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Master of Aerospace Engineering
Research Project

GAUSSIAN PROCESS FOR STRUCTURAL ANALYSIS AND OPTIMIZATION

S2 Project report

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This assignment is entirely my own work. Quotations from literature are properly indicated with appropriated references in the text. All literature used in this piece of work is indicated in the bibliography placed at the end. I confirm that no sources have been used other than those stated.

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24/06/2019	Sneha Dwivedi
Date	Signature

Abstract

Numerical methods are a favorable substitution to physical experiments carried out on the system which generally prove to be substantially expensive and requires good amount of man power. Numerical analysis deals with the approximation based machine learning algorithms. Numerical models are build with the help of these algorithms. Gaussian process,considered in machine learning algorithm, is a process of deriving an approximate function by measuring the similarity between the sample points and hence defining the data on the non-sampling points for optimizing the derived function.

The project deals with the incorporation of a method of Moving Least Squares(MLS) as a kernel function in the Gaussian process which is rather a very unconventional approach to follow. There are various kernel functions that are used in the Gaussian process but the most common one is MLS. As a part of this project, the basic requirement was the understanding of the kernel functions and the following report summarize the usage of kernel functions in a certain application domain. The final aim of this project will be to propose a kernel function that works effectively on comparing to conventional methods and can be incorporated as a new technique to solve complex engineering problems in various domains.

Keywords: Gaussian process, Kernel functions, Mesh less methods, Moving Node approach, Element Free Galerkin (EFG) method.

1 Introduction

1.1 Mesh based Methods-

The mesh based method are used dominantly in the field of engineering. The most known method, Finite Element Method (FEM) has been used widely for the purpose of numerical analysis. The FEM technique is based on the division of the domain into finite number of elements specified with a finite number of parameters. The solution of the problem is obtained on solving the equations at the nodes of the elements. The discretization process under finite element is its foremost advantage. With time, the application of mesh based method expanded and hence put the major drawbacks into light. [3]
In case of mesh based methods, the data is transferred from the physical domain to the computational domain for all the calculations are performed in the computational domain. Once the calculations are complete, mapping is done and the entire data is again transferred back to the physical domain. The entire process is complex and the method often face problem with the distortion of the mesh in case of large deformation. Also, in case of a extremely complex geometries and discontinuities, the mesh based methods suffers heavily due to extensive re-meshing requirements.

1.2 Mesh less Methods-

In the case of Meshless Methods, all the calculations are performed in the physical domain. The interpolation in this method is free from mesh and hence the calculations takes place at the nodes, therefore discarding the possibility of re-meshing and distortion.[4] The meshless methods works upon the technique of interaction of the nodes with their neighbours. The extensive properties are not assigned to the mesh but rather to every single nodes in the domain. One of the oldest meshfree method is Smoothed Particle Hydrodynamics (SPH); in 1977 discovered by Gingold, Monaghan and Lucy; which initially dealt with the astronomical problems.[4] Later on, Libersky applied the SPH in solid mechanics. This initial method simplified the model implementation due to lack of meshing and also worked ideally for the complex boundary dynamics.[8] Although a hurdle in this method of analysis has been experienced. It was seen that setting the boundary condition in SPH has been more difficult than in the mesh based methods. The method was instead readily used in solid mechanics. Many methods with more advancement and improve in the accuracy has been in developed with time and are now working in various domains on several applications.

1.3 Gaussian process -

It is a process in which every finite linear combination of the random variables is normally distributed.[12] The process can also be used in prediction, or kriging where a non sampled point can be predicted by observing the prior function distribution and the sampling points. Basically gaussian processes are an alternative approach to regression problem.

1.4 Moving Least Square approximation -

With time, substantial improvements took place in the field of mesh less methods and then Moving Least Square approximation came into use. The Moving least square is a method to reconstruct a function from a set of non-sampled points by calculating the weighted least square. This approximation played a vital role in the establishment of various other meshless methods. The method taken into major consideration in this report will be the Element Free Galerkin (EFG) method which is being studied and applied on thin plates deflection[2] and shells earlier.[3] [4] In this report, the nodal analysis of a cantilever beam is studied in order to understand the execution of the kernel functions in EFG. The kernel functions can be changed in order to attain more accuracy.

1.5 Kernel Functions -

With the study of various applications of kernels, the aim of optimising the structure becomes clearer. The upcoming section will lay the discussion upon the learning outcomes of the referred code on Moving Node Approach[1] and also the changes incurred in the code in order to study the distinct use and implementation of kernel functions and meshless methods in the domain. The possible application of meshless methods like in thermal conductivity[5] and elasticity and fracture[7] is also studied for providing the future research direction to this project.

2 Semester 2 section

The project work began with the literature survey of the various components of the project. The fact that there were no subsequent prerequisite available for our project and hence we have to start by studying the most basic part of the project. After the allocation of the project, the major work relied upon learning the Surrogates models and the Gaussian process. The project demanded a strong hold on the basics, so a significant amount of time has been allotted on the literature survey.

The literature survey included the study of various applications of meshless method through research paper. The research papers provide a broad domain of the working of various meshless methods including thin plates[2] and shells[3], transient heat conduction problems[5], elasticity and fracture problems[7],etc. These papers allowed better understanding of various methods that can be opted to numerically analyse a problem. Along with the various meshless methods, the study of kernel functions[1][8] used in Gaussian process are also very important as changing the kernels will help us change the error terms in the problem.

The literature survey will be continued throughout in order to inculcate the fundamentals and keep updates with the possible applications. The next major step in the project will be the study of various test cases and understand the working of their conventional methods.

During the S2 section of the project, a test case has been studied in order to understand the working of the conventional kernel functions employed in the code and then change the kernel function in order to observe the effectiveness of the results. The test case is based upon the field of topology optimization wherein an optimal material layout based upon specific performance targets is the desired output.[1] The use of meshless methods has been deployed for the optimization and henceforth all the equations are constructed on the nodes.

The major outcome during this session was the understanding of the MATLAB code provided for the above mentioned problem in order to observe the working of kernel functions in MATLAB and the way they can be altered in order to achieve different results. The main aspect of this session was to understand a significant test case and observe the alterations that can be readily taken into consideration in future while working on a particular application.

2.1 Context and key issues

The use of meshless methods has been very prominent since the discovery of SPH in 1977 and it boosted the engineering field with more accuracy in the results.[8] The meshless method changes the way numerical analysis has been used and hence it becomes readily important that the field keeps on expanding with new ways of solving complex geometries. Hence working upon the meshless methods allows a vast range of methods to work with that produces different results and put forward the most accurate results for a certain problem.

The key issue concerned with this project is to work upon a kernel function in order to optimise a particular body. The use of kernel functions in MATLAB is the foremost importance and keen observing point of the project. The main issue for this semester was the use of the kernel functions used in the particular test case of moving node analysis.

2.2 State of the Art

The bibliography content of the project was pretty vast as the study of the basic elements of the gaussian process needs to be studied. The surrogates model, meshless methods, kernel functions has been the most important aspect of the literature survey. Furthermore, the literature survey expanded taking into consideration the various research papers provided for the study of the meshless methods that include varied applications of the methods.

The paper based upon the meshless methods of laminated and functionally graded plates and shells[4] has been of significant advantage on kernels and in particular, Element Free Galerkin (EFG)[14] as it is the required kernel for our test case on moving node approach. The EFG method[3] with linear MLS approximation[8] was used in this test case to study the crack growth and propagation. The EFG method was more suitable for crack propagation as it does not require re-meshing and neglects the use of finer refinements near the cracks.

The subsequent method used for the crack analysis was Reproducing Kernel Particle Method (RKPM).[3] Earlier the method was employed by Li et al.[9] for the simulation of dynamic shear band propagation and failure mode transition. Also it becomes feasibly easy to solve application of structure dynamics[11] and metal forming as well[10]. The major advantage of using RKPM is that non-linear structural analysis of large deformation is carried out potentially. The method not only shows good performance but also the results are very accurate.

A yet another application of meshless method came for transient heat conduction problems by using EFG.[5] In this scenario, EFG is applied to compute two-dimensional unsteady state problem where a square plate is taken into consideration with three sides of the plate maintained at a constant temperature of 100° C and the upper side is subjected to 500° C. Secondly the use of mass lumping technique is taken into consideration in this problem. Mass lumping techniques helps in deriving a matrix free formulation by “lumping” the mass matrices. This technique helps in saving computational time and cost as well. The results obtained from this technique were compared with the FEM results and it was observed and hence concluded that EFG combining mass lumping technique provides an effective and an accurate result. Along with the result, it was stated that the MLS shape function does not coincide with Kronecker delta scheme and hence it becomes slightly difficult interpolating the boundary conditions.

The bibliography was not constrained to only literature survey but substantial help was extracted from the tutorial videos on topics like kernel functions[13] and gaussian process.

The major bibliographic content was from the research paper of Moving node approach in topology optimization.[1] The two methods employed here were EFG and Meshless Local Petrov-Galerkin (MLPG).[15] The results from both the methods were compared and the paper provide in-depth knowledge about the methods. Through this paper, the difference between the working of two meshless methods had come to light. It also provide a proper in-sight of implementation of the meshless methods in this test case.

The problem revolves around a two-dimensional cantilever beam which is being subjected to a transverse load on the right edge which is a parabolic traction and its fixed on the left. The essential boundary conditions are provided and the EFG and MLPG method are applied to discretize the linear elasticity problem.

2.3 Justification of the potential degree of novelty

The main aspect of study in this project has been the kernel functions and their usage in order get accurate results. As mentioned in the State of the Art, our main bibliography context was the Moving Node Approach in Topology optimization[1] wherein a certain kernel function is used to solve the cantilever beam problem. Although the kernel function used in this problem is providing us accurate results but in order to understand more about the working of kernels and their selection, the use of other kernel functions was extremely necessary. There are many kernels functioning for various applications and there is always a scope to use different kernels for one problem and that is being discussed in this report.

As in the initial stage of the project, the understanding of kernels has been of foremost importance and hence working upon a described problem with our main kernels will allow us to understand it easily than sticking to just bibliography content. Hence the usage of different kernels in the MATLAB code in order to attain new results is the justified potential degree of novelty.

2.4 Aims and objectives

The aims and objectives are as follows-

- **Kernels –** The main objective of this entire project is using a certain kernel function to solve various problems based in any domain. The project in its later part will focus on any one problem related to solid mechanics that will be solved using an unconventional kernel function. There has been many disruptions as discussed above following the cracks in thin plates and shells which are readily solved using kernels and hence the aim is to figure yet another possible concern which can be solved through kernel functions.

- **Comparison of methods –** During this semester, the study of the previous works took place and hence number of meshless methods were introduced. The report will be focusing more upon the EFG and MLPG methods as these two are the most used ones. Hence one of the major objective turns out to be the selection of the methods for different analysis purposes. There are many differences in EFG and MLPG which helps us select the most optimal one for a certain analysis.

- **Error norm and Compliance -** The error norm is a scalar quantity calculated by the relative error between the elastic energy of the approximated solution and analytical solution whereas the compliance is the inverse of the global stiffness of the structure. The error norm and compliance is calculated in both the methods and are compared. It is observed with the distinctive values of these parameters that which method is more reliable and accurate in getting results. Moreover the value of error norm varies with change in parameters like number of nodes, domain size and number of integration points. Compliance can be very helpful in understanding the nodal distribution and its largely focused to minimize the compliance for topology optimization.

3 Investigation Methods

3.1 Predefined Parameters

As the MATLAB code for the problem of cantilever beam has already been provided to us,[1] the aim as mentioned above was to observe the outcomes while changing the kernels. The cantilever beam has been made initially with the boundary conditions wherein the left edge has been fixed and a transverse load is applied on the right edge as shown in the figure below-

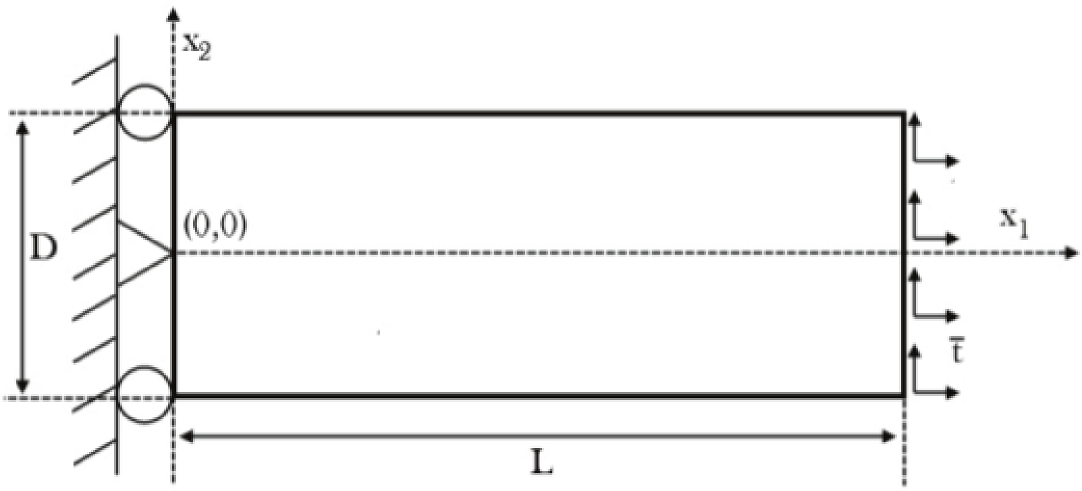


Fig 1- A cantilever beam of length L and height D is subjected to traction force on right and fixed condition on left. *Cited in [1]*

3.2 Meshless methods EFG and MLPG

The number of nodes defined in the x and the y directions are 10 and domain shape is considered to be rectangular. Further more, the problem is sub-divided into two possible ways of analysis, EFG method and MLPG method.

The Element Free Galerkin (EFG) method uses the shape functions of moving least square as their test function by employing a global weak formulation of the model. Due to lack of the Kronecker delta criteria, the displacement vector is expressed in terms of the virtual nodal displacement and hence it prevents the application of essential boundary conditions. Hence the boundary conditions in EFG method is enforced using the Lagrange multiplier. In order to calculate the integrals, Gauss quadrature scheme is used which can numerically find an approximation of the integral by assembling the weighted value at the integration points marked at every integration cell.

The one big difference between EFG and MLPG was derived during the nodal distribution in both the cases. Although both the cases are of mesh less methods, the EFG technique still uses a background mesh or integration cells where the integration of the equations is evaluated.[16] The integration cells consists of integration points and there are 4 defined integration points in every cell as shown in the figure below-

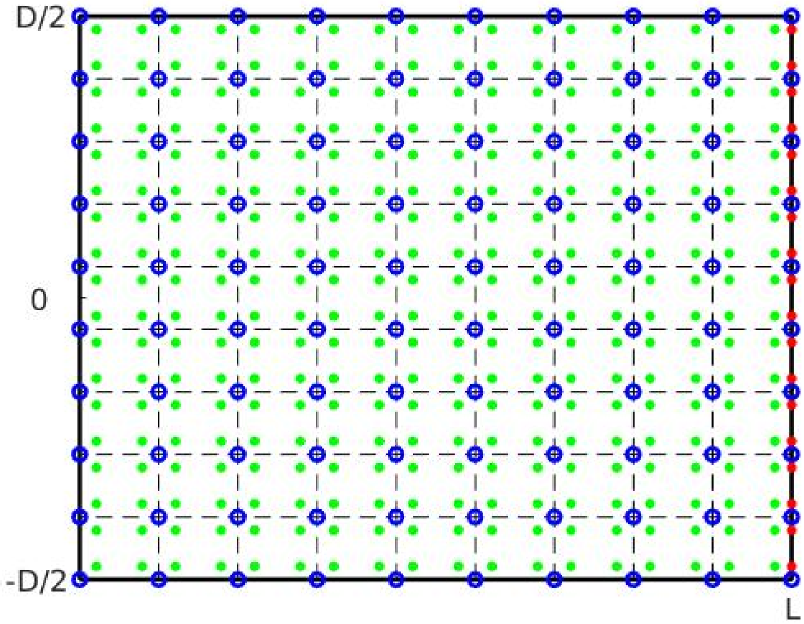


Fig 2 – The discretized domain and the boundary condition on a cantilever beam using EFG method. *Cited in [1]*

The Meshless Local Petrov-Galerkin(MLPG), which is based on a local weak form also works on the MLS approximation. In the case of MLPG mixed collocation method[17], no such background mesh is created and all the integration is done by nodal integration i.e. the equations are solved at the nodes. The integrals are present in local weak form and are evaluated on domains.

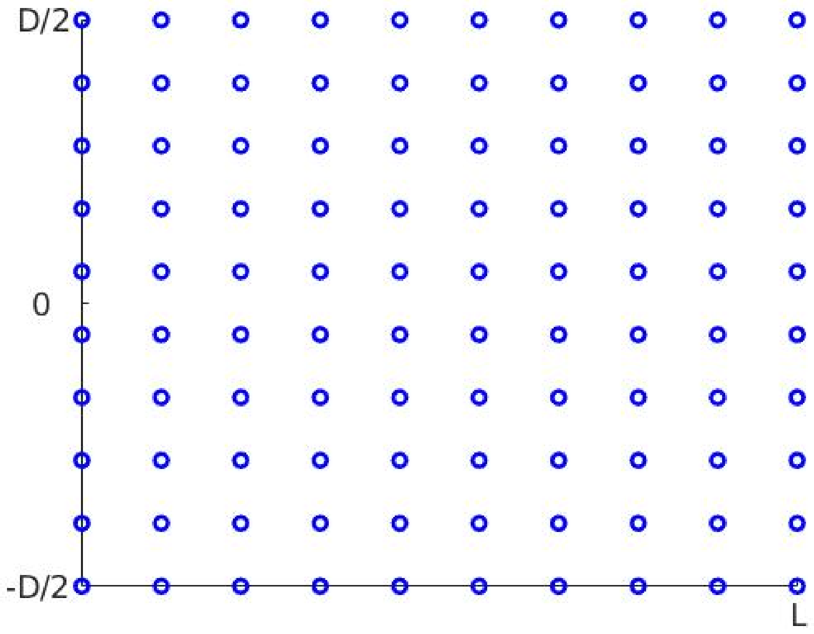
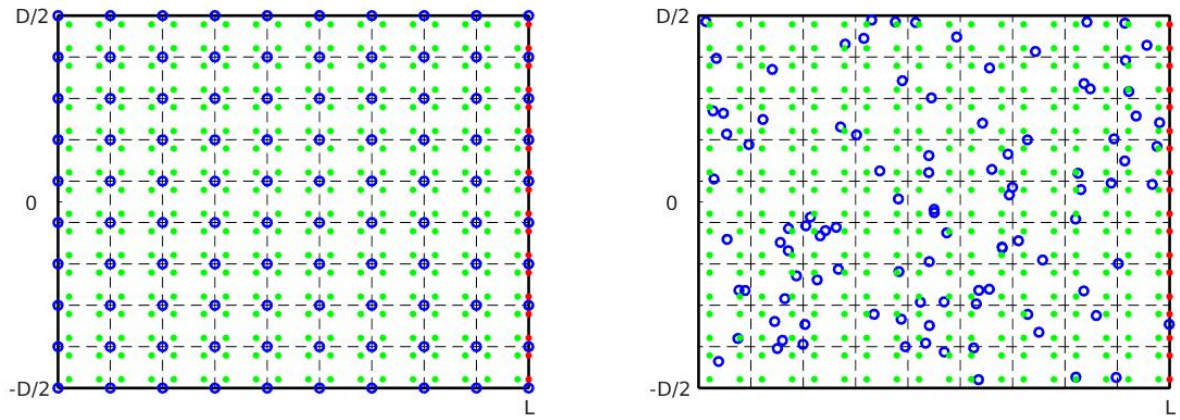


Fig 3 – The nodes created on a cantilever beam using MLPG method. *Cited in [1]*

3.3 Nodal distribution

There are two different nodal distribution methods that are taken into consideration in EFG method namely; random and regular/pattern nodal distribution. The difference between the two distribution will be striking in the output and as shown in the figure 4(a) and 4(b). While the nodes are well positioned as a rectangular grid in case of the regular nodal distribution, the nodes are scattered in the domain in case of random distribution where the random nodal distribution can have infinite number of possibilities.



(a) (b)

Fig 4 – (a)shows the regular distribution of the nodes inside the domain and (b) shows the random distribution of nodes in a domain in EFG method. *Cited in [1]*

Apparently, the value of error norm and compliance is affected by the nodal distribution. In the case of a random nodal distribution it is observed that there is an increase in the value of the error norm as well as a small increase in the compliance.

3.4 Displacement Boundary conditions

The changes are incurred while the boundary conditions are changed. There are two types of displacement boundary conditions provided. One is the essential boundary conditions and the other is the collocated boundary condition. The type showed in figure 4 is the collocated boundary condition where the number of collocated boundary particles along the fixed edge of the beam is equal to the number of nodes along the height.
In case of the essential boundary conditions, the number of nodes will be equal to the number of cells along the height + 1. [1] On analysis, it is been observed that the error norm and the compliance does not vary much in case of regular nodal distribution, but instead observed a significant change in these parameters in the case of random distribution.

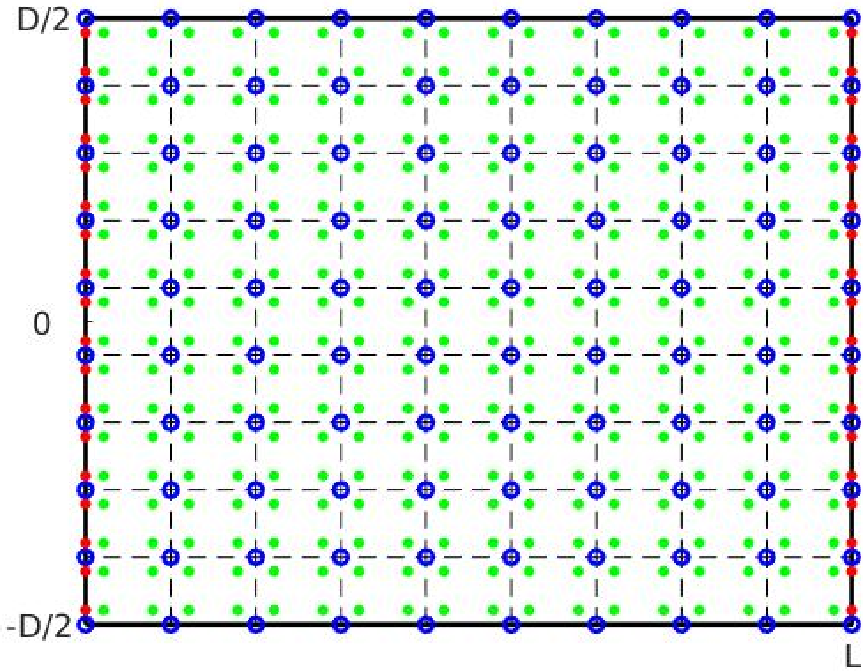


Fig 5 – The essential boundary condition signified with red dots denoting the boundary integration points. *Cited in [1]*

3.5 Smoothing length

In a grid, a certain domain is selected which has a node along with its neighbouring nodes. The distance of the node from the boundary of the domain is the smoothing length that allows the method to work on that defined area in order to generate parameters on non sampled nodes. The value of smoothing length has been provided in the code initially but on bringing changes in the value of the length, certain changes has been observed and noted.

3.6 Kernel Functions

The two methods employed to solve the problem were EFG and MLPG. In both the methods, the linear equations are discretized by MLS approximation approach. A MLS approximation has been defined which contains the kernel functions for the analysis. There are many kernel functions that can be employed to get accurate results.

In the given scenario, a cubic spline weight function has been used to get the results which holds (*Cited in [1])-*

$$W(x-x^I,d)=W(r)=\alpha(\frac{2}{3}-4r^2+4r^3) if\, 0\leqslant r\leqslant\frac{1}{2}$$

$$W(r)=\alpha(\frac{4}{3}-4r+4r^2-\frac{4}{3}r^3) if\,\frac{1}{2}\leqslant r\leqslant 1$$

$$W(r)=\alpha(0) otherwise$$

where r is the normalized distance and α is a constant depending on the dimension and shape of the kernel. The normalized distance r is chosen such that:

$$r=\frac{|x-x_i|}{d}$$

Also the derivative of this spline weight function is given as:

$$W_{,}(r)=\alpha(-8r+12r^2) if\, 0\leqslant r\leqslant\frac{1}{2}$$

$$W_{,}(r)=\alpha(-4+8r-4r^2) if\,\frac{1}{2}\leqslant r\leqslant 1$$

$$W_{,}(r)=\alpha(0) otherwise$$

The same is carried out for a two-dimensional domain and hence yet another similar terms and their derivatives will be there for the y axis as well. Hence for a particular domain, the kernel function will be (*Cited in [1])-*

$$W(x-x^I,d)=W(r_1,d_1 d)W(r_2,d_2 d)$$

and the derivatives will be equal to-

$$W_{,1}(x-x^I,d)=W_{,1}(r_1)W(r_2)=W_{,r1}(r_1)r_{1,1}W(r_2)$$

$$W_{,2}(x-x^I,d)=W(r_1)W_{,2}(r_2)=W(r_1)W_{,r2}(r_2)r_{2,2}$$

The above kernel function using the spline weighted function has been initially used to obtain the result under the prescribed parameters mentioned above. The results from this kernel function has been kept to compare the results when the changes will be called in the kernels.

3.7 Changes carried out in the kernels

Now the changes are carried out in the kernel function. The given code on moving node approach has been of spline kernel function. As our primary aim had been the better understanding of the working of kernel and the effects of different kernel on the same problem, the kernels are changed.

- **Radial Basis Function (RBF) –** A radial basis function has a value depending upon the distance from the origin or some particular point. For example if the value is depending upon the origin distance, then the function is -

$$\Phi(x)=\Phi(||x||)$$

-

  But if the distance is considered from any point “p”, the function is -

$$\Phi(x,p)=\Phi(||x-p||)$$

  Hence the RBF kernel also need two samples and is represented as -

$$K(x,x')=\exp\left(\frac{-||x-x'||^2}{2\sigma^2}\right)$$

where $||x-x'||^2$ is the squared Euclidean distance between the two vectors and $\frac{1}{2\sigma^2}$ can be replaced with a parameter У. As the problem is in two dimensions, the variance will be calculated in terms of x and y.

- **Gaussian kernel –** Gaussian kernel is a type of a Radial Basis kernel.

$$W(r)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left(\frac{-x-\bar{x}^2}{2\sigma^2}\right)$$

  The above equation is needed to run the code in terms of Gaussian function. Hence in order to make the code work, additional inputs has been given. The term variance and mean for the variable x and y are being added as they are important enough to change the magnitude of the output. Gaussian functions are usually used in statistics to define normal distribution.

With the addition of these two different kernels, it is observable as a problem can be solved through various kernel functions. The results are obtained in various conditions by changing the nodal distribution, number of nodes and boundary conditions.

3.8 MATLAB coding

As it has been mentioned during the progress report, the use of MATLAB will be of foremost importance and hereby it required understanding of the kernels to inculcate them into MATLAB code. The spline kernel function has already been written in the code and the changes were made in the kernel functions as per the equations given above for the Radial Basis Kernel and the Gaussian kernel respectively.

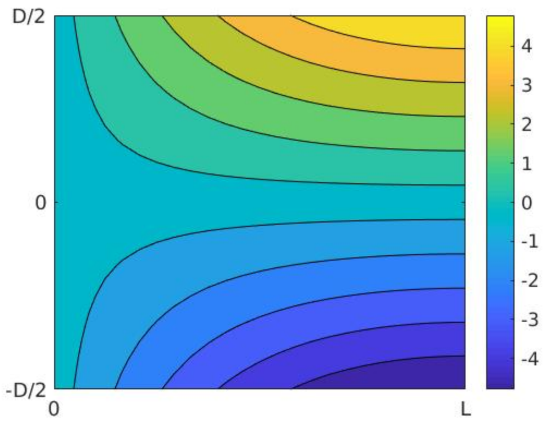
Further changes were made in the code to input the variance as well as the mean. For radial basis function, the value of variance is needed and hence it was being defined under the function. Similarly for the Gaussian kernel, the value of mean has been defined. Both variance and mean has been defined for two dimensional problem.

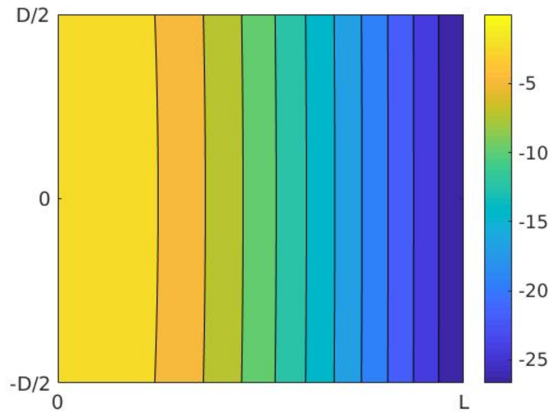
4 Results and analysis

The analysis is performed in order to obtain the displacement and the stress on the cantilever beam due to a traction force. Also the aim will be to keep the error norm minimum which is the relative error between the elastic energy of the approximate and the analytical solution and compliance is also kept minimum.

The initial analysis and results are based on the spline kernel function using the two methods of EFG and MLPG.

In case of EFG, the displacement in both the axis are obtained as-

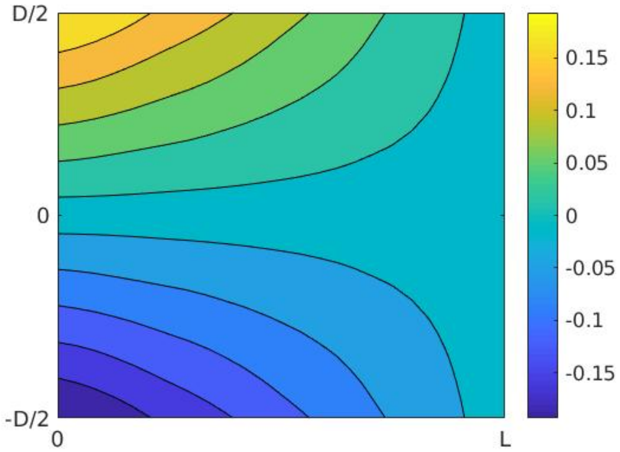




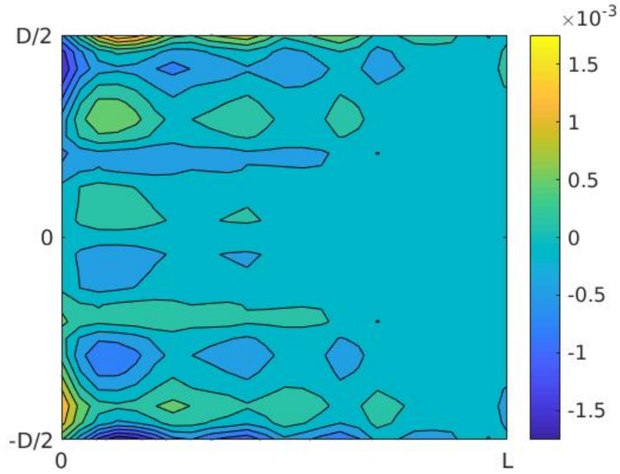
(a) (b)

Fig 6- (a) shows the displacement in x direction while (b) shows the displacement in y direction. *Cited in [1]*

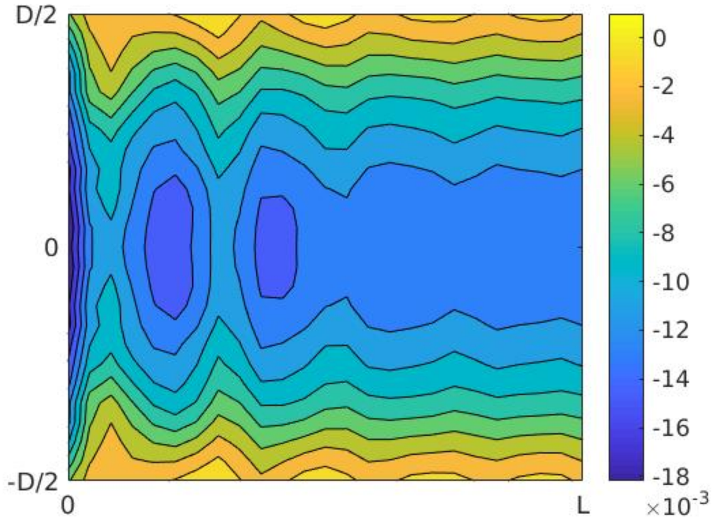
The stress obtained are as -



(a)



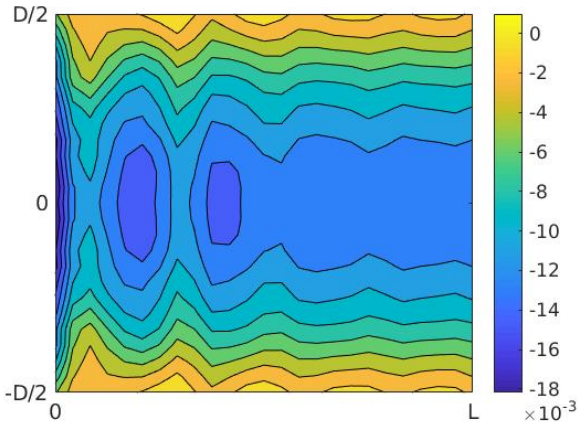
(b)



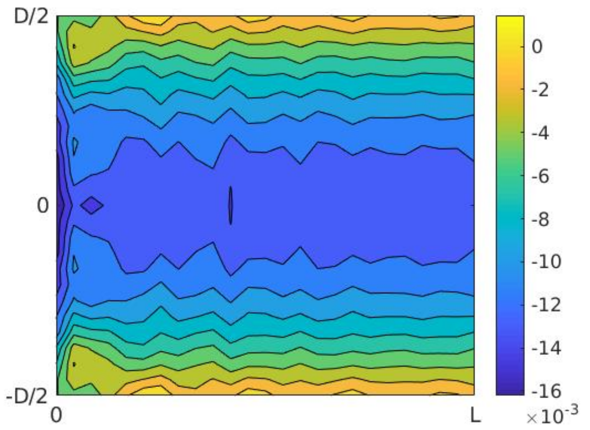
(c)

Fig 7- (a) and (b) represent the normal stress concentration in both the axis and (c) shows the shear stress. *Cited in [1]*

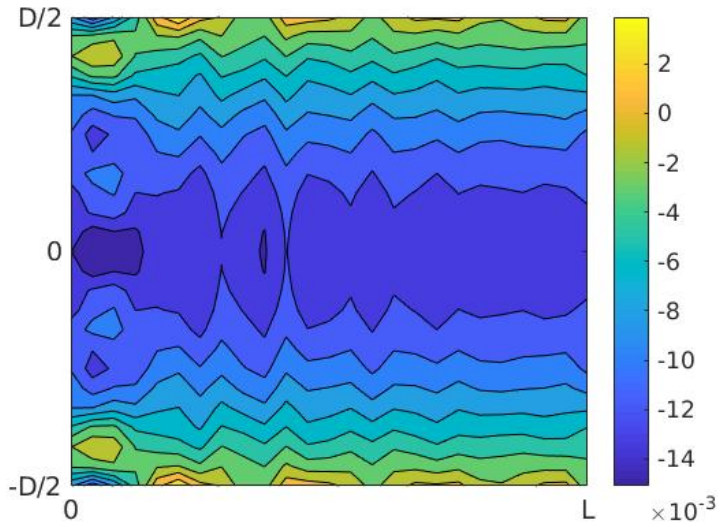
On changing the smoothing length from 2.5 to 3 and 3.5, the number of oscillations in the case of sheer stress went on increasing along which a substantial increase in the error has also been observed.



(a)



(b)



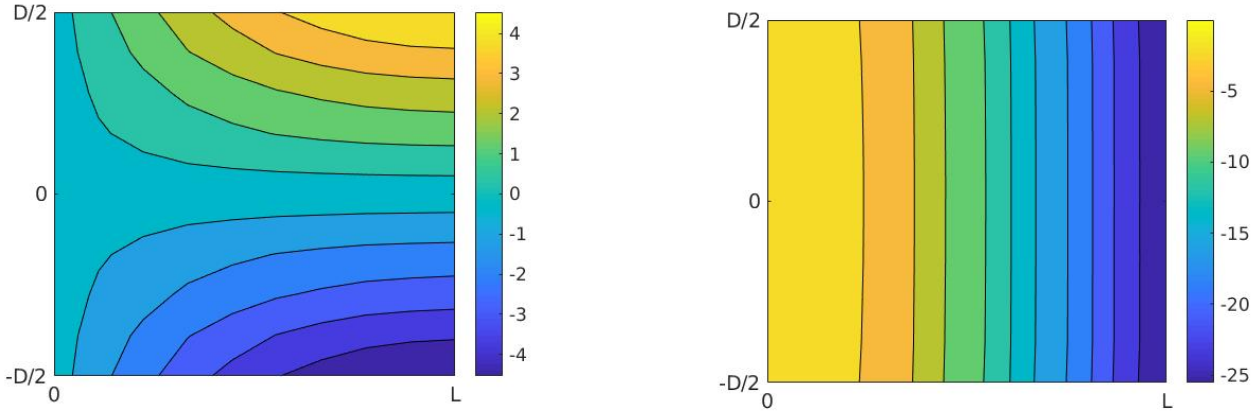
(c)

Fig 8- The change in the oscillations due to increase in the smoothing length (a) at 2.5 *Cited in [1]* , (b) at 3 and (c) at 3.5.

Error norm	0.0169 or 1.69%
Compliance	2.6674

Table 1 – The error norm and compliance achieved using EFG method for collocated boundary condition. *Cited in [1]*

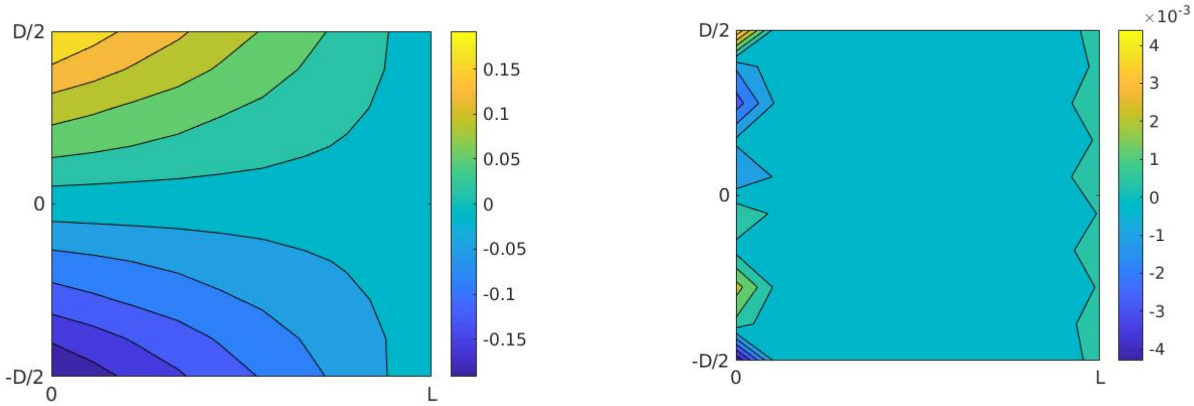
In case of MLPG, the displacement in both the axis are obtained as-



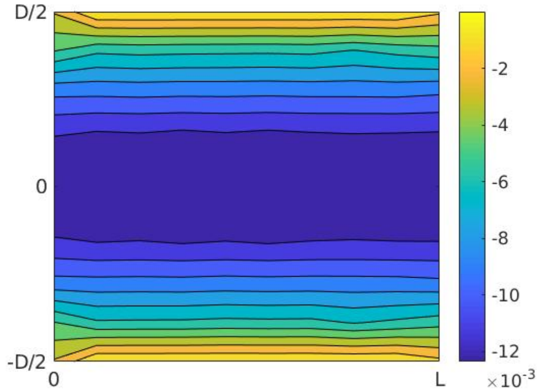
(a) (b)

Fig 9- (a) shows the displacement in x direction while (b) shows the displacement in y direction. *Cited in [1]*

Similarly for the stresses, the MLPG method provides-



(a) (b)



(c)

Fig 10- (a) and (b) represent the normal stress concentration in both the axis and (c) shows the shear stress. *Cited in [1]*

The error norm achieved after analysis by MLPG method was 0.0483 or 4.83% which is more than that obtained from EFG method.

It is further observed that on increasing the number of nodes, the error reduces. In the previous results, the number of results on both the axis were 10. Now we increased the number of nodes from 10 to 20. The error norm is bought down to 0.0056 or 5.6% whilst the compliance remain nearly the same. The reduce in the error has been backed by the number of oscillations in the stress representation.

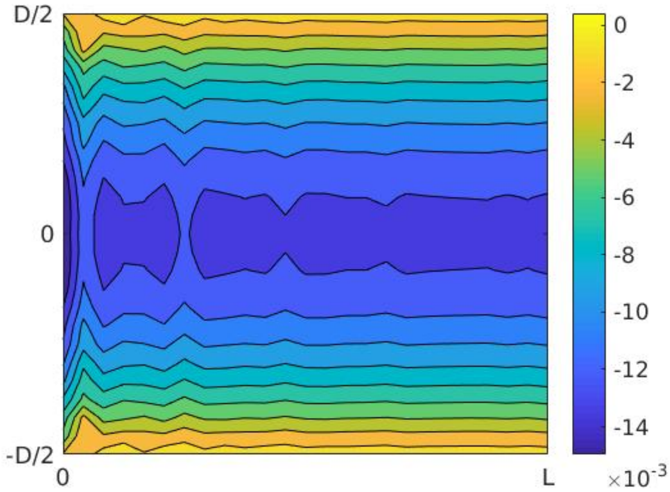
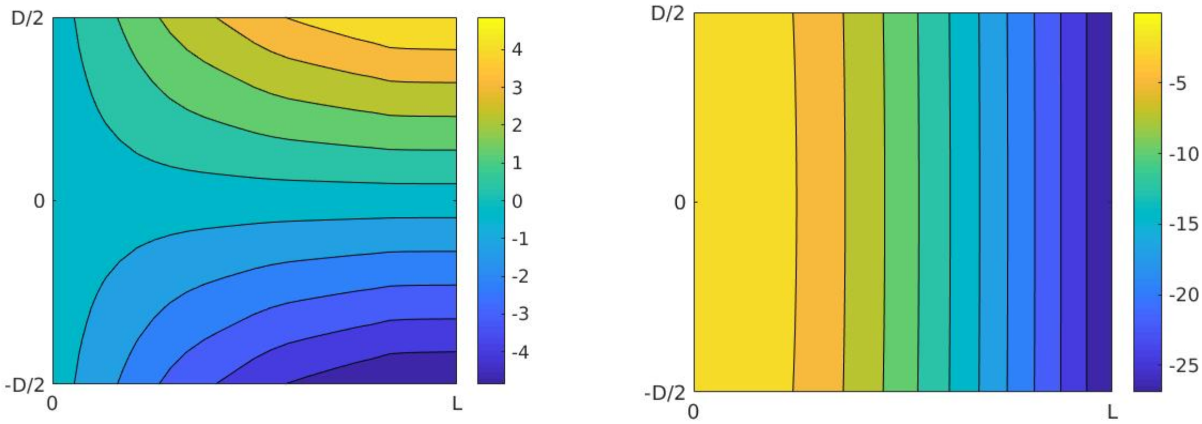


Fig 11- The oscillations are reduced as compared to the previous stress representation.

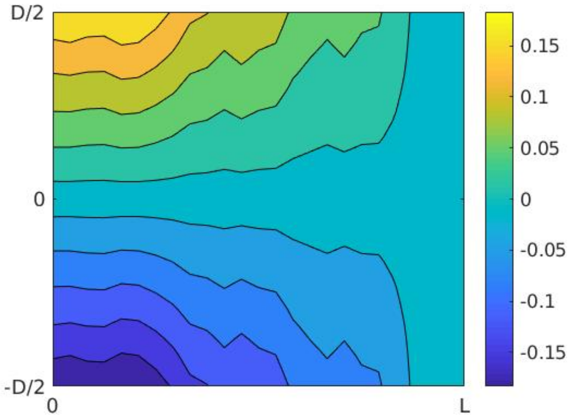
The next results are observed for the condition using RBF kernel in the code. Hence for the case in EFG method under the collocated boundary condition, the displacement in both the axis are obtained as-



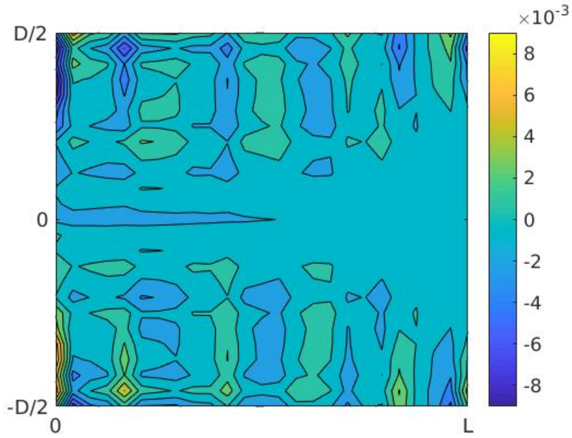
(a) (b)

Fig 12- (a) shows the displacement in x direction while (b) shows the displacement in y direction.

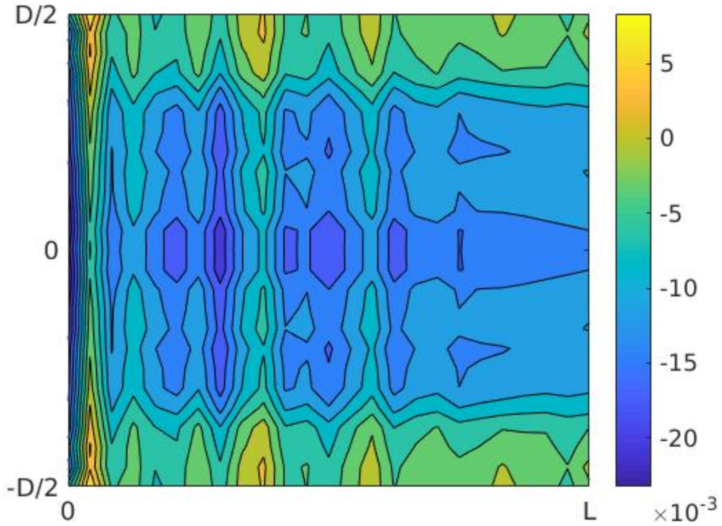
The stress obtained are as -



(a)



(b)



(c)

Fig 13- (a) and (b) represent the normal stress concentration in both the axis and (c) shows the shear stress.

Error norm	0.0976 or 9.76%
Compliance	2.6888

Table 2 – The error norm and compliance achieved using EFG method for collocated boundary condition using RBF kernel.

On changing the number of nodes from 10 to 20, the error norm has reduced to 0.0718 or 7.18%.

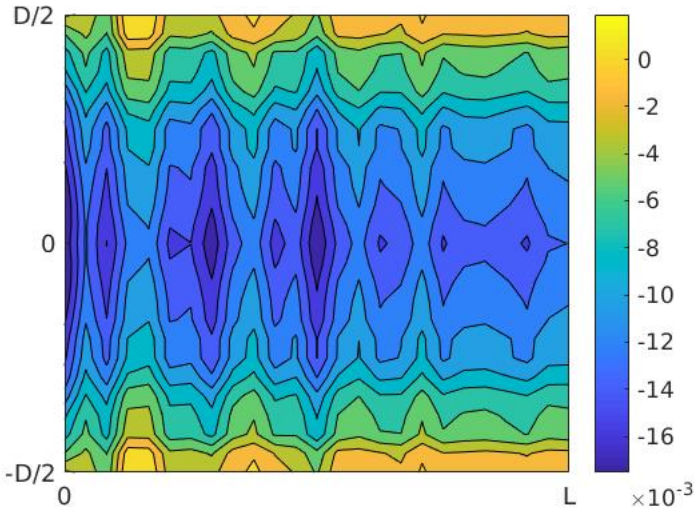
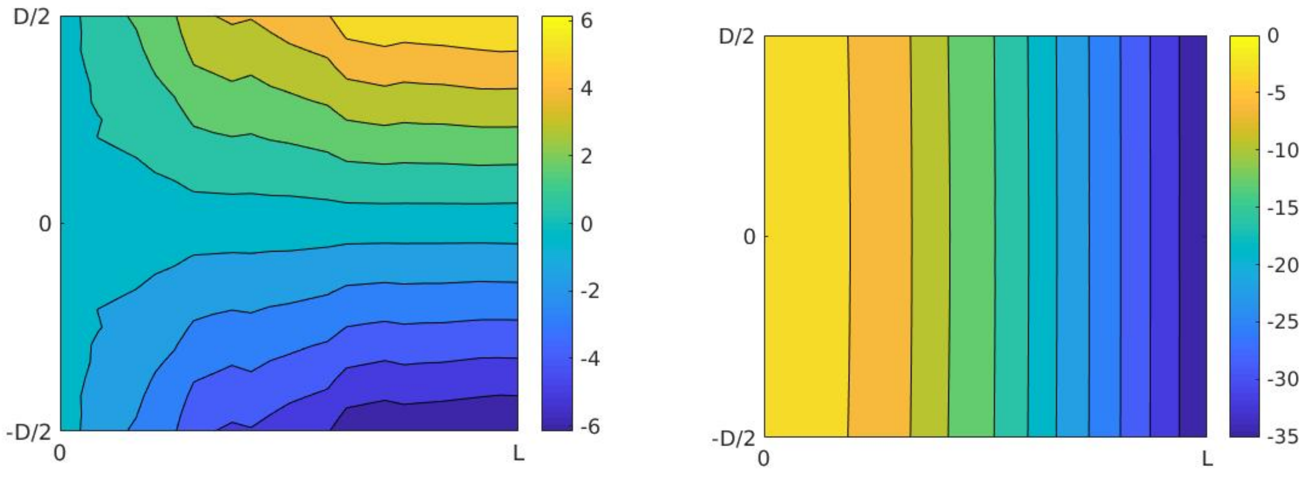


Fig 14- The oscillations are reduced as compared to the previous stress representation on increasing the number of nodes.

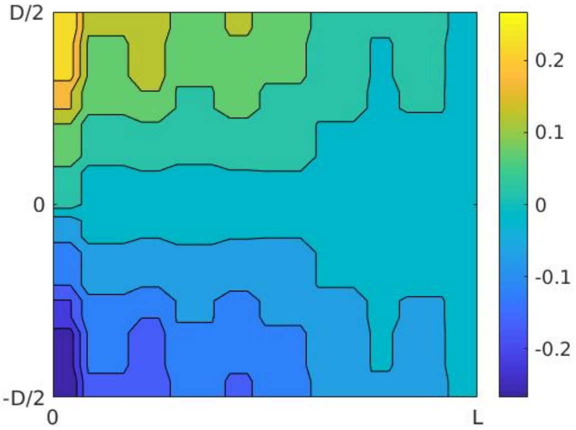
The next results are observed for the condition using gaussian kernel in the code. Hence for the case in EFG method under the collocated boundary condition, the displacement in both the axis are obtained as-



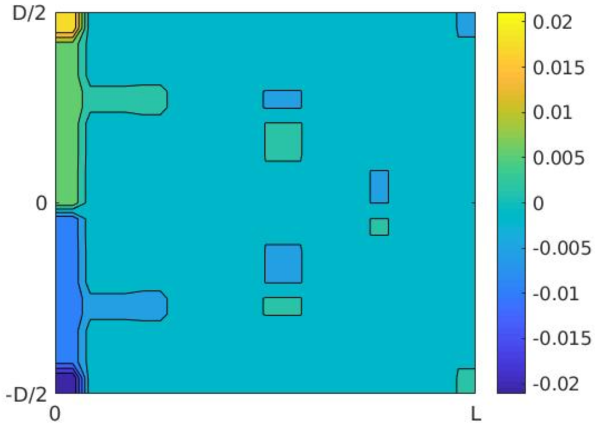
(a) (b)

Fig 15- (a) shows the displacement in x direction while (b) shows the displacement in y direction.

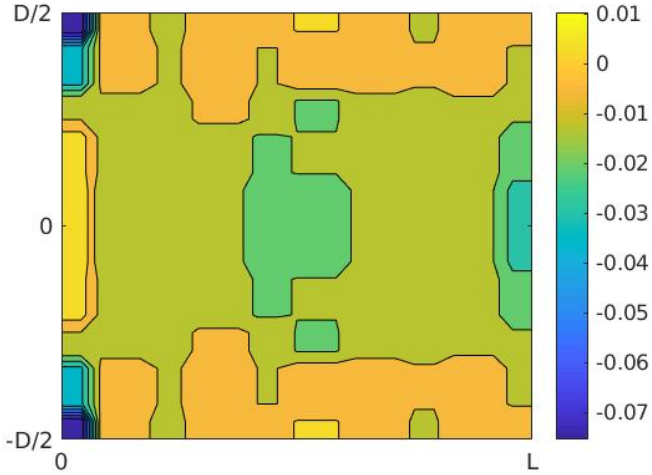
The stress obtained are as -



(a)



(b)



(c)

Fig 16- (a) and (b) represent the normal stress concentration in both the axis and (c) shows the shear stress.

Error norm	0.3533 or 35.33%
Compliance	3.5077

Table 3 – The error norm and compliance achieved using EFG method for collocated boundary condition using Gaussian kernel.

Hence in this case as well, it was observed that o increasing the number of nodes by twice the original, there is a decrease in the error norm to 0.2713 or 27.13% but is still more than our previous kernel.

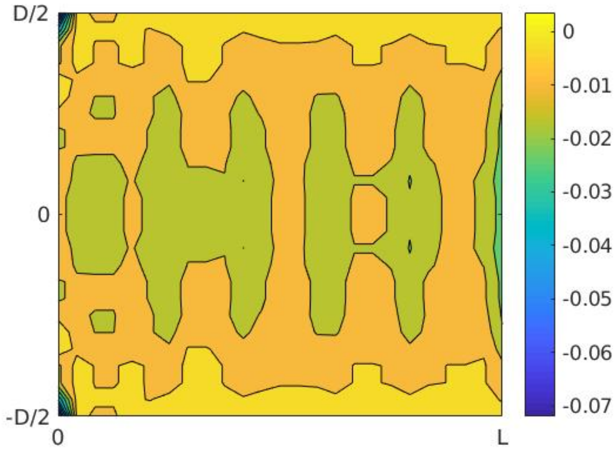


Fig 17- The oscillations are reduced as compared to the previous stress representation on increasing the number of nodes.

5 Conclusion and perspectives

In light of the research objective of this project and considering it as the initial phase, conclusions and recommendations are drawn from the study for the future work of this project.

The main achievements drawn from the study were-

- **Use of kernel functions –** The study helped to understand the kernels and the way of implementing them into MATLAB codes. Te usage of three different kernels over a same set of problem also helped us understand the suitability of the kernel in various applications. As suggested in the degree of novelty, that existing kernels will be used in order to solve the same problem, and hence the usage of RBF kernel and Gaussian kernel takes place. The major achievement to look forward to here was the initialization of these different kernels.

- **Comparison of meshless methods –** The two methods used in the analysis were EFG and MLPG, and hence by changing various parameters, we understand that EFG was a more reliable and accurate method for analysis of a cantilever beam as the error norm achieved in the case of MLPG was nearly thrice that was achieved in EFG.

The upcoming tasks for the next semester will be initializing one of these kernels to solve problems based on different domains. While going through the bibliography content of the project, it became readily clear that these kernels can be used in many different applications. Two of the major applications that came in light the most were, transient heat conduction problems and deformation of thin plates. Further applications of kernel could be image clarification and retrieval or modelling of geofluid flow and also for for free and forced vibration analyses for solids.

There are many domains still untouched from the numerical analysis which might be a very interesting scope of research. Although from the initial study and research it is very evident that now the replacement of kernels can be employed to compare the results and select the most accurate one.

The degree of novelty as mentioned above has been justified by replacing the kernels in order to obtain different results for comparison and understanding purpose. Moreover the aim of this project has been to lay the learning foundation for kernels and meshless method which could be helpful for the coming semester’s study and application.

6 References

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7 List of figures

8. List of Tables