  $(x, y)$  is  $\pi/5$  radians ( $36^\circ$  degrees) then the ahead neighbor is at pixel  $(x+1, y+1)$  and the behind neighbor is at pixel  $(x - 1, y - 1)$ , whereas if the gradient direction is  $9\pi/10$  ( $162^\circ$ ) then the ahead neighbor is at pixel  $(x - 1, y)$  and the behind neighbor is at pixel  $(x + 1, y)$ .
- For thresholding, start from the pixel locations that remain as possible edges after non-maximum suppression and eliminate those having a gradient magnitude of lower than 1.0. Then, for the remaining pixels, compute the mean,  $\mu$ , and the standard-deviation,  $s$ , of the gradient magnitudes. The threshold will be the minimum of  $\mu + 0.5s$  and  $30/\sigma$ , the former value because in most images, most edges are noise, and the latter value to accommodate clean images with no noise. Dividing by  $\sigma$  is because Gaussian smoothing reduces the gradient magnitude by a factor of  $\sigma$ .

The command-line should be

```
python p2_edge.py sigma in_img
```

where

- `sigma` is the value of  $\sigma$  used in Gaussian smoothing, and
- `in_img` is the input image.

(I have posted an example on line with `sigma = 2` and `in_img = disk.png`)

The text output from the program will be:

- The number of pixels that remain as possible edges after the non-maximum suppression step.
- The number of pixels that remain as possible edges after the gradient threshold of 1.0 has been applied.
- $\mu$ ,  $s$  and the threshold, each on a separate line and accurate to 2 decimal places.
- The number of pixels that remain after the thresholding step.

Three output images will be generated, with file names created by adding a four character string to the file name prefix of the input image. Examples below assume that the image is named `foo.png`. Here are the three images:

- The gradient directions of all pixels in the image encoded to the following five colors: red (255, 0, 0) for pixels whose gradient direction is primarily east/west; green (0, 255, 0) for pixels whose gradient direction is primarily northwest/southeast; blue (0, 0, 255) for pixels whose gradient direction is primarily north-south; white (255, 255, 255) for pixels whose gradient direction is primarily northeast/southwest; and black (0, 0, 0) for any pixel on the image border (first or last row or column) and for any pixel, regardless of gradient direction, whose gradient magnitude is below 1.0. The file name should be `foo_dir.png`.
- The gradient magnitude before non-maximum suppression and before thresholding, with the maximum gradient mapping to the intensity 255. The file name should be `foo_grd.png`.
- The gradient magnitude after non-maximum suppression **and** after thresholding, with the maximum gradient mapping to the intensity 255. The file name should be `foo_thr.png`.

#### Notes:

- Be sure that your image is of type float32 (or float64) **before** Gaussian smoothing.
- At first it will seem a bit challenging — or at least tedious — to convert the initial gradient direction, which is measured in radians in the range  $[-\pi, \pi]$ , into a decision as to whether the gradient magnitude is primarily west/east, northwest/southeast, north/south, or northeast/southwest. For example, the ranges from  $[-\pi, -7\pi/8]$ ,  $[-\pi/8, \pi/8]$ , and  $[7\pi/8, \pi]$  are all east-west. You could write an extended conditional to assign these directions, or you write **one or two expressions**, using Numpy's capability for floating-point modular arithmetic, to simultaneously assign 0 to locations that are west/east, 1 to locations that are northwest/southeast, etc. Think about it!
- This problem is a bit harder than previous problems to solve without writing Python for loops that range over the pixels, but good solutions do exist. Full credit will be given for a solution that does not require for loops, while up to 36 of 40 for students in 4270 will be given for a solution that requires for loops. (For students in 6270 this will be 32 out of 40.) In other words, we've provided mild incentive for you to figure out how to work solely within Python (and numpy) without for loops. Examples that have been given in class and even worked through on homework can help. You'll have to consider each direction (somewhat) separately.
- A final word of wisdom: build and test each component thoroughly before using it in a larger system - I know it's hard to force yourself to move this slowly, but I promise it will make this (and future) problems easier!

3. (35 points) Here's our first problem where you get to design the algorithm — not just implement one that is given to you. I suggest you review techniques used in the change detection example from Lecture 6 and 7. The solution should be relatively short.

A colleague of mine who I work with on aerial surveys was visiting Arizona this past summer and took photos of bats coming out of a cave at night. Here is one of them, show the original image and a cropped region of the sky where there are bats.



As you can see, the bats are very small. The original images are 4,000 x 6,000 and the bats might be about 10 pixels across. This means, for example, that we can't downsize the image prior to working with it. We must crop it.

Your job is to write a python script that takes in a cropped color image showing only regions from the sky and outputs the following:

- (a) The number of bats found.
- (b) For each bat, the row and column (y and x) position of the center of the bat in the image, and the area (number of pixels). There should be one line of output and three values on that line per bat.
- (c) A copy of the original cropped color image with a circle (`cv2.circle`) drawn in red around each bat found. The radius of the circle should be proportional to the `sqrt` of the area. You can decide the constant of proportionality.

The command-line is simply

```
python p3_bats img
```

the output image should have `_bats` appended to its prefix. For example if the input file is `foo.png` the output file should be `foo_bats.png`.

Grading will be divided approximately equally between:

- Quality of results, judged by comparison to our results and those of other students.
- Quality of algorithm design and implementation
- Clarity of explanation. For this, please include comments at the start of your .py file that outline the choice of methods you made, why you made them, what assumptions you made about the data that allowed your methods to work, and what changes you might need if the conditions of the data changed.

## Written Problem

1. **(15 points)** Evaluate the quality of the results of seam carving on several images. In particular, find images for which the results are good, and find images for which the results are poor. Show these images before and after applying your code. What causes poor results? Please explain.