Gaussian Process for Structural Analysis and Optimization

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1 Introduction

The mesh based method are used dominantly in the field of engineering. The most known method,Finite Element Method (FEM) has been used widely for the purpose of numerical analysis. The FEM technique is based on the division of the domain into finite number of elements specified with a finite number of parameters. The solution of the problem is obtained on solving the equations at the nodes of the elements. The discretization process under finite element is its foremost advantage.With time, the application of mesh based method expanded and hence put the major drawbacks into light.[3] In case of mesh based methods, the data is transferred from the physical domain to the computational domain for all the calculations are performed in the computational domain. Once the calculations are complete,data is again transferred back to the physical domain. The entire process is complex and the method often face problem with the distortion of the mesh in case of large deformation. In the case of Meshless Methods, all the calculations are performed in the physical domain. The interpolation in this method is free from mesh and hence the calculations takes place at the nodes, therefore discarding the possibility of re-meshing and distortion.[3] The extensive properties are not assigned to the mesh but rather to every single nodes in the domain. One of the oldest mesh-free method is Smoothed Particle Hydrodynamics (SPH); in 1977 discovered by Gingold, Monaghan and Lucy; which initially dealt with the astronomical problems.[3] The meshless method changes the way numerical analysis has been used and hence it becomes readily important that the field keeps on expanding with new ways of solving complex geometries.

2 State of the Art

The bibliography content of the project was pretty vast as the study of the basic elements of the gaussian process needs to be studied. The surrogates model, meshless methods, kernel functions has been the most important aspect of the literature survey. The paper based upon the meshless methods of laminated and functionally graded plates and shells[3] has been of significant advantage on kernels and in particular, Element Free Galerkin(EFG).[6] The EFG method was more suitable for crack propagation. The subsequent method used for the crack analysis was Reproducing Kernel Particle Method(RKPM).[2] Earlier the method was employed by Li et al.[5] for the simulation of dynamic shear band propagation and failure mode transition. A yet another application of meshless method came for transient heat conduction problems by using EFG.[4] In this scenario, EFG is applied to compute two-dimensional unsteady state problem where a square plate is taken into consideration with three sides of the plate maintained at a constant temperature of 100 C and the upper side is subjected to 500 C.The major bibliographic content was from the research paper of Moving node approach in topology optimization.[1] The two methods employed here were EFG and Meshless Local Petrov-Galerkin(MLPG).[7]

3 Development

As the MATLAB code for the problem of cantilever beam has already been provided to us,[1] the aim as mentioned above was to observe the outcomes while changing the kernels. The cantilever beam has been made initially with the boundary conditions wherein the left edge has been fixed and a transverse load is applied on the right edge. The number of nodes defined in the x and the y directions are 10 and domain shape is considered to be rectangular. Further more, the problem is sub-divided into two possible ways of analysis, EFG method and MLPG method. There are two different nodal distribution methods that are taken into consideration in EFG method namely; random and regular/pattern nodal distribution. The two methods employed to solve the problem were EFG and MLPG. In both the methods, the linear equations are discretized by MLS approximation approach. A MLS approximation has been defined which contains the kernel functions for the analysis. There are many kernel functions that can be employed to get accurate results. The given code on moving node approach has been of spline kernel function. As our primary aim had been the better understanding of the working of kernel and the effects of different kernel on the same problem, the kernels are changed. The two types of kernel used are Radial Basis kernel and Gaussian kernels.

4 Results

The analysis is performed in order to obtain the displacement and the stress on the cantilever beam due to a traction force. The results are obtained for the displacement and stress concentration in the beam. Although the study of the shear stress approximation could be great importance as the variation of error is justified through that results.

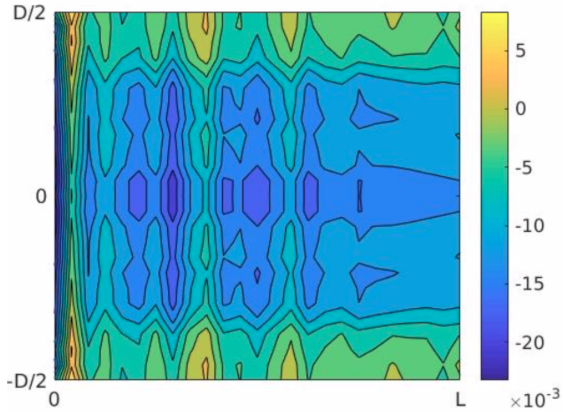

Figure 1: Radial Basis Function

The error norm and compliance achieved using EFG method for collocated boundary condition using RBF kernel were 0.0976 and 2.6888 respectively.

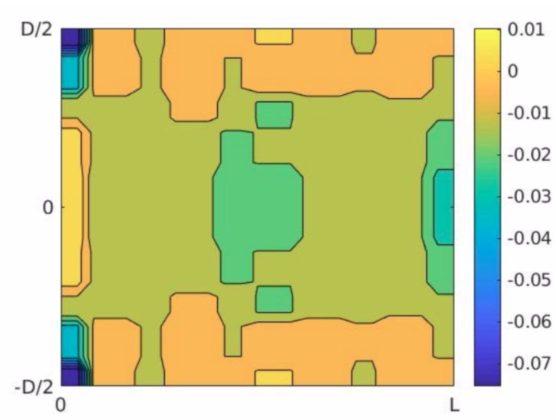

Figure 2: Gaussian Function

Similarly, on using the Gaussian kernel,the error norm and the value of compliance achieved were 0.3533 and 3.5077 respectively.
The results shows varied oscillations in both the cases, which signifies the level of error norm achieved using different kernels. It came into observation that there are many parameters that affected the error norm. The changes in the number of nodes, nodal distribution and the different boundary conditions also provide some changes in the error norm and slight changes in the compliance for both the methods.

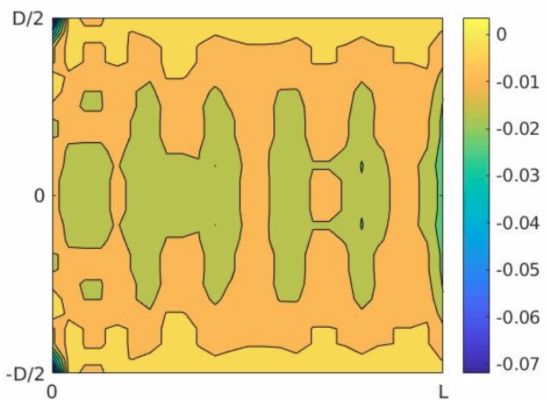

Figure 3: Gaussian Function at 20 nodes

In the figure above, it is readily observed that with the increase in number of nodes, there has been a significant drop in the number of oscillations which can be validated with the fact that the error norm reduces drastically to 0.2735. Hence a possible way to decrease the error norm could be to increase the number of nodes. A significant change is seen during the change of smoothing length in various conditions. On increasing the smoothing length in the problem, we noticed the increase in the oscillations andhereby the error norm increases.

5 Conclusion

In light of the research objective of this project and considering it as the initial phase, conclusions and recommendations are drawn from the study for the future work of this project. The study helped to understand the kernels and the way of implementing them into MATLAB codes. Te usage of three different kernels over a same set of problem also helped us understand the suitability of the kernel in various applications. The two methods used in the analysis were EFG and MLPG, and hence by changing various parameters, we understand that EFG was a more reliable and accurate method for analysis of a cantilever beam as the error norm achieved in the case of MLPG was nearly thrice that was achieved in EFG. The upcoming tasks for the next semester will be initializing one of these kernels to solve problems based on different domains. While going through the bibliography content of the project, it became readily clear that these kernels can be used in many different applications. Two of the major applications that came in light the most were, transient heat conduction problems and deformation of thin plates. Further applications of kernel could be image clarification and retrieval or modelling of geofluid flow and also for for free and forced vibration analyses for solids. There are many domains still untouched from the numerical analysis which might be a very interesting scope of research. Although from the initial study and research it is very evident that now the replacement of kernels can be employed to compare the results and select the most accurate one. The degree of novelty has been justified by replacing the kernels in order to obtain different results for comparison and understanding purpose. Moreover the aim of this project has been to lay the learning foundation for kernels and meshless method which could be helpful for the coming semester's study and application.

6 References

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