Aeroelastic Analysis and Optimization of a Highly Flexible Aircraft and Application at X-HALE-BR

(Final report)

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***Abstract*—This work addresses the formulation on a mathematical model based in unsteady aerodynamics with strip theory and a nonlinear beam model to analyze the aeroelastic properties of highly flexible aircrafts. This formulation will be used to optimize its characteristics, aiming at its improvement.**

I. Context

Aeroelasticity is the subject that describes the interaction of aerodynamic, inertia and elastic forces in a flexible structure and the phenomena that it can result in. Since some aeroelastic phenomena (as flutter and divergence) can cause structural failure, it has been influencing the design of airplanes, bridges, wind turbines, helicopters, etc. The first recorded flutter problem to be modeled and solved was the Handley-Page O/400 bomber in 1916 [9].

The studies about aeroelasticity have greatly evolved since 1916 and today it is possible to analyze and design very light, and highly flexible flying wing configurations, which is of interest for the development of the next generation of high-altitude, long endurance (HALE) unmanned aerial vehicles. The flexibility of such aircraft leads to large deformation, what makes linear theories not relevant for their analysis. The deformed shape is significantly different from the undeformed shape [4].

One example of this highly flexible HALEs is the X-HALE-BR aircraft, that may be seen in Fig 1. This specific aircraft will be of great importance for the following work and the object for its application. It is a unmanned aerial vehicle that was born in the ITA (Instituto Tecnológico de Aeronautica/ Technological Institute of Aeronautics), a Brazilian Institute.

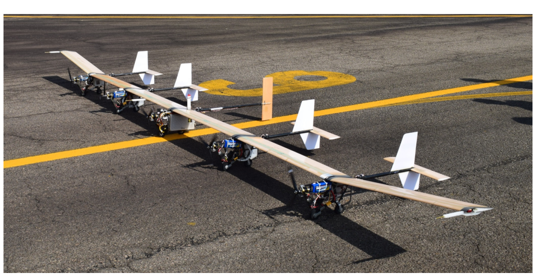


Fig. 1. X-HALE-BR Aircraft

Combined with recent studies, it is possible to combine the aeroelastic studies in highly flexible aircraft and aeroelastic optimization to improve its aeroelastic characteristics, as the flutter speed [3]. This may be done using computational tools as the python's package *openMDAO* [2].

II. Past works

Nowadays there are some recent modelings for the aeroelastic model of highly flexible aircrafts.

In [7], Su makes a complete analysis of flexible aircrafts based in a reduced-order, nonlinear, strain-based finite element framework combined with finite-state unsteady subsonic aerodynamics to compute airloads along the lifting surfaces. An important characteristic of his work is that all members of the vehicle are considered flexible. In the joints of the structure the Lagrange Multiplier Method is applied to model the nodal displacement constraints. The model is then applied in real fully flexible aircrafts for validation.

In [1] and [5], Ribeiro et al makes a mathematical formulation to model highly flexible airplanes along with a self-made computational tool, a MatLab's Toolbox, called AeroFLex. A nonlinear beam model for large displacements was applied to represent the structural dynamics combined with the strip theory for the aerodynamics including three bidimensional modeling approaches: a quasi-steady, a quasi-steady with apparent mass and a full unsteady. A large aspect ratio flying wing was considered as a test case.

In [8], van Schoor et al. makes an aeroelastic model for airplanes with very flexible wings in which the structural dynamics of the airplane were obtained from its finite element model. In an assumed mode approach, a sub-set of the natural mode shapes were used to calculate the quasi-steady generalized modal forces using a two-dimensional strip model, which included unsteady drag and leading edge suction forces.

In [3] Guo et al. investigated the optimization to maximize the flutter speed through different optimization methods and considering parameters like the orientations of the fiber of a composite wing box structure.

For the aeroelastic analysis, the model that was chosen for the following work was the one from Ribeiro et al., because of the following:

- Originally made for the same aircraft in study;

- Come with a Matlab's Toolbox;
- Recent and updated work;
- It has very precise results in comparison with other works and experimental data;
- It is possible to take out some functionalities of the toolbox to simplify the study.

For the optimization, the analysis of Guo et al. were used to chosen the optimization methods and what parameters could be optimized.

III. Problem statement

To create the aeroelastic model, the model from [1] was used with some simplifications: the toolbox has some functionalities that are not essential for this work, as rigid body dynamics and stress and the possibility to choose between different aerodynamic models. Only the flexible body and unsteady aerodynamics will be used. This is justified by the fact that the work is preferable to start more simple and non-essential functionalities may consume computational power and increase the time of the optimization.

A. Structural dynamics model

The equations that describe the aeroelastic model of the airplane, result of the mathematical model, may be seen in (1),(2) and (3), in which M is the flexible structure mass matrix, β represents the linear and rotational speeds, $\tilde{\epsilon}$ is the vector of deformations and $\tilde{X} = \dot{\tilde{\epsilon}}$, $\tilde{\lambda}$ the lag states from the unsteady aerodynamic model and $\tilde{k}$ is the vector of kinematics variables shown in Section III-A, with ϕ being angle of bank, θ being angle of pitch, ψ being angle of heading and H being the vertical velocity of the aircraft. $\tilde{\delta}_{u,i}$ is the flap deflection and $\tilde{\pi}_{u,i}$ is the engine throttle. M_{ij} is the mas matrix, C_{ij} the damping matrix, K_{FF} structural rigidity matrix. Since the mass matrix are not diagonal, one can see that the rigid body states (β) are inertially coupled with structural states(ϵ). R_F and R_B represents the generalized forces, that are applied in the airplane. They come from the aerodynamic, gravitational and propulsive forces applied to each structural node. As described in section III-B, the strip theory is applied to calculate the aerodynamic forces and moments with bidimensional models in each node.

The Jacobian matrices $J_{h\epsilon}$ and $J_{\theta\epsilon}$ represents the relationship between structural deformations (ϵ) and nodal displacements and rotations. J_{hb} and $J_{\theta b}$ represents represents the relationship between rigid body degrees of freedom and nodal displacement and rotations. The Jacobian matrices are nonlinear functions of ϵ. The way to obtain they is described in [1] and [5].

Finally, K is the structural rigidity matrix and C is the structural damping matrix. A linear relationship between C and K is used as follow: $C = cK$, where c is the damping ratio.

$$\begin{bmatrix} M_{FF} & M_{FB} \\ M_{BF} & M_{BB} \end{bmatrix} \begin{bmatrix} \ddot{\epsilon} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} C_{FF} & C_{FB} \\ C_{BF} & C_{BB} \end{bmatrix} \begin{bmatrix} \dot{\epsilon} \\ \beta \end{bmatrix} + \\ + \begin{bmatrix} K_{FF} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon \\ \vec{b} \end{bmatrix} = \begin{bmatrix} R_F \\ R_B \end{bmatrix} \quad (1)$$

$$\begin{gathered} M_{FF}(\epsilon) = J_{h\epsilon}^T M J_{he} \\ M_{FB}(\epsilon) = J_{h\epsilon}^T M J_{hb} \\ M_{MF}(\epsilon) = J_{hb}^T M J_{h\epsilon e} \\ M_{BB}(\epsilon) = J_{hb}^T M J_{hb} + M_{RB} \\ C_{FF}(\epsilon, \dot{\epsilon}) = J_{h\epsilon}^T M \dot{J}_{h\epsilon} + C \\ C_{FB}(\epsilon, \dot{\epsilon}, \beta) = J_{h\epsilon}^T M H_{hb} + 2 J_{h\epsilon}^T M \dot{J}_{hb} + C_{RB} \\ C_{BF}(\epsilon, \dot{\epsilon}) = J_{hb}^T M \dot{J}_{h\epsilon} \\ C_{BB}(\epsilon, \dot{\epsilon}, \beta) = J_{hb}^T M H_{hb} + 2 J_{hb}^T M \dot{J}_{hb} \\ K_{FF} = K \end{gathered} \quad (2)$$

$$\begin{bmatrix} R_F \\ R_B \end{bmatrix} = \begin{bmatrix} J_{p\epsilon}^T \\ J_{pb}^T \end{bmatrix} F^{pt} + \begin{bmatrix} J_{\theta\epsilon}^T \\ J_{\theta b}^T \end{bmatrix} M^{pt} + \begin{bmatrix} J_{p\epsilon}^T \\ J_{pb}^T \end{bmatrix} B^F F^{dist} + \\ + \begin{bmatrix} J_{\theta\epsilon}^T \\ J_{\theta b}^T \end{bmatrix} B^M M^{dist} + \begin{bmatrix} J_{h\epsilon}^T \\ J_{hb}^T \end{bmatrix} N \vec{g} + \begin{bmatrix} 0 \\ R_{RB}^{ext} \end{bmatrix} \quad (3)$$

The Euler angles ϕ, θ and ψ describe the bank, pitch and heading of the aircraft. They time rate derivative are related with the angular speeds (P,Q,R) as may be seen in (4), (5) and (6).

$$\dot{\theta} = Qcos(\phi) - Rsin(\phi) \quad (4)$$

$$\dot{\phi} = P + tan\theta(Qsin(\phi) + Rcos(\phi)) \quad (5)$$

$$\dot{\psi} = \frac{(Qsin(\phi) + Rcos(\phi))}{cos(\theta)} \quad (6)$$

Furthermore, the relationship between speeds in the Body Frame (U,V,W) and Inertial Frame ($\dot{H}$,$\dot{x}$,$\dot{y}$) is represented in (7), (8) and (9).

$$\dot{H} = Usin\theta - Vsin\phi cos\theta - Wcos\phi cos\theta \quad (7)$$

$$\dot{x} = Ucos\theta cos\psi + V(sin\phi sin\theta cos\psi - cos\phi sin\psi) \\ + W(cos\phi sin\theta cos\psi + sin\phi sin\psi) \quad (8)$$

$$\dot{y} = Ucos\theta cos\psi + V(sin\phi sin\theta sin\psi - cos\phi cos\psi) \\ + W(cos\phi sin\theta sin\psi + sin\phi cos\psi) \quad (9)$$

B. Aerodynamic Model

Even though [1] describes 3 options of aerodynamic models to be used in the AeroFlex Toolbox (quasi-steady, quasi-steady with apparent mass and unsteady), only the unsteady one was chosen in this work, because it is closer to reality, and it has an oscillatory character that is essential for the flutter phenomenon, greatly influencing in the flutter speed.

The expressions for the lift L, moment M_{ea} and drag D may be seen in (10), (11) and (12).

$$L = \pi\rho b^2(-\ddot{z} + \dot{y}\dot{\alpha} - d\ddot{\alpha}) + 2\pi\rho b\dot{y}^2\left[-\frac{\dot{z}}{\dot{y}} + \left(\frac{1}{2}b - d\right)\frac{\dot{\alpha}}{\dot{y}} - \frac{\lambda_0}{\dot{y}}\right] \tag{10}$$

$$M_{ea} = Ld + 2\pi\rho b^2\left(-\frac{1}{2}\dot{y}\dot{z} - \frac{1}{2}d\dot{y}\dot{\alpha} - \frac{1}{2}\dot{y}\lambda_0 - \frac{1}{16}b^2\ddot{\alpha}\right) \tag{11}$$

$$D = -\rho b\dot{z}^2 C_{d_0} \tag{12}$$

Where ρ is the air density, α the local angle of attack, b is the airfoil semichord, a is the distance between elastic axis and the half chord normalized by b and C_{d_0} if the drag coefficient at 0 lift. λ_0 is described by (13).

$$\lambda_0 = \frac{1}{2}\sum_{N}^{n=1} b_n\lambda_n \tag{13}$$

Where b_n can be obtained from (14).

$$\begin{array}{ll} b_n = (-1)^{n-1}\frac{(N_A+n-1)!}{(N_A-n-1)!} & 1 < n < N_A - 1 \\ b_{N_A} & \end{array} \tag{14}$$

The lag states λ_n can be obtained from the system of differential equations (15).

$$\dot{\lambda} = E_1\lambda + E_2\ddot{z} + E_3\ddot{\alpha} + E_4\dot{\alpha} \tag{15}$$

N_A is the number of aerodynamic lag states λ_n, E_1, E_2, E_3, E_4 are matrices presented in [1].

The trailing edge deflection is implemented by adding incremental values to the airfoil aerodynamic forces and moments as shown in (16) and (17). $C_{L,\delta}$ and $C_{m,\delta}$ can be obtained through experimental data or airfoil analysis softwares. δ_u is the airfoil flap deflection.

$$\begin{array}{c} L' = L + L^\delta \\ M' = M + M^\delta \end{array} \tag{16}$$

$$\begin{array}{c} L^\delta = \rho b\dot{y}^2 C_{L,\delta}\delta_u \\ M^\delta = \rho b^2 C_{m,\delta}\delta_u \end{array} \tag{17}$$

C. Control inputs

Two types of control inputs are used: flap deflection $\delta_i = \delta_u = \delta_{u,i}$ and engine throttle = π_i. Propulsion forces are modeled as point forces attached to a structural node.

D. Linearization and stability

In the progress of the modeling it is possible to verify that the system is nonlinear and coupled. The system of equations can be represented as (18). f is a nonlinear function which dimension is equal to the total number of system's states. $\vec{k}$ is the vector of kinematic variables $\vec{k} = \begin{bmatrix}\phi & \theta & \psi & H\end{bmatrix}$. To solve the equations, first of all the system is numerically linearized, which gives the equation (19).

$$f(\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \lambda, \dot{\lambda}, \beta, \dot{\beta}, \vec{k}, \dot{\vec{k}}, \delta_{u,i}, \pi_i) \tag{18}$$

$$M\begin{bmatrix}\dot{\tilde{X}} \\ \dot{\tilde{\epsilon}} \\ \dot{\lambda} \\ \dot{\tilde{k}}\end{bmatrix} = A\begin{bmatrix}\tilde{X} \\ \tilde{\epsilon} \\ \lambda \\ \tilde{k}\end{bmatrix} + B\begin{bmatrix}\tilde{\delta}_{u,i} & \tilde{\pi}_i\end{bmatrix} \tag{19}$$

The linearization is done numerically. By analyzing the eingenvalues of $M^{-1}A$ it is possible to verify if the system is stable.To determine the instability speed (flutter, divergence or other), the following procedure is applied: the airplane speed is increased; for each speed, a new equilibrium condition is obtained; the system is linearized; the largest real part of the eigenvalues of $M^{-1}A$ is taken. Once one of the eigenvalues has a positive real part, the system is unstable. The imaginary part of this eigenvalue gives the frequency associated with the unstable aeroelastic mode.

E. Optimization

Once the instability speed for the aircraft can be calculated, it is possible to change parameters trying to increase this speed to improve the airplane's resilience to the aeroelastic phenomena. The strategy to optimize the aircraft is to change its general parameters using the openMDAO package [2]. One important constraint of the optimization is not to remove the flexible characteristics of the aircraft, to not heavily increase its mass.

1) Creation of the simplified model: First of all, a simplified model were created and the parameters to the optimization were chosen. The model is a flat plate pinned on the left and free on the right, with the parameters c, that is equivalent to the wing's chord, t, the thickness, L, the half wing span, the sweep angle and the twist angle, both only in the root for simplification purpose. In this model, $c >> t$ and $L >> c$, so the plate may be closer to the reality and the approximations of the theory may be applied. The diagram of the model may be seen in Fig. 2. One can notice that this model is similar to the second case of study in the Section IV-A

The material chosen were aluminum 7075, largely used in aircrafts. Its structural properties may be verified in the Tab. I.

In addition, as shown in [6], the Inertia matrix I and the rigidity matrix K of each node may be calculated as shown in (20) and (21), in which A is the sectional area $A = ct$. The damping ratio chosen were 0.04, so $C = 0.04K$.

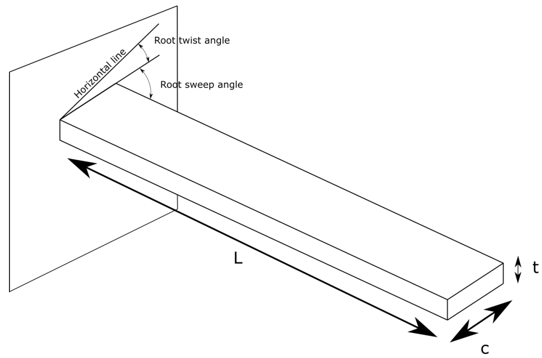


Fig. 2. Diagram of the simplified model

TABLE I
ALUMINUM 7075 PROPERTIES

Property	Value
Young Modulus (E)	71.7 GPa
Shear Modulus (G)	29.9 GPa
Volumetric Density (ρ)	2810 Kg/m^3

$$I = \begin{bmatrix} \rho A\frac{tc^3}{12} + \rho A\frac{ct^3}{12} & 0 & 0 \\ 0 & \rho A\frac{tc^3}{12} & 0 \\ 0 & 0 & \rho A\frac{ct^3}{12} \end{bmatrix} \tag{20}$$

$$K = \begin{bmatrix} EA & 0 & 0 & 0 \\ 0 & GJ & 0 & 0 \\ 0 & 0 & E\frac{tc^3}{12} & 0 \\ 0 & 0 & 0 & E\frac{ct^3}{12} \end{bmatrix} \tag{21}$$

In addition, the following aerodynamic parameters were fixed as may be seen in Tab. II based in the literature about thin plates [9].

TABLE II
AERODYNAMIC PROPERTIES

Aerodynamic Parameters	Value
N_A	4
C_{d_0}	0.02
C_{m_0}	0
α_0	$-\frac{5\pi}{180}$
C_{L_0}	$\frac{2\pi}{180}$
$C_{L,\delta}$	0
$C_{m,\delta}$	0

2) The optimization problem: With the model ready and the parameters chosen, now it is possible to prepare the optimization problem. In this work, the goal is to increase the flutter speed of the aircraft and minimize its weight. Therefore, the proposed objective function to be minimized with its constraints may be seen in (22) with its constraints in (23). The first constraint is a way to guarantee the condition $c >> t$. As L is naturally bigger than c in the chosen range, an additional condition is not necessary. The other conditions were arbitrarily chosen, based on amount of material spent, aerodynamic performance and literature.

$$J(c, t, L, sweep, twist) = -\frac{V_{flutter}}{V_0} + \frac{m}{m_0} \tag{22}$$

Subject to:

$$\begin{cases} c > 10t \\ 0.1 < c < 1 \\ 0.001 < t < 0.05 \\ 3 < L < 15 \\ 0 < sweep < 10^\circ \\ 0 < twist < 10^\circ \end{cases} \tag{23}$$

V_0 and m_0 are constants to make the problem dimensionless. They were arbitrarily chosen as *mean expected values* $V_0 = 10\ m/s$ and $m_0 = \rho Lct = 2810 \times 0.5 \times 0.005 \times 5 = 35.125\ Kg$. As an additional constraint, it is necessary to verify if the wing tip displacement is bigger than 15%, so the wing may be considered highly flexible. The flutter speed is saturated at 100 m/s to either lower the time searching for the flutter speed and because it is not desired to achieve greater speeds. The optimizer used in openMDAO was "COBYLA".

IV. FIRST RESULTS AND FUTURE WORK

The diagram of the phases of the project may be seen in Fig. 3.

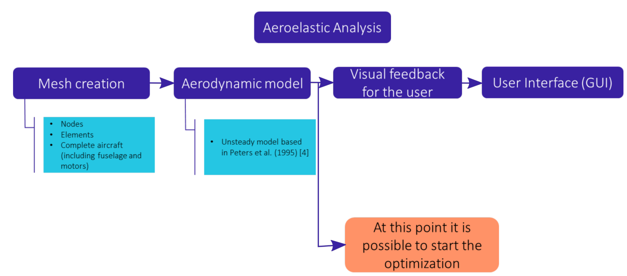


Fig. 3. Project phases diagram

To start the modeling, the python classes of the nodes, elements and structures of the airplane were made, following the Fig. 4. With the mesh, the other elements of the aircraft as engines and fuselage need to be modeled, following the AeroFlex model. Then, it is possible to calculate the matrix of the model and calculate the critic speed for the further optimization.

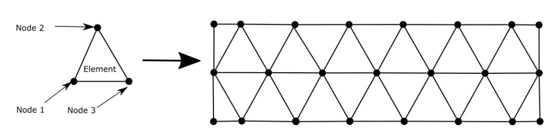


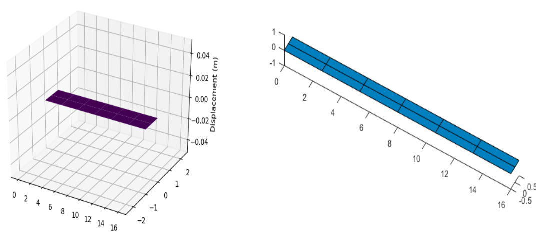
Fig. 4. Project phases diagram

A. Results of the aeroelastic model

In order to validate the code and the aeroelastic model two outputs of the problem were used: the wing tip displacement in meters and the flutter speed in meters per second. The outputs of the program were compared with the outputs of the AeroFlex toolbox and real data or literature. The cases used for study were the following:

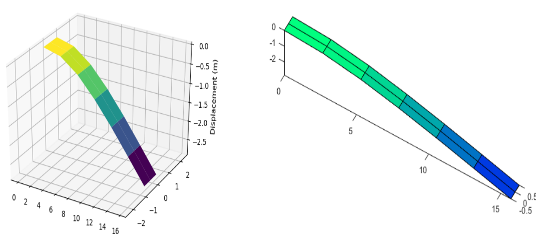
- Case 1: Rigid beam pinned in the left, wind speed of 10m/s
- Case 2: Flexible beam pinned in the left deformed by its weight, wind speed of 10m/s
- Case 3: Free body analysis of a highly flexible flying wing, flying speed of 10 m/s, wind speed of 0 m/s

The visual outputs of the deformation of the structures may be seen in Figs. 5, 6 and 7. The data comparison may be seen in the Tab. III in which D means wing tip deformation and F flutter speed.



(a) Present work result. (b) AeroFLex result.

Fig. 5. Results comparison in case 1.



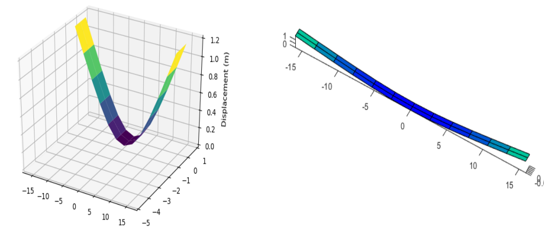
(a) Present work result. (b) AeroFLex result.

Fig. 6. Results comparison in case 2.

TABLE III
RESULTS COMPARISON

Case	Present work code		AeroFlex		Real data
	D (m)	F (m/s)	D (m)	F (m/s)	F (m/s)
1	0	32.8	0	32.6	32.2
2	-2.86	23.8	-2.85	23.4	23.2
3	1.20	63.3	1.18	63.5	63.5

Since the outputs from the code of the present work, the AeroFlex toolbox and the literature are close, with less than 0.05 (m) and 1 (m/s), which is considered tolerable for the present intents, the code is considered validated and it is possible to follow to the step of the optimization.



(a) Present work result. (b) AeroFLex result.

Fig. 7. Results comparison in case 3.

B. Results of the optimization

The results from the optimization may be seen in the Tab. IV.

TABLE IV
RESULTS OF THE OPTIMIZATION

Optimization Results	Value
J	-7.6436
Number of function evaluations	80
c	$0.3911\ m$
t	$0.0010\ m$
L	$4.8047\ m$
$Sweep$	$2.7937°$
$Twist$	$1.0130°$
$Mass$	$5.2814\ Kg$
$V_{flutter}$	$77.9395\ m/s$
Tip displacement	$-4.5331\ m$

Graphically, the results may be seen as the Fig. 8, with the parameters that minimizes the optimization functions.

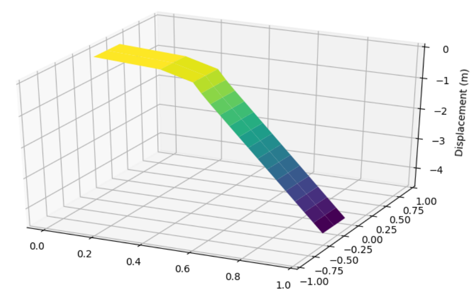


Fig. 8. Plate model that resulted from the optimization

As expected, the optimization returned a aircraft configuration with a high flutter speed combined with a very low mass. These are good results that greatly increase the performance of the aircraft. With this, it is possible to conclude that the optimization was effective.

C. Future works

Talking about the code, a good improvement is to include the flight dynamics in the analysis, so it is possible to analyze the wing tip displacement during the flight. It is needed to model the problem with composites and include some of its characteristics in the optimization, as material, orientation and number of layers. It is planned also in future versions a creation of a GUI (graphic user interface) so it becomes easier to model the aircraft in the code and generate the desired analysis. About the optimization, it is possible to analyze which type of algorithm is better to the present situation, create a pareto to have a greater quantity of aircrafts to compare, increase the number of parameters that are modifiable and even transform the problem in a multidisciplinary one. It is possible to connect the code with an software of aerodynamics simulations like XFOIL to generate and compare a large amount of airfoils and data. It is still possible to use genetic algorithms for example to increase the performance and the generation of aircrafts to analyze.

In a far future, it is desired to use the optimized aircraft and the aeroelastic model to design an active flutter suppress controller to greatly improve its resilience against the flutter phenomenon.

V. Conclusion

First of all, a model of the problem of the highly flexible wing was made based on an existing model. In it, the structural dynamics and the aerodynamics were modeled and than coupled, generating nonlinear coupled equations of structures with high deformations. As soon as the modeling phase ended, the code to solve the problem was built in Python 3.5 and some examples were solved with either the original code, the AeroFLex Toolbox, and the code of the present work, reaching in the same results. The code was then considered valid to be used to solve this problem.

Then, a simple model of a aluminum plate representing a half highly flexible wing. Some geometric parameters were chosen to be optimized and the optimization problem was built, with its objective function and its well defined constraints. Then a code was designed with the openMDAO python package with the optimization problem. Its results were then collected and analyzed, generating a configuration that proved to be efficient and in accord with the requisites of the project.

Since the project is to be an initiation in research, a highly complex problem with an extremely complex model was studied with simple techniques and simplifications to make it reasonable to be solved. The results were considered effective for the present problem, but physical and more complexes analyses are still important to be made to make it useful in a future more complex project.

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