

Un cadre de conception pour l'impression des structures minces multi-échelles, incluant une analyse de flambement non intrusive

How to check local buckling in
multiscale topology optimization ?

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Pantel 3, @ , Joseph Morlier 2, *, @ , Miguel
Charlotte 4, *, @

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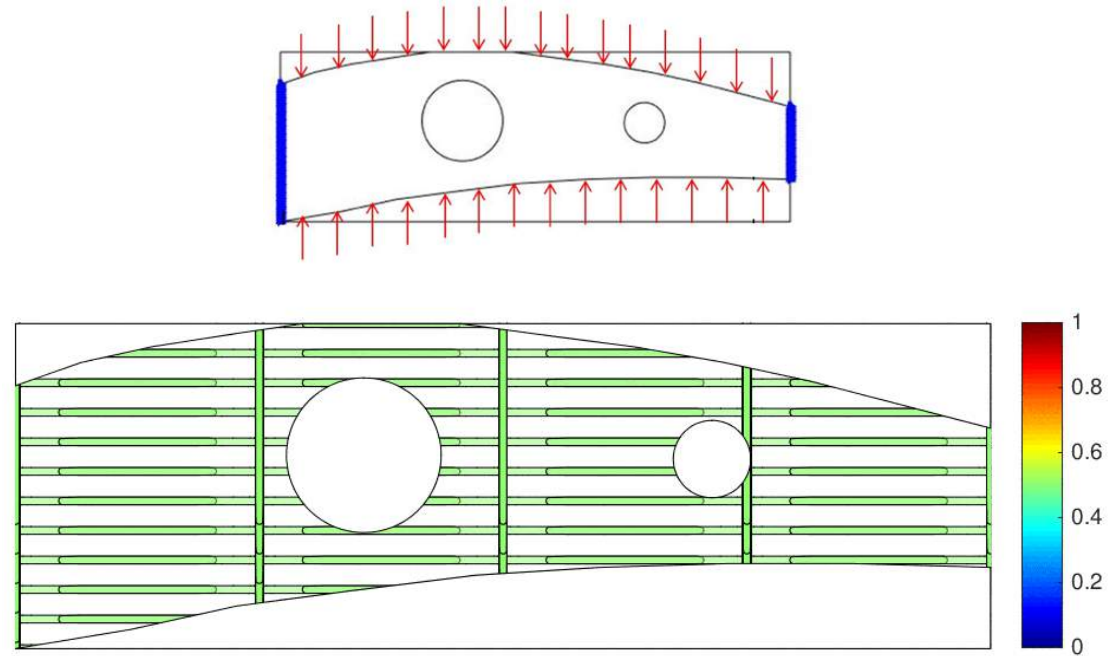


About Me?

<https://ica.cnrs.fr/en/author/jmorlier/>



- Prof in Structural and Multidisciplinary Optimization



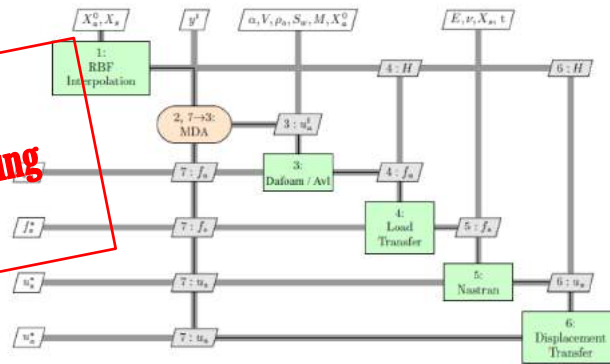
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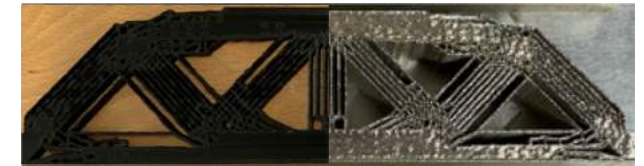
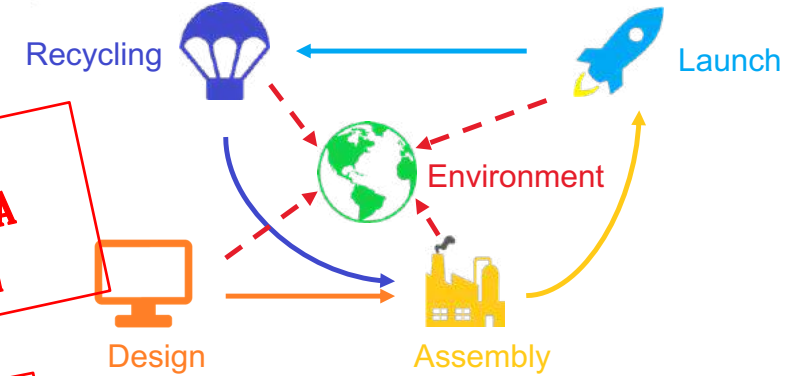
- 6 PhDs, 3 MsCs

MDO for Aeroelastic Wing design



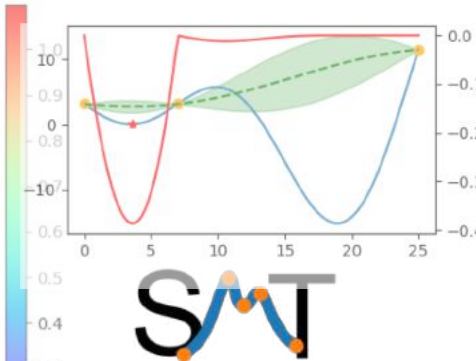
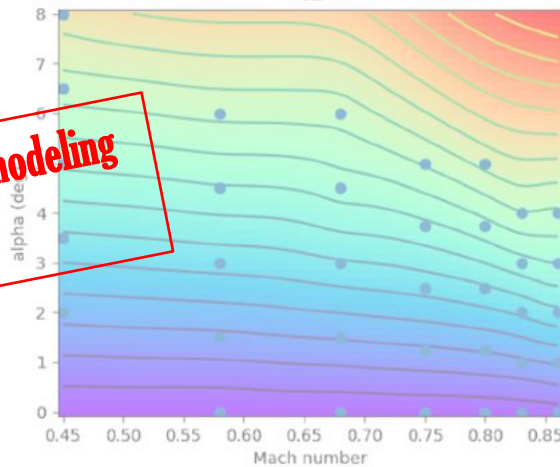
**MDO for Aerospace systems Including LCA
:= EcoOptimization**

Digital fabrication



<https://github.com/SMTorg/SMT>

**Surrogate modeling
AI4E**



CSMA 2024





Au programme

- **Part1: EMT0**
- **Part2: Buckling mitigation**
- **Conclusions**



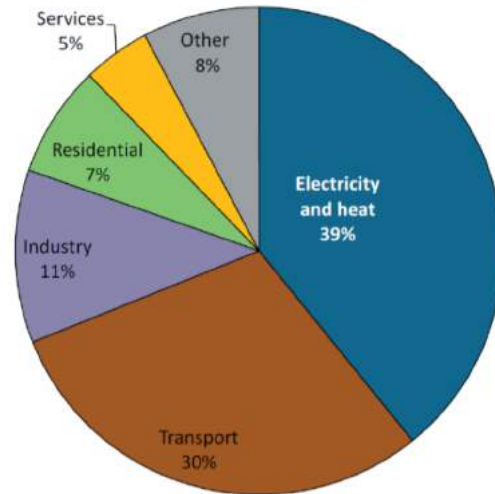
Au programme

**Thanks to Edouard
Duriez**

• **Part1: EMTO**

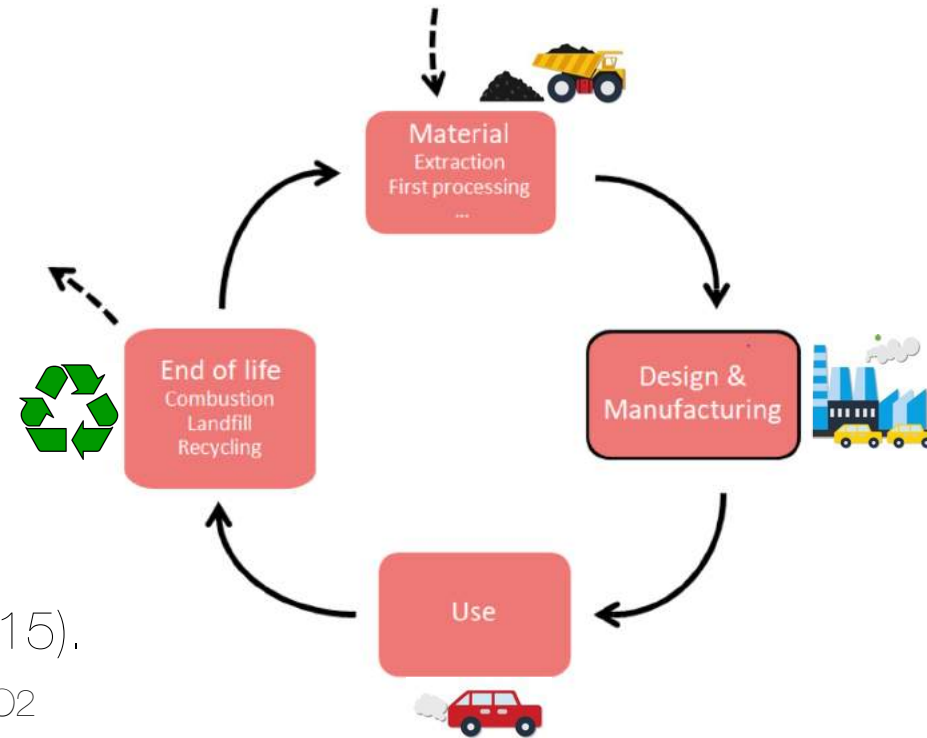
- **Part2: Buckling mitigation**
- **Conclusions**

Overview



CO₂ emissions of the OECD (2015).

International Energy Agency IEA. Energy and CO₂ emissions in the OECD. 2017

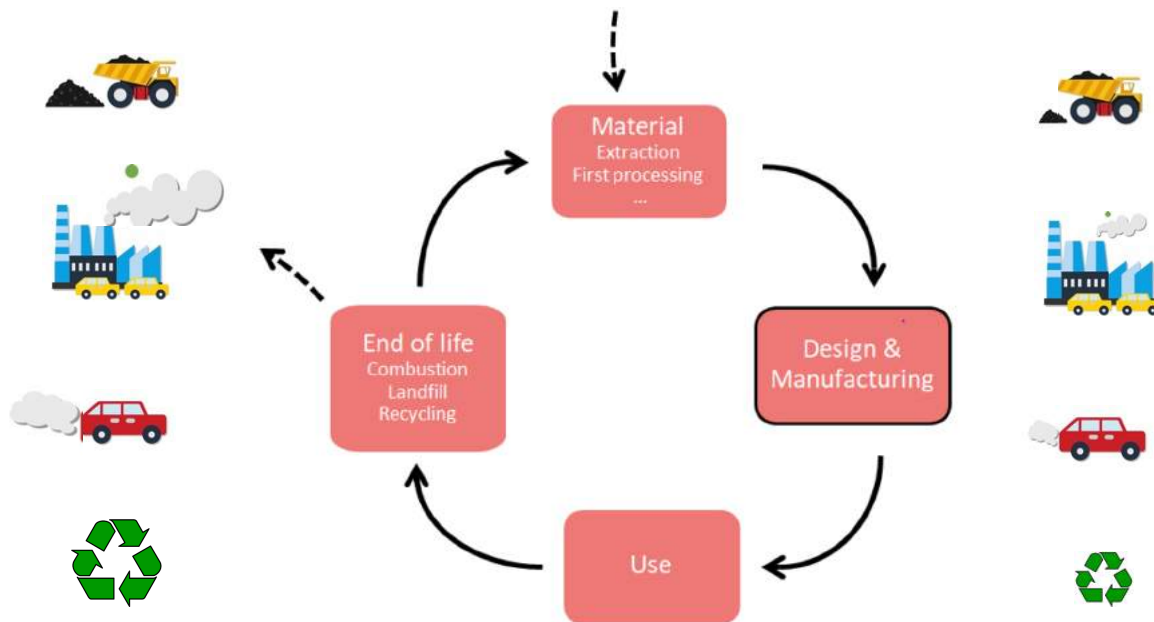


Vehicle life cycle

Q : How to find structural designs, materials and additive manufacturing processes with the lowest life-cycle CO₂ footprint?

Hypothesis 1

- CO_2 emissions minimization of parts
 - If material choice is **imposed** => mass minimization



Vehicle life cycle

If not... more complicated

ELSEVIER

Procedia CIRP
Volume 109, 2022, Pages 454-459

Ecodesign with topology optimization

Edouard Duriez ^a, Joseph Morlier ^a, Catherine Azzaro-Pantel ^b, Miguel Charlotte ^a

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ELSEVIER

A fast method of material, design and process eco-selection via topology optimization, for additive manufactured structures

Edouard Duriez ^a, Catherine Azzaro-Pantel ^b, Joseph Morlier ^a, Miguel Charlotte ^a

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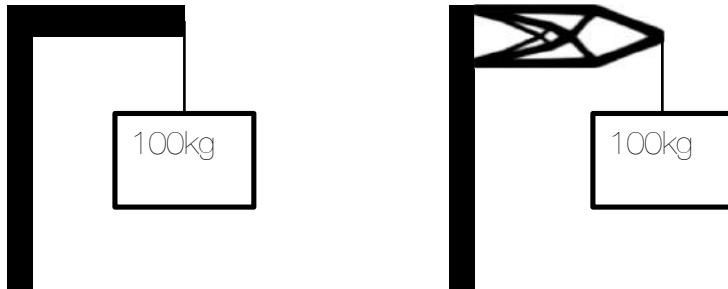
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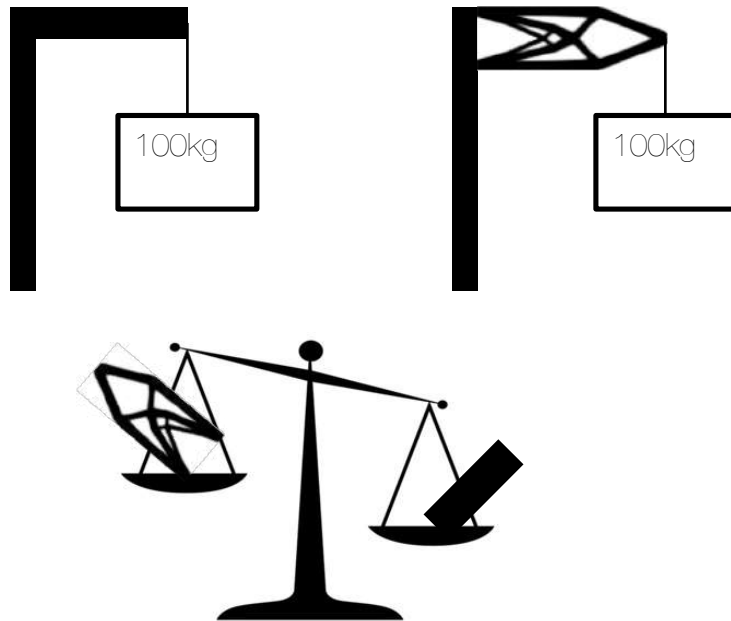
Mass minimization of parts

- Redesign through topology optimization
=> same performance



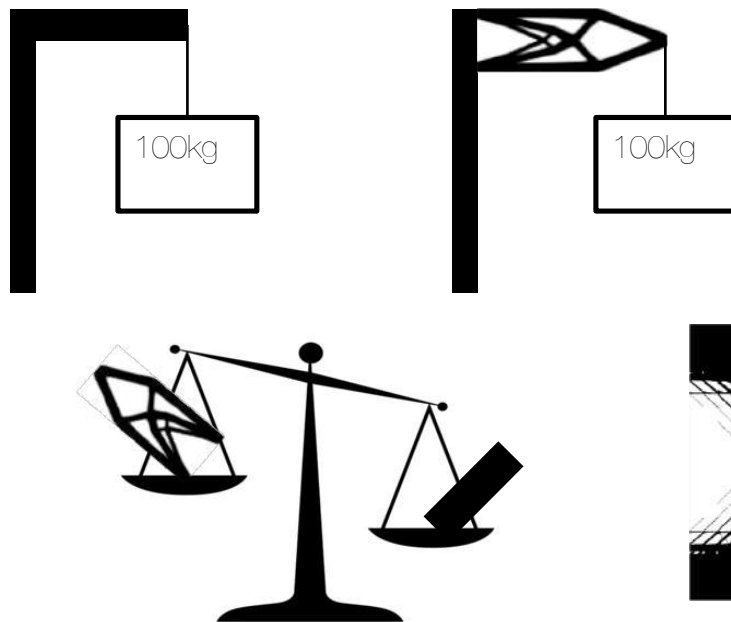
Mass minimization of parts

- Redesign through topology optimization
=> same performance but lower mass

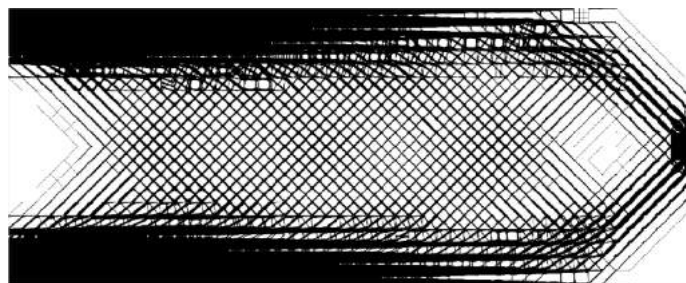


Mass minimization of parts

- Redesign through topology optimization
=> same performance but lower mass



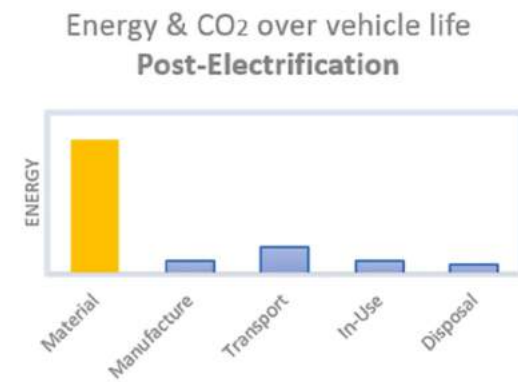
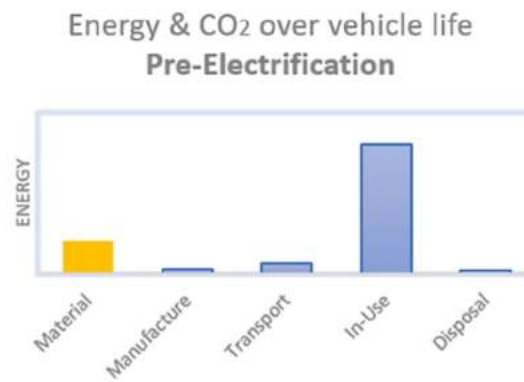
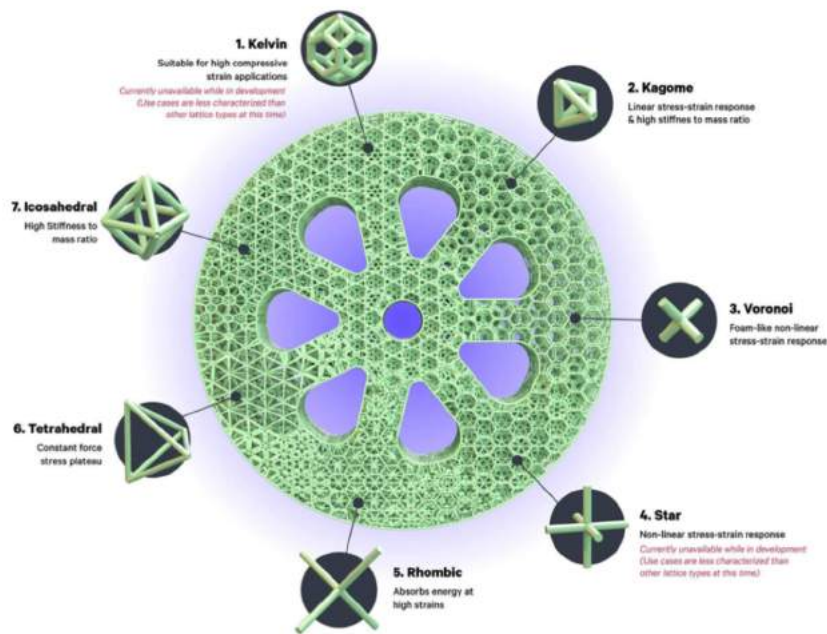
— One step further :
multiscale topology
optimization



Unit cell/material/process

as new design variables in Structural and Multidisciplinary Optimization

Eco Material selection
Eco Process selection

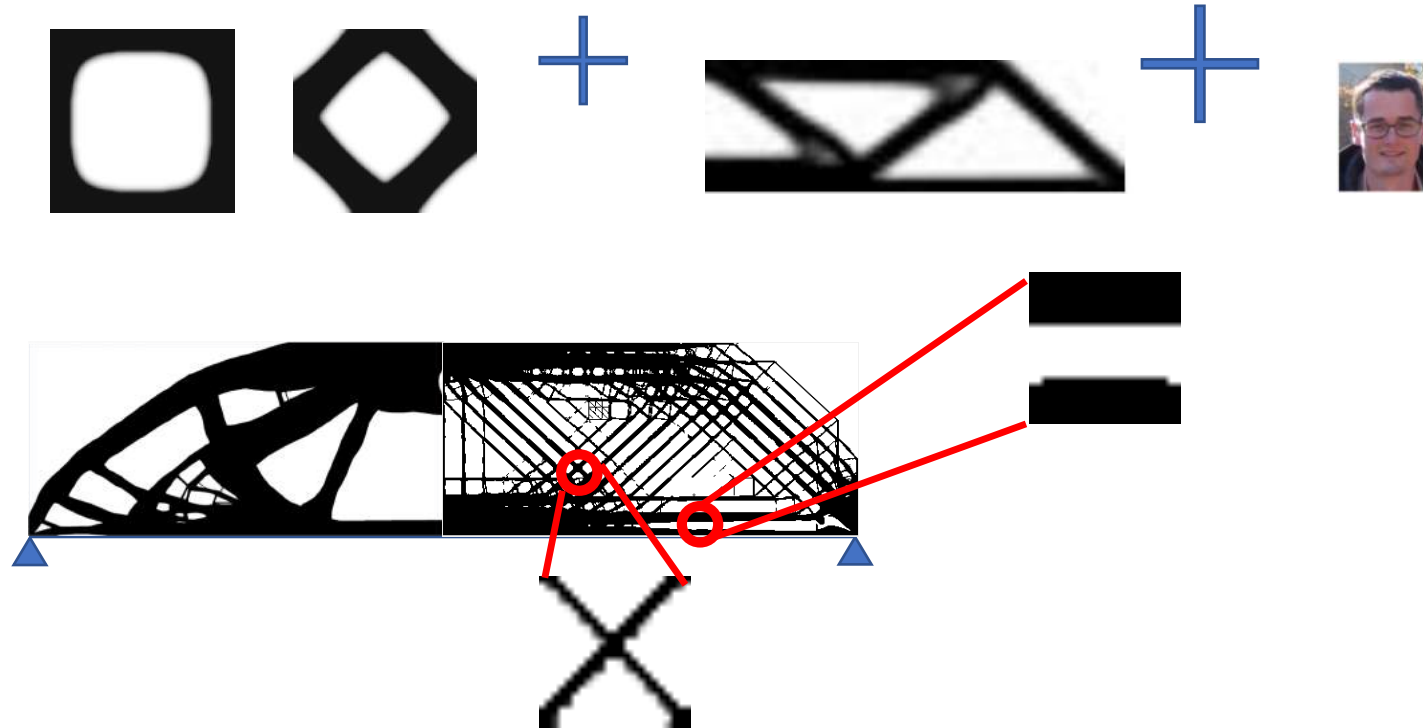


<https://www.ansys.com/blog/the-impact-of-materials-on-sustainability-part-2>

Unit cell design (anisotropy)
Digital materials

Multi-scale TO (well connected+ locally-oriented)

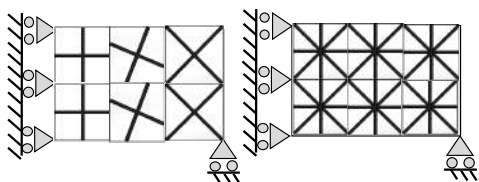
A two level optimization that combines Unit cell design & Topology Optimization



Xia L, Breitkopf P (2015) Design of materials using topology optimization and energy-based homogenization approach in Matlab. Struct Multidisc Optim 52(6):1229–1241.

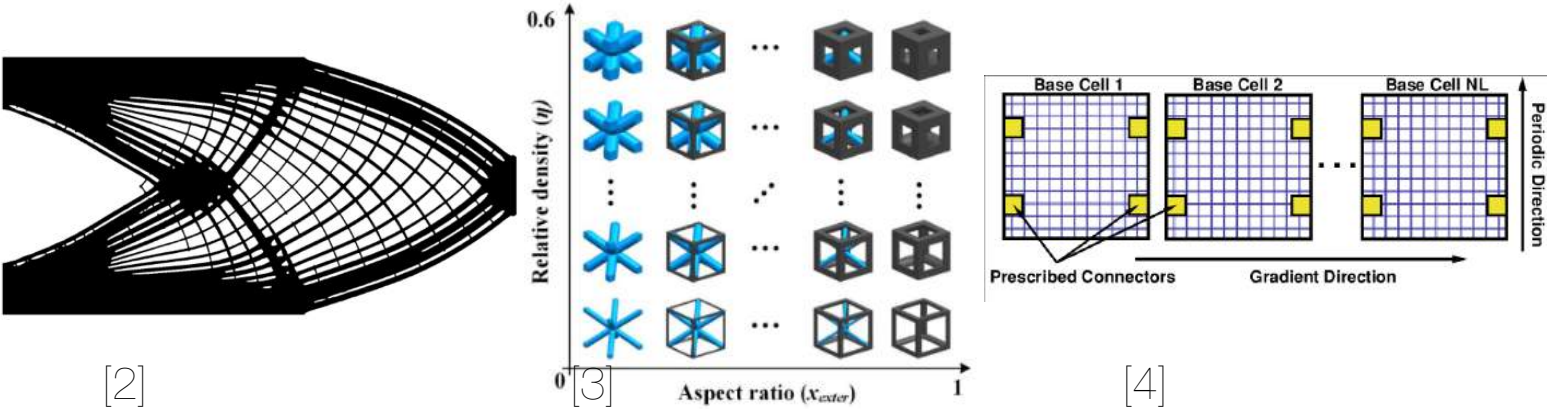
Wu, Jun, Ole Sigmund, and Jeroen P. Groen. "Topology optimization of multi-scale structures: a review." Structural and Multidisciplinary Optimization 63.3 (2021): 1455-1480.

Main MTO methods



15

Approach	Examples	Connectivity	Locally adapted	Speed	Manufacturability
De-homogenization	[1],[2]				
Parametrized lattice	[3]				
Connectors	[4]				



[1] Grégoire Allaire, Perle Geoffroy-Donders et Olivier Pantz. « Topology optimization of modulated and oriented periodic microstructures by the homogenization method ». en. In : *Computers & Mathematics with Applications*.

[2] Groen, Jeroen P., and Ole Sigmund. "Homogenization-Based Topology Optimization for High-Resolution Manufacturable Microstructures." *International Journal for Numerical Methods in Engineering*

[3] Wang, Chuang, et al. "Concurrent Design of Hierarchical Structures with Three-

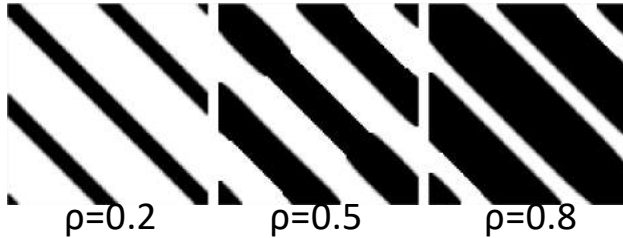
Dimensional Parameterized Lattice Microstructures for Additive Manufacturing." *Structural and Multidisciplinary Optimization*

[4] Zhou S, Li Q (2008) Design of graded two-phase microstructures for tailored elasticity gradients. *Journal of Materials Science*

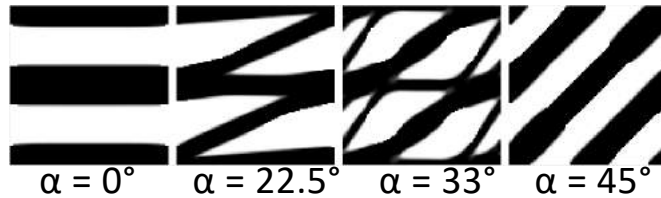
[5] Wu, Jun, et al. "Topology Optimization of Multi-Scale Structures: A Review." *Structural and Multidisciplinary Optimization*

Scale-bridging variables

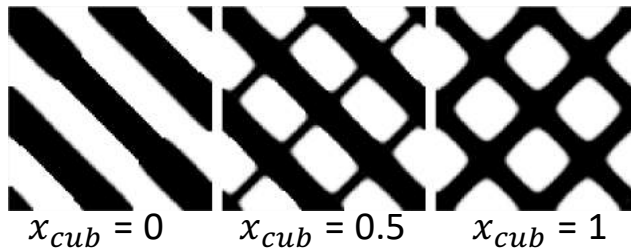
- Density : ρ



- Orientation : α



Cubicity : x_{cub}



→ relative importance of the two principal directions. A value of 1 means the two principal directions are equivalent, while a value of 0 means the first principal direction alone is considered.

Microscale Problem

Since the objective is to create micro-structure with optimal properties towards specific directions, the objective function is a weighted function of the two components $E\alpha_{1111}$ and $E\alpha_{2222}$

the problem being solved at the micro-scale to obtain the i^{th} macrocell.

$$\begin{aligned} \underset{\rho_{i,j}}{\text{minimize}} \quad & c_i = E_{1111}^{\alpha} * \left(1 - \frac{x_{cub}^i}{2}\right) + E_{2222}^{\alpha} * \frac{x_{cub}^i}{2} \\ \text{subject to} \quad & K_i u_i^{A(pq)} = f_i^{(pq)} \\ & \sum_{j=1}^m \rho_{i,j} \leq m * x_{dens}^i \\ & \epsilon < \rho_{i,j} < 1 \end{aligned}$$

where K_i is the i^{th} -macrocell assembled stiffness matrix, $u_i^{A(pq)}$ and $f_i^{(pq)}$ are the global displacement vector and the external force vector of the i^{th} -macrocell for the case (pq) respectively, $\rho_{i,j}$ is the density of the j^{th} micro-element of the i^{th} -macrocell.

In all the results presented in Section 2.4, micro-structures of size 100*100 are used.

Rotated homogenized stiffness tensor

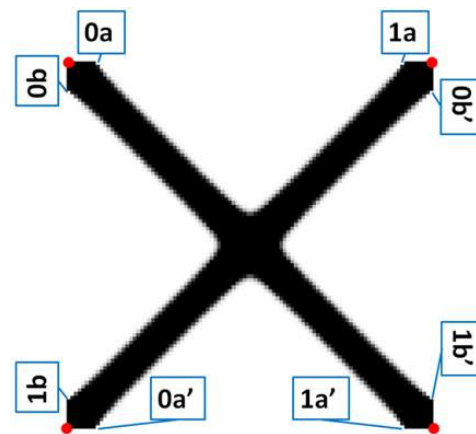
$$\mathbf{E}^{\alpha} = \mathbf{M}_{\alpha}^T * \mathbf{E} * \mathbf{M}_{\alpha} = (E_{klmn}^{\alpha})_{k,l,m,n \in \{1,2\}}$$

Transmission zones

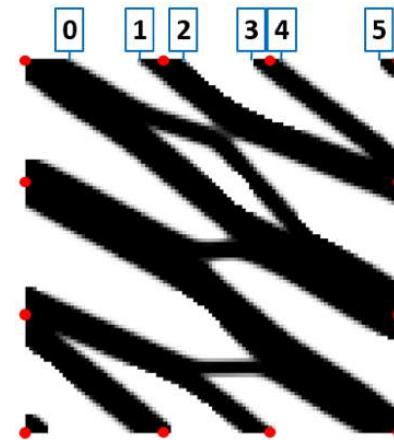
17

- To address connectivity issue
 - Impose location of strain transmission from one cell to another
- ⇒ Periodic boundary conditions only in those locations

4 transmission
Zones (TZ):



12TZ:



- Difference to Kinematical Connective constraints : absence of non-design zones

Zhou S, Li Q (2008) Design of graded two-phase microstructures for tailored elasticity gradients. *Journal of Materials Science* 43:5157–5167. <https://doi.org/10.1007/s10853-008-2722-y>

Multiscale Topology Optimization

Macroscale Problem

need to solve



Microscale Problem

$$\text{minimize}_{x_{\text{dens}}^i, x_a^i, x_b^i, \dots} c = u^T K u$$

$$\text{subject to } K u = f$$

$$\sum_{i=1}^n \sum_{j=1}^m \rho_{ij} \leq n \times m \times v_f$$

$$\epsilon < \rho_{ij} < 1$$

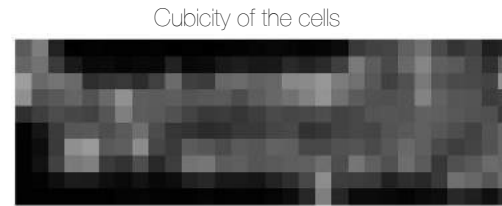
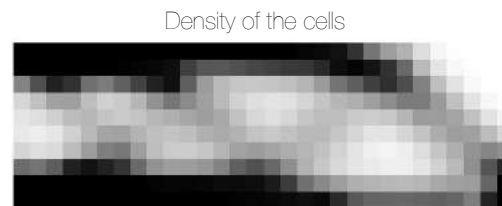
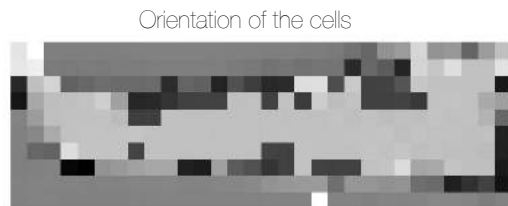
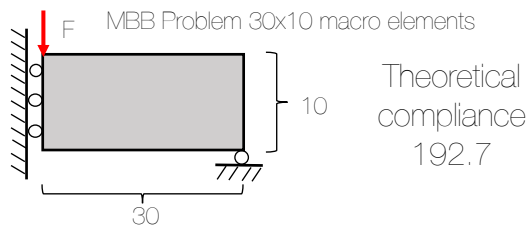
$$x^i = [x_{\text{dens}}^i, x_{\text{or}}^i, x_{\text{cub}}^i]$$

best unit cell per quad

$$\text{minimize}_{\rho_{ij}} c_i = E_{1111}^{i\alpha} \times \left(1 - \frac{x_{\text{cub}}^i}{2}\right) + E_{2222}^{i\alpha} \times \frac{x_{\text{cub}}^i}{2}$$

$$\text{subject to } K_i u_i^{A(pq)} = f_i^{(pq)}$$

$$\sum_{j=1}^m \rho_{ij} \leq m \times x_{\text{dens}}^i$$



Since the objective is to create micro-structure with optimal properties towards specific directions, the objective function is a weighted function of the two components $E\alpha_{1111}$ and $E\alpha_{2222}$



Comparison

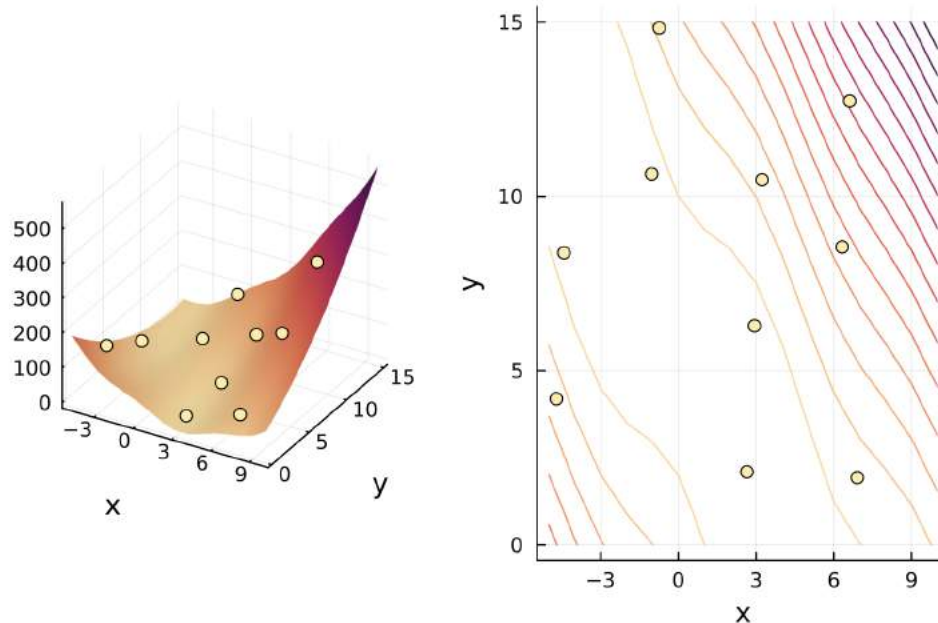
- Top88 versus EMTO



How to speed up?

22

- To address speed issue ($t_{tot} = t_{cell} * n_{cell} * n_{it}$; $t_{cell} = 10'$)



<https://github.com/SMTorg/SMT>

database:

<https://data.mendeley.com/datasets/b5hyzxcg7fv/1>

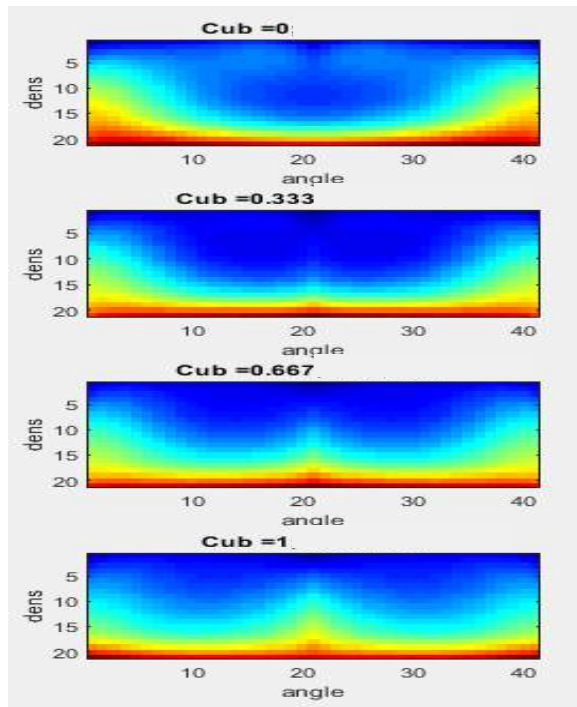
DOI: 10.17632/b5hyzxcg7fv.1

- $t_{tot} = 10''$ on 200-300 macro-element design

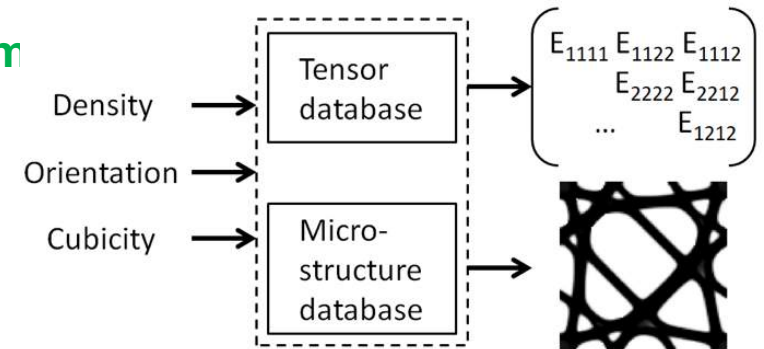
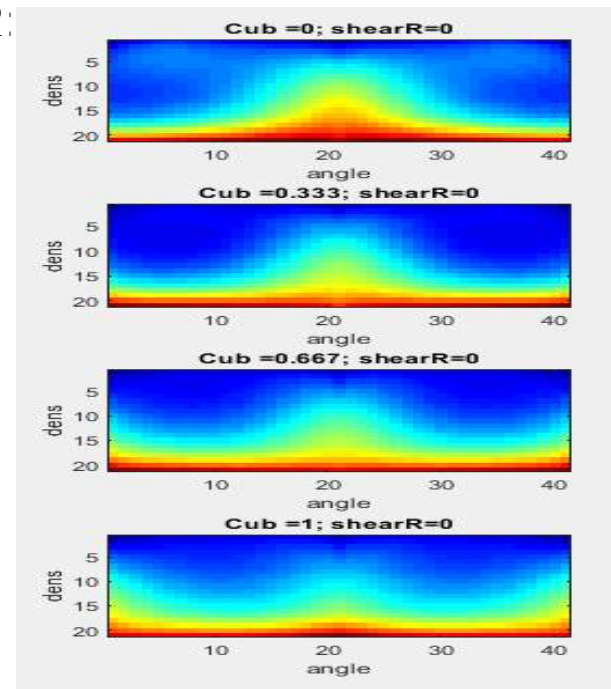
Elastic tensor's surrogate

to Avoid Nelement*cellOptimization at each macroOptim step

- 3 inputs : macro-density, angle, cubicity
- 6 outputs : elastic tensor values
- Gaussian interpolation : capture local effects but mitigate noise
- E_{1111}



E_{2222} :



Efficient Multiscale Topology Optimization

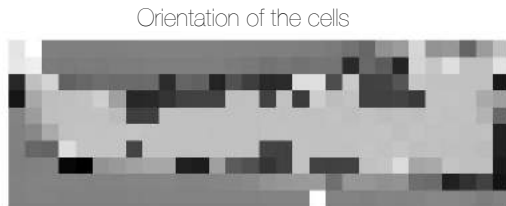
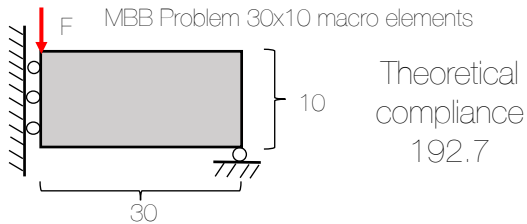
Macroscale Problem

$$\underset{x_{\text{dens}}^i, x_a^i, x_b^i, \dots}{\text{minimize}} \quad c = u^T K u$$

$$\text{subject to} \quad K u = f$$

$$\sum_{i=1}^n \sum_{j=1}^m \rho_{ij} \leq n \times m \times v_f$$

$$\epsilon < \rho_{ij} < 1$$

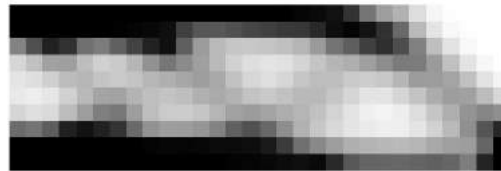


$$x^i = [x_{\text{dens}}^i, x_{\text{or}}^i, x_{\text{cub}}^i]$$

Nearest Optimal

$$x^i = [x_{\text{dens}}^i, x_{\text{or}}^i, x_{\text{cub}}^i]$$

Density of the cells



Cubicity of the cells



Gaussian Process Regression

$$\mathbf{E}_{\text{pred}}(x^i) = \frac{\sum_{l=1}^k G(x^i, x_l) \mathbf{E}_{\text{db}}(x_l)}{\sum_{l=1}^k G(x^i, x_l)}$$

$$G(x^i, x_l) = \exp\left(\frac{-d_{\text{eucl}}(x^i, x_l)^2}{2b^2}\right)$$

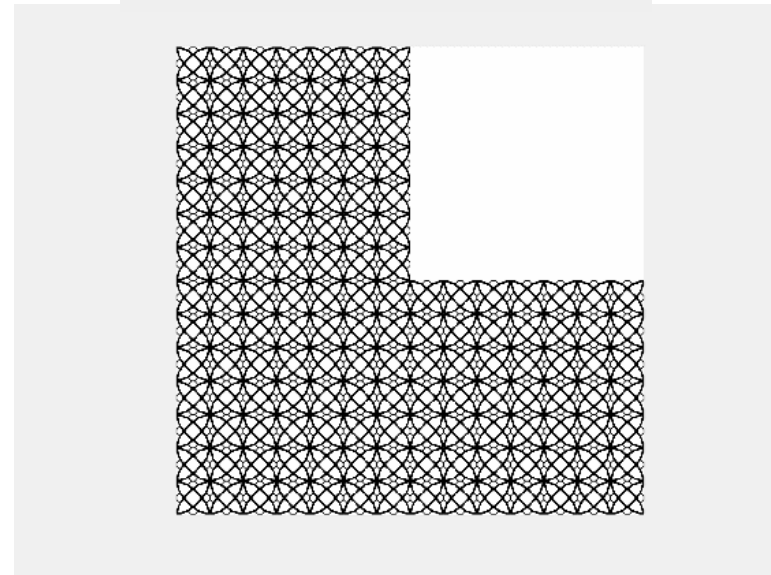
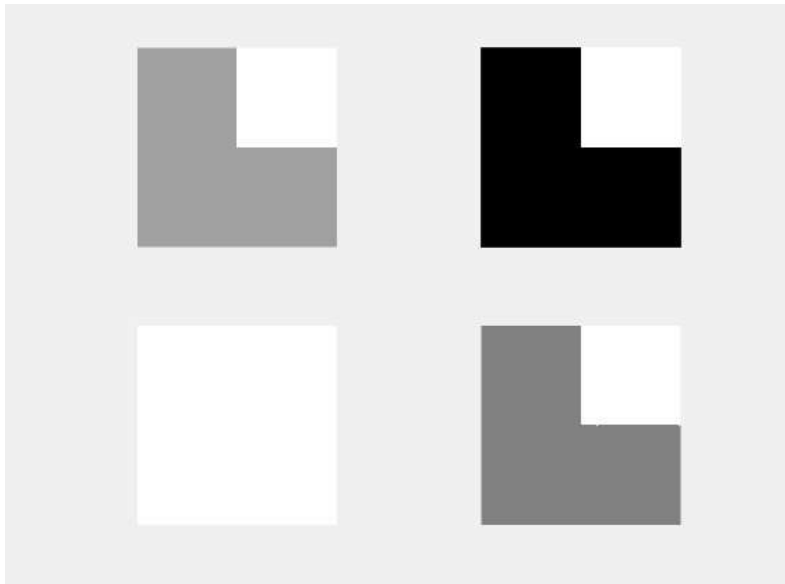
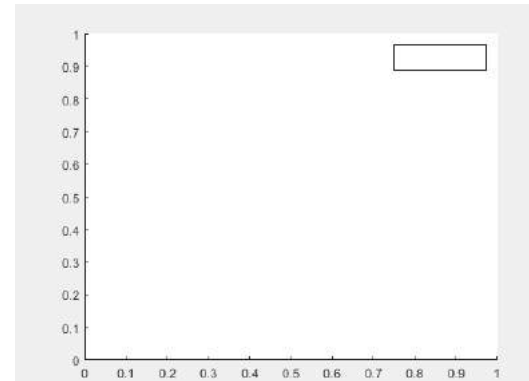
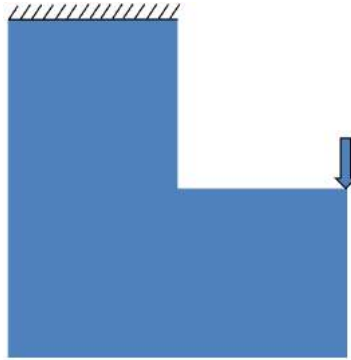
$$x^{i'} = [x_{\text{dens}}^i + \Delta, x_{\text{or}}^i, x_{\text{cub}}^i] \quad \Delta = 0.01$$

$$\frac{\partial \mathbf{E}_{\text{pred}}}{\partial x_{\text{dens}}}(x^i) \approx \frac{\mathbf{E}_{\text{pred}}(x^{i'}) - \mathbf{E}_{\text{pred}}(x^i)}{\Delta}$$



Result on classical test cases

- Validation on small grid
⇒ Evaluate full-scale design



- 4x14*14 design variables; stopping criteria : $\text{tolfun} < 10^{-3}$

More info

Duriez, E., Morlier, J., Charlotte, M., & Azzaro-Pantel, C. (2021). A well connected, locally-oriented and efficient multi-scale topology optimization (EMTO) strategy. Structural and Multidisciplinary Optimization, 1-24.

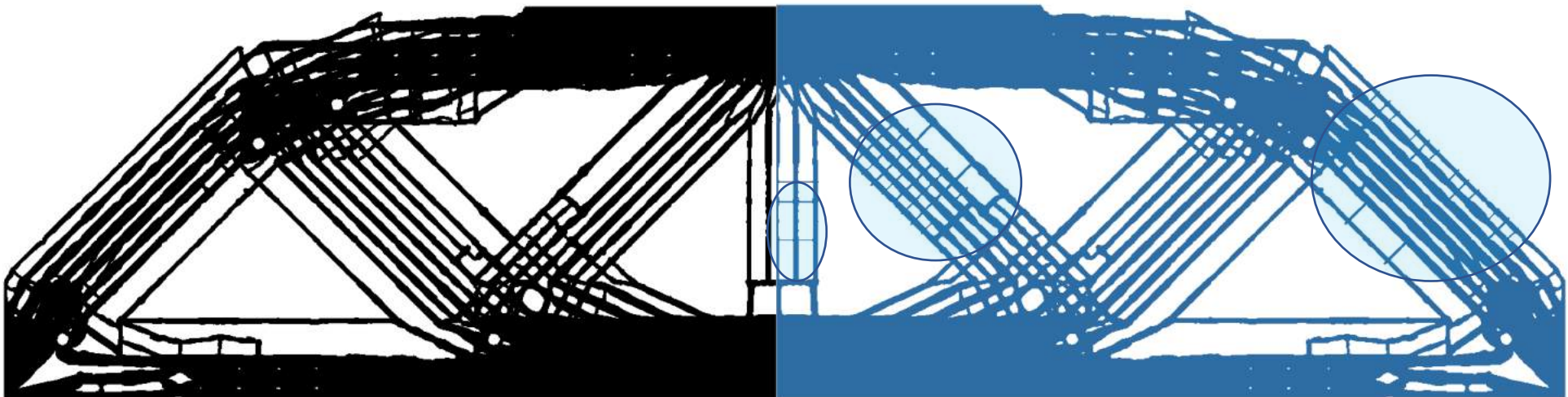


Au programme

*Thanks to Edouard
Duriez*

- Part1: EMT0
- **Part2: Buckling mitigation**
- Conclusions

Do you see a difference (Left2Right)?



Avoid buckling constraint in the TopOpt loop

- Is there a « cheap » post treatment capable of detecting micro-structures responsible for the buckling ??
- We start by diminishing the size of the design to be studied by regrouping elements 4 by 4.
- We then retrieve the buckling load factors (BLF) and the associated eigen vectors of this design for the first 12 modes, using part of the code from

Ferrari, F., Sigmund, O. & Guest, J.K. Topology optimization with linearized buckling criteria in 250 lines of Matlab. Struct Multidisc Optim (2021).

```
>>topBuck250(480,240,3,4,2,'N',0.5,2,[0.1,0.7,1.2],300,2,0,0,0,['V','C'],2.5))).
```

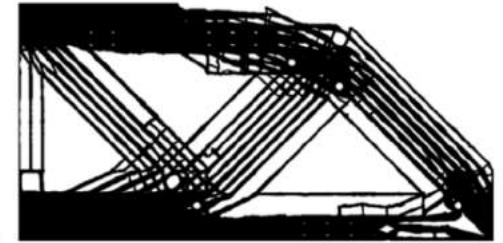
Need a score

$$b_j = \sum_{i=1}^{12} \lambda_i \times \frac{d_{ij}}{\max_j(d_{ij})}$$

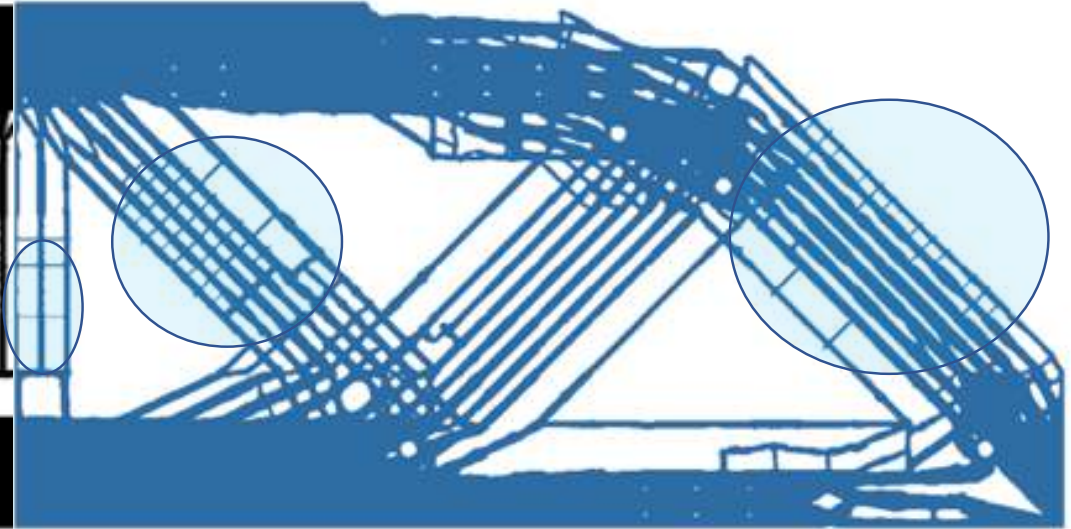
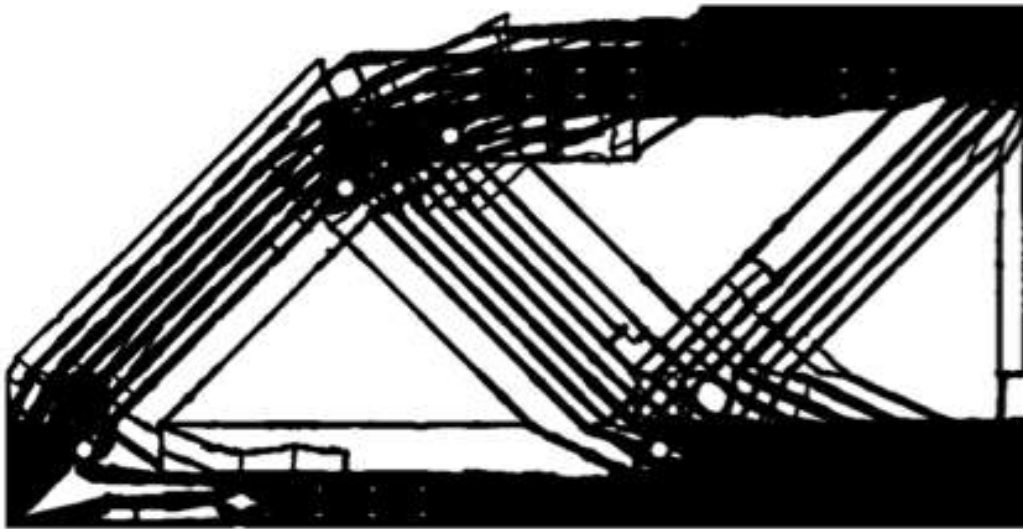
"buckling score" (b_j) for each cell, capturing how its elements are affected by buckling
The mean of this weighted displacement over each cell (j) is noted d_{ij} .



(a) The MBB beam problem.

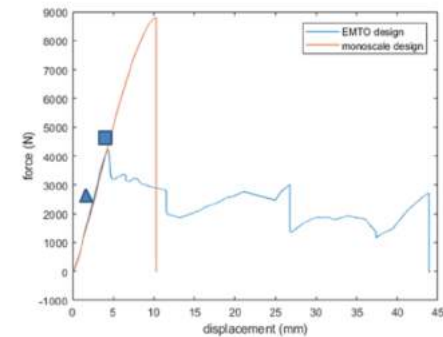


(b) Output of EMTO.

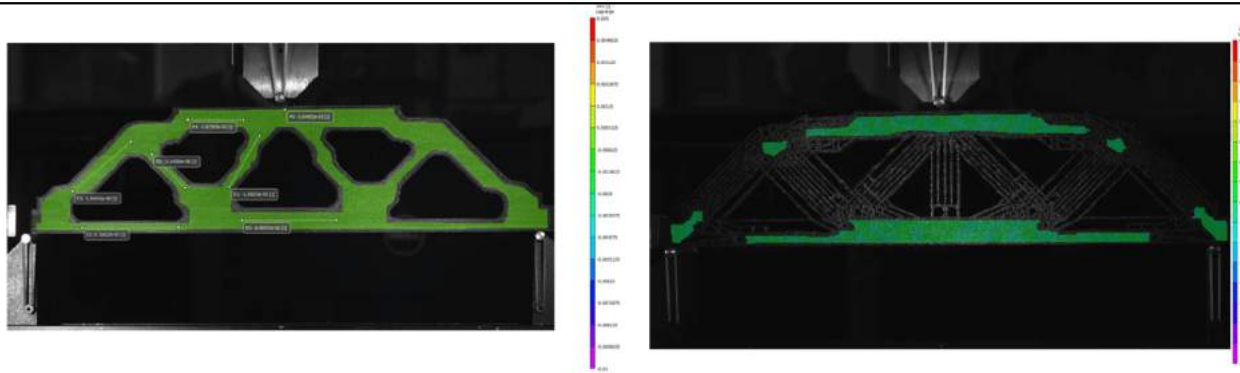


Experimental validation

- Comparison to top88 (smoothed) on same grid and same total volume fraction.
- Planar stiffness (N/m^2) : $S = \frac{F}{\delta_e}$



Method	planar stiffness (N/m^2)	$F_{lim} / t(N/m)$
EMTO + BMPT	6.48×10^7	$2.67e5$
EMTO	6.48×10^7	$1.44e5$
top88 smoothed	6.17×10^7	$2.67e5$



- => EMTO takes advantage of printing anisotropy



Au programme

- Part1: EMT0
- Part2: Buckling mitigation
- **Conclusions**

Conclusions

- VS monoscale:
 - Lower compliance
 - Lower maximum load
 - Higher energy absorption
- Design acceleration through SMT
- Framework enabling to easily mitigate local buckling
 - Keep stiffness advantage while having buckling limit load similar to monoscale
- Directional properties similar to composites (CFRP), but using only one material (and void) => easier recyclability?

Opensource initiatives

<https://github.com/topggp/blog>

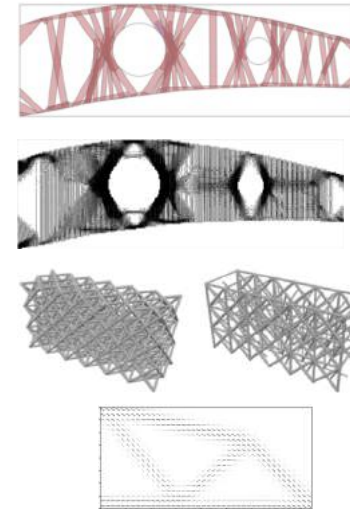
<https://github.com/mid2SUPAERO/EMTO>

<https://github.com/mid2SUPAERO/TTO>

https://github.com/mid2SUPAERO/SOMP_Ansys

<https://github.com/mid2SUPAERO/EMTO/tree/main/buckling>

<https://github.com/SMTorg/SMT>



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<https://smt.readthedocs.io/en/latest/>



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<https://www.linkedin.com/pulse/possible-build-aircraft-wing-lego-joseph-morlier/?articleId=6627240732975480832>



https://www.tripadvisor.fr/LocationPhotoDirectLink-g187529-d574612-i349532022-Museum_of_Natural_Science_Museo_de_Ciencias_Naturales-Valencia_Province_o.html

Is it possible to build an aircraft wing in LEGO® ?

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joseph morlier

Professor in Structural and Multidisciplinary Design Optimization, ... any idea?

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