Recent advances in structural and multidisciplinary optimization @SUPAERO

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Thanks for the invitation

My Research Group (Joint research with ONERA on MDO)

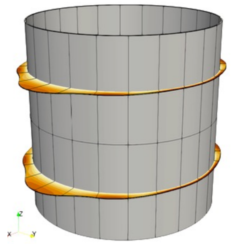
http://www.institut-clement-ader.org/pageperso.php?id=jmorlier

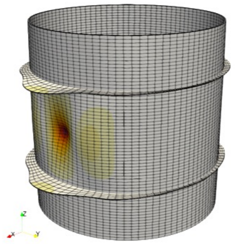
- 5 PhDs, 1 postdoc, 4 MsC

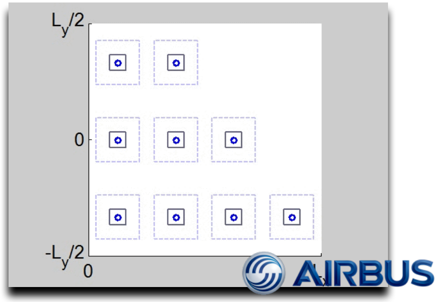
Structural Optimization

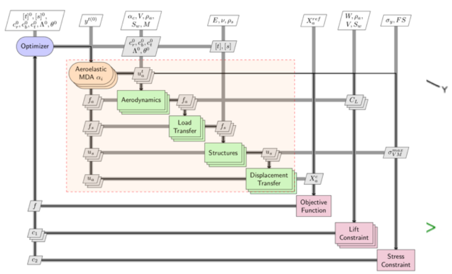
$$\min\ w(a, c)$$
$$a \in \mathbb{R}^{10}$$
$$c \in \Gamma^{10}$$
$$s.t.\ \ s(a, c) \le 0$$
$$d(a, c) \le 0$$
$$\underline{a} \le a \le \overline{a}$$

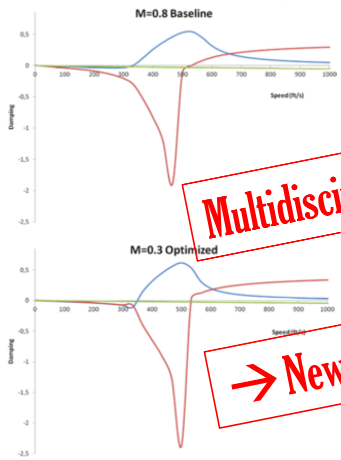
AIRBUS





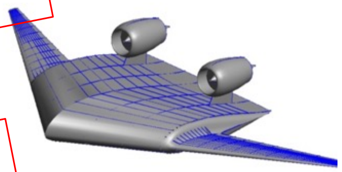




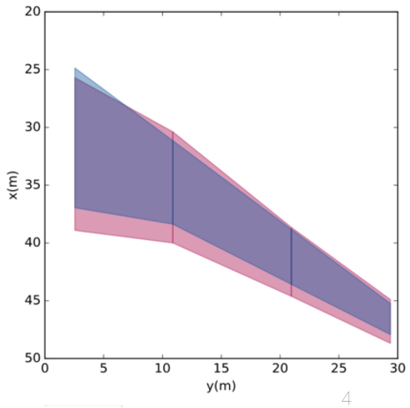


Multidisciplinary Optimization

→ New Aircraft Concept



AIRBUS



Optimized planform (red) and baseline (blue).

CHAIR FOR ECO DESIGN OF AIRCRAFT

Our Goals: new optimization process in the design loop of Aerostructures (flexible)

- Reduce in a « smart way » the computation time of optimization for coupled simulations
- Global Optimization using surrogate modeling → fixed budget (enriching process) to deal with INDUSTRIAL problems
- Specialized surrogates for HD (engineering) problems and UQ
- Taking into account different levels of fidelity

N. Bartoli et al, Improvement of efficient global optimization with mixture of experts: methodology developments and preliminary results in aircraft wing design, Proceedings of AIAA 2016

→ Methods applied to AD Aircraft Design,: Put the aircraft structure / aeroelasticity in the loop at the early stage of MDO process

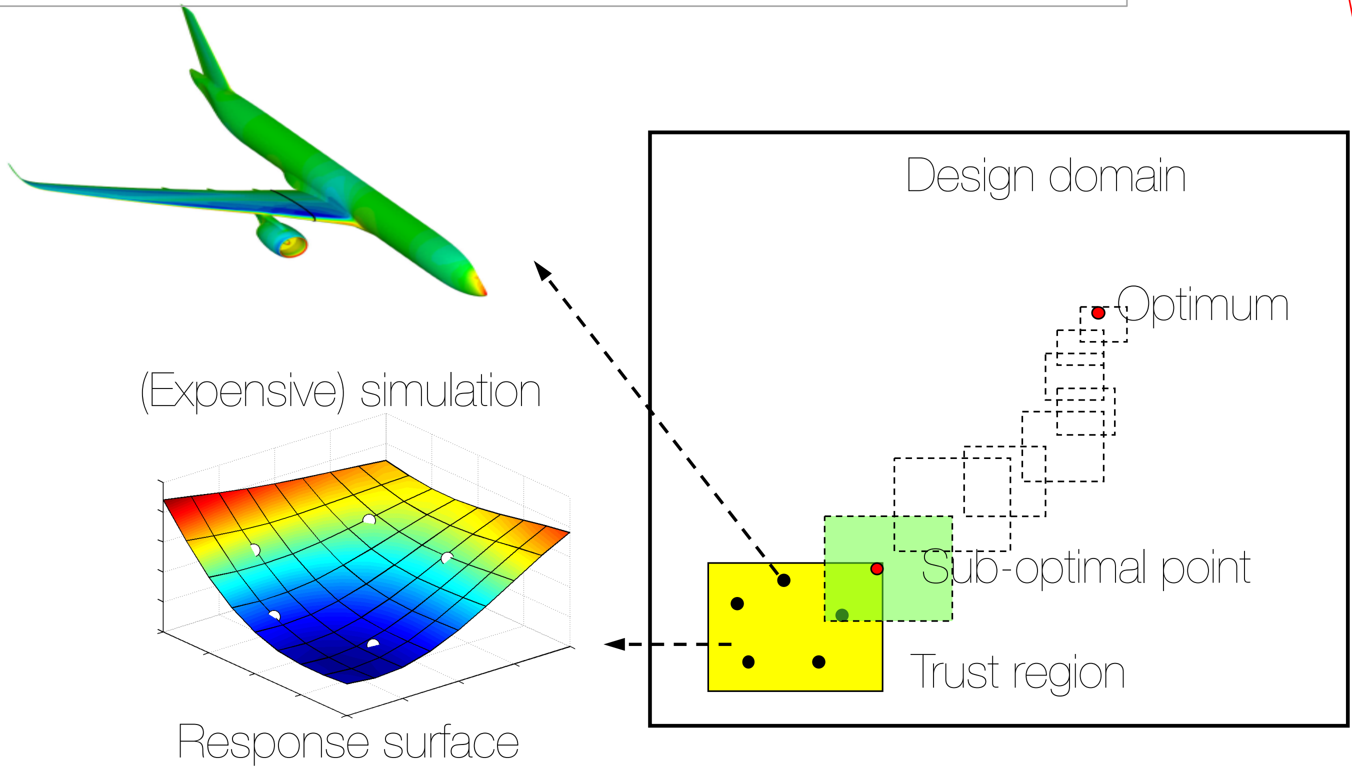
→ compatible with openMDAO

http://openmdao.org

SURROGATE MODELING (learning for Optimizing)

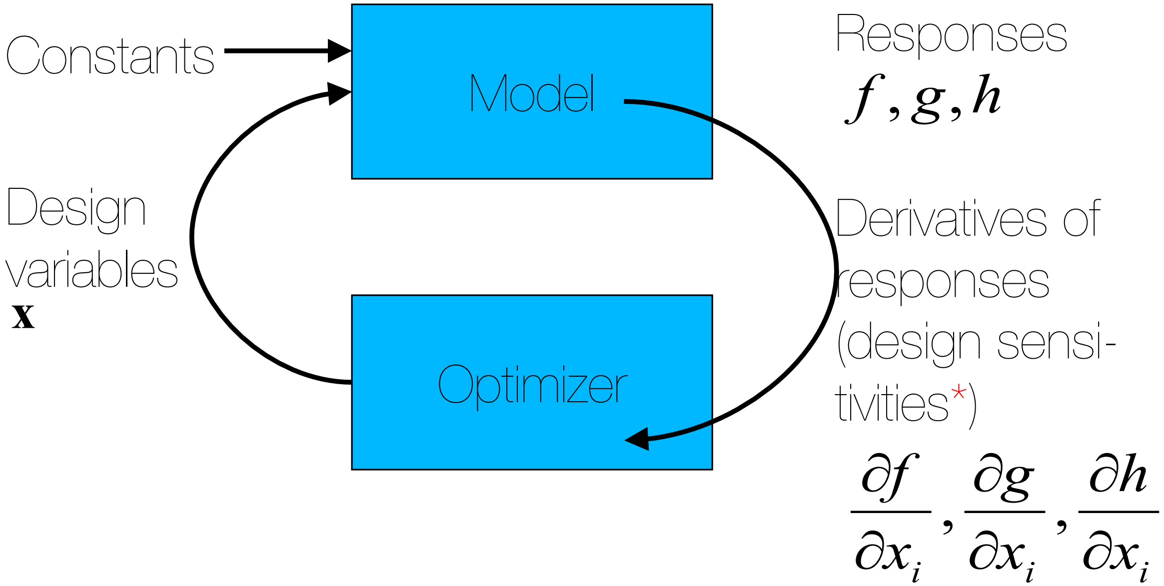
Jacobs, J. H., et al. "Framework for sequential approximate optimization." Structural and Multidisciplinary Optimization 27.5 (2004): 384-400.

But Why?



Gradient Based Optimization

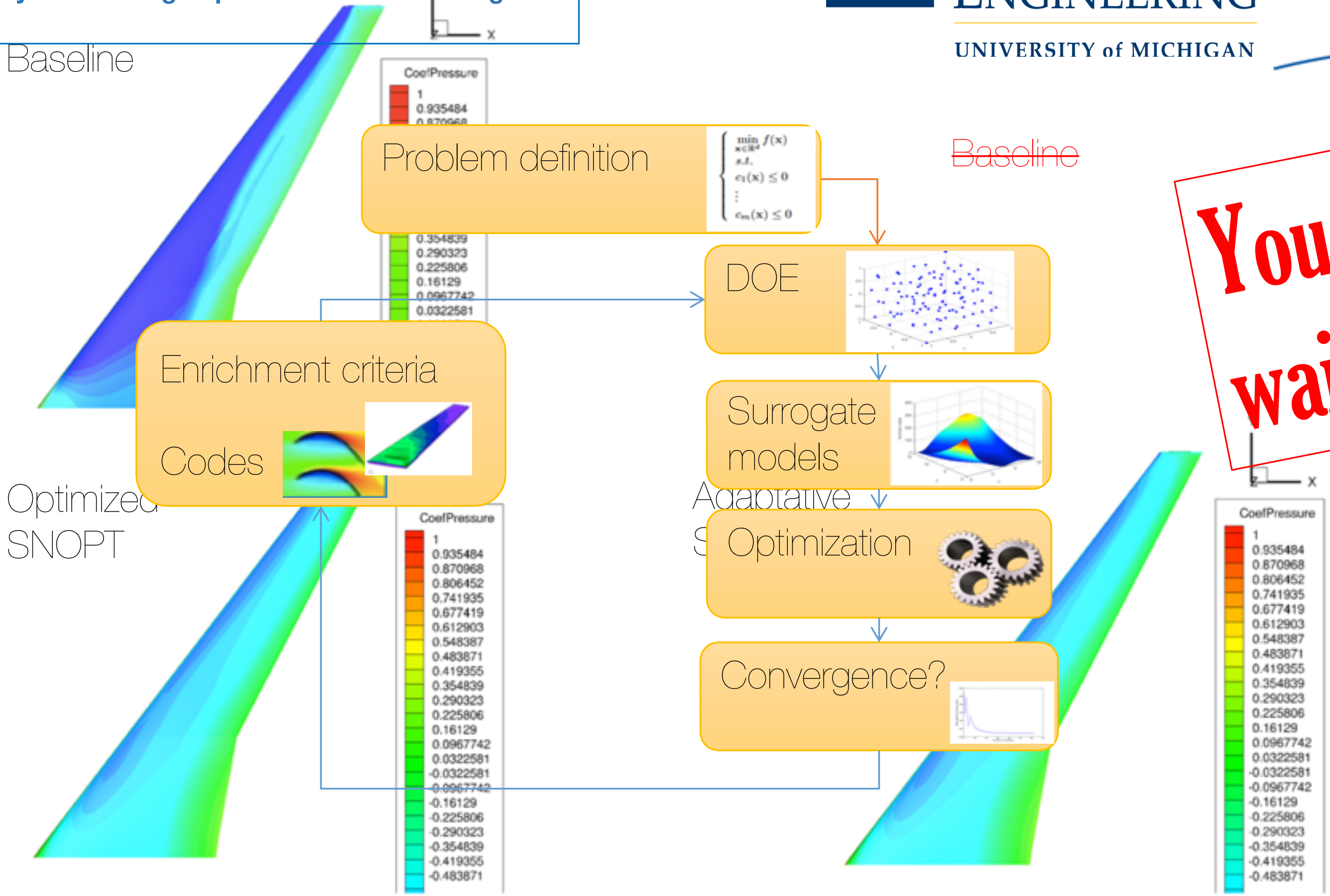
is costly /difficult to implement industrial code
And also sensitive to discontinuity/sensible to X_0



*SOL200 in MSC Nastran for example

Is it possible to resume in one slide ?

N. Bartoli, et al. An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization. Proceedings of AIAA AVIATION 2017



You have to wait ;)

Outlines for today

1. GP aka Kriging
2. Kriging for Global Optimization
3. New developments in topology optimization
4. Add control law in the design loop

1. GP aka Kriging

2. Kriging for Global Optimization
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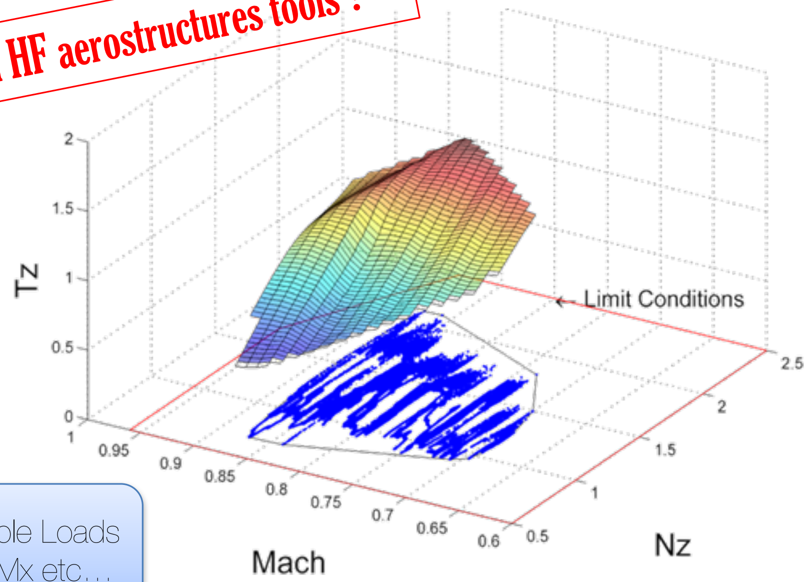
Machine learning for load estimation (Ankit Chiplunkar, AIRBUS FUND)

Kriging (Pionneer)	Gaussian Processes (link with AI)
Developed by Daniel Krige – 1951; formalized by Georges Mathéron in the 60's (Mines Paris)	Neural network with infinite neurons tend to Gaussian Process 1994
Evaluation: minimize error variance	Evaluation: Marginal Likelihood

Loads identification

First flight ready → Flight test of Maneuvers → Capture aero/ structural data → Extract loads → Certification of limit loads

Can we extrapolate limit loads using both measurements and HF aerostructures tools ?



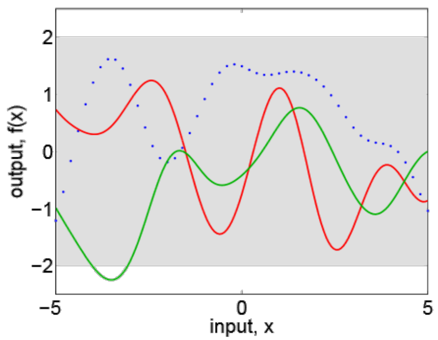
Can we automate the process?

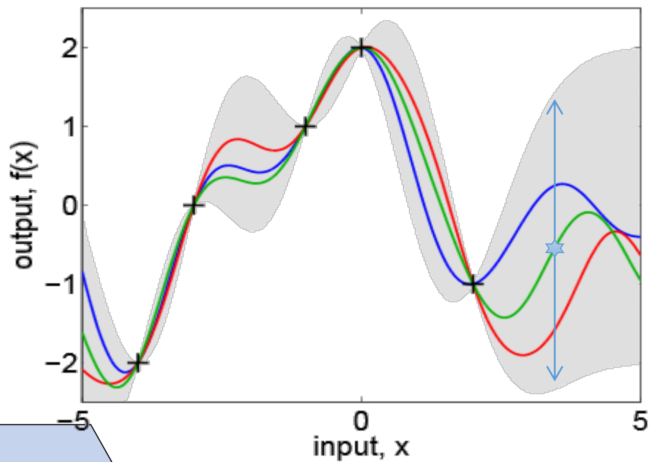
Statistical step

A/C states α, β, M, δ's → Distributed Gaussian Process → Flexible Loads Tz, Mx etc…

Gaussian Process Regression

Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/





Prior
f|x
$GP(0, K)$

X

Y

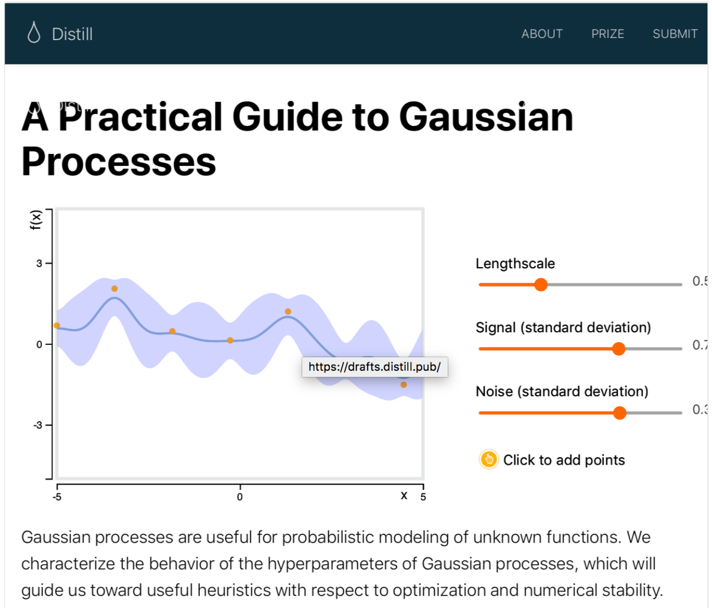
Training
y|x
$GP(0, K + \sigma_n^2 I)$
Optimize: for a better hyperparameters and better prior

Posterior on y
$\boldsymbol{y}^* | \, \boldsymbol{x}^*, \boldsymbol{y}, \boldsymbol{x}$

GP ($K_*[K + {\sigma_n}^2 I]^{-1}$y, $K_{**} + \sigma_n^2 I \; - \; K_*^T [K + {\sigma_n}^2 I]^{-1} K_*$)

A good starting point x_0=Rasmussen's book

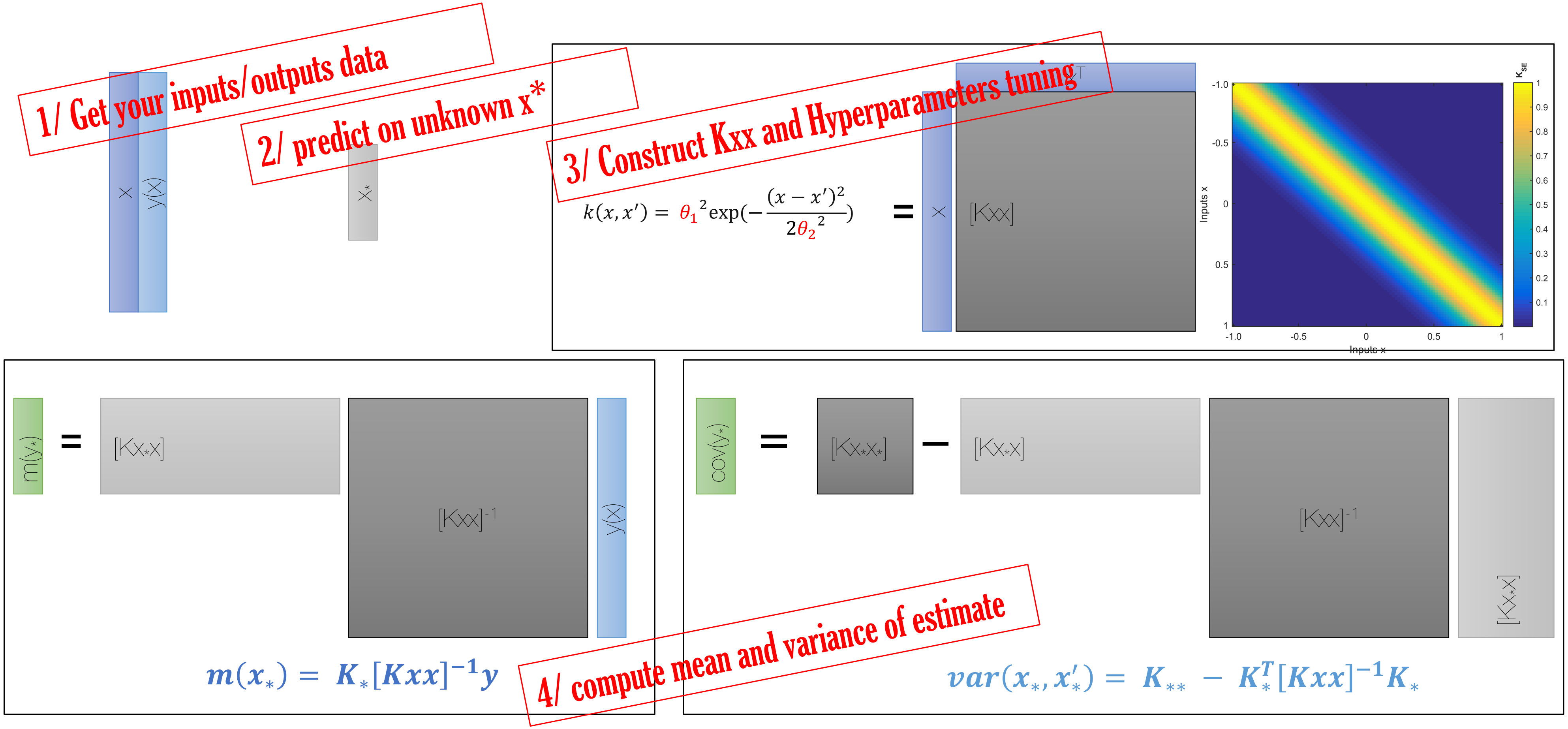
- https://drafts.distill.pub/gp/



C. E. Rasmussen & C. K. I. Williams, Gaussian Processes for Machine Learning, the MIT Press, 2006, ISBN 026218253X. © 2006 Massachusetts Institute of Technology. www.GaussianProcess.org/gpml

Gaussian Processes for Machine Learning

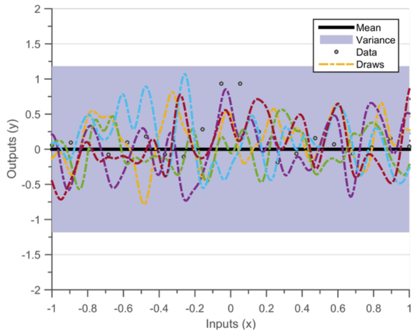
Matrix view of Gaussian Process

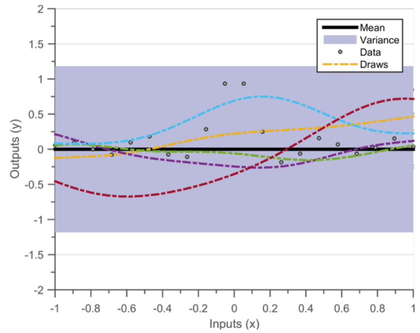


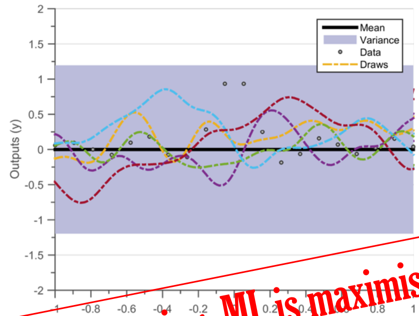
Optimizing marginal likelihood (ML)

$$\text{ML} = log\big(p(y|X,\theta)\big) = -\frac{1}{2}y^T K^{-1} y - \frac{1}{2}log|K| - \frac{n}{2}log(2\pi)$$

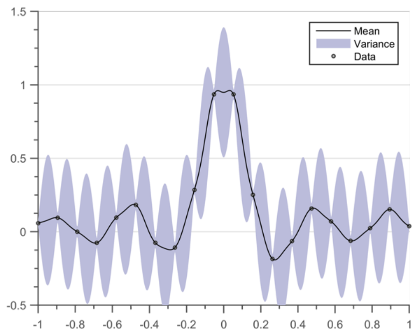
- It is a combination of **data-fit term**, a **complexity penalty** term and a **normalization term**



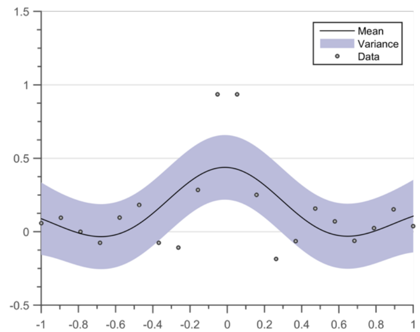




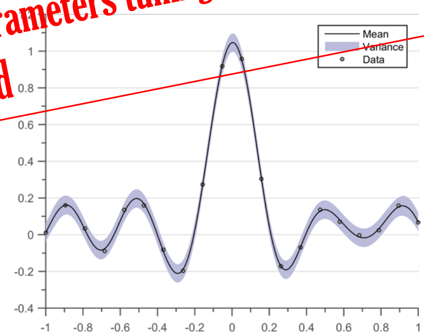
3/ Hyperparameters tuning: ML is maximised, θ^* is found



ML = -8.2

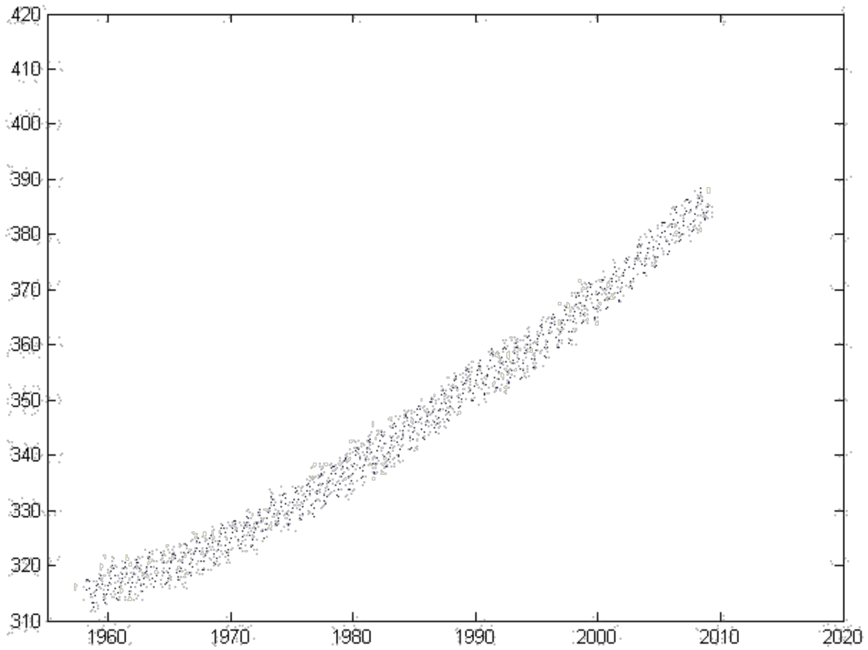


ML = -35.3



ML = 6,04

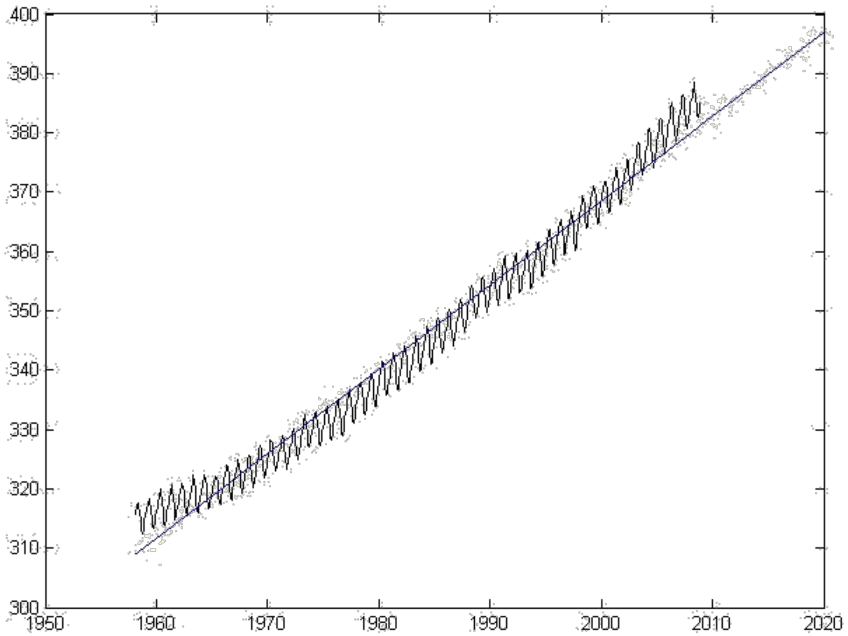
A SIMPle Example



Month-wise data of Co2 concentration in atmosphere at Hawaii

Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/

Example – Linear Regression



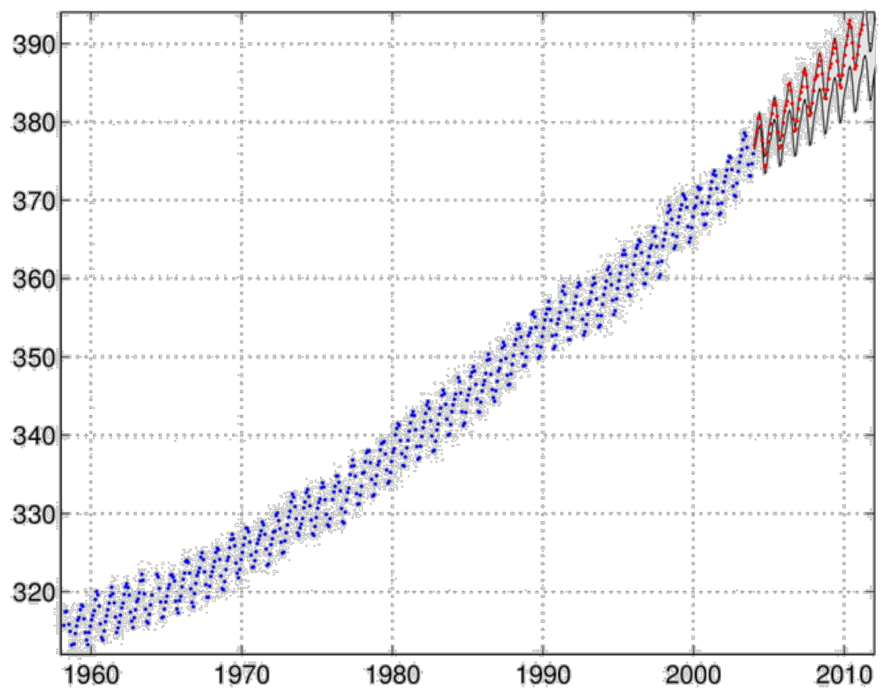
Should we choose a polynomial?

What degree of polynomial should we choose? (overfitting)

For a given degree, what parameters of polynomial should we choose

Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/

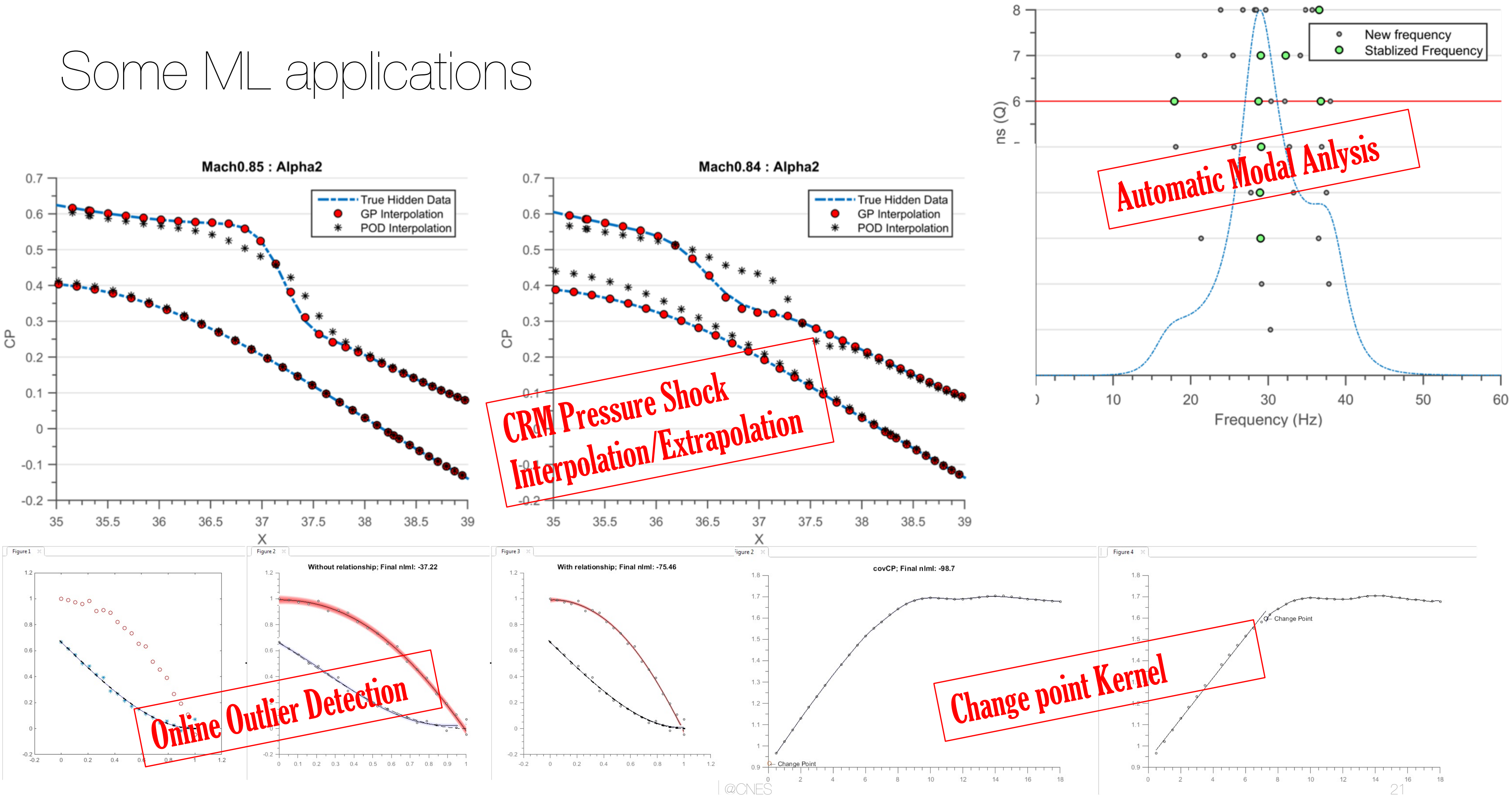
Example – Gaussian Process



Predicted variance after year 2005 in grey, real data-points in red

Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/

Some ML applications



Multi-Output Gaussian Process – Flight Test examples

Given: $f_1 = g(f_2, x)$

- Earlier examples include **Gradient Enhanced Kriging** [$] (GEK) or **Co-kriging** *

- But we want to expand this to integral enhanced kriging, double differential, or any functional relationship between outputs

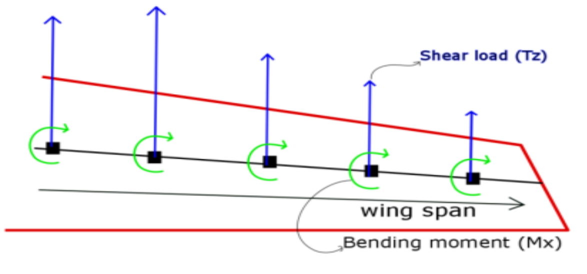
*** Forrester *et al* (2007) Multi-fidelity optimization via surrogate modelling. Proceedings of the Royal Society A, 463(2088), 3251–3269, (doi:10.1098/rspa.2007.1900).**

[$] Liu. Development of gradient-enhanced kriging approximations for multidisciplinary design optimization. 2003.

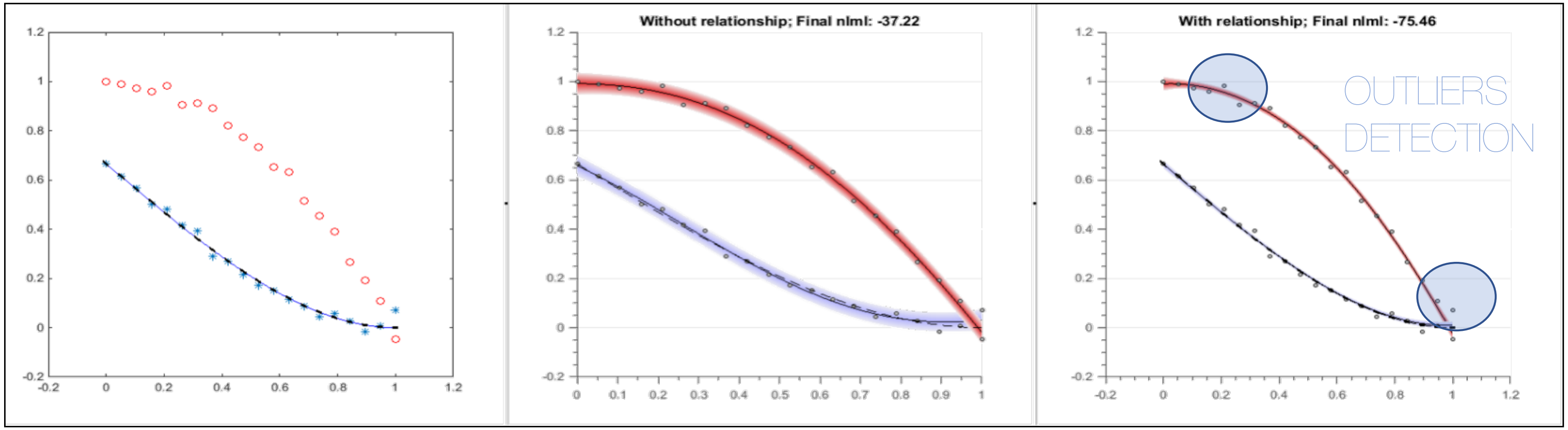
- Add physics[£] as constraints in the mathematical problem

[£]Constantinescu, Physics-based covariance models for Gaussian processes with multiple outputs." International Journal for Unceret *al*tainty Quantification 3.1 (2013).

Example 1: use the Relationship between Tz and Mx permits to reduce uncertainties

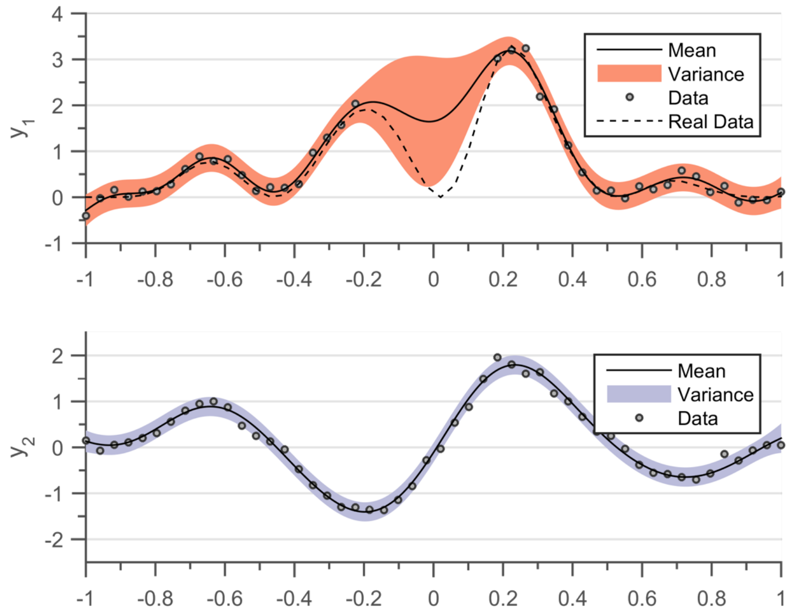


$$Mx = \int_{\eta}^{\eta_{edge}} (x - \eta)\, Tz\, dx$$

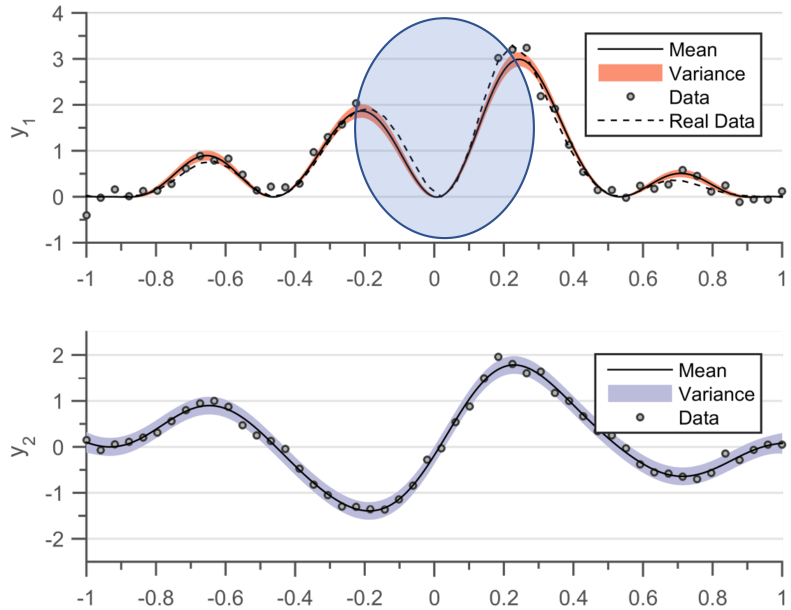


Example 2: Faulty sensors (using synthetic data)

$$y_1 = (y_2)^2$$

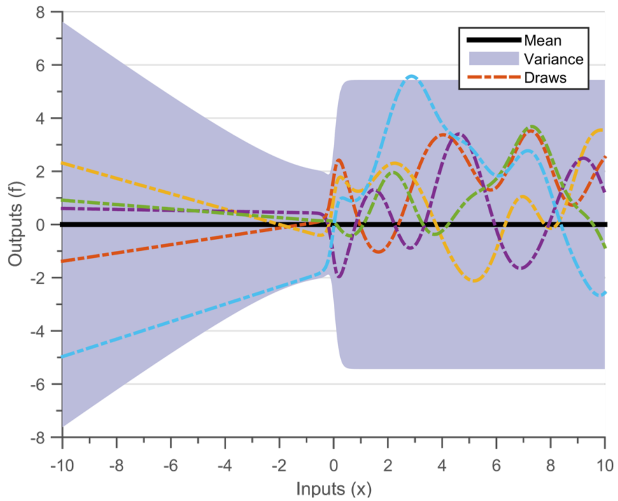


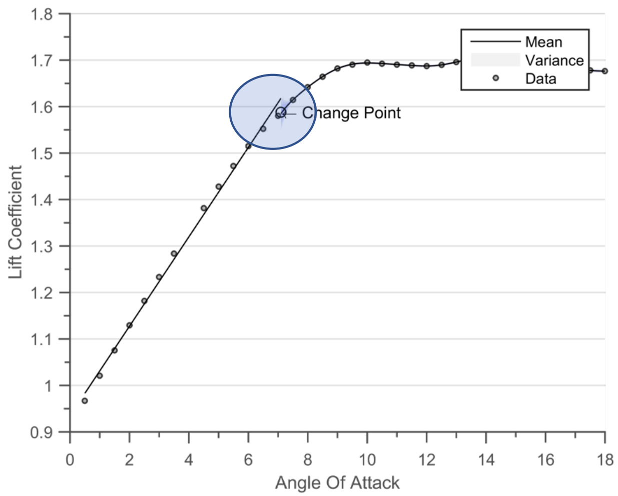
Independent GPs



Related GPs

Example 3: Identifying onset of non-linearity





$$k_{CP}(k_1, k_2, x_1, x_2) = sigm(x_1)k_1 sigm(x_2) + (1 - sigm(x_1))k_2(1 - sigm(x_2))$$

- Estimate change in pattern
- Use global optimization to identify the non-linearity automatically

Papers & conf on this topic

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Approximate Inference in Related Multi-output Gaussian Process Regression. Lecture Notes in Computer Science. 10163, 88-103. 2017

Chiplunkar and J. Morlier. Operational Modal Analysis in Frequency Domain using Gaussian Mixture Models . Proceedings of IMAC XXXV, 2017

Chiplunkar, E. Bosco and J. Morlier. Gaussian Process for Aerodynamic Pressures Prediction in Fast Fluid Structure Interaction Simulations. Proceedings of WCSMO12 2017

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Adding Flight Mechanics to Flight Loads Surrogate Model using Multi-Output Gaussian Processes. AIAA AVIATION 2016

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Sparse Physics-Based Gaussian Process for Multi-output Regression using Variational Inferenc. Proceedings of ICPRAM 2016 2016

Several Papers in preparation

What if

We use Surrogate models to develop Global Optimization Algorithm ?

→EGO* (unconstrained problem)
→SEGO$ (constrained problem

*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. Journal of Global optimization, 13(4), 455-492.

$Sasena, M. J., Papalambros, P., & Goovaerts, P. (2002). Exploration of metamodeling sampling criteria for constrained global optimization. Engineering optimization, 34(3), 263-278.

1. GP aka Kriging

2. Kriging for Global Optimization

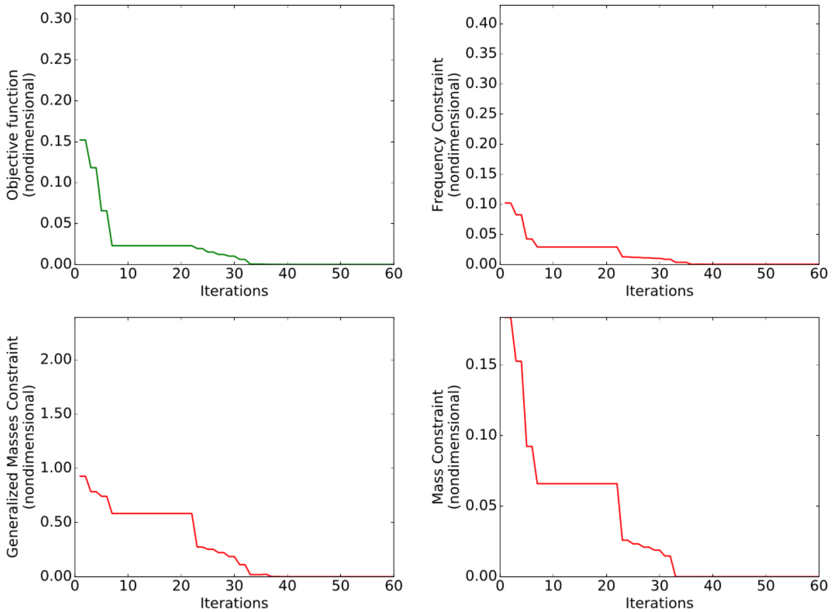
3. New developments in topology optimization

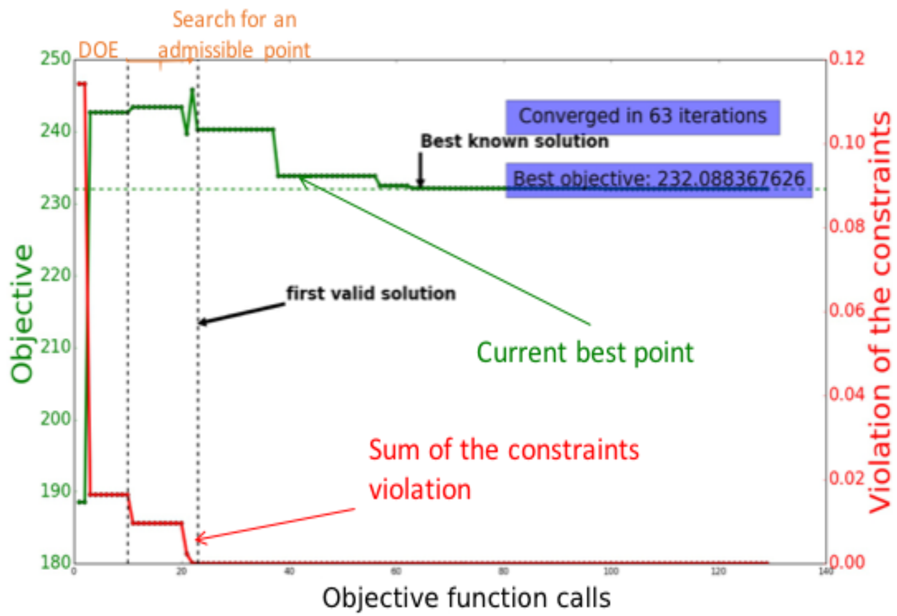
4. Add control law in the design loop

Joint Work since 2007 with N. Bartoli (Onera)

New paradigm for Surrogate Based Optimization (SBO)

Gradient based Optimality, Feasibility





Stopping criteria: tolfun, tolx, maxiter

Surrogate based Global optimization :
Max Budget (Function calls)

A good starting point X_0=Forrester's book

Engineering Design via Surrogate Modelling

A Practical Guide

Alexander I. J. Forrester, András Sóbester and Andy J. Keane

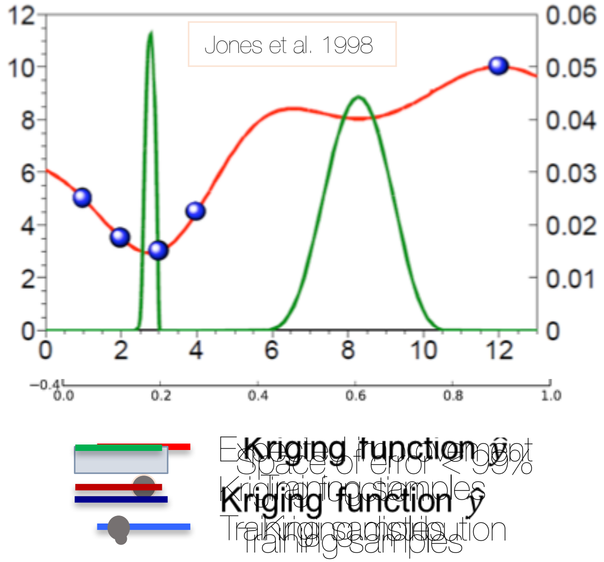
University of Southampton, UK

Focus on (Enrichment) infill sampling criteria

$\hat{y}$ is the kriging prediction of y

$\hat{s}$ is the estimation error (standard deviation) of the prediction

We use the –analytical- variance estimation to enrich the prediction



Expected Improvement

$$EI(x) = \begin{cases} \overbrace{(f_{min} - \hat{y}(x))\Phi\left(\frac{f_{min} - \hat{y}(x)}{\hat{s}(x)}\right)}^{\text{Local exploitation}} + \overbrace{\hat{s}(x)\phi\left(\frac{f_{min} - \hat{y}(x)}{\hat{s}(x)}\right)}^{\text{Exploration}} & if\ \hat{s}(x) > 0 \\ 0 & if\ \hat{s}(x) = 0 \end{cases}$$

Φ is the cumulative distribution function for the standard normal distribution

ϕ is the Probability density function for the standard normal distribution

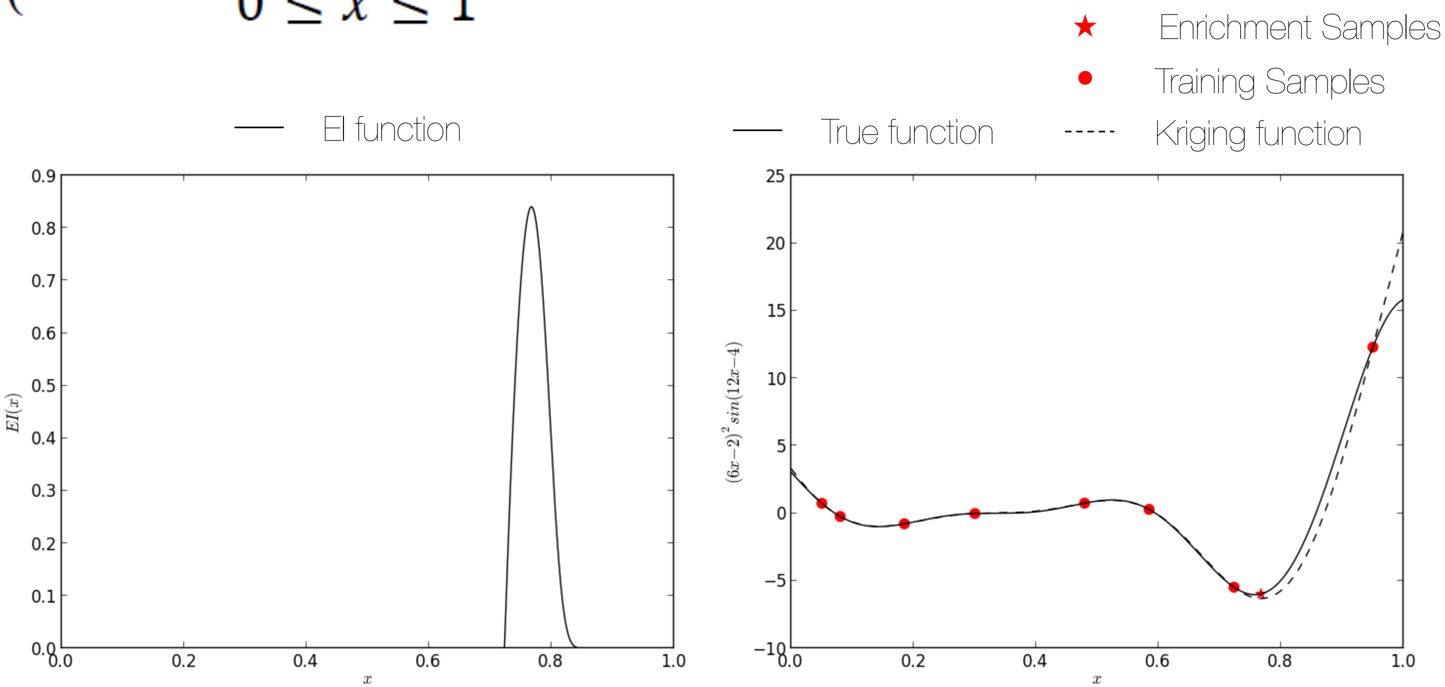
$$\begin{cases} \max_{x \in \mathbb{R}^d} \quad \mathrm{EI}(x) \\ \text{s.t.} \\ \hat{c_1}(x) \geq 0 \\ . \\ \hat{c_i}(x) = 0 \\ . \\ \hat{c_n}(x) \geq 0 \end{cases}$$

➔ Other criteria exist ....

*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. Journal of Global optimization, 13(4), 455-492.

Illustration on 1D example

$$
\begin{cases}
\min \ (6x-2)^2 \sin(12x-4) \\
\qquad\qquad s.t. \\
\qquad\quad 0 \le x \le 1
\end{cases}
$$



Infill sampling criteria

Expected Improvement criterion (EI) (to maximize)

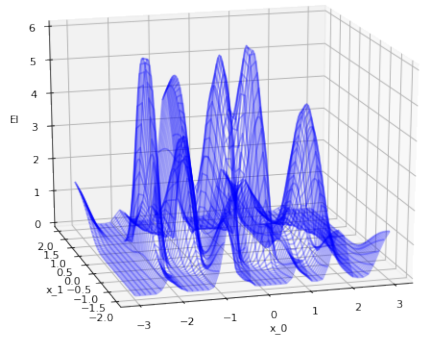
$$\mathrm{EI}(x) = \mathrm{E}[\max(0, y_{\min} - \hat{y}(x))]$$

Φ: cumulative distribution function $\mathcal{N}(0,1)$ ϕ: probability density function $\mathcal{N}(0,1)$

$$\mathrm{EI}(x) = \underbrace{(y_{\min} - \hat{y}(\boldsymbol{x}))\,\Phi\left(\frac{y_{\min} - \hat{y}(\boldsymbol{x})}{s(\boldsymbol{x})}\right)}_{\text{Exploitation}} + \underbrace{s(\boldsymbol{x})\phi\left(\frac{y_{\min} - \hat{y}(\boldsymbol{x})}{s(\boldsymbol{x})}\right)}_{\text{Exploration}}$$

+ Analytical formula ➡ Quick to evaluate
(Criteria and gradient)

- Highly multimodal ➡ Hard to optimize



EI example

*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. Journal of Global optimization, 13(4), 455-492.

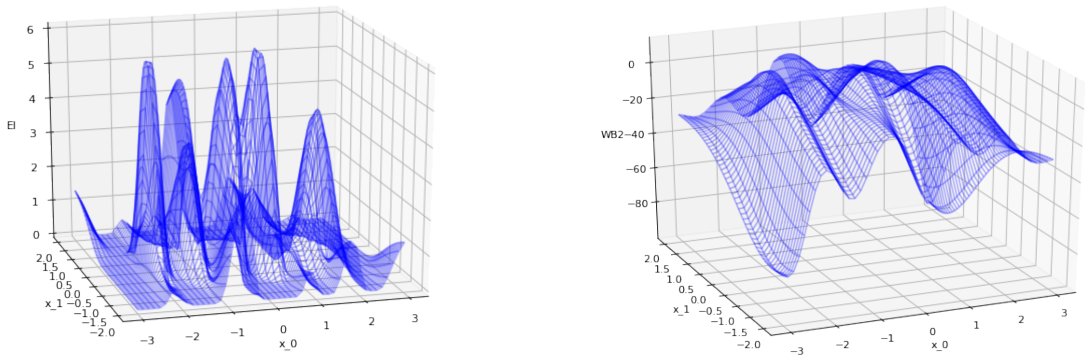
Enrichment infill sampling criteria

WB2 criterion

$$\mathrm{WB2}(x) = \mathrm{EI}(x) - \hat{y}(x)$$

Influence of surrogate model prediction

+ 'Smoother' ➡ Easier to optimize
+ Quicker convergence
- Lack of normalization ➡ Reduce the global aspect



EI and WB2 criteria computed on same Kriging surrogate

$Sasena, M. J., Papalambros, P., & Goovaerts, P. (2002). Exploration of metamodeling sampling criteria for constrained global optimization. Engineering optimization, 34(3), 263-278.

New criteria

N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, " An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization", 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433,Denver, USA, June 2017

WB2S scaled criterion

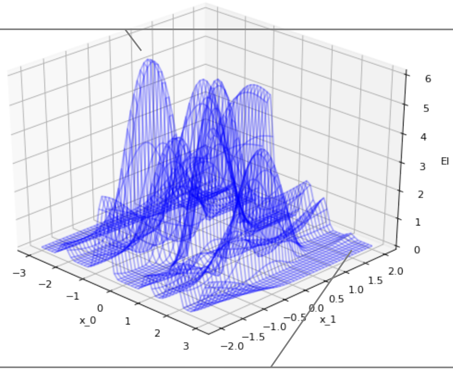
$$\mathrm{WB2S}(x) = s\ \mathrm{EI}(x) - \hat{y}(x)$$

- EI numerically improved
- 'dynamical' normalization

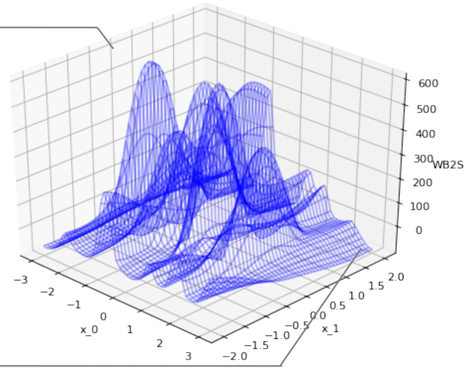
+ Exploration similar to EI

+ non negligeable value in the area where EI is low

EI



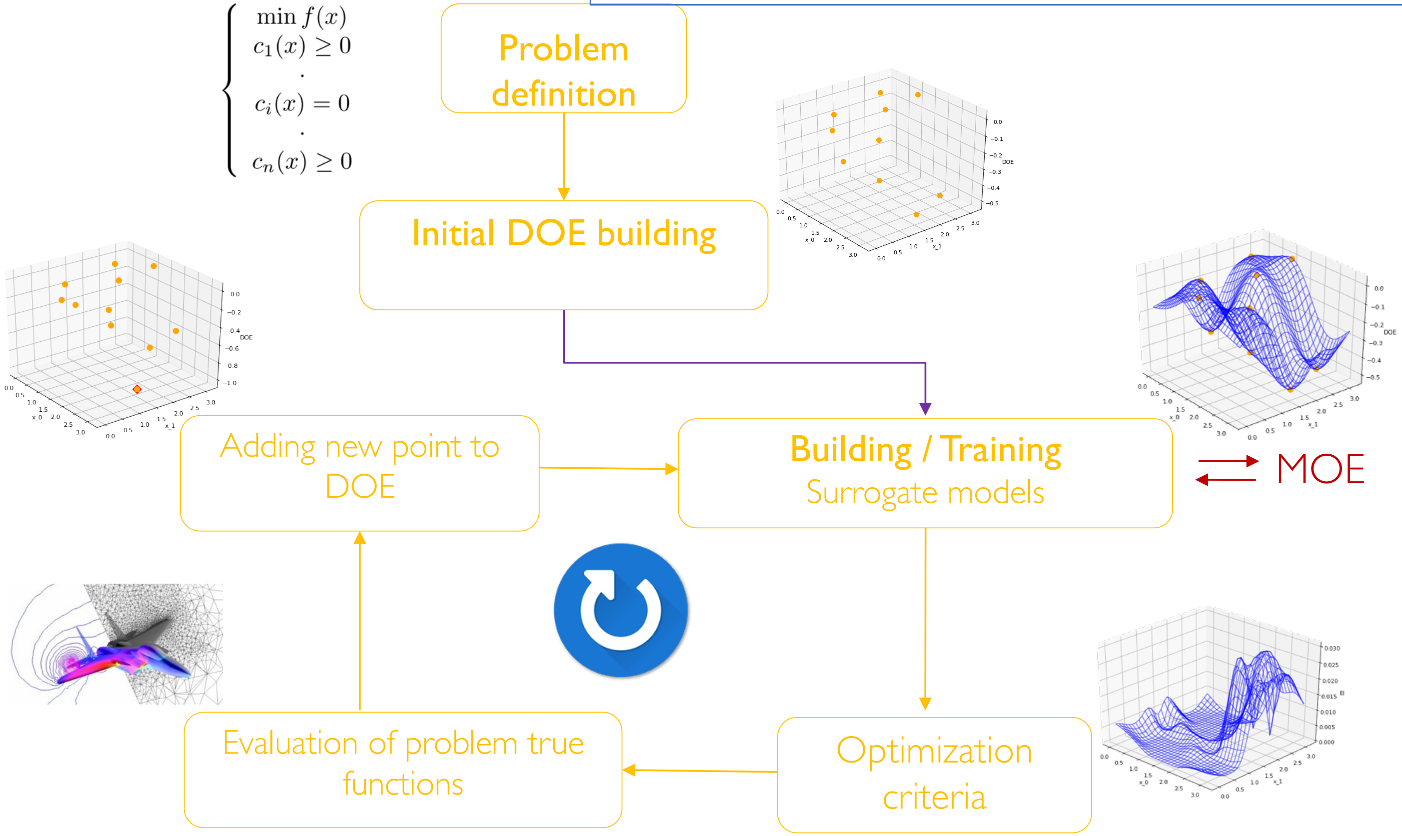
WB2S



SEGOMOE algorithm

N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, " An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization", 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433, Denver, USA, June 2017

$$\begin{cases} \min f(x) \\ c_1(x) \geq 0 \\ \vdots \\ c_i(x) = 0 \\ \vdots \\ c_n(x) \geq 0 \end{cases}$$



Enrichement step

N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, " An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization", 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433,Denver, USA, June 2017

Super EGO Formulation

Costly initial problem

$$\begin{cases} \min f(x) \\ c_1(x) \geq 0 \\ \quad . \\ c_i(x) = 0 \\ \quad . \\ c_n(x) \geq 0 \end{cases}$$

Cheap enrichment problem

$$\begin{cases} \max\limits_{x \in \mathbb{R}^d} \quad \mathrm{EI}(x)/\mathrm{WB2}(x)/\mathrm{WB2s}(x) \\ \text{s.t.} \\ \hat{c}_1(x) \geq 0 \\ . \\ \hat{c}_i(x) = 0 \\ . \\ \hat{c}_n(x) \geq 0 \end{cases}$$

n +1 metamodels

Possibly Multimodal

Multimodal

Global optimization method

ADODG6* testcase

CFD guys know very well the multimodality of this problem...

Wing drag minimization problem (subsonic, Euler equations)

	Function/variable	Description	Quantity	Range
minimize	C_D	Drag coefficient	1	
with respect to	α	Angle of attack	1	$[-3.0, 6.0]$ (°)
	θ	Twist	8	$[-3.12, 3.12]$ (°)
	δ	Dihedral	8	$[-0.25, 0.25]$ (unit of chord)
		Total variables	17	
subject to	$C_L = 0.2625$	Lift coefficient	1	
		Total constraints	1	

Can SEGOMOE help us to reach the global optimum ?

Is it less dependant on X0 compared to SNOPT £?

*AIAA, Aerodynamic Design Optimization Discussion Group
http://mdolab.engin.umich.edu/content/aerodynamic-design-optimization-workshop

£ https://web.stanford.edu/group/SOL/snopt.htm

ADODG* 6 TOOLS & GOALS

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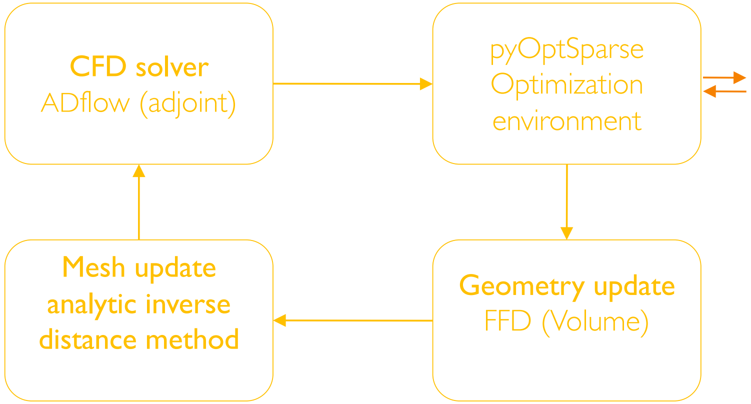
UCSD/ Stanford

isae SUPAERO

ONERA THE FRENCH AEROSPACE LAB

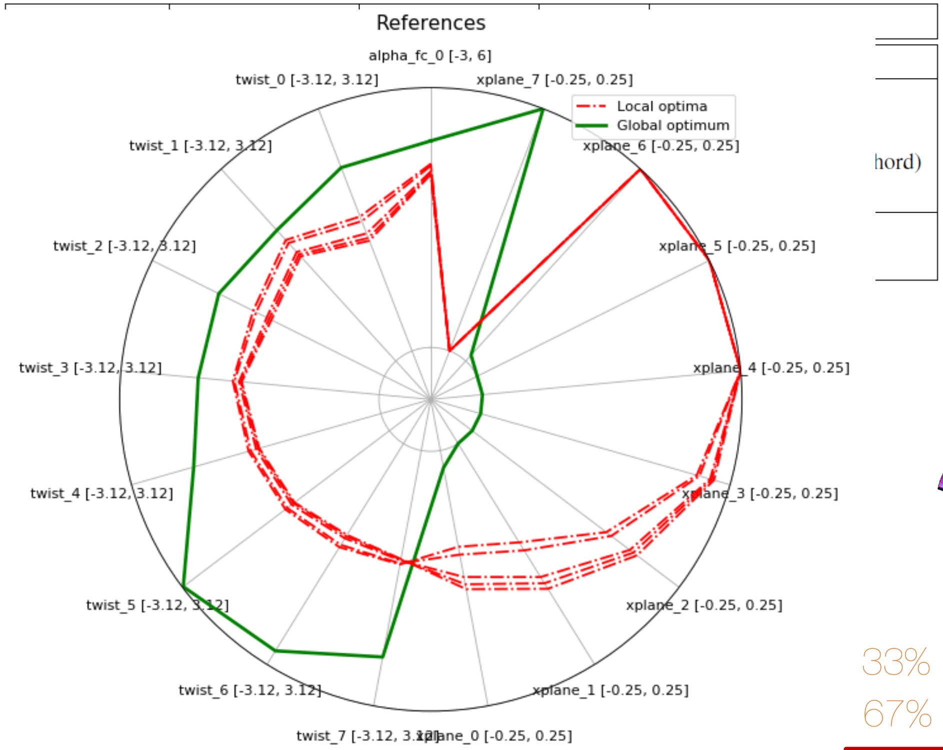


SNOPT | SEGOMOE

Compare the robustness of the solution with respect to X_0

Multimodal optimization problem (SNOPT Benchmark)

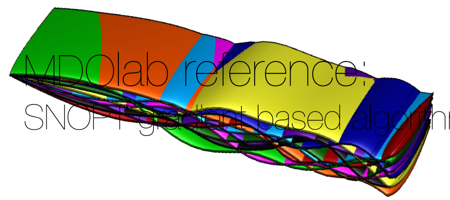
Wing drag minimization problem (subsonic, Euler equations with ADFlow solver)



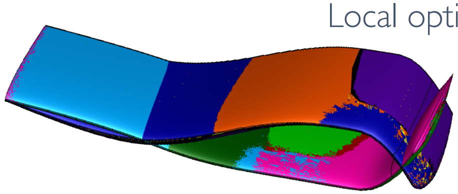
→ 5/15 runs found the global optimum

Multi start with DOE of 15 (LHS)

MDOlab reference:
SNOPT gradient-based algorithm



Local optima



Global optimum

33% Cx : 39.909 points
67% Cx : 40.19x points

(Mesh 180K cells)

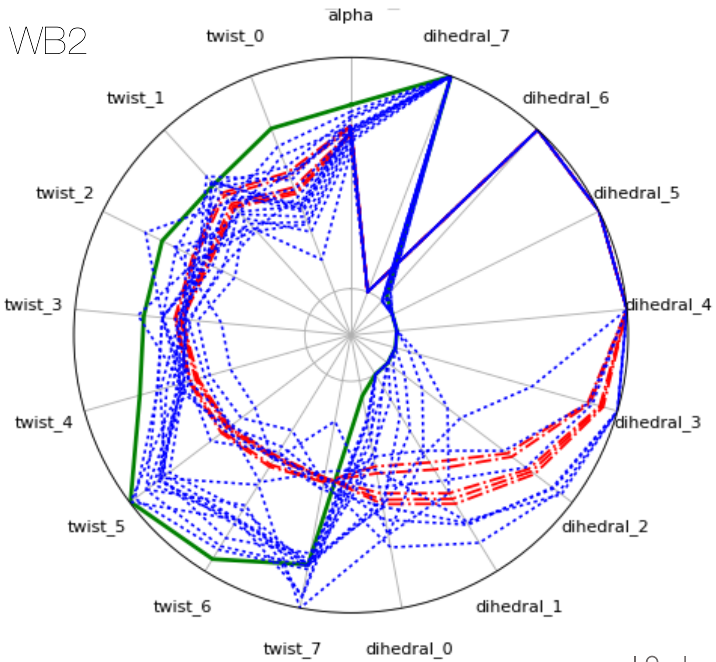
Accuracy Cz ~$1e^{-10}$ – 300 calls to ADflow

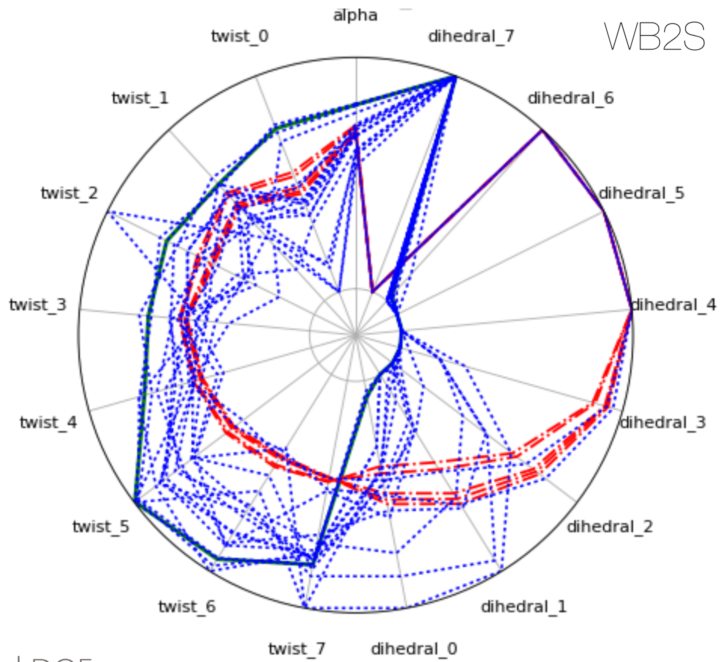
Multimodal optimization problem (SEGOMOE 1)

initial DOE = 17 points (1xd)

Frozen budget: 500 evaluations

Surrogate models: KPLSK





18 shared DOE

- Global optimum 11 / 18
  Best point ~ 382 evaluations
  Accuracy Cz ~ $3.9e^{-6}$

- Global optimum 16 / 18
  Best point ~ 399 evaluations
  Accuracy Cz ~ $3.5e^{-6}$

Multimodal optimization problem (SEGOMOE 2)

Initial DOE= 68 points (4xd)

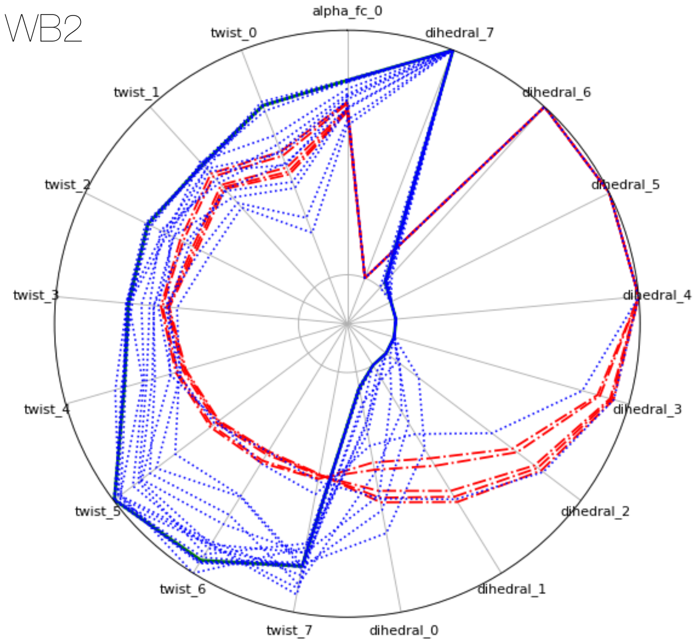
DOE=17 n_runs=18
Local optima
Global optimum

Frozen budget: 500 evaluations

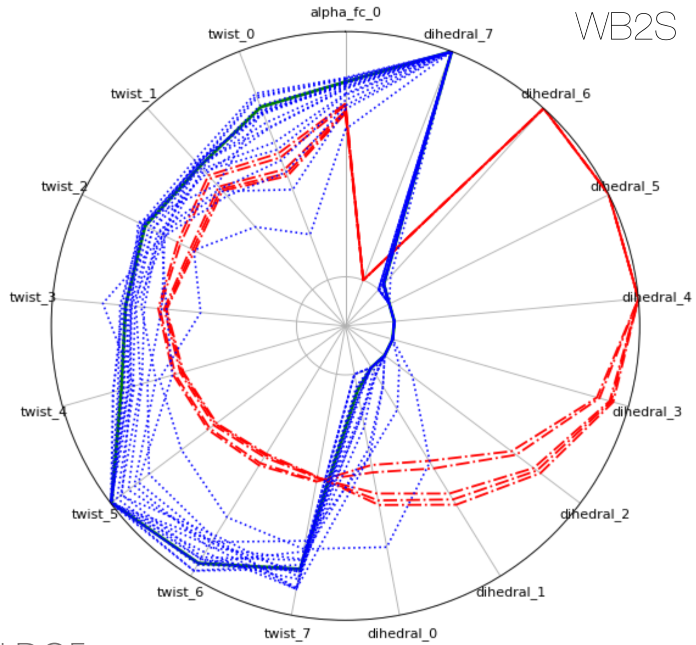
68

Surrogate models : KPLSK

WB2



WB2S



18 shared DOE

Global optimum 16 / 18
Best point ~ 409 evaluations
Accuracy Cz ~ $2.97e^{-6}$

Global optimum 18 / 18
Best point ~ 477 evaluations
Accuracy Cz ~ $3.24e^{-6}$

Conclusions on: ADODG6 results

N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, R. Priem,N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, Adaptive modeling strategy for multimodal constrained global optimization, submitted to Elsevier's AST.

Criterion	Results	DOE 17 $= d$	DOE 34 $= 2d$	DOE 51 $= 3d$	DOE 68 $= 4d$	DOE 85 $= 5d$	DOE 102 $= 6d$
WB2	**Global**	11/18	14/18	14/18	16/18	15/18	14/18
	$\mu_{\text{prox-glob}}$	71.4%	78.5%	80.1%	89.19%	84.41 %	80.15%
	Mean evaluations	**382**	**426**	**414**	**409**	**428**	**368**
	Max evaluations	500	500	500	497	498	489
	Min evaluations	225	273	189	270	254	248
	Mean violation	3.9e-6	4.0e-6	3.2e-6	2.97e-6	2.64e-6	3.22e-6
WB2S	**Global**	**16/18**	**17/18**	**18/18**	**18/18**	**18/18**	**18/18**
	$\mu_{\text{prox-glob}}$	**80.8%**	**85.6%**	**88.8%**	**95.45%**	**93.4 %**	**94.87%**
	Mean evaluations	399	458	468	477	478	468
	Max evaluations	497	500	500	500	498	500
	Min evaluations	256	312	335	406	370	408
	Mean violation	3.5e-6	3.5e-6	4.5e-6	3.24e-6	2.52e-6	2.76e-6

WB2S: more robust

Best compromise: initial DOE~ 3 - 4d

A surrogate model toolbox: SMT

SMT 0.2 documentation »

An open source python toolbox for surrogate models (since July 2017)

Amine Bouhlel, M., Bartoli, N., Regis, R. G., Otsmane, A., & Morlier, J. (2018). Efficient global optimization for high-dimensional constrained problems by using the Kriging models combined with the partial least squares method. Engineering Optimization, 1-16.

- Including the models KPLS and KPLS-K (for processing High Dimensionnal input variables)
- Focus on derivatives:
  - training derivatives used for gradient-enhanced modeling,
  - prediction derivatives,
  - derivatives with respect to the training data

SMT: A Python Surrogate Model Toolbox

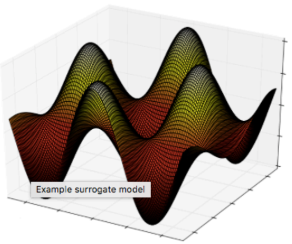
preamble

SMT, surrogate model toolbox, is a Python toolbox that includes four types of surrogate models---the least square (ls), square polynomial (pa2), inverse distance weighting (idw) and kriging-based models.

The toolbox supports Linux and Microsoft Windows, except the idw model which is available only with Linux.

This package is devoted to gathering several types of surrogate models within the same platform, for providing a benchmark to the engineering field. Another purpose of the SMT is for research purposes.

SMT is typically used as a surrogate model for a (time-consuming) computer model.



https://github.com/SMTorg/SMT

Papers & conf on this topic

Amine Bouhlel, M., Bartoli, N., Regis, R. G., Otsmane, A., & Morlier, J. (2018). Efficient global optimization for high-dimensional constrained problems by using the Kriging models combined with the partial least squares method. Engineering Optimization, 1-16.

Bouhlel, M. A., Bartoli, N., Otsmane, A., & Morlier, J. (2016). An improved approach for estimating the hyperparameters of the kriging model for high-dimensional problems through the partial least squares method. Mathematical Problems in Engineering, 2016.

Amine Bouhlel, M., Bartoli, N., Regis, R. G., Otsmane, A., & Morlier, J. (2018). Efficient global optimization for high-dimensional constrained problems by using the Kriging models combined with the partial least squares method. Engineering Optimization, 1-16.

Bartoli, N., Lefebvre, T., Dubreuil, S., Olivanti, R., Bons, N., Martins, J., ... & Morlier, J. (2017). An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization. In 18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4433).

Bartoli, N., Bouhlel, M. A., Kurek, I., Lafage, R., Lefebvre, T., Morlier, J., ... & Regis, R. (2016). Improvement of efficient global optimization with application to aircraft wing design. In 17th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4001).

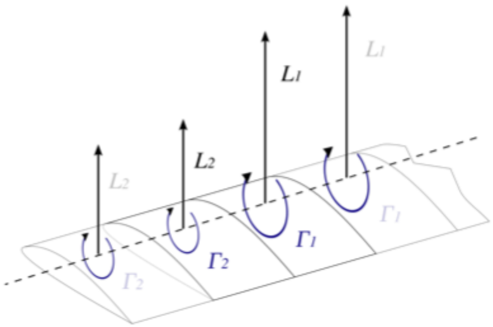
Bartoli, N., Bouhlel, M. A., Kurek, I., Lafage, R., Lefebvre, T., Morlier, J., ... & Regis, R. (2016). Improvement of efficient global optimization with application to aircraft wing design. In 17th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4001).

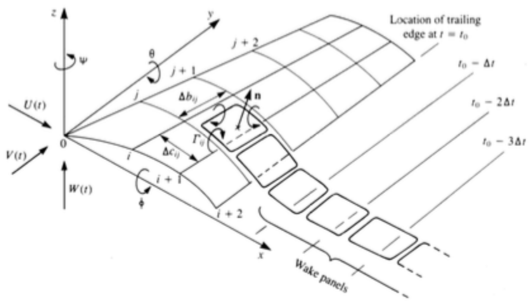
Several Papers in preparation

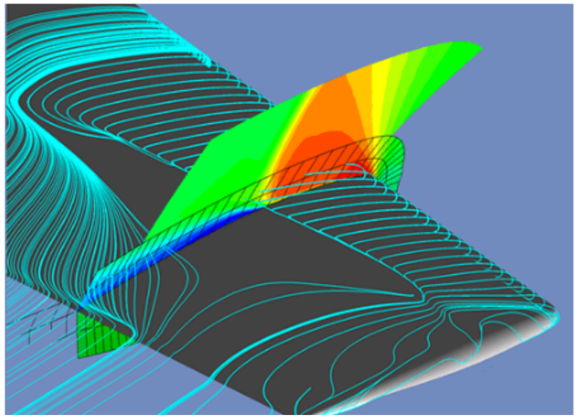
What if ?

- Several levels of fidelity of the same simulation are available

→ For example in aerodynamics: Lifting line theory, Vortex lattice method, and RANS CFD code

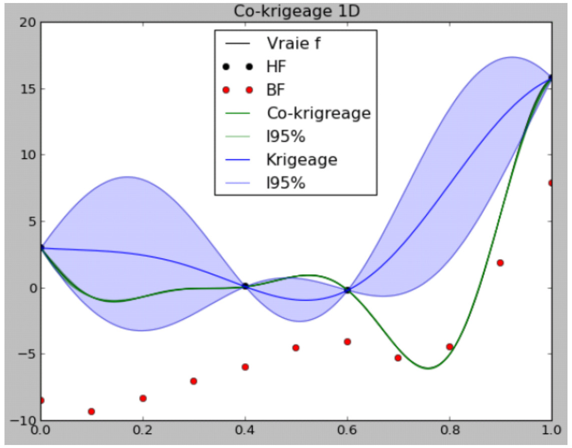


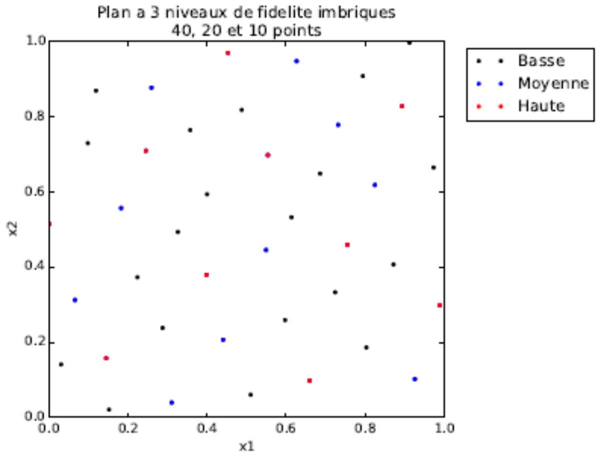




Multi-fidelity kriging

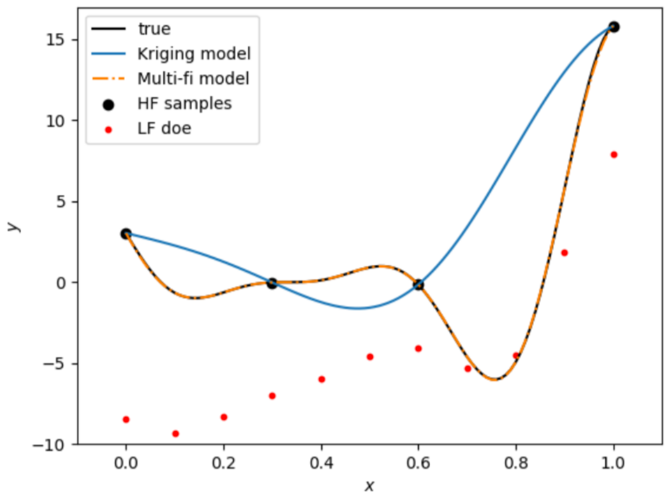
- Surrogate adapted for multi-fidelity data (co-kriging with recursive approach)
- Design of experiments adapted to multi-fidelity (nested DOE)
- Developments of modules integrated in the OpenMDAO Framework (since Jan 2015) and now in SMT (since 2018)

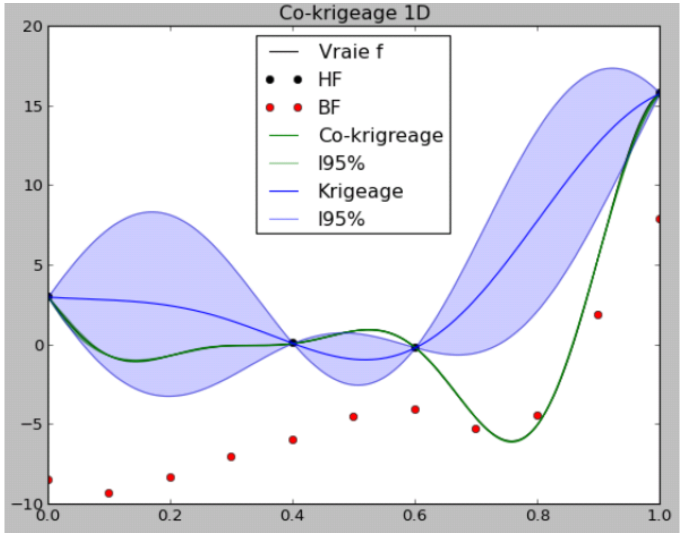




Le Gratiet, L. and Garnier, J., "Recursive co-kriging model for Design of Computer experiments with multiple levels of fidelity," International Journal for Uncertainty Quantification, 2014, pp. 365–386
Vauclin, R., "Développement de modèles réduits multi-fidélité en vue de l'optimisation de structures aéronautiques," Tech. rep., ISAE-SUPAERO, July 2014

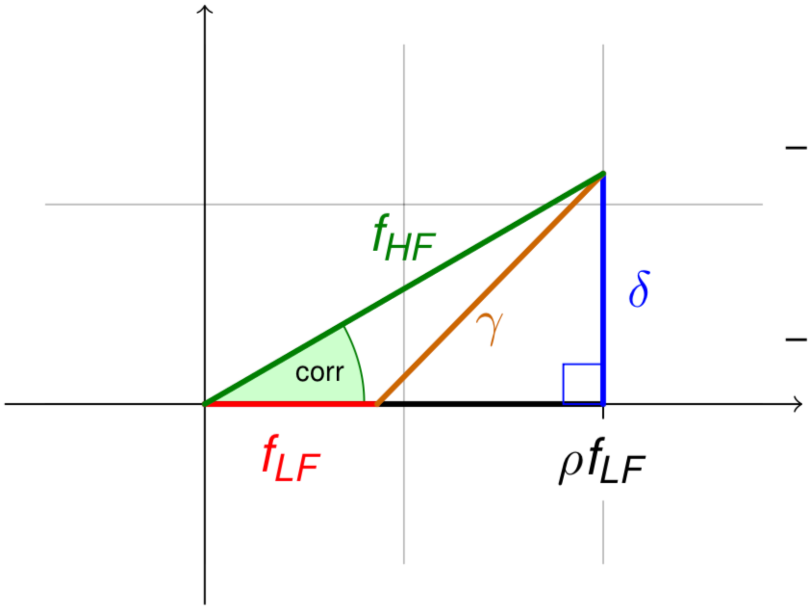
How to best use low-fidelity information to enhance the high-fidelity model?





Remember Co-Kriging is the way to learn the difference between HF & LF …

Co Kriging



– Additive formulation [Lewis 2000]

$$f_{HF}(x) = f_{LF}(x) + \gamma(x)$$

– Kennedy-O'Hagan [Kennedy 2001]

$$\begin{cases} f_{HF}(x) = \rho f_{LF}(x) + \delta(x) \\ f_{LF}(\cdot) \perp \delta(\cdot) \end{cases}$$

The addition of the term ρ makes the multi-fidelity learning more robust to poor correlation as well as differences in modelization.

$Alexandrov, N., Lewis, R., Gumbert, C., Green, L., & Newman, P. (2000, January). Optimization with variable-fidelity models applied to wing design. In 38th Aerospace Sciences Meeting and Exhibit (p. 841).

Kennedy, M. C., & O'Hagan, A. (2001). Bayesian calibration of computer models. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 63(3), 425-464.

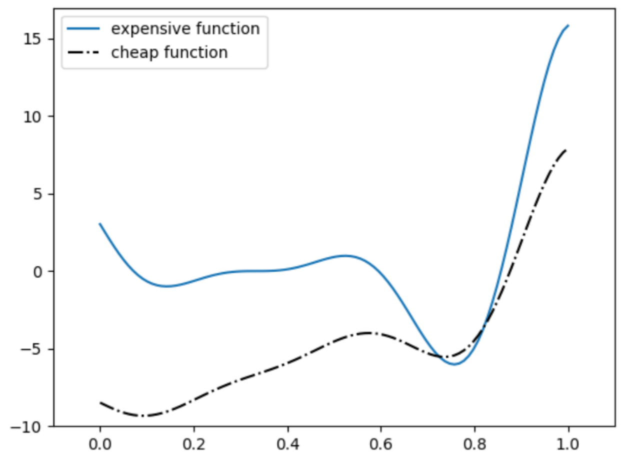
MFEGO

- most promising point: EI criterion

$$x^* = \underset{x}{\text{argmax}}\,(E[I(x)])$$

- choice of levels of enrichment: trade-off information gain/cost

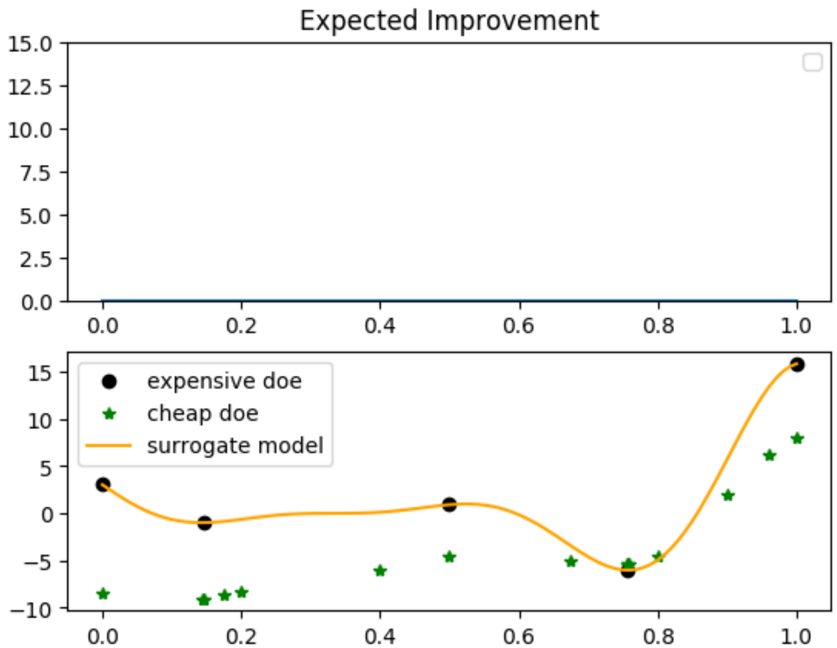
$$k^* = \underset{k \in (0,\ldots,l)}{\text{argmax}} \quad \frac{\sigma^2_{red}(k, x^*)}{f(c_k)}$$



$\Rightarrow$ By using low-fidelity to reduce the uncertainty we reduce the Exploration contribution to the EI criterion
$\Rightarrow$ High-fidelity is used for Exploitation and model enhancement

$$f_{HF}(x) = (6x - 2)^2 \times \sin(2(6x - 2))$$
$$f_{LF}(x) = 0.5 f_{HF} + 10(x - 0.5) - 5$$

Results (Toy problem)



Cost ratio: 1/1000

	HF	LF	Cost
MFEGO	3+2	6+9	5.015
EGO	4+11	-	15

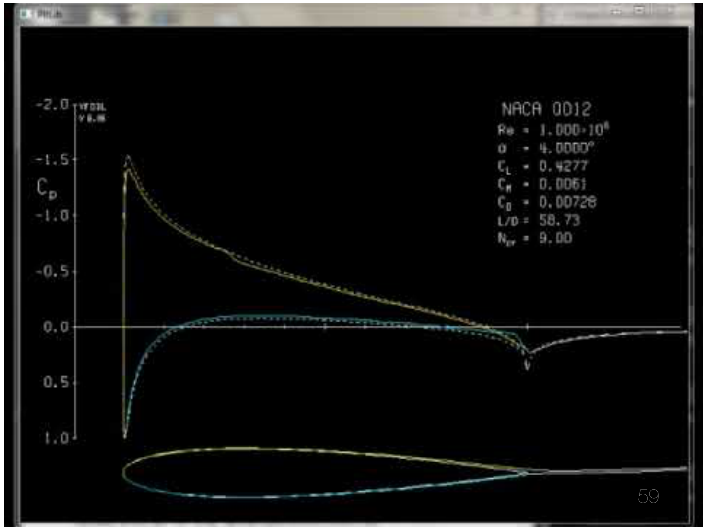
First application: Unconstrained problem

* Maximize L/D (Lift-to-Drag ratio)
* 9 Design Variables:
  – 4 thickness modes
  – 4 camber modes
  – Angle of Attack
* Constants of the problem:
  – Mach number = 0.25
  – Reynolds number = 6×10^6

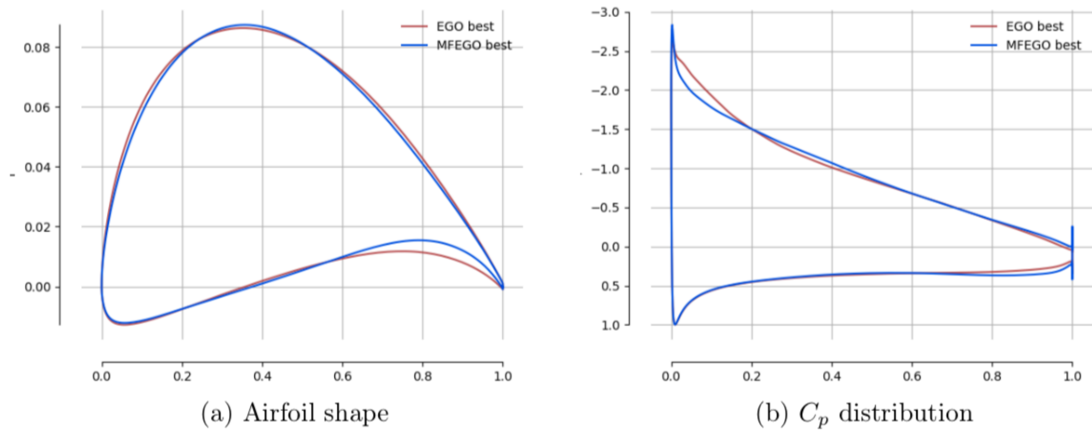
* https://web.mit.edu/drela/Public/web/xfoil/
$ http://mdolab.engin.umich.edu

* HF:CFD RANS solver (Adflow $)
* LF: PANEL solver (Xfoil *)

Estimated COST RATIO: 1/200



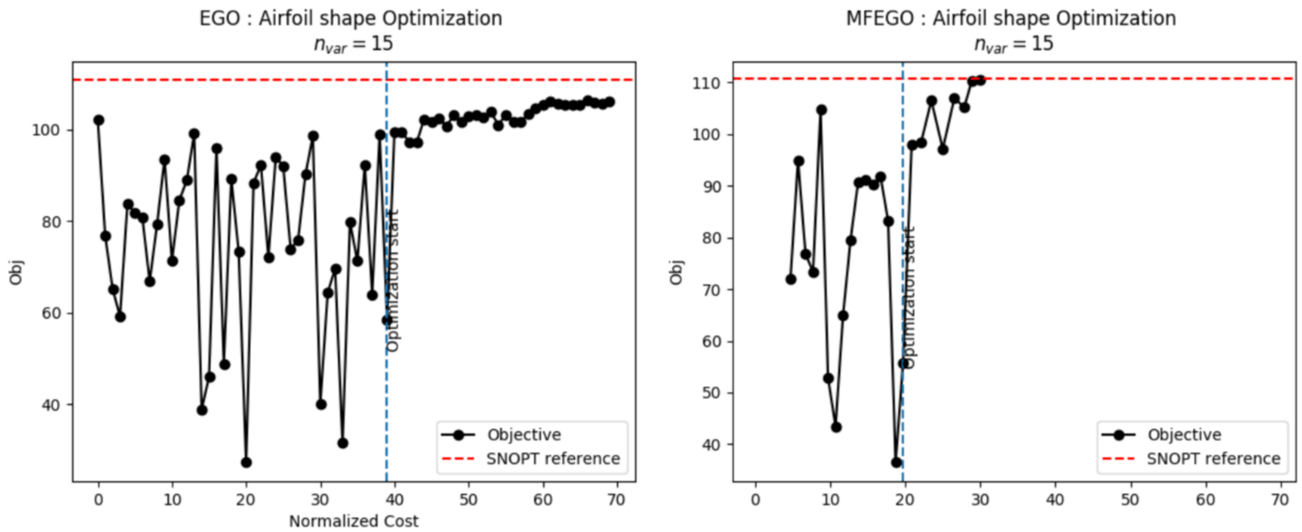
Airfoil Optimization (1)



Comparison of EGO and MFEGO resulting airfoil shapes and C_p distributions for an unconstrained L/D maximization

	HF DOE	LF DOE	HF Opt	LF Opt	Cost	Obj
EGO	30	-	40	-	70	108.44
MFEGO	28	58	8	214	37.41	109.94

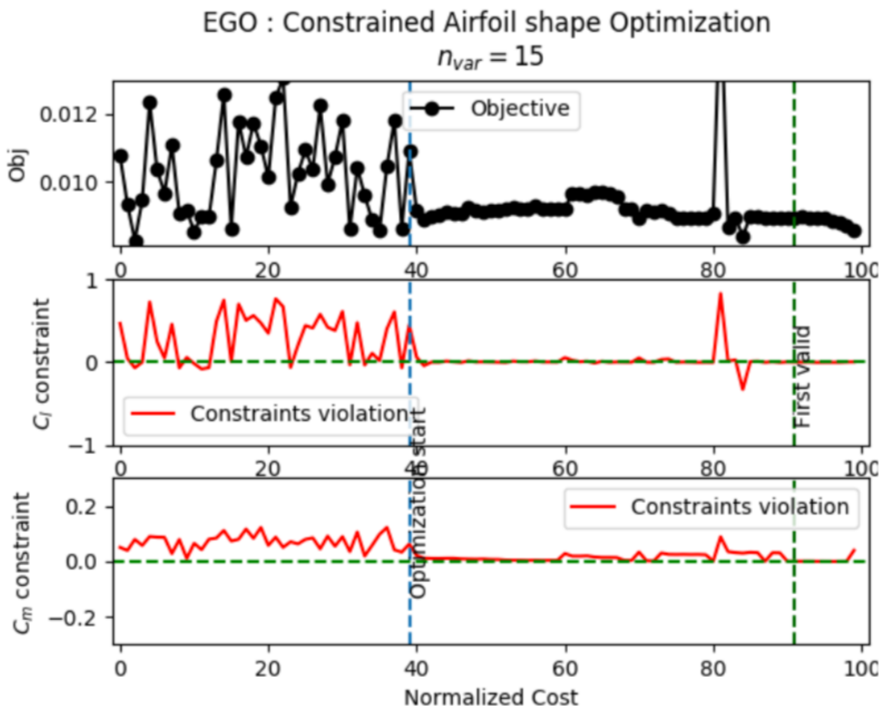
Airfoil Optimization (2)

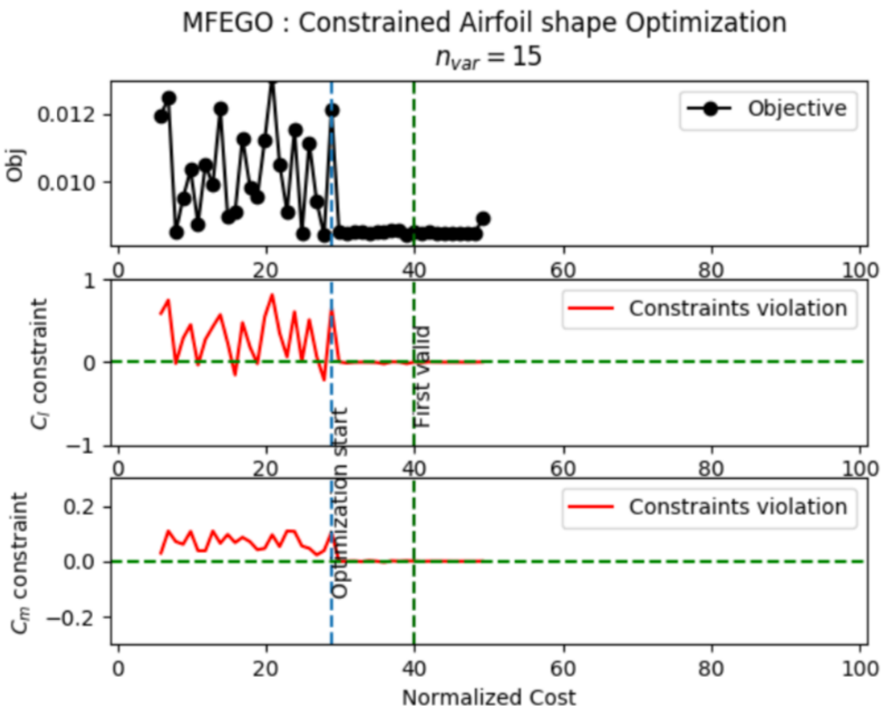


- L/D maximization
- 15 design variables (7 camber + 7 thickness modes + AoA)

	HF	LF	Cost	Obj
EGO	40 + 30	-	70	104.9
MFEGO	16 + 8	744 + 437	29.89	110.5
SNOPT	21	-	21	110.7

Second application: Constrained Optimization





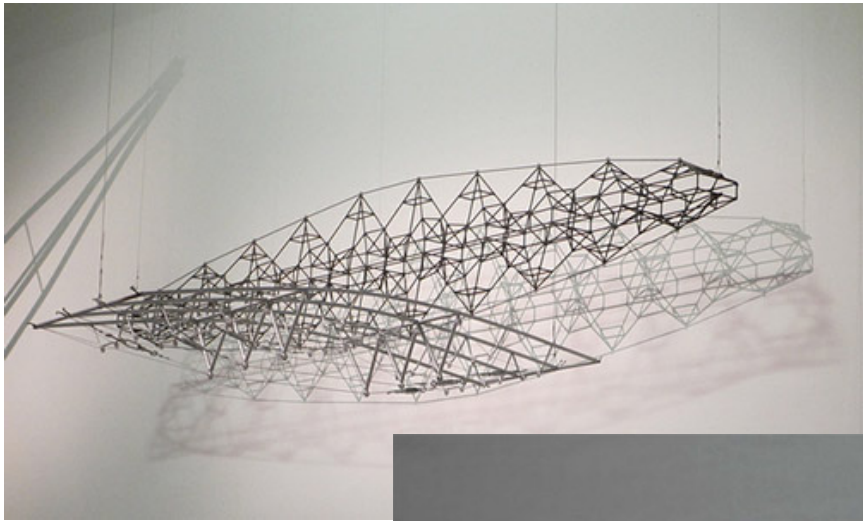
1. GP aka Kriging
2. Kriging for Global Optimization

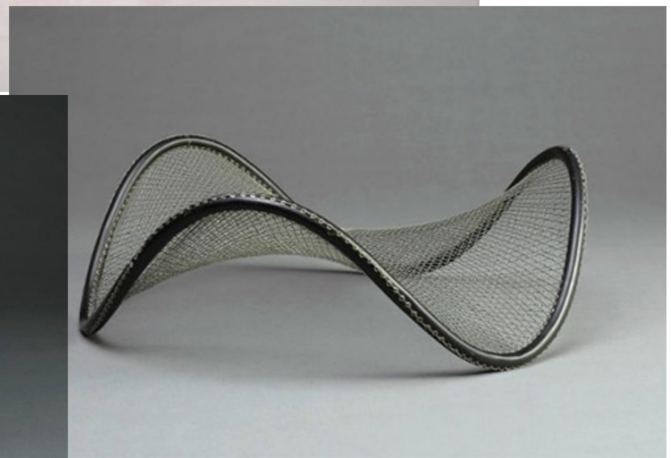
3. New developments in topology optimization

4. Add control law in the design loop

“The art of structure is where to put the holes.”

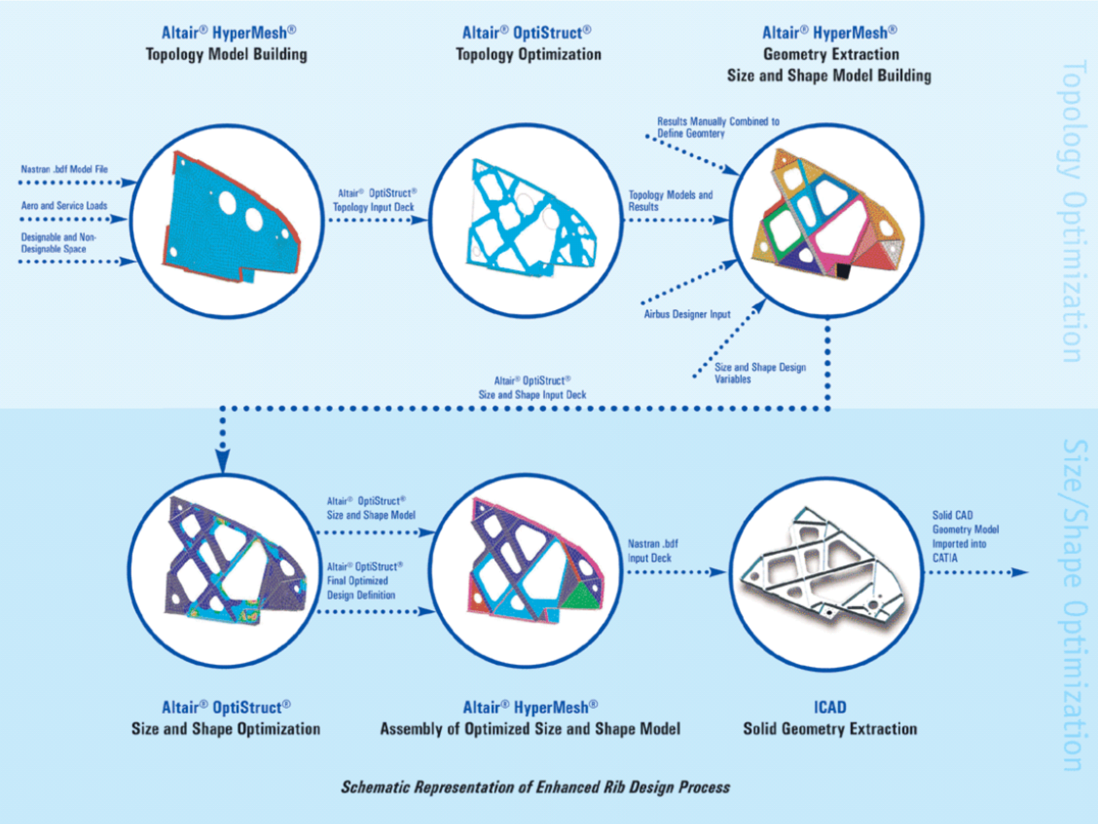
~Robert Le Ricolais (1894-1977)





http://www.dataisnature.com/?p=2053

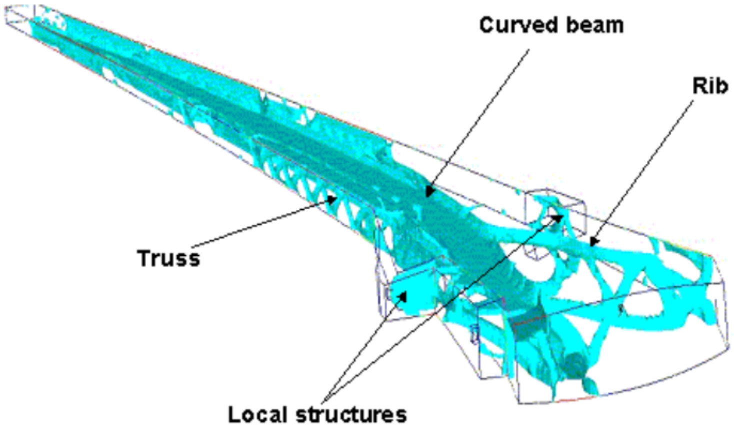
INDUSTRIAL PROCESS



Altair Engineering

Starting point*, Open at least 2 questions ??

*L.Krog, S.Grihon, A.Marasco, Smart design of structures through topology optimisation, 8th World Congress on Structural and Multidisciplinary Optimization, June 1 - 5, 2009, Lisbon,



Identified features:
- **Curved beam** for carrying bending loads
- Large **rib**-like structure connecting the gear rib area to the forward root joint
- **Truss**-like spar structures
- **Local structures** around gear-rib and pylon areas

Can we do pattern (structural element) recognition?

Can we process an optimization from an industiral catalog of structural elements / materials?

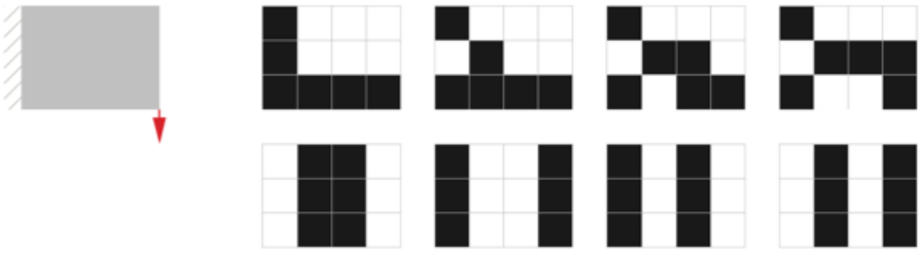
Pixels?

When the size of the FE model is increasing, the SIMP optimization problem is …

Chris Columbus et al, Pixels, movie 2015

Pixels

- Finding a solution by checking all the possible combinations IS impossible since the number of topologies nT increases exponentially with the number of finite elements n
- nT = 2^n,



The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

Division into elements (pixels or voxels) and binary decision for each
or example 10,000 elements --> 210,000 possible configurations!

Intuitive Problem? Quadratic Form

- Objective function: Strain energy

$$\min c(\mathbf{x}) = \mathbf{U}^T\mathbf{F} = \mathbf{U}^T\mathbf{K}\mathbf{U} \qquad x_e = \frac{\rho_e}{\rho_0} \text{ with} \qquad (4)$$

with $\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^{N} x_e^p$ one can write:

$$\min c(\mathbf{x}) = \sum_{e=1}^{N} (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \qquad (5)$$

Transform discrete variables continuously
(TO USE gradient-based algorithms)
Cheap derivatives!

- Contraints: mass target

$$\frac{V(\mathbf{x})}{V_0} = f = const \iff \sum_{e=1}^{N} V_e x_e - V_0 f = 0 = h(\mathbf{x})$$

$$0 < \rho_{\min} \leq \rho_e \leq 1$$

$$\frac{\partial c}{\partial \rho_e} = -p(\rho_e)^{p-1}\mathbf{u}_e^T\mathbf{k}_0\mathbf{u}_e$$

Visiting scholar at UoM (Thanks Prof MARTINS)

Fondation ISAE-SUPAERO
Reconnue d'utilité publique

AEROSPACE ENGINEERING
UNIVERSITY of MICHIGAN

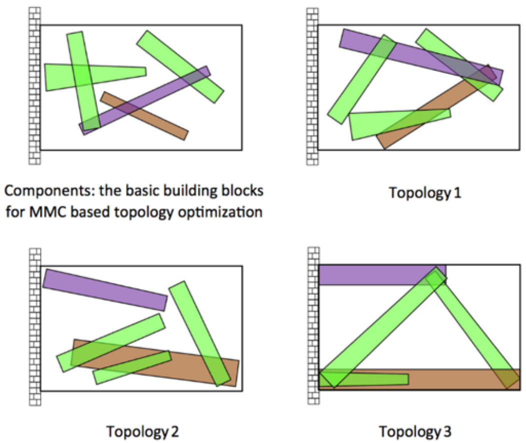
Can we do pattern (structural element) recognition?

Struct Multidisc Optim (2016) 53:1243–1260
DOI 10.1007/s00158-015-1372-3

RESEARCH PAPER

A new topology optimization approach based on Moving Morphable Components (MMC) and the ersatz material model

Weisheng Zhang[1] · Jie Yuan[1] · Jian Zhang[1] · Xu Guo[1]

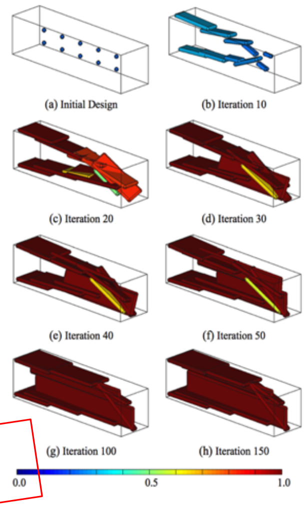


Struct Multidisc Optim
DOI 10.1007/s00158-016-1466-6

RESEARCH PAPER

A geometry projection method for the topology optimization of plate structures

Shanglong Zhang[1] · Julián A. Norato[1] · Arun L. Gain[2] · Naesung Lyu[3]



YES… using Explicit Topology Optimization

Optimization algorithm

Nodal movement

- Optimization algorithm based on decoupling
- Movement of nodes
  - Direction of the movement

- Measure performance of structure by compliance
  - Move mass nodes in the direction of decreasing compliance

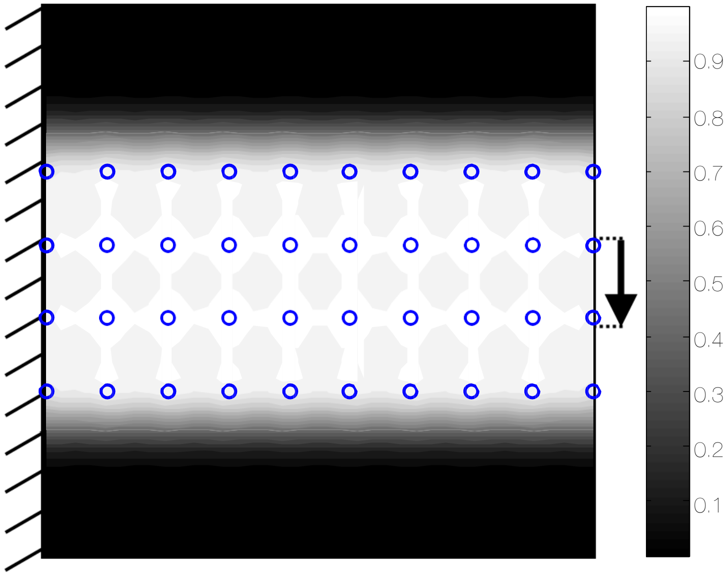
$$C = f^T u \qquad \frac{\partial C}{\partial x_j^l}$$

Original Work →

J. T. Overvelde, "The moving node approach in topology optimization", Master's thesis, TU Delft, Delft University of Technology, 2012.

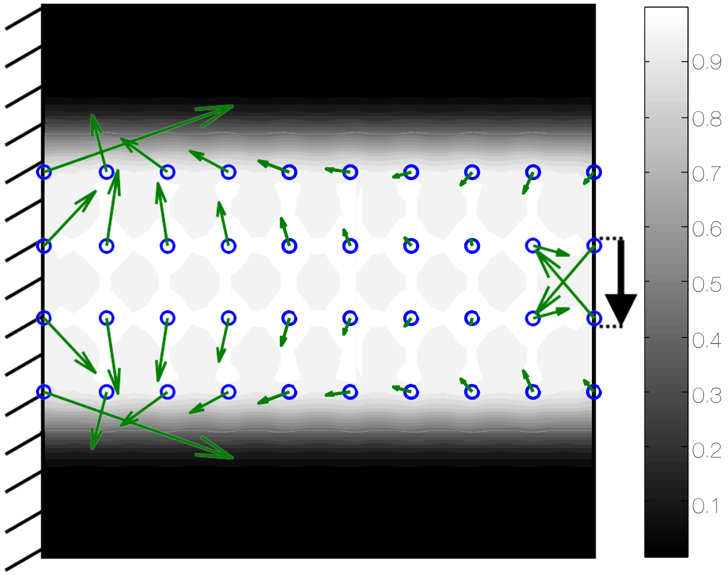
Optimization algorithm

Example of compliance sensitivity



Optimization algorithm

Example of compliance sensitivity



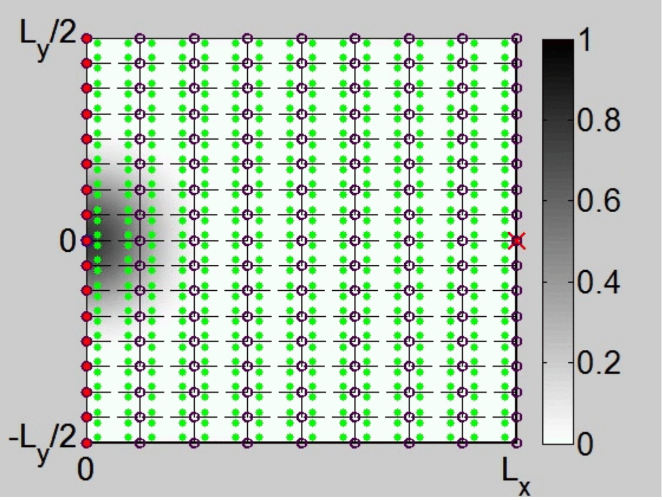
Moving Node Approach (MNA)

- Key idea:

Move material => optimal layout

- Regular discretization = precision

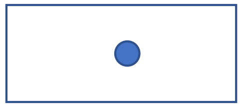
- Material distribution -> mass nodes



The variables (5 per Node)

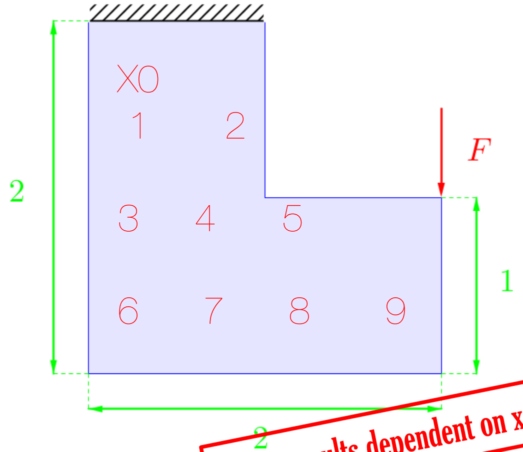
Optimization variables :

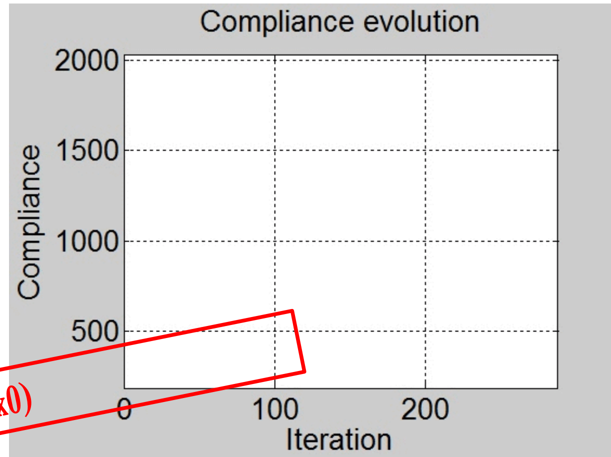
- Positions (x,y)
- Orientation (θ)
- Dimensions (Lx,Ly)



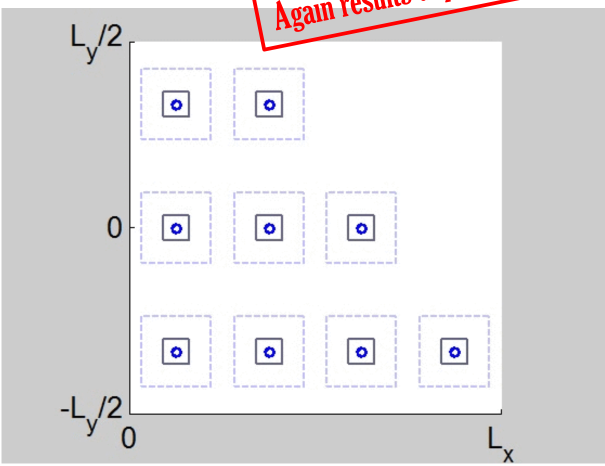
Deformable Structural Members

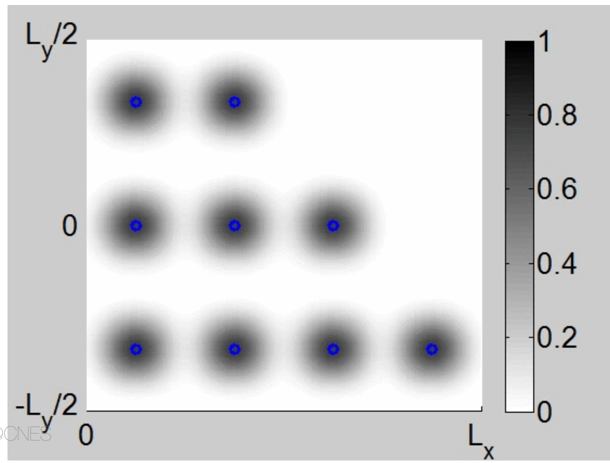
Results on L-Shape 9*5 variables





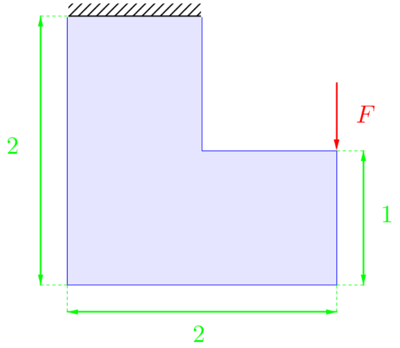
Again results dependent on x0, length(x0)



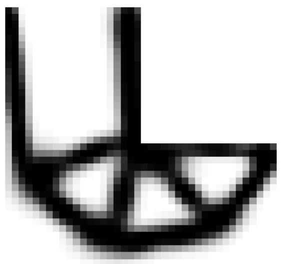


Results on L-Shape
(Best solution using a multistart approach)

TopMNA include Few hyperparameters, starting from a « dense » regular grid with Fusion/merging/Recognition of beam elements



MNA

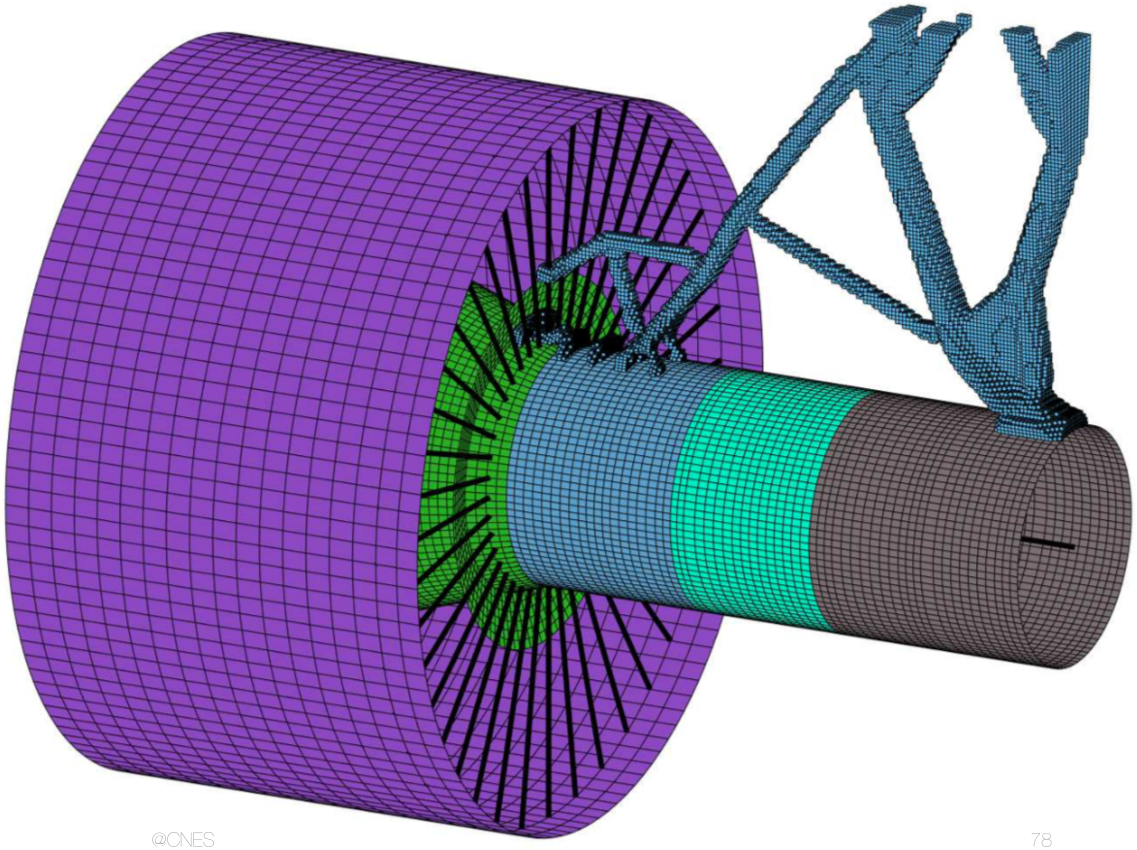


SIMP (REFERENCE)

```
disp('MNA')
topmna(x0,nelx,nely,volfrac,3,[ratio;aspect],tolchange);

disp('SIMP')
top88(nelx,nely,volfrac,3,2,1)
```

Of course we are working on bigger problem with industrial constraints…



Coniglio, S., Morlier, J., Gogu, C., & Amargier, R. (2018). Original Pylon Architecture Design Using 3D HPC Topology Optimization. In 2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference (p. 1388).

Papers & conf on this topic

Coniglio, S., Gogu, C., Amargier, R., & Morlier, J. (2017, June). Pylon and engine mounts performance driven structural topology optimization. In World Congress of Structural and Multidisciplinary Optimisation (pp. 1349-1363). Springer, Cham.

Coniglio, S., Gogu, C., & Morlier, J. (2018). Weighted Average Continuity Approach and Moment Correction: New Strategies for Non-consistent Mesh Projection in Structural Mechanics. Archives of Computational Methods in Engineering, 1-29.

Coniglio, S., Morlier, J., Gogu, C., & Amargier, R. (2018). Original Pylon Architecture Design Using 3D HPC Topology Optimization. In 2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference (p. 1388).

Barjhoux, P. J., Diouane, Y., Grihon, S., Bettebghor, D., & Morlier, J. (2017, June). Mixed variable Structural optimization: toward an efficient hybrid algorithm. In World Congress of Structural and Multidisciplinary Optimisation (pp. 1880-1896). Springer, Cham.

Barjhoux, P. J., Diouane, Y., Grihon, S., Bettebghor, D., & Morlier, J. (2018). A Bilevel Methodology for solving a Structural Optimization Problem with both Continuous and Categorical Variables. In 2018 Multidisciplinary Analysis and Optimization Conference (p. 3579).

G. Raze et al, Optimisation topologique sans maillage : vers la reconnaissance d'éléments structuraux, CSMA 2017

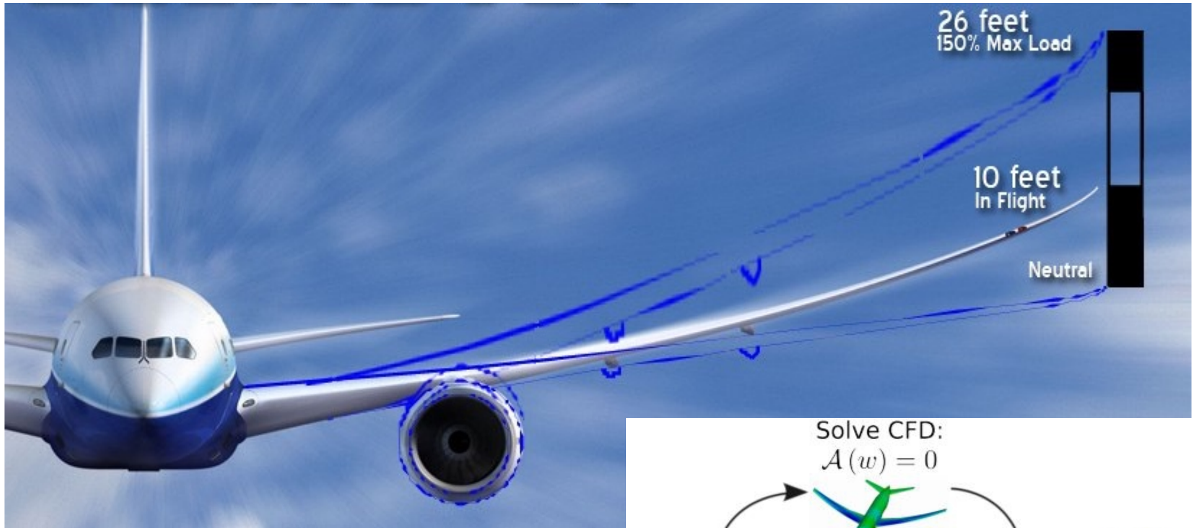
T. Hirshler et al., Analyse Isogéométrique pour les problèmes d'Optimisation de Forme des Structures Coques, , CSMA 2017

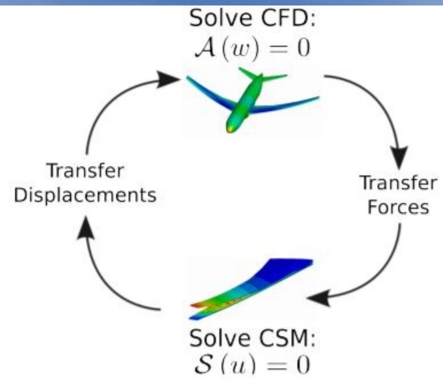
Several Papers in preparation

Outlines for today

1. GP aka Kriging
2. Kriging for Global Optimization
3. New developments in topology optimization
4. Add control law in the design loop

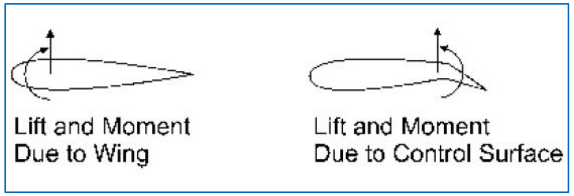
The importance of aerostructural coupling





$\mathcal{A}$: Aerodynamic residuals
w: Aerodynamic states
$\mathcal{S}$: Structural residuals
u: Structural states

+ control law*:

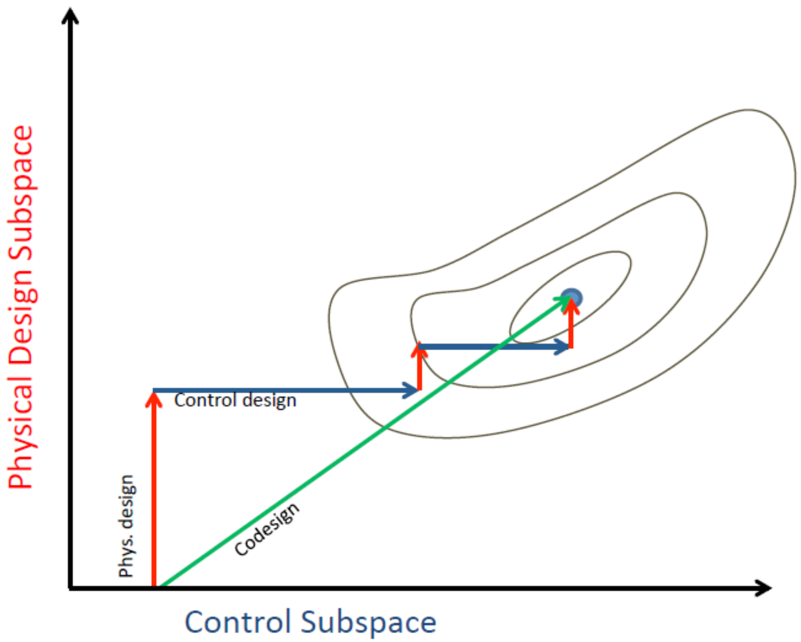


*J. R. Wright, J. E. Cooper. Introduction to Aircraft Aeroelasticity and Loads. 2007.

Co-Design:
Integrated Physical and Control System Design *

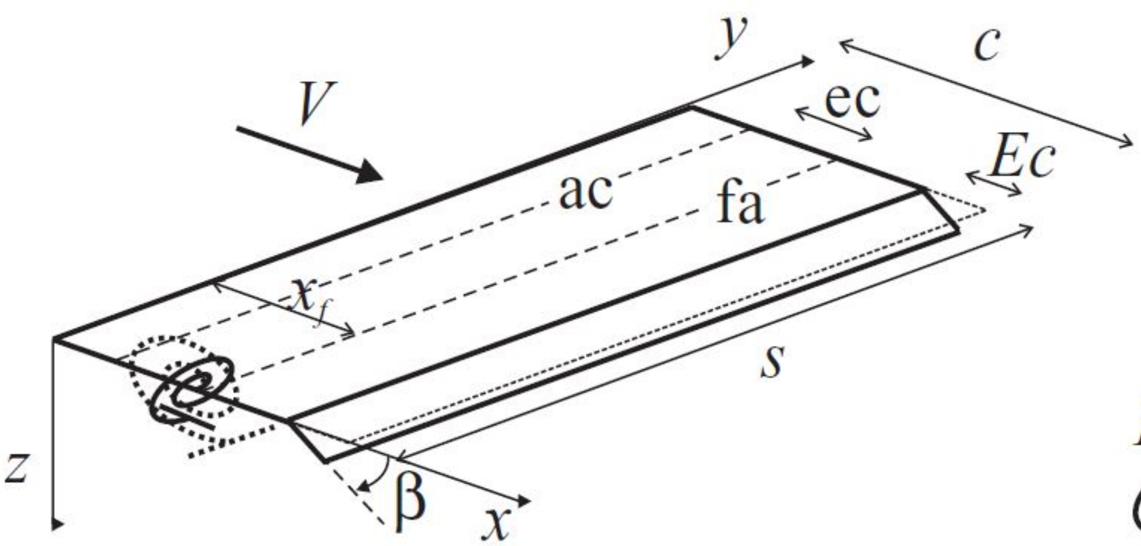
Navigate in physical and control design subspaces simultaneously.

➞ Tailor structural/mechanical/control system designs: system optimality

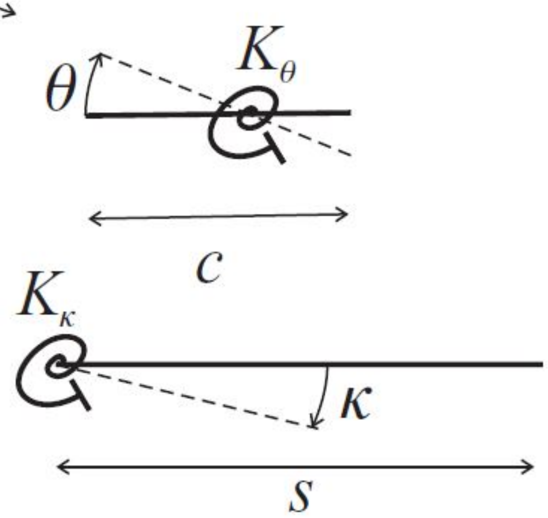


*J. T. Allison, A. Deshmukh. Integrated Physical and Control System Design for Horizontal Axis Wind Turbines. University of Illinois at Urbana-Champaign.

Example: Wing model*



Degrees of freedom:
pitch θ and flap κ



*J. R. Wright, J. E. Cooper. Introduction to Aircraft Aeroelasticity and Loads. 2007.

Mathematical modelling

$$\boxed{\begin{bmatrix} I_k & I_{k\vartheta} \\ I_{k\vartheta} & I_\vartheta \end{bmatrix}}\begin{Bmatrix} \ddot{k} \\ \ddot{\vartheta} \end{Bmatrix} + \rho V \boxed{\begin{bmatrix} \frac{cs^3 a_w}{6} & 0 \\ -\frac{c^2 s^2 e a_w}{4} & -\frac{c^3 s}{8} M_\vartheta \end{bmatrix}}\begin{Bmatrix} \dot{k} \\ \dot{\vartheta} \end{Bmatrix} + \left(\rho V^2 \boxed{\begin{bmatrix} 0 & \frac{cs^2 a_w}{4} \\ 0 & -\frac{c^2 s e a_w}{2} \end{bmatrix}} + \boxed{\begin{bmatrix} K_k & 0 \\ 0 & K_\vartheta \end{bmatrix}}\right)\begin{Bmatrix} K \\ \vartheta \end{Bmatrix} = \rho V^2 cs \begin{Bmatrix} -\frac{s a_c}{4} \\ \frac{c b_c}{2} \end{Bmatrix}\boxed{\beta} + \rho V cs \begin{Bmatrix} \frac{s}{4} \\ \frac{c}{2} \end{Bmatrix}\boxed{w_g}$$

structural inertia

aerodynamic damping

aerodynamic stiffness

structural stiffness

control surface angle

gust term

$$A\ddot{q} + \rho V B \dot{q} + (\rho V^2 C + E) q = g\beta + h w_g$$

State space modelling

$$\begin{bmatrix}\dot{\mathbf{q}}\\ \ddot{\mathbf{q}}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{A}^{-1}(\varrho \mathbf{V}^2\mathbf{C}+\mathbf{E}) & -\mathbf{A}^{-1}(\varrho \mathbf{V}\mathbf{B})\end{bmatrix} + \begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \mathbf{A}^{-1}\mathbf{g}\end{bmatrix}\boldsymbol{\beta} + \begin{bmatrix}\mathbf{0}\\ \mathbf{A}^{-1}\mathbf{h}\end{bmatrix}\mathbf{w_g}$$

$$\dot{\mathbf{x}} = \mathbf{A_s x} + \mathbf{B_s u} + \mathbf{E_s w_g}$$

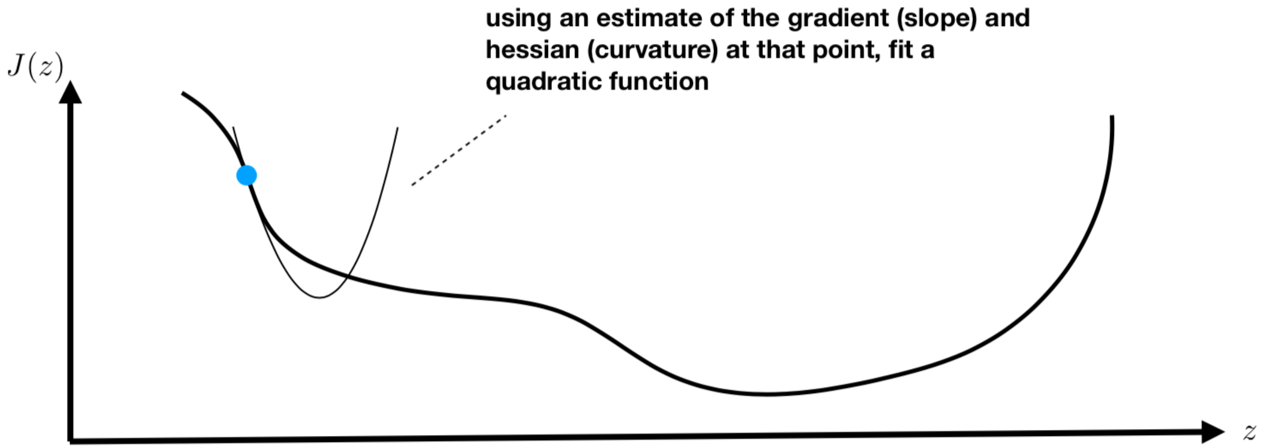
State variables: $x = \begin{bmatrix}q\\ \dot{q}\end{bmatrix} = \begin{bmatrix}k\\ \vartheta\\ \dot{k}\\ \dot{\vartheta}\end{bmatrix}$ Control variable: u=β

Transcription Methods in Optimal Control

- Transcribe the Optimal Control Problem into a Non-linear Program, and then use a general NLP solver
- Direct Transcription" because the full state trajectory is optimised directly in the NLP as part of the decision variable

SNOPT is a popular commercial solver that uses Sequential Quadratic Programming (SQP):

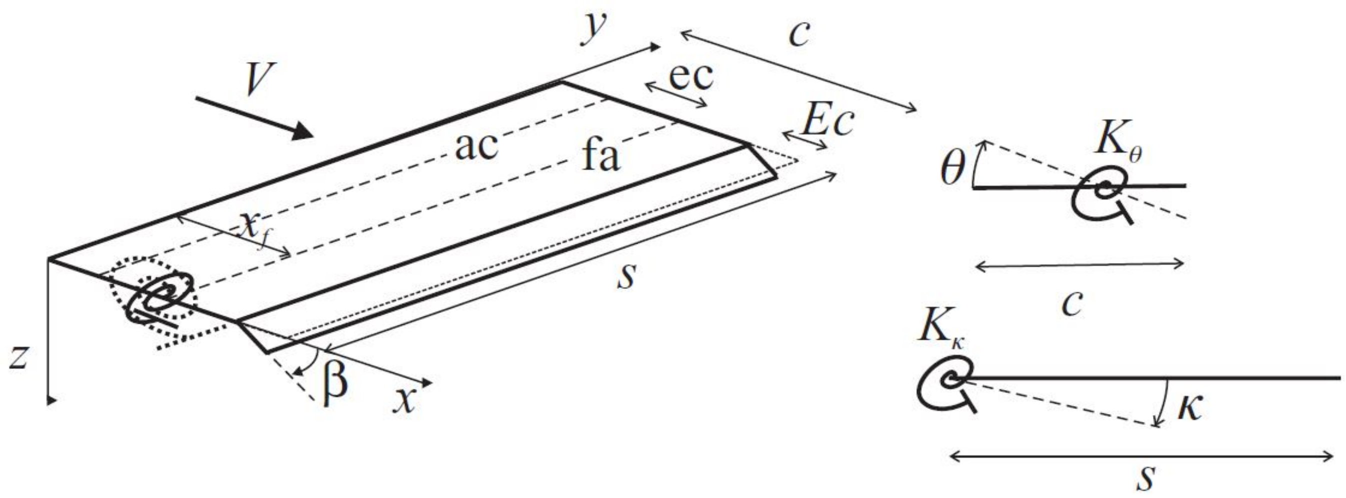
***Very* rough idea of SQP:**



Design variables

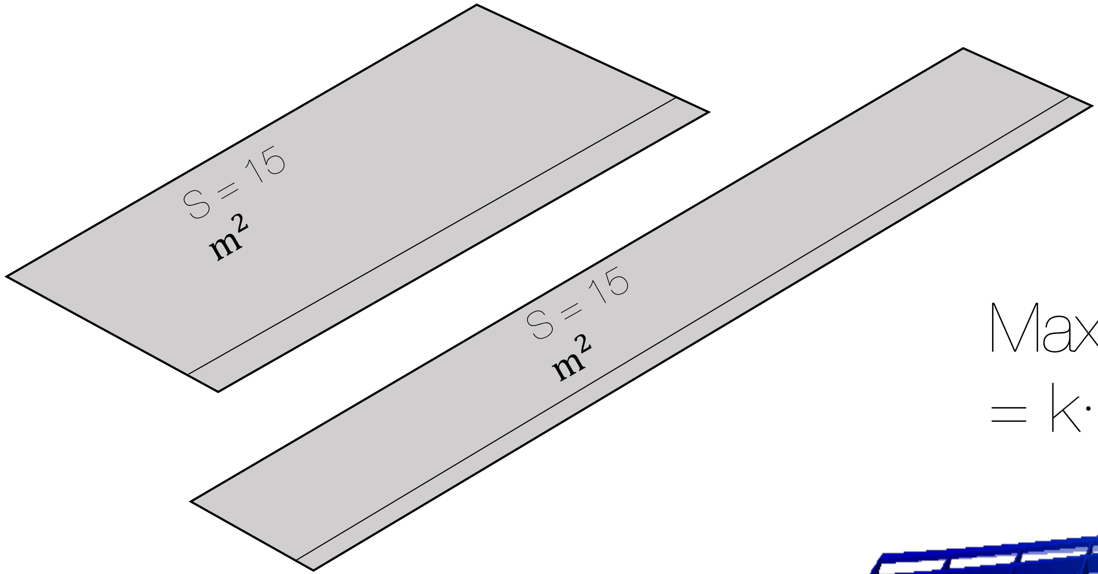
s: semi span [4-10] (m)
c: chord [1-3] (m)
mass/area of wing [70-130] (Kg/$\mathbf{m^2}$)
$\mathbf{f_k}$: flapping frequency [3-7] (Hz)

$\mathbf{f_\vartheta}$: pitch frequency [8-12] (Hz)
perc_xcm: position of center of mass from nose [0,1c-0,9c] m: unit
perc_xf: position of flexural axis from nose [0,1c-0,9c]



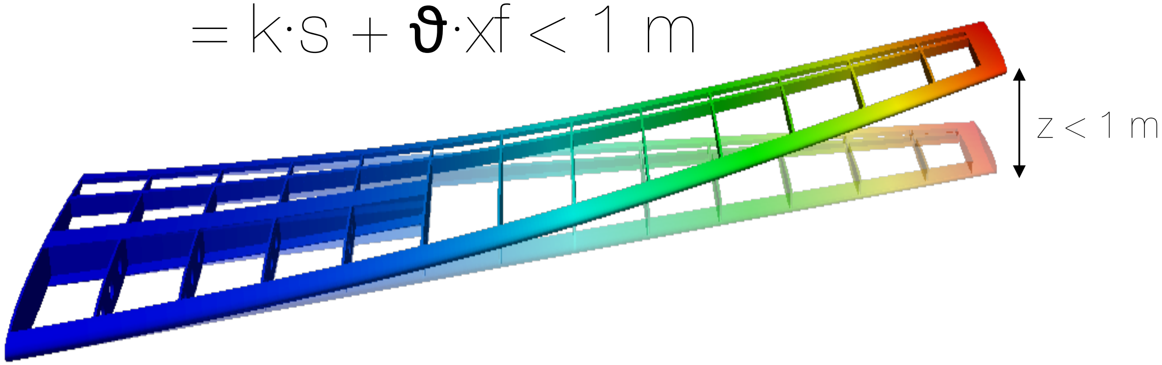
Constraints

Wing area: $S = c \cdot s = \text{constant} = 15\ \mathbf{m^2}$



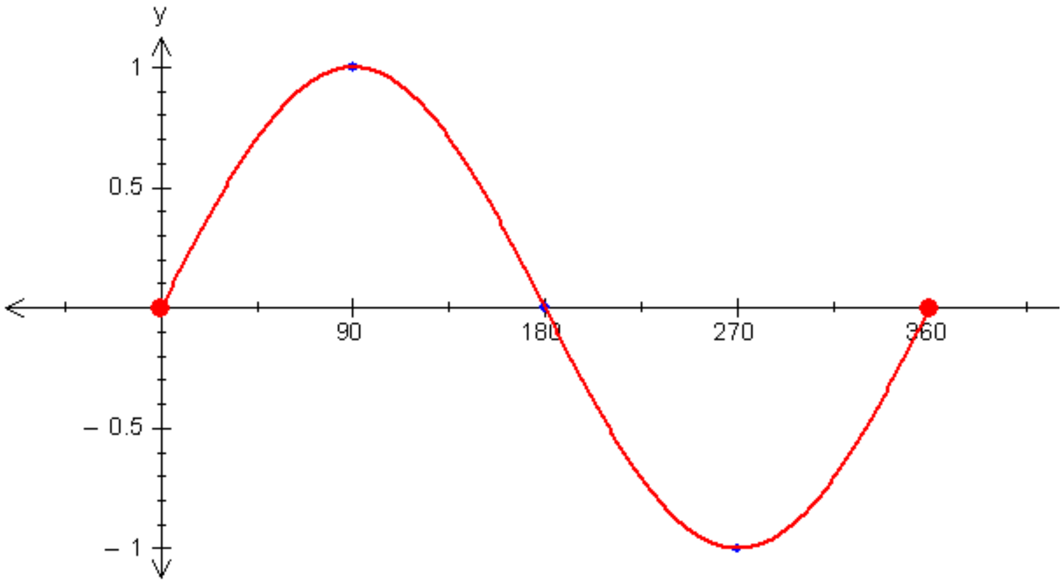
Eccentricity between flexural axis and aerodynamic center: $0{,}1 < e < 1$

Maximum vertical displacement: $z = k \cdot s + \boldsymbol{\vartheta} \cdot xf < 1\ \text{m}$

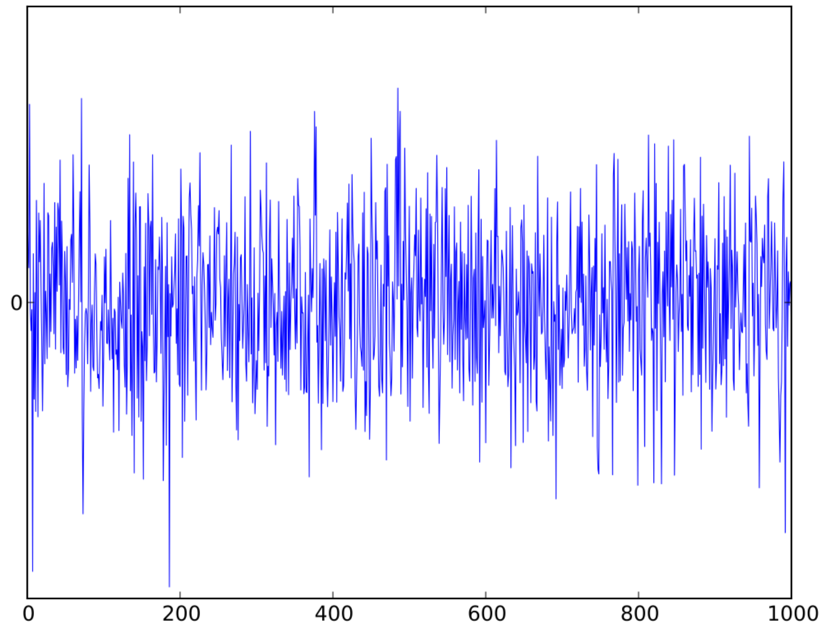


Input excitation

gust '1-cosine'



random turbulence

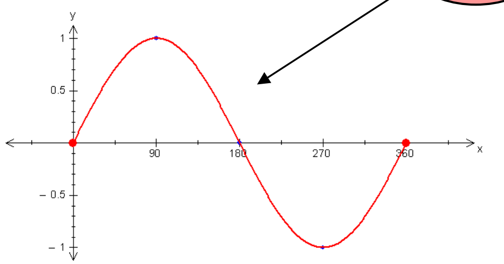


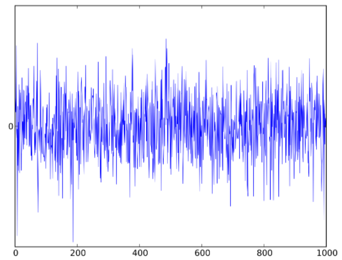
Objective function

handling + comfort + control cost

$$J = \int_{0}^{t_F} \left(r_1 z^2 + r_2 \ddot{z}^2 + r_3 u^2 \right) dt$$

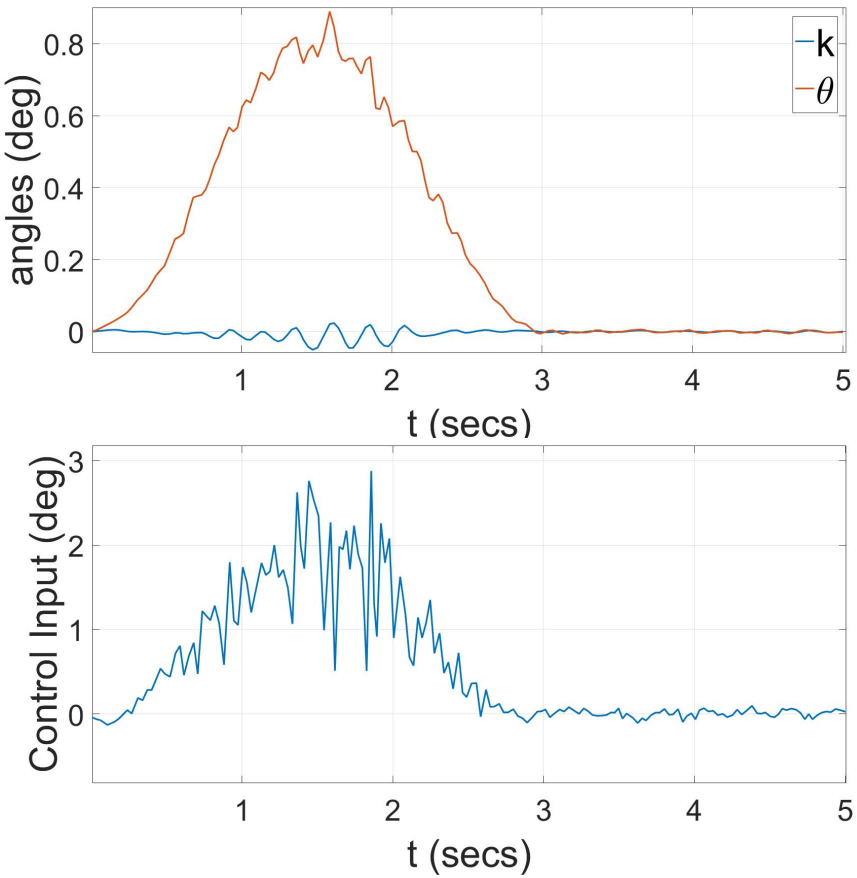
$$J = J_{gust} + J_{turb}$$



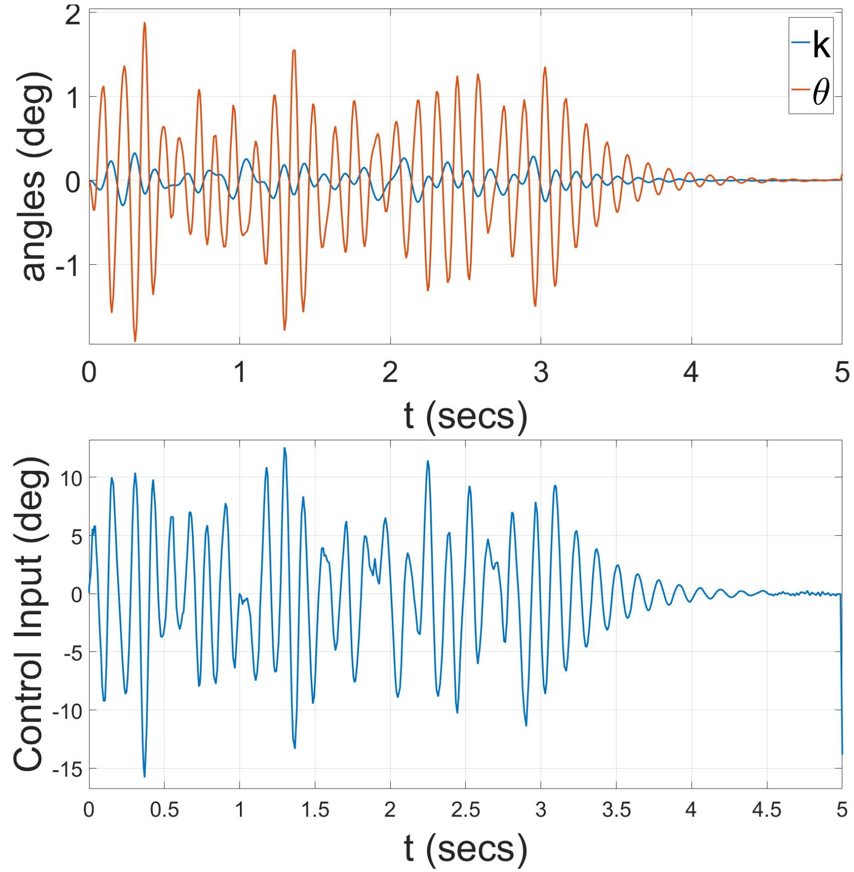


System response - sequential optimization

"1-cosine" gust input

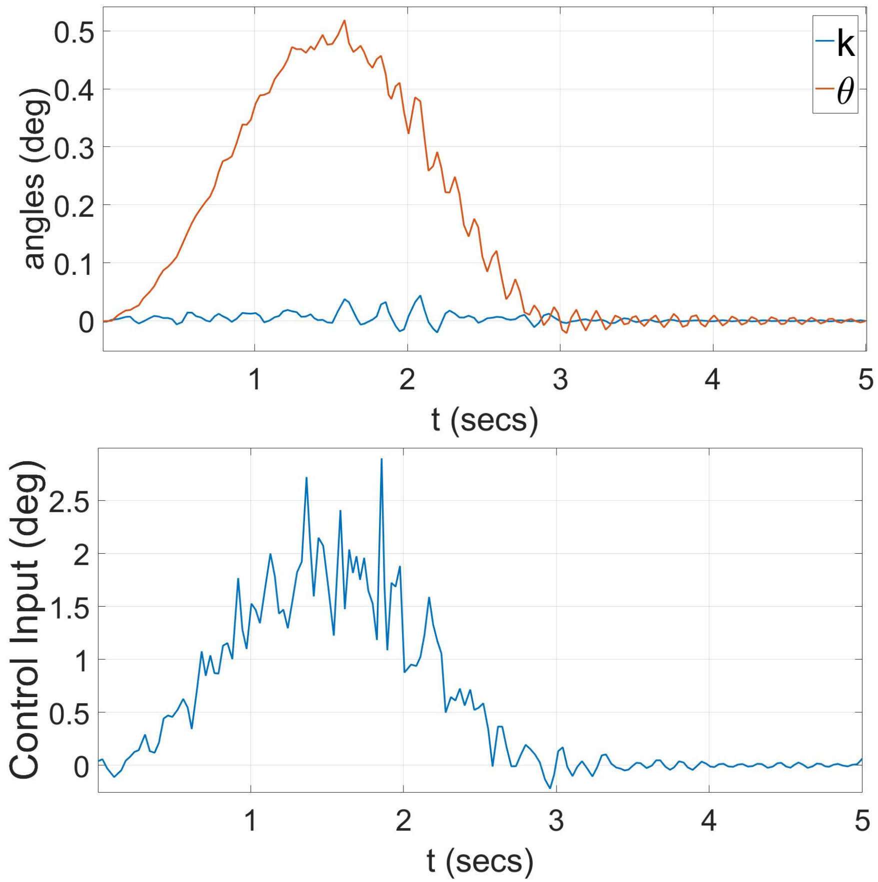


Random turbulence input

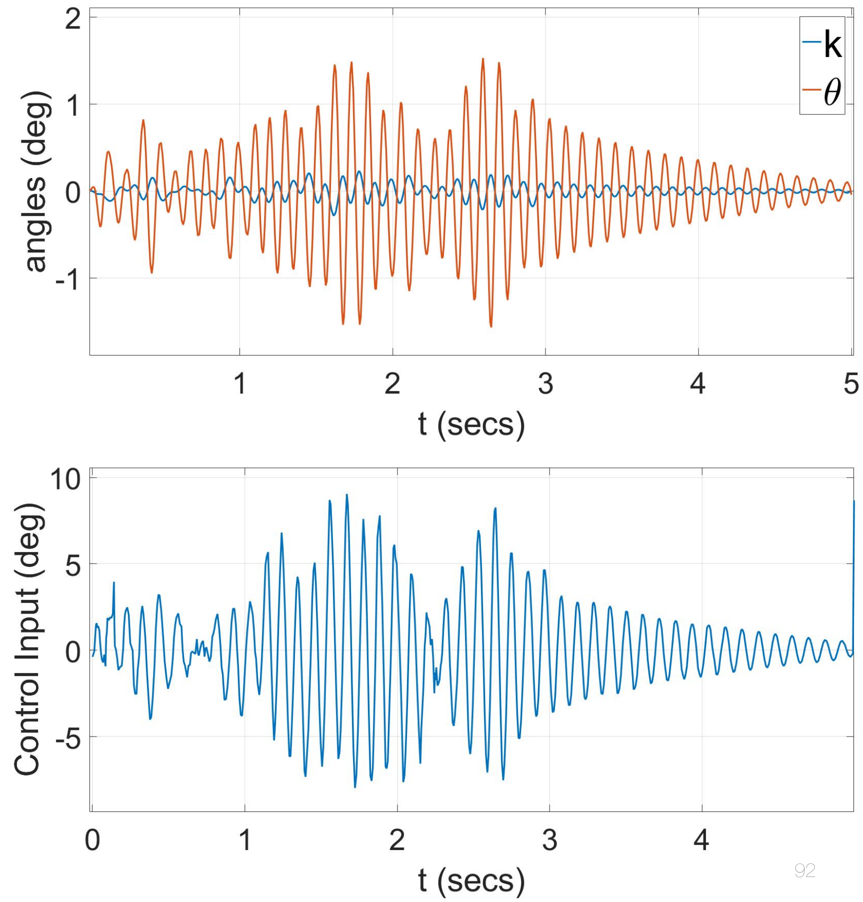


System response – simultaneous optimization

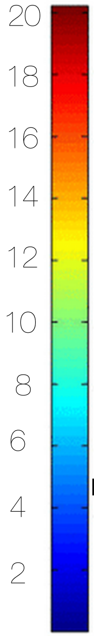
"1-cosine" gust input



Random turbulence input



Objective function values

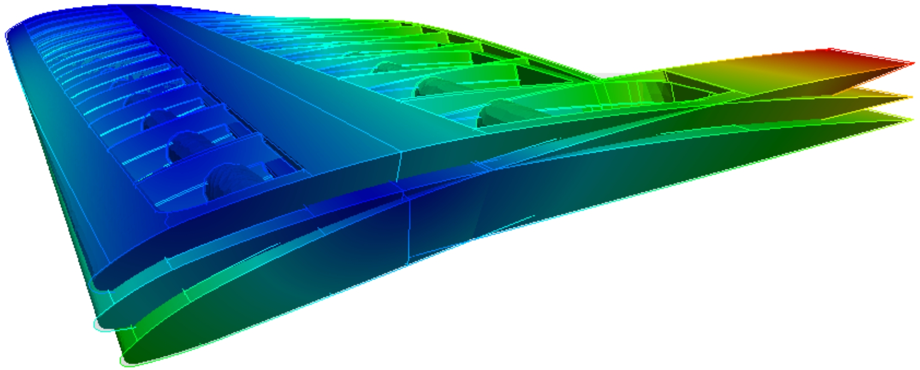


Sequential optimization: J = 16.48

Simultaneous optimization: J = 4.67

Gain: 71,52 %

Design optimal point

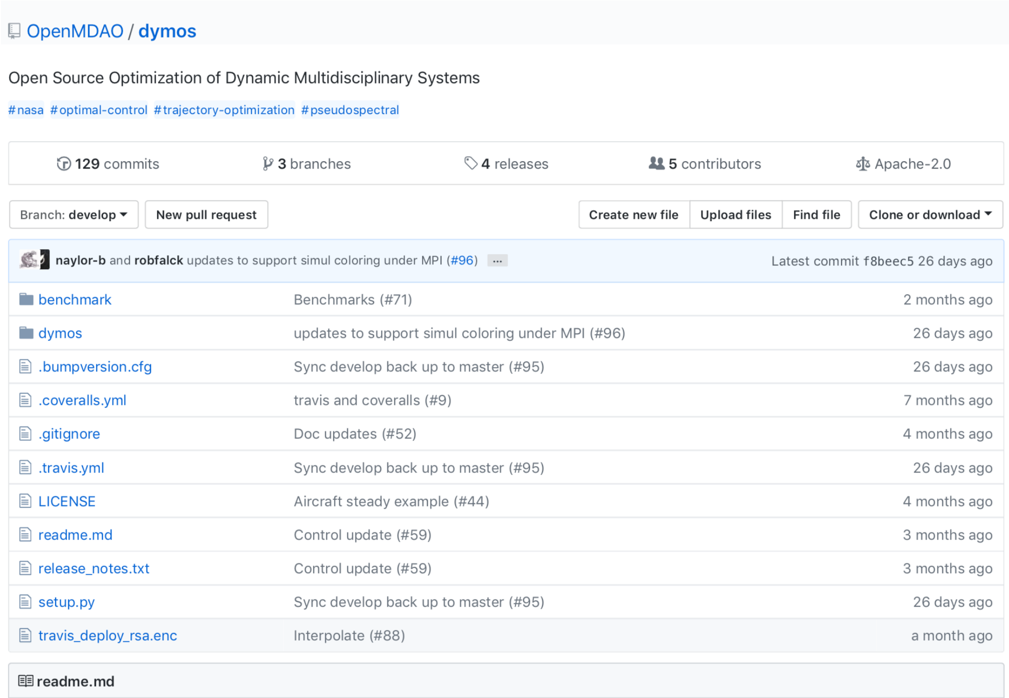


semi span	s = 6.85 m
chord	c = 2.18 m
unit mass / area of wing	m = 122 kg/$\mathbf{m^2}$
flapping freq	$\mathbf{f_k}$ = 3.277 Hz
pitch freq	$\mathbf{f_\vartheta}$ = 9.318 Hz
position of centre of mass	perc_xcm: 0.5
Position of flexural axis	perc_xf: 0.35

Conclusions

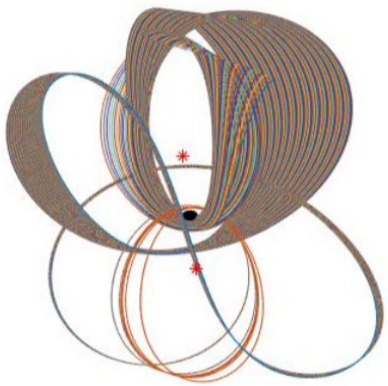
- New Surrogate and ML technics for an automated optimal design process
- SEGOMOE offers a standard solution for global optimization for a constrained problem (up to 100 design variables and several hundred of constraints)
- Opensource solution due to collaboration Nasa, University of Michigan and ONERA
- Researches in Structural optimization explore mixed optimization (continuous, discrete/categorial variables) and Stress constraint aggregation (up to Millions of FE)
- topMNA, A new step toward a explicit topology optimization (shortcut the CAD/CAE link)

Codesign is the current trend
→OpenMDAO/dymos
https://github.com/OpenMDAO/dymos



New collaboration, new PhD (laurent Beauregard)

Codesign is a current trend in research at SUPAERO for optimal design of reusable launcher (design+ Optimal control of Trajectory)



SAC LAB

AIRBUS

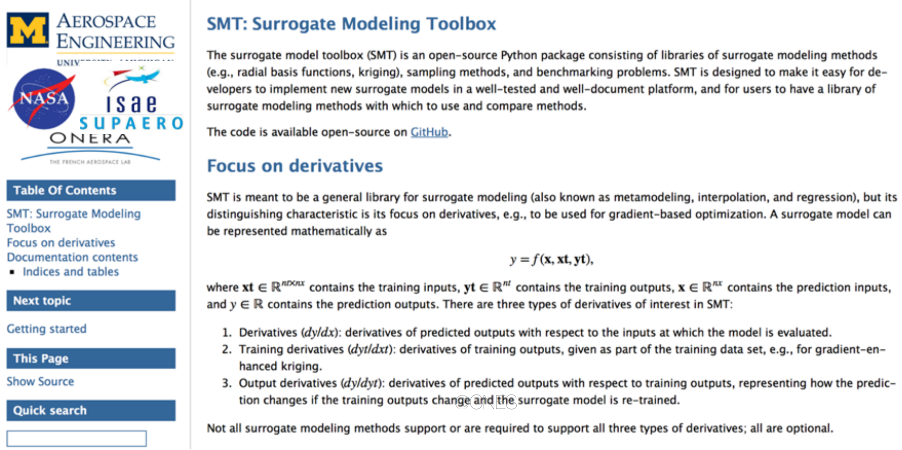
arianeGROUP

NB: Since 2013 new course at SUPAERO : MDO [Structural&Multidisciplinary Design Optimization, 2*30H] (MsC level] with ONERA/AIRBUS

Please Visit :
https://github.com/SMTorg/SMT
https://github.com/mid2SUPAERO for student's project

- Thanks to My co-workers: Joaquim Martins, Nathalie Bartoli, Thierry Lefevbre, Emmanuel Benard, Claudia Bruni, Emmanuel Rachelson, Nicolas Gourdain, John Hwang, Mohamed Bouhlel, Peter Schmolgruber, Youssef Diouane, Sylvain Dubreuil, Christian Gogu, Stephanie Lisy Destrez and PhDs Pierre-Jean Barjhoux, Simone Coniglio, Elisa Bosco, Joan Mas Colomer, Ankit Chiplunkar, Alessandro Sgueglia, Laurent Beauregard and MsC Mostafa Meliani, Mahfoud Herraz Gabriele Capasso, Ghislain Haine, Giovane Filippi…
- At Airbus: S. Grihon, A Gazaix, M. Colombo, R. Amargier S. Trapier, A. Lucchetti, F. Vetrano ….

Surrogate modeling in HD, focus on derivatives



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Focus on derivatives
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Getting started

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SMT: Surrogate Modeling Toolbox

The surrogate model toolbox (SMT) is an open-source Python package consisting of libraries of surrogate modeling methods (e.g., radial basis functions, kriging), sampling methods, and benchmarking problems. SMT is designed to make it easy for developers to implement new surrogate models in a well-tested and well-document platform, and for users to have a library of surrogate modeling methods with which to use and compare methods.

The code is available open-source on GitHub.

Focus on derivatives

SMT is meant to be a general library for surrogate modeling (also known as metamodeling, interpolation, and regression), but its distinguishing characteristic is its focus on derivatives, e.g., to be used for gradient-based optimization. A surrogate model can be represented mathematically as

$$y = f(\mathbf{x}, \mathbf{xt}, \mathbf{yt}),$$

where $\mathbf{xt} \in \mathbb{R}^{nt \times nx}$ contains the training inputs, $\mathbf{yt} \in \mathbb{R}^{nt}$ contains the training outputs, $\mathbf{x} \in \mathbb{R}^{nx}$ contains the prediction inputs, and $y \in \mathbb{R}$ contains the prediction outputs. There are three types of derivatives of interest in SMT:

1. Derivatives (dy/dx): derivatives of predicted outputs with respect to the inputs at which the model is evaluated.
2. Training derivatives (dyt/dxt): derivatives of training outputs, given as part of the training data set, e.g., for gradient-enhanced kriging.
3. Output derivatives (dy/dyt): derivatives of predicted outputs with respect to training outputs, representing how the prediction changes if the training outputs change and the surrogate model is re-trained.

Not all surrogate modeling methods support or are required to support all three types of derivatives; all are optional.