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Optimization of Composite Wing Structures for Maximum Flutter Speed

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This paper is aimed at investigating aeroelastic tailoring of composite wing structures for maximum flutter speed by using different optimization methods. A gradient-based deterministic method (GBDM) and genetic algorithm (GA) have been employed in the investigation. It has been noted from previous research that a significant increase in flutter speed can be achieved by tailoring the fiber orientations of a composite wing box structure using the former method. However the solution largely depends upon the initial laminate lay-up. The current investigation shows that the optimized solution from GA is much less dependent upon the starting lay-up. It is much more robust in searching an optimization solution although more computing time is normally required and a global solution is not guaranteed. Sensitivity of flutter speed to optimized lay-ups has been considered when employing GA. A filter and local search algorithm has been introduced into the GA optimization to assess and identify a feasible solution from the optimized lay-ups. A uniform and a tapered composite wing structure of different swept angles have been considered for demonstration purpose in this paper. The results show that by using the two different optimization methods, same level of flutter speed increment but slightly different optimized lay-ups for the wing structures have been achieved.

Nomenclature

EI, GJ, CK	=	bending, torsion and bending-torsion coupling rigidities respectively
H, Φ	=	transverse displacement and rotation of a wing box beam
m, I_p	=	mass and polar mass moment of inertia per unit length of a wing box beam
$X\alpha$	=	distance between the mass and geometric elastic axes of the wing box cross-section
ω	=	frequency
$[K_D(\omega)]$	=	frequency dependent generalized dynamic stiffness matrix
$\{q\}$, [D]	=	generalized coordinate and damping matrix of the structure
$[QA]_R$, $[QA]_I$	=	real and imaginary parts of the generalized unsteady aerodynamic matrix
b, α	=	semi-chord, fiber orientation
ρ, V	=	air density and velocity
$k=\omega b/V$	=	reduced frequency
V_f	=	flutter speed
$f_V(\alpha)$	=	objective functions
$F(\alpha_i)$	=	fitness of an individual variable
$P(\alpha_i)$	=	probability of $F(\alpha_i)$ of an individual variable

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1. Introduction

Because of the favourable properties of high specific strength and directional stiffness, composite materials offer a great potential and many advantages in the optimisation of aircraft structures. In this investigation, attention has been paid to the aeroelastic tailoring of composite wing structures. Some of the early works in this field has shown that warping restraint and elastic coupling have positive effect on the divergence speed of forward swept composite wings[1-4]. The elastic coupling produced by unsymmetrical laminate lay-ups could also have significant effect on the aeroelastic behaviour of a composite wing[5, 6]. Therefore investigation has been made into optimising laminate lay-up of composite wing structures for desirable aeroelastic behaviour[7-10]. In the previous research[11], it has shown that the aeroelastic tailoring solution by using a gradient-based deterministic method (GBDM) is dependent on the initial design variables with which the optimisation is set to start. This is because the search of a set of better design variables relies on the variables and the objective function gradient of the current point. It is tricky and difficult to find a starting point, which would lead to the best achievable optimization solution. On the other hand however, this method offers a stable solution in a sense that the solution is based on the existence of a continuous and relatively small derivative of objective function. A stable solution has tolerance for the difference between optimised and manufactured laminate lay-ups and hence can be considered as a practical design option. As an alternative tool for the optimisation analysis, a code based on genetic algorithm (GA) has also been developed and used in the investigation. Comparing with the former, the GA method is much more robust, although less efficient, in finding an optimised lay-up solution. The major concern with the use of GA is that for some optimised lay-ups the flutter speed is highly sensitive to a variation of the lay-up. To tackle this problem, a sensitivity index has introduced into the GA process as a constraint to make sure an optimised solution is stable and acceptable.

In this current investigation, a generic uniform and a tapered composite wing of different swept angles have been taken as examples. To minimise the computing demand in numerical modelling and aeroelastic optimisation, the wing structure has been simplified to a single-cell composite box beam. Analysis for rigidity values of a composite thin-walled box beam has been carried out by a number of investigators in recent years[12-16]. Based on an asymptotic analysis of two-dimensional shell theory, Berdichevsky et al.[15] developed the variational asymptotical theory to derive the governing equations of anisotropic thin-walled beams. Later Armanios and Badir[16] extended this theory to the free vibration analysis of anisotropic thin-walled beams. In the present paper, the method from the above work has been adapted to determine the bending, torsion and bending-torsion coupling rigidities of the thin-walled composite box beam. The stiffness and mass matrices in the governing equations are then established by using the dynamic stiffness method[17-18]. The Wittrick-Williams algorithm[19] is used to calculate the wing structural modes for high accuracy and computational efficiency. The flutter calculation is carried out within the range of incompressible airflow and subsonic speed. Both the strip theory and lifting surface theory [20] have been used to calculate the unsteady aerodynamic forces. The V-g method is used as a solution technique in the flutter analysis. In the investigation, the wing box is divided into four laminated thin-walled panels (skins and spar webs) along the circumference in each of the five spanwise sections. The fibre orientations in each ply of the 4x5 laminated panels are taken as optimisation variables to search for the maximum flutter speed of the wing. From the analysis results, it has been noted that although the optimised lay-ups are not necessarily the same, almost the same increments of flutter speed can be achieved by tailoring the panel fibre orientations of a composite wing structure without weight penalty using either the GBDM or GA.

2. Vibration and Flutter Analysis

In the demonstration example as illustrated in Figures 2 and 3, the composite thin-walled box section acts as the primary structure of the wing carrying principle load. The calculation of structural properties is therefore based on the wing box. The components forward of the front spar and behind the rear spar are assumed to contribute only to mass and inertia properties and aerodynamic forces on the wing.

In this current investigation, the wing primary structure is idealized as a single-cell box beam clamped at the wing root and divided into five spanwise sections. For each of the box sections, the bending, torsion and bending-torsion coupling rigidities *EI, GJ* and *CK* have been obtained based on its geometry, properties and laminate lay-ups using the method from Armanios and Badir[16]. For vibration analysis of a composite beam with material bending-torsion coupling, the dynamic stiffness matrix (DSM) method[17, 18] has been used. In this method, the motion of each box section modeled as a beam element neglecting the shear deformation and warping effect can be represented as follows

$$EI\frac{\partial^4 h}{\partial y^4}+CK\frac{\partial^3 \phi}{\partial y^3}+m\frac{\partial^2 h}{\partial t^2}-mX_\alpha\frac{\partial^2 \phi}{\partial t^2}=0 \tag{1}$$

$$GJ\frac{\partial^2 \phi}{\partial y^2}+CK\frac{\partial^3 h}{\partial y^3}+mX_\alpha\frac{\partial^2 h}{\partial t^2}-I_p\frac{\partial^2 \phi}{\partial t^2}=0 \tag{2}$$

By solving the differential equations, an exact solution for displacement function is obtained. A dynamic stiffness matrix is determined by relating the displacements to the force vectors at both ends of each beam element. The assembly of all element stiffness matrices along the wingspan gives the dynamic stiffness matrix of the whole wing structure. The dynamic stiffness matrix is actually a combination of both stiffness and mass matrices and a function of frequency. This results in a non-standard eigenvalue problem, which can be solved by using the Wittrick and William algorithm[19].

Using the normal mode method, the flutter equation for an oscillating wing can be written in generalized coordinates as:

$$\left[[K_D(\omega)]-\frac{1}{2}\rho V^2[QA]_R+i\omega\cdot[D]+i\frac{1}{2}\rho V^2[QA]_I\right]\{q\}=0 \tag{3}$$

3. Optimization Methods

3.1 Using GBDM for maximum flutter speed

In the optimization, effort is primarily focused on achieving a maximum flutter speed by tailoring the laminate fiber orientations. Since the wing weight will not be affected, the analysis can be expressed as an unconstrained optimization problem as follows:

$$\text{Minimize } f_v(\alpha)=[1-\frac{V_f(\alpha)-V_f(\alpha_0)}{V_f(\alpha_0)}]^2 \quad \text{within } \{\alpha_l\}\le\{\alpha\}\le\{\alpha_u\} \tag{4}$$

where $f_v(\alpha)$ is objective function, $V_f(\alpha)$ is the wing flutter speed, α is a vector containing the fiber orientations set as design variables with the lower and upper bounds $\{\alpha_l\}$ and $\{\alpha_u\}$, α_0 represents a set of specified initial fiber orientations.

To solve the optimization problem, the Davidson-Fletcher-Powell (DFP) variable metric method[21] is used as optimizer whereas the Golden Section method[22] based on polynomial interpolation is employed for the one-dimensional search.

3.2 Using GA for maximum flutter speed

The optimization model in GA may be represented by

$$\text{Maximize } V_f(x) \quad \text{subject to } x\in\{A \mid (\alpha_1,\alpha_2\ \dots\ \alpha_n)\},\ \alpha_n\in[-90, 90] \tag{5}$$

The GA is a computational model of natural selection and evolution from Darwinian theory. During the evolution, individuals with good characteristics, which help an individual of the population survive, will gradually dominate the population as the individuals with bad characteristics die off. Unlike GBDM, which is deterministic based on a set of criteria, GA is probabilistic because it works with coding of the entire design spaces rather than actual variables. For this study, a binary grey coding of design variables are chosen. Each variable is discretized based on its minimum and maximum value and its resolution into N-bit binary unsigned integer called substrings. Each substring, representing a design variable, is strung together to form a single string which, named chromosome, represents an individual of the genetic algorithm population.

In the beginning of GA process, an initial population of the design variables is formed at random and represented in a binary unsigned integer of length l_i. In the process of reproduction, the selection of mating pool for next generation is performed based on the probability $P(\alpha_i)$ of fitness $F(\alpha_i)$ of an individual variable, which is represented below in a proportional selection method.

$$P(\alpha_i) = F(\alpha_i) / \sum_{i=1}^{n} F(\alpha_i) \tag{6}$$

Following the selection of mates, a double-spot crossover proceeds based on a random selection of crossing sites of the mated string couples through an adaptive probability of crossover. Mutation, which can keep the diversity of species to avoid premature, is then performed according to an adaptive probability of mutation for each string on a bit-by-bit basis. Both processes can be illustrated below.

crossover example $\begin{matrix} 101\overset{\leftrightarrow}{1}1000\overset{\leftrightarrow}{0}1 \\ 011\overset{\leftrightarrow}{0}0011\overset{\leftrightarrow}{1}0 \end{matrix} \rightarrow \begin{matrix} 101\overset{\leftrightarrow}{0}1000\overset{\leftrightarrow}{1}1 \\ 011\overset{\leftrightarrow}{1}0011\overset{\leftrightarrow}{0}0 \end{matrix}$

$\begin{matrix} 10111\ \overset{\leftrightarrow}{0}0001 \\ \downarrow \\ 10111\ \overset{\leftrightarrow}{1}0001 \end{matrix}$ before and after mutation

Once the above processes have been completed, a set of optimized variables and their fitness values can be determined for the selection of next generation. In the process, elitist strategy is used to ensure the elitist individual can survive in the next generation. The above procedure is performed in an iterative manner until the following convergence condition is satisfied:

$$(F_{max} - F_{avg}) / F_{max} < \varepsilon \qquad or \qquad t > t_{max} \tag{7}$$

where F_{max} and F_{avg} are the maximum and average fitness values of the selected generation; ε is a user specified small positive value; t and t_{max} represent the generation number and its upper bound, respectively.

To assess the sensitivity of optimized lay-ups to the flutter speed, a filter and local search algorithm is added in the GA and operated when the convergence condition is satisfied. If the sensitivity of an optimum solution is satisfied, the optimization can be terminated. Otherwise, it operates the jump algorithm, which generates 40 percents individuals of the population randomly and form the new generation together with the other 60 percents individuals. Fig.1 shows the GA procedure.

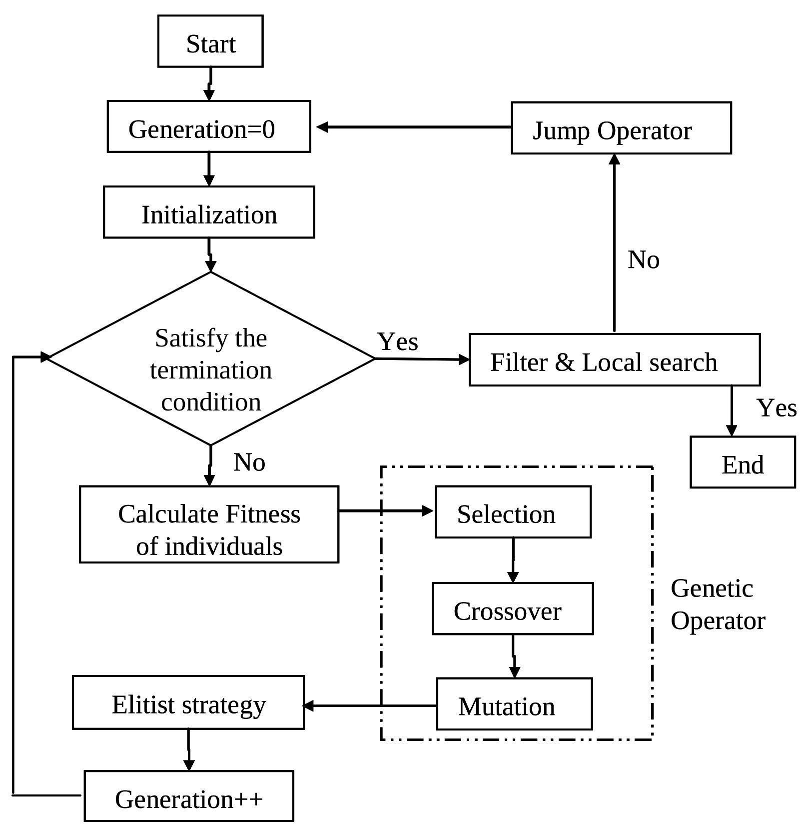

Figure 1. A flow-chart of the GA procedure

4. Examples and Discussion

4.1 Wing box model and design variables

In this paper, a wing model as illustrated in Fig. 2 has been taken as demonstration example. For each of the four cases considered in the demonstration, the swept angle, taper ratio and geometric data at the wing root section are listed in Table 1.

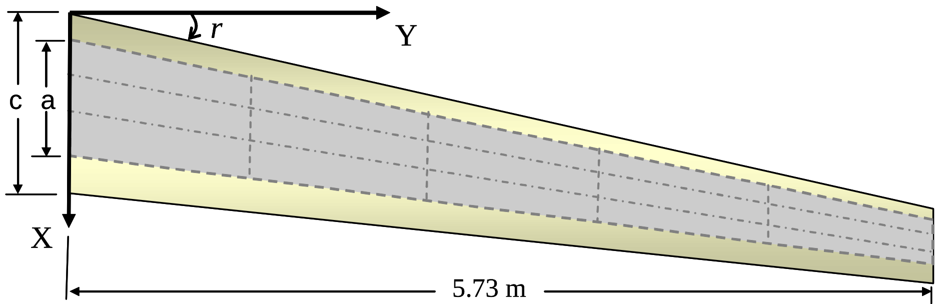

Figure 2. General layout and dimension of the swept-back composite wing

Table 1. Geometric data at root section of the wing models

Case No.	swept angle *r* (degree)	a (m)	b (m)	c (m)	h_1 (m)	h_2 (m)	taper ratio
1 2	0 20	0.8	0.37	1.22	0.07	0.05	1
3 4	0 20	0.97	0.55	1.45	0.105	0.075	0.57

In the analysis, the wing model is divided into five sections along its span. In each of the sections, the wing box is divided into four thin-walled panels, each made of 8 layers CFRP laminates, along its circumference representing the skins and spar webs as illustrated in Fig. 3. The material properties of a high strength CFRP used in this example are E_1=140 GPa, E_2=9.5 GPa, G_{12}=5.8 GPa and density ρ=1562 Kg/m^3. The fibre orientations in each ply of the 5x4 laminated panels are taken as design variables in the optimization process searching for the maximum flutter speed of the wing. Restricted to symmetric lay-up, the number of independent design variables for each wing box section is 16 and the total number is 80 for five sections of the whole wing model.

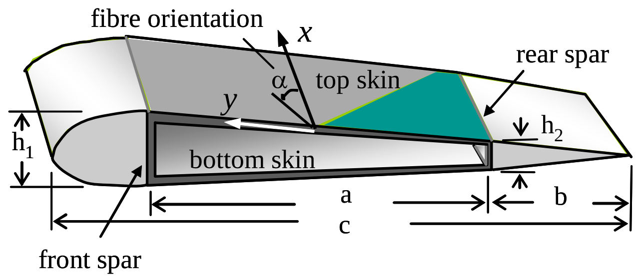

Figure 3. Cross-section details of the composite wing box

Since the torsional rigidity *GJ* of a wing box structure normally has dominant effect on the flutter speed, the lay-up associated with the maximum *GJ* value is chosen to compare with the optimized results in the demonstration example.

4.2 Aeroelastic tailoring of wing boxes using GBDM

In the first example, a non-swept uniform wing with geometric data shown in Table 1 case-1 has been considered. The results presented in Table 2 case 1.1 show that the laminate lay-up associated with the maximum *GJ* leads to a flutter speed V_f =176 m/s. Comparing with it, an increase in flutter speed by 5% can be achieved by optimizing the laminate lay-ups of the wing root section as shown in case 1.2. In this case, the optimized lay-ups in the five spanwise wing sections are simply kept uniform and identical. When each of the spanwise sections is optimized independently, a multi-section optimization is performed and an increase in V_f up to 20% has been achieved as shown in case 1.3. The results show that although the optimized lay-ups lead to a significant reduction of *GJ* a bending-torsional coupling rigidity *CK* has been produced and plays significant role for the increase in V_f.

Table 2. Optimized results for a uniform wing *r* =0 using GBDM (case-1)

Optimised results	Maximum *GJ* (case 1.1)	Uniform-section (case 1.2)	Multi-section § (case 1.3)
Lay-up in top skin:	$[45 / -45 / 45 / -45]_S$	$[50.4 / -45.0 / 46.5 / -45.0]_S$	$[79.4 / -29.0 / 90.0 / -36.6]_S$
bottom skin:	$[-45 / 45 / -45 / 45]_S$	$[-45.1 / 46.3 / -45.2 / 46.2]_S$	$[-71.5 / 43.5 / -71.8 / 19.4]_S$
front spar:	$[-45 / 45 / -45 / 45]_S$	$[-44.9 / 44.8 / -45.2 / 44.8]_S$	$[-48.9 / 41.1 / -48.9 / 37.6]_S$
rear spar:	$[45 / -45 / 45 / -45]_S$	$[47.2 / -43.3 / 46.8 / -43.6]_S$	$[41.9 / -51.6 / 37.4 / -47.3]_S$
Rigidity (*MN.m*^{2})			
in bending	EI = 0.1217	EI = 0.1160	EI=0.2393
torsion	GJ = 0.7766	GJ = 0.7727	GJ=0.4408
coupling	CK = 0.00	CK = -0.0092	CK=-0.1987
Flutter speed (*m/s*)	V_f = **176.0**	V_f = **185.0**	V_f = **211.0**
& frequency (*rad/s*)	ω_f = 101.8	ω_f = 85.7	ω_f = 43.3

§ lay-up and rigidities shown at root section only

In the second case where the uniform wing is swept back at *r*=20 degree, the effect of *CK* on V_f appears to be less significant. Comparing with the maximum *GJ* case 2.1 in Table 3, only 2.7% and 6% increase in V_f as shown in cases 2.2 and 2.3 has been achieved by performing the uniform and multi-section optimisation respectively.

Table 3. Optimized results for a uniform wing *r* =20 using GBDM (case-2)

Optimised results	Maximum *GJ* (case 2.1)	Uniform-section (case 2.2)	Multi-section § (case 2.3)
Lay-up in top skin:	$[45 / -45 / 45 / -45]_S$	$[33.5 / -44.3 / 33.5 / -44.3]_S$	$[49.0 / -35.0 / 74.0 / -35.0]_S$
bottom skin:	$[-45 / 45 / -45 / 45]_S$	$[-35.6 / 39.8 / -35.6 / 39.8]_S$	$[-60.0 / 43.0 / -66.0 / 43.0]_S$
front spar:	$[-45 / 45 / -45 / 45]_S$	$[-30.0 / 30.0 / -30.0 / 30.0]_S$	$[-30.0 / 30.0 / -30.0 / 30.0]_S$
rear spar:	$[45 / -45 / 45 / -45]_S$	$[30.0 / -30.0 / 30.0 / -30.0]_S$	$[30.0 / -30.0 / 30.0 / -30.0]_S$
Rigidity (*MN.m*^{2})			
in bending	EI = 0.1217	EI = 0.1934	EI = 0.1667
torsion	GJ = 0.7766	GJ = 0.7058	GJ = 0.6260
coupling	CK = 0.00	CK = 0.070	CK = -0.1518
Flutter speed (*m/s*)	V_f = **182.2**	V_f = **186.7**	V_f = **192.7**
& frequency (*rad/s*)	ω_f = 94.8	ω_f = 76.8	ω_f = 69.2

§ lay-up and rigidities shown at root section only

For a tapered wing box with geometry data listed in Table 1 under case-3 and case-4, the effect of *CK* on V_f appears to be largely reduced. In case-3 for example, the results presented in Table 4 show that little increase in V_f beyond that associated with the maximum *GJ* can be obtained by a uniform-section optimisation and only 1.5% increase by multi-section optimisation. In case-4 where a sweptback tapered wing is considered, only 3.9% increase in V_f has been achieved by multi-section optimisation as shown in Table 5. To summarise, a comparison of the optimised results in the above cases is shown in Fig.4.

Table 4. Optimized results for a tapered wing *r*=0 using GBDM (case-3)

Optimised results	Maximum *GJ* (case 3.1)	Uniform-section § (case 3.2)	Multi-section § (case 3.3)
Lay-up in top skin:	$[45 / -45 / 45 / -45]_S$	$[45.2 / -45.0 / 45.0 / -45.1]_S$	$[45.5 / -44.5 / 45.4 / -44.6]_S$
bottom skin:	$[-45 / 45 / -45 / 45]_S$	$[-45.0 / 45.0 / -45.0 / 45.0]_S$	$[-44.6 / 45.4 / -44.6 / 45.4]_S$
front spar:	$[-45 / 45 / -45 / 45]_S$	$[-49.5 / 44.4 / -45.6 / 44.5]_S$	$[-44.6 / 44.7 / -45.3 / 44.9]_S$
rear spar:	$[45 / -45 / 45 / -45]_S$	$[45.0 / -45.0 / 45.0 / -45.0]_S$	$[45.5 / -44.6 / 45.4 / -44.5]_S$
Rigidity ($MN.m^2$)			
in bending	EI = 0.2167	EI = 0.2125	EI = 0.2166
torsion	GJ = 1.3825	GJ = 1.378	GJ = 1.382
coupling	CK = 0.0	CK = -0.020	CK = 0.001
Flutter speed (*m/s*)	V_f = **202.6**	V_f = **203.0**	V_f = **206.3**
& frequency (*rad/s*)	ω_f = 123.0	ω_f = 123.4	ω_f = 117.0

§ lay-up and rigidities shown at root section only

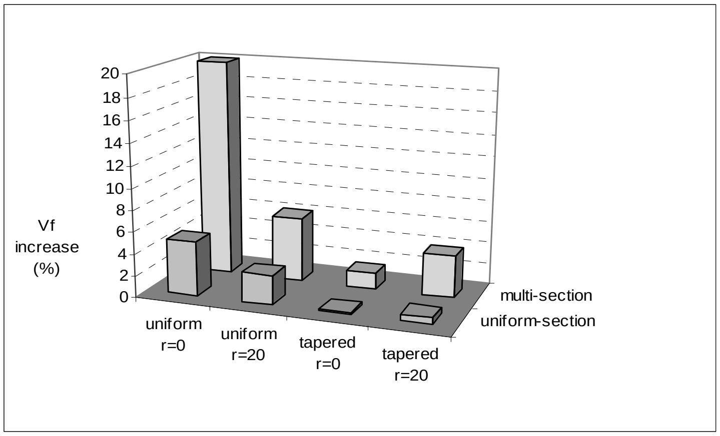

Figure 4. Increase in flutter speed in cases 1-4

Table 5. Optimized results for a tapered wing *r* =20 using GBDM (case-4)

Optimised results	Maximum *GJ* (case 4.1)	Uniform-section§ (case 4.2)	Multi-section§ (case 4.3)
Lay-up in top skin:	$[45 / -45 / 45 / -45]_S$	$[43.6 / -56.2 / 44.9 / -51.5]_S$	$[44.9 / -45.1 / 45.1 / -44.9]_S$
bottom skin:	$[-45 / 45 / -45 / 45]_S$	$[-45.1 / 46.3 / -45.1 / 46.3]_S$	$[-45.1 / 44.9 / -45.1 / 44.8]_S$
front spar:	$[-45 / 45 / -45 / 45]_S$	$[-45.0 / 45.0 / -45.0 / 45.0]_S$	$[-45.0 / 45.0 / -45.0 / 45.0]_S$
rear spar:	$[45 / -45 / 45 / -45]_S$	$[45.0 / -45.0 / 45.0 / -45.0]_S$	$[45.0 / -45.0 / 45.0 / -45.0]_S$
Rigidity ($MN.m^2$)			
in bending	EI = 0.2167	EI = 0.2048	EI = 0.2168
torsion	GJ = 1.3825	GJ = 1.3492	GJ = 1.382
coupling	CK = 0.0	CK = 0.0814	CK = -0.0021
Flutter speed (*m/s*)	V_f = **207.2**	V_f = **208.5**	V_f = **215.2**
& frequency (*rad/s*)	ω_f = 116.2	ω_f = 112.5	ω_f = 104.2

§ lay-up and rigidities shown at root section only

4.3 Aeroelastic tailoring of wing boxes using GA

As an alternative approach, the genetic algorithm (GA) has also been employed for aeroelastic tailoring of the wing. Taking the cases 1.2-4.2 as examples, a set of optimised results has been obtained and shown in Table 6.

Table 6. Optimal solutions by GA (case-1.2 – 4.2)

Optimised results	Case 1.2	Case 2.2	Case 3.2	Case 4.2
Lay-up in top skin:	$[(45.1 / -52.1)_2]_S$	$[(-39.4 / 39.2)_2]_S$	$[(45.0 / -47.0)_2]_S$	$[(44.3 / -50.8)_2]_S$
bottom skin:	$[(-44.8 / 47.3)_2]_S$	$[(-35.1 / 52.2)_2]_S$	$[(-55.0 / 49.0)_2]_S$	$[(63.9 / -40.0)_2]_S$
front spar:	$[(-53.3 / 54.6)_2]_S$	$[(-42.6 / 43.0)_2]_S$	$[(43.0 / -46.0)_2]_S$	$[(-47.7 / 41.0)_2]_S$
rear spar:	$[(35.9 / -37.7)_2]_S$	$[(-44.1 / 25.3)_2]_S$	$[(-37.0 / 43.0)_2]_S$	$[(-41.1 / 23.9)_2]_S$
Rigidity (*MN.m*^{2})				
in bending	EI = 0.11309	EI = 0.16267	EI = 0.2169	EI = 0.22900
torsion	GJ = 0.76002	GJ = 0.71043	GJ = 1.3730	GJ = 1.22050
coupling	CK = 0.04206	CK = 0.08606	CK = -0.1341	CK = 0.22283
Flutter speed (*m/s*)	V_f = **185.0**	V_f = **188.0**	V_f = **202.3**	V_f = **212.7**
& frequency (*rad/s*)	ω_f = 102.5	ω_f = 70.0	ω_f = 122.9	ω_f = 81.6

In the optimisation, it is noted that there actually exist optimised lay-ups that give significantly higher V_f values beyond the range achievable by GBDM. However those results could be misleading and unfeasible in practical design since the increase in V_f is very dramatic in a small local region and too sensitive to the lay-up variation. Following a process of sensitivity assessment added in the GA, those results have been treated as unacceptable and filtered off. The result in Table 6 for each of the cases only presents one of the optimum and acceptable results obtained by using GA. Comparing with the results by GBDM, it is noted that although these optimised lay-ups are different the increase in V_f is within the same range in all the cases as shown in Fig.5.

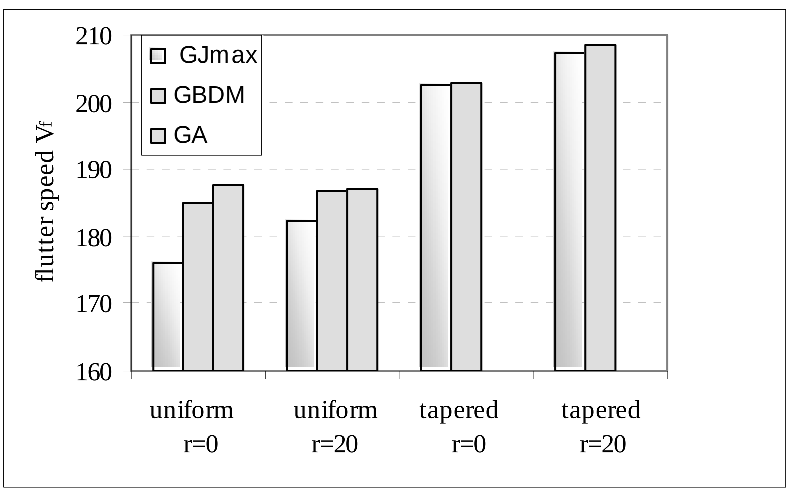

Figure 5. Comparison of optimized results in cases 1-4

5. Conclusions

From the investigation, it has been noted that significant increase in flutter speed of a composite wing box may be achieved by optimising the fibre orientations without weight penalty. This is a great advantage of laminated composite structure over its metallic counterpart in aeroelastic tailoring. In addition, the following conclusions can be drawn.

- For a uniform composite wing box, there is a great potential to maximise its flutter speed by optimising the laminate lay-ups. The optimum flutter speed can be significantly higher than that associated with the maximum torsion rigidity *GJ*. In such case, the effect of bending-torsion coupling rigidity *CK* on flutter speed can be significant and beneficial;
- For a tapered composite wing box, the potential to achieve a feasible optimum lay-up with significant increase in V_f seems much less than a uniform wing box. The flutter speed is basically dominated by torsion rigidity. In such case, the laminate lay-up associated with the maximum *GJ* offers a feasible and approximately global optimum for aeroelastic stability;
- In the tapered wing case, the computing time required in an aeroelastic tailoring can be largely reduced by taking torsion rigidity instead of flutter speed in the objective function. In such case, the GBDM is preferable to the GA;
- In a general case of aeroelastic tailoring, the GA is preferable for its robustness of achieving an optimum solution although it normally demands much more computing time comparing with the GBDM.

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