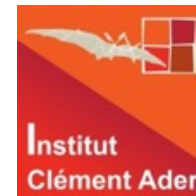


# Methods for Flutter Analysis

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Supaéro, Toulouse  
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# Flutter: Dynamic Aeroelastic Instability

Cause: Interaction between **Inertia**, **Elasticity**, and Unsteady **Aerodynamic** Forces

Consequence: Unstable oscillations that may lead to structural failure



[\[NASA\]](#)

Certification requires aircraft to be flutter-free for all nominal conditions

# Flutter Solutions in Nastran (MSC or Nastran95\*)

## Solution Sequence: Flutter Analysis (SOL 145)

- Frequency-based
- Predict flutter speed, the point from which oscillations become undamped
- Outputs: Eigenmodes and eigenfrequencies of the flutter modes

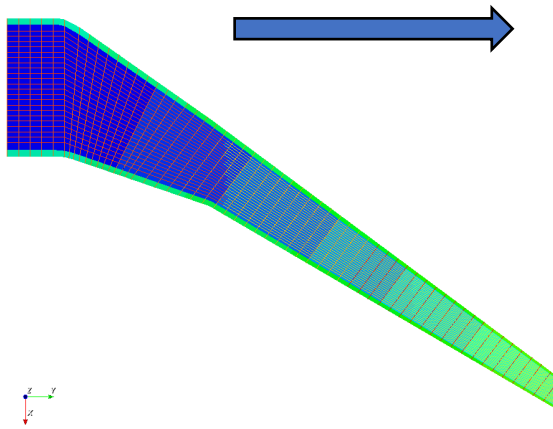
## 2 Types of Methods:

- British method: PK
- American methods: K and KE

\*[\[https://github.com/nasa/NASTRAN-95\]](https://github.com/nasa/NASTRAN-95)

# Physical Models for Flutter

Interaction between **Inertia**, **Elasticity**, and **Aerodynamics** requires 2 physical models:



Finite Element Model: Inertia and Elasticity

$$[M]\{\ddot{u}_s\} + [K]\{u_s\} = \{F_A\}(\dot{u}_s, u_s)$$

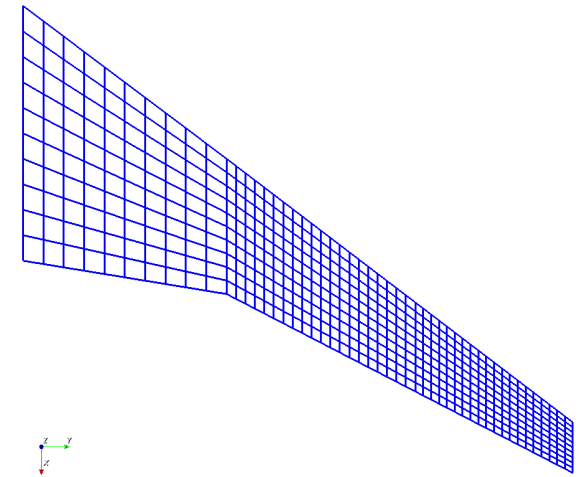
Aerodynamic Models:

- Doublet Lattice Method (Subsonic)
- ZONA51 (Supersonic, MSC only)

$$\{w_j\}(\dot{u}_s, u_s) = [A_{jj}]\{f_j/q\}$$

$$q = \frac{1}{2}\rho V^2$$

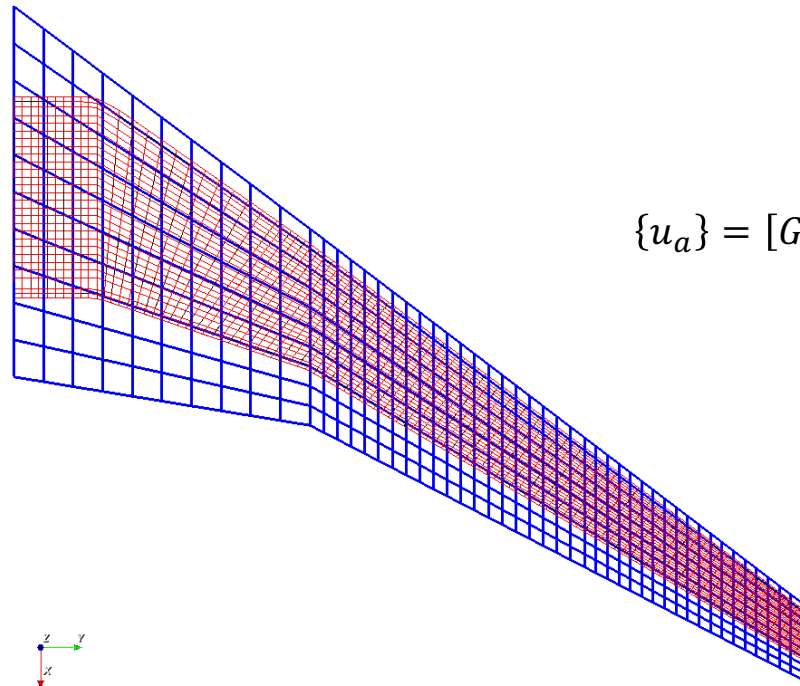
$f$ : panel pressure



[\[Nastran Aeroelastic Analysis User's Guide\]](#)

# Connection Between Models: Interpolation

Surface splines are used to relate the structural grid point deflections  $\{u\}$  to the ones of the aerodynamic grid  $\{u_a\}$ .



$$\{u_a\} = [G_{as}]\{u_s\}$$

# Linear Aeroelastic Equation

$$[M]\{\ddot{u}_s\} + [K]\{u_s\} = [A_c]\{\dot{u}_s\} + [A_k]\{u_s\}$$

Assuming a harmonic solution:

$$(-\omega^2[M] + [K] - i\omega[A_c] - [A_k])\{\eta\} = 0$$

$$Q = \frac{i\omega[A_c] + [A_k]}{\frac{1}{2}\rho V^2}$$

Reduced frequency:

$$\kappa = \frac{\omega c}{2V}$$

By using the natural modal basis  $[\Phi]$ :

$$\{u_s\} = [\Phi]\{\eta\}$$

$$[\bar{K}] = [\Phi]^T [K] [\Phi]$$

$$[\bar{M}] = [\Phi]^T [M] [\Phi]$$

$$[\bar{Q}] = [\Phi]^T [Q] [\Phi]$$

$$\left(-\omega^2[\bar{M}] + [\bar{K}] - \frac{1}{2}\rho V^2[\bar{Q}](m, \kappa)\right)\{\eta\} = 0$$

Mach number:  $m$

[\[J. R. Wright and J. E. Cooper, Introduction to Aircraft Aeroelasticity and Loads, 2014\]](#)

[\[Nastran Aeroelastic Analysis User's Guide\]](#)

# Solution of the Flutter Equation (K-Method)

Strategy: An artificial structural damping  $g$  is introduced:

$$\left( (1 + ig)[\bar{K}] - \omega^2[\bar{M}] - \frac{1}{2}\rho V^2[\bar{Q}](m, \kappa) \right) \{\eta\} = 0$$

Reason: Formulate a complex eigenvalue problem

Consequence: The solution is physically meaningful **only** when  $g = 0$  (Flutter point)

$$\left( -\frac{(1 + ig)}{\omega^2}[\bar{K}] + [\bar{M}] + \frac{1}{2}\frac{\rho V^2}{\omega^2}[\bar{Q}](m, \kappa) \right) \{\eta\} = 0 \quad [F] = [\bar{M}] + \frac{1}{2}\frac{\rho V^2}{\omega^2}[\bar{Q}](m, \kappa) \quad \lambda = \frac{(1 + ig)}{\omega^2}$$

Complex eigenvalue problem:

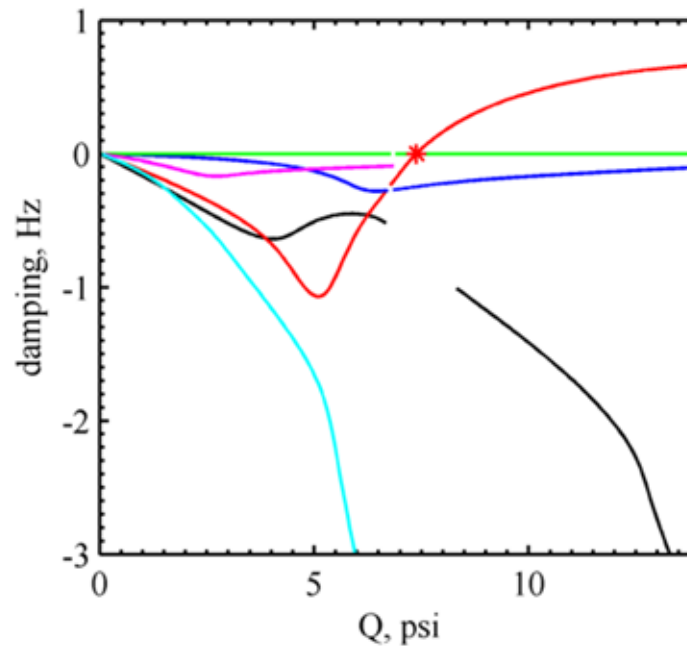
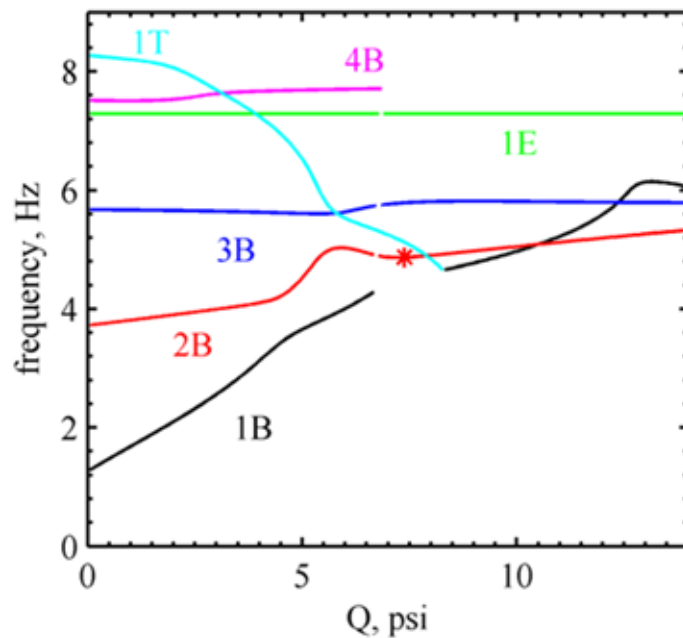
$$([F] - \lambda[\bar{K}])\{\eta\} = 0$$

[\[J. R. Wright and J. E. Cooper, Introduction to Aircraft Aeroelasticity and Loads, 2014\]](#)

[\[Nastran Aeroelastic Analysis User's Guide\]](#)

# In practice: Interpretation of the Results

V-g and V-f plots are used to obtain the flutter speed and frequency (\*)



Note:  $g$  is **not** a physical damping, it is artificial. That means  $g > 0$  implies unstable oscillations. This indicates a positive damping would be required for the oscillations to be neutrally stable.

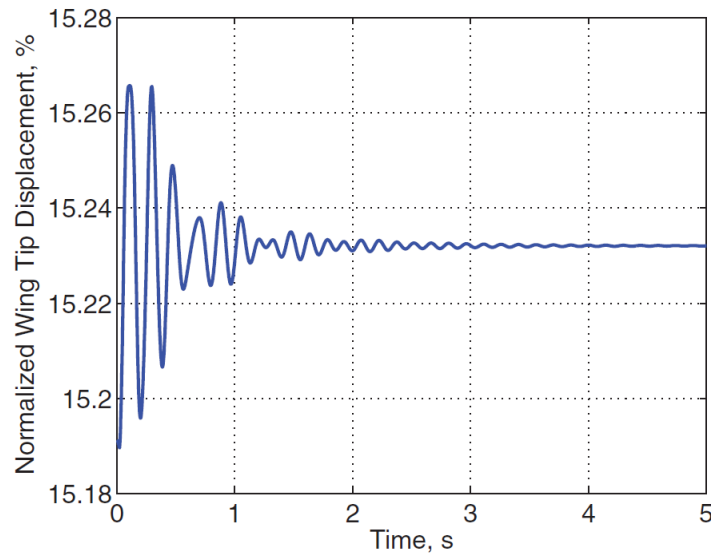
[NASA]



# Other Methods for Flutter (outside Nastran)

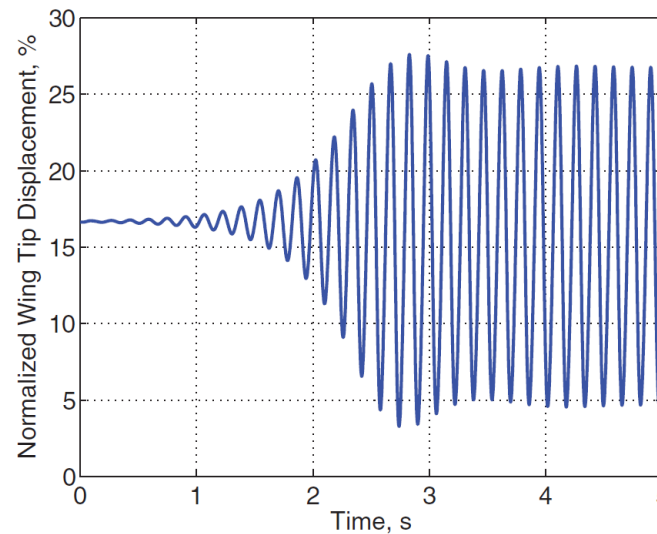
Time-Domain Methods : The time variable is discretized and the aeroelastic equations of motion are integrated to obtain the transient response

Before Flutter:



a) Speed: 147 m/s

After Flutter:



b) Speed: 162 m/s

✓ Can be applied to many physical models

x Expensive to sweep across velocities to find the flutter point

[\[W. Su and C. Cesnik, Journal of Aircraft, 2010\]](#)

[\[E. Jonsson et al., Progress in Aerospace Sciences, 2019\]](#)

# Conclusion

- DLM Frequency-Domain Methods (Nastran SOL 145):
  - Inexpensive
  - Well known and widely used
  - Accuracy limited by the physical models available
  - Solution of the linearized equations
- Time-Domain methods:
  - Flexibility: They can be applied to many physical models of several fidelities (e.g., CFD)
  - Possibility to consider nonlinearities
  - Expensive to sweep through velocities to find the flutter point

Thanks for your attention !