Recent advances in structural and multidisciplinary optimization @SUPAERO

Prof. Joseph Morlier

My Research Group (Joint research with ONERA on MDO)

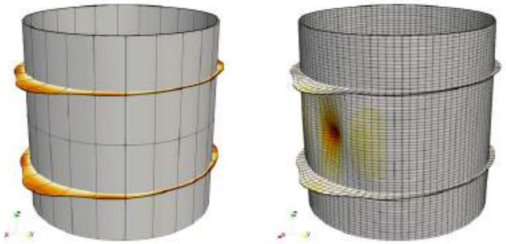
http://www.institut-clement-ader.org/pageperso.php?id=jmorlier

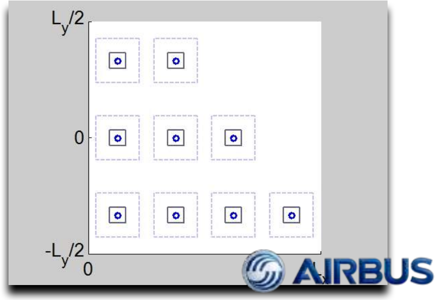
- 4 PhDs, 1 postdoc, 4 MsC

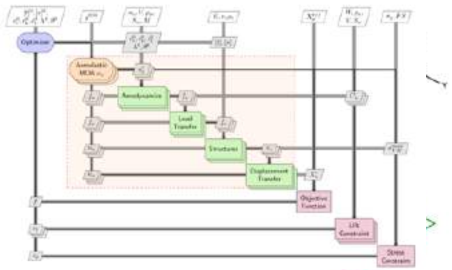
$$\min_{\substack{a \in \mathbb{R}^{10} \\ c \in \Gamma^{10}}} w(a, c)$$

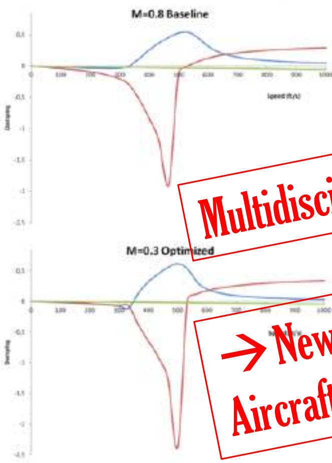
$$s.t.\ \ s(a,c) \leq 0$$
$$d(a,c) \leq 0$$
$$\underline{a} \leq a \leq \overline{a}$$

Structural Optimization



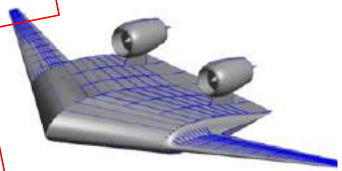


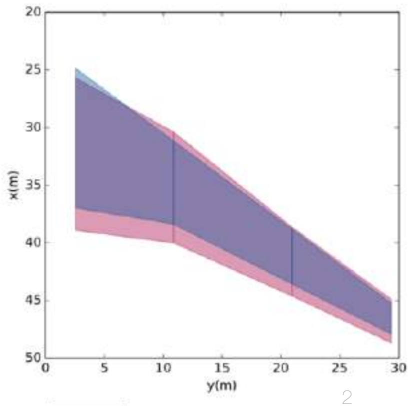




Multidisciplinary Optimization

→ New Aerostructures/ Aircraft Concept





Optimized planform (red) and baseline (blue).

CHAIR FOR ECO DESIGN OF AIRCRAFT

Our Goals: new optimization process in the design loop of Aerostructures (flexible)

- Reduce in a « smart way » the computation time of optimization for coupled simulations
- Global Optimization using surrogate modeling → fixed budget (enriching process) to deal with INDUSTRIAL problems
- Specialized surrogates for HD (engineering) problems and UQ
- Taking into account different levels of fidelity

Bartoli, N., Lefebvre, T., Dubreuil, S., Olivanti, R., Bons, N., Martins, J. & Morlier, J. (2017). An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization. In 18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4433).

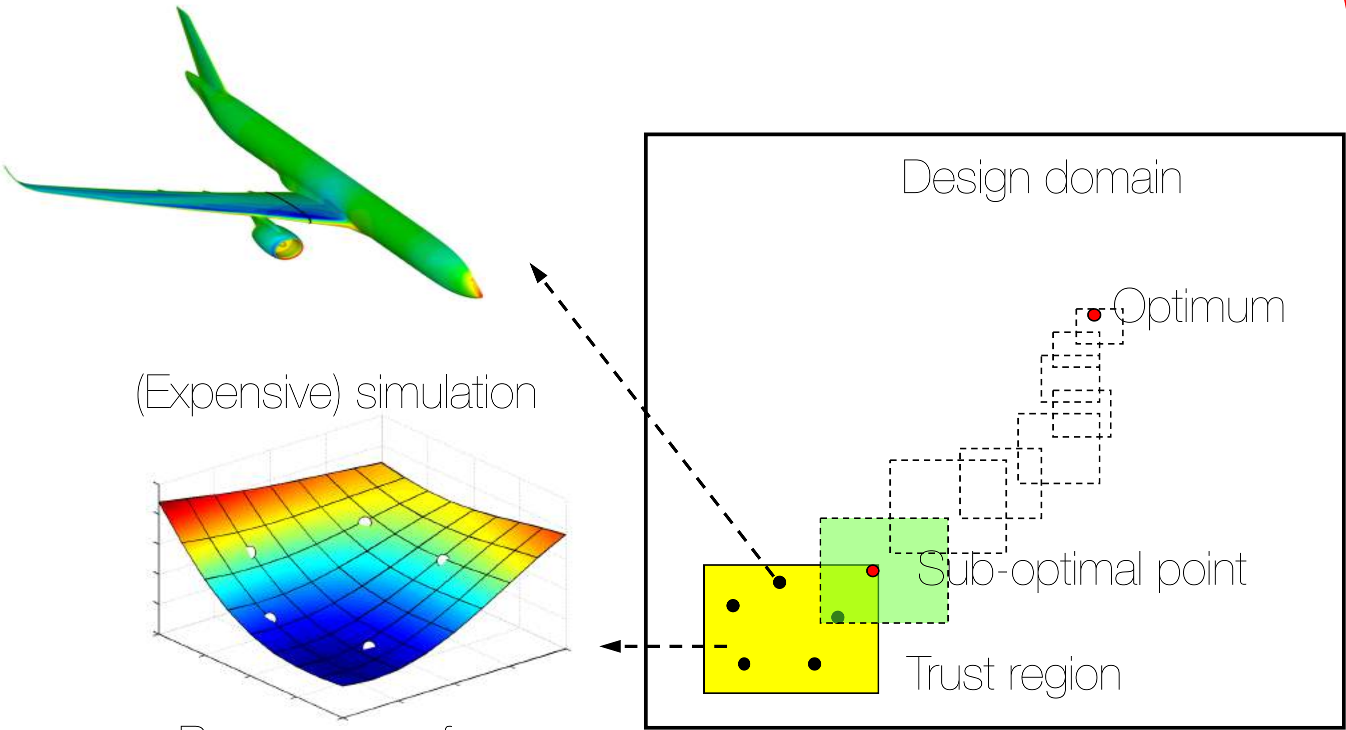
→ Methods applied to AD Aircraft Design: Put the aircraft structure / aeroelasticity in the loop at the early stage of MDO process

→ compatible with openMDAO

http://openmdao.org

SURROGATE MODELING (learning for Optimizing)

Jacobs, J. H., Etman, L. F. P., Van Keulen, F., & Rooda, J. E. (2004). Framework for sequential approximate optimization. Structural and Multidisciplinary Optimization, 27(5), 384-400.

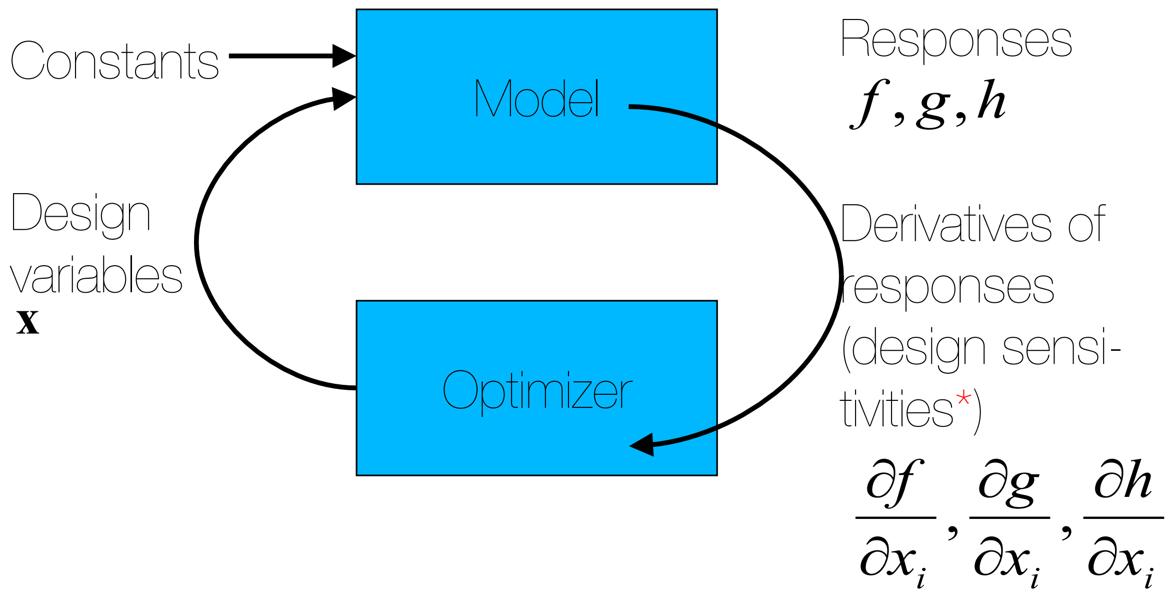


Response surfaces
Or metamodels, surrogate models etc…

But Why?

Gradient Based Optimization

Costly if FD/difficult to implement Adjoint in industrial code
And also sensitive to discontinuity/sensible to X_0



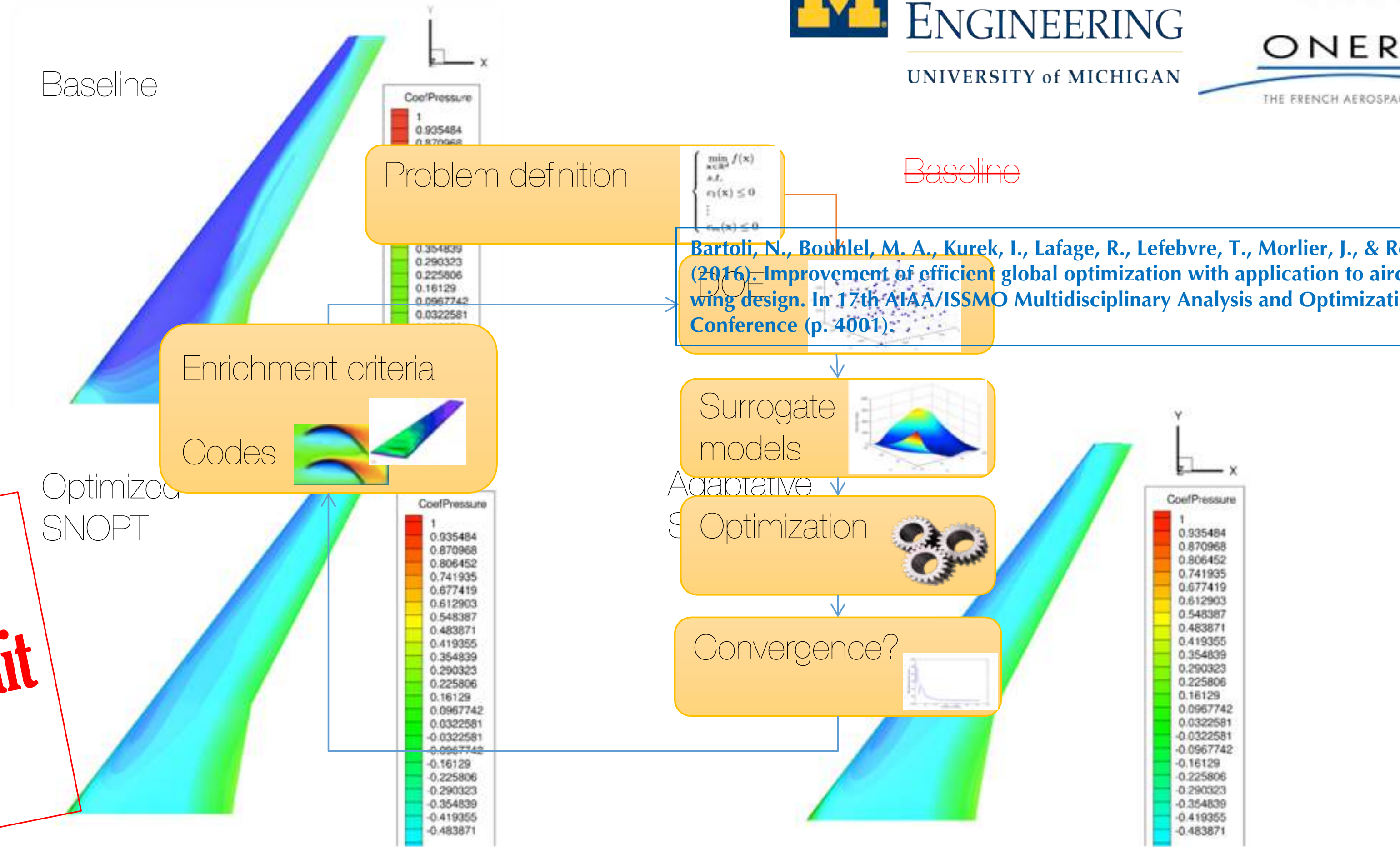
*SOL200 in MSC Nastran for example

Is it possible to resume in one slide ?

AEROSPACE ENGINEERING
UNIVERSITY of MICHIGAN

isae SUPAERO

ONERA
THE FRENCH AEROSPACE LAB



Bartoli, N., Bouhlel, M. A., Kurek, I., Lafage, R., Lefebvre, T., Morlier, J., & Regis, R. (2016). Improvement of efficient global optimization with application to aircraft wing design. In 17th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4001).

No…You have to wait ;)

Outlines for today

1. GP aka Kriging
2. Kriging for Global Optimization
3. New developments in topology optimization
4. Codesign is MDO?

1.GP aka Kriging

2. Kriging for Global Optimization
3. New developments in topology optimization
4. Codesign is MDO?

Machine learning for load estimation (A. Chiplunkar, PhD)

Kriging (Pionneer)	Gaussian Processes (link with AI)
Developed by Daniel Krige – 1951; formalized by Georges Mathéron in the 60's (Mines Paris)	Neural network with infinite neurons tend to Gaussian Process 1994
Evaluation: minimize error variance	Evaluation: Marginal Likelihood

Krige, D. G., 1951, A statistical approach to some basic mine valuation problems on the Witwatersrand: J. Chem. Metal. Min. Soc. South Africa, v. 52, p. 119–139.

Matheron, G., 1963b, Principles of geostatistics: Economic Geol., v. 58, p. 1246–1266.

Neal, R. Priors for infinite networks. Tech. rep., University of Toronto, 1994.

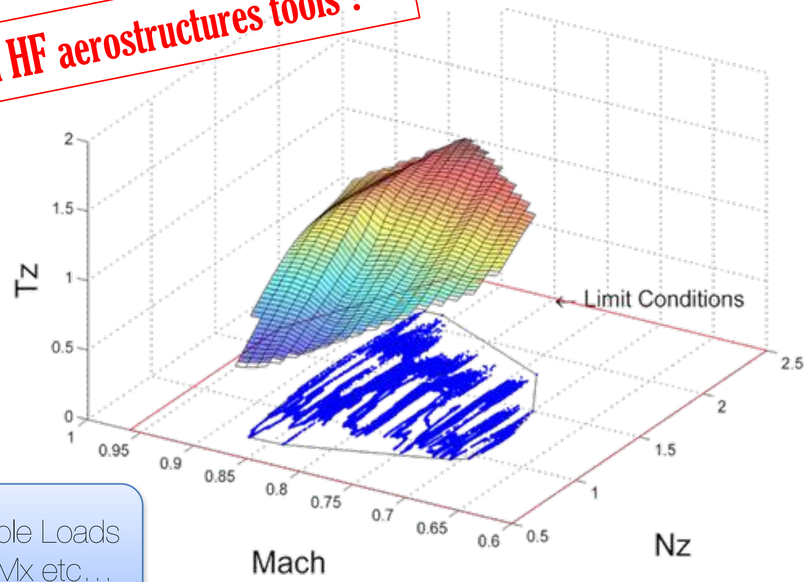
Williams, C. K. I., and Rasmussen, C. E. Gaussian processes for regression. *Advances in Neural Information Processing Systems 8* (1996), 514–520.

http://extrapolated-art.com

Loads estimation

First flight ready → Flight test of Maneuvers → Capture aero/ structural data → Extract loads → Certification of limit loads

Can we extrapolate limit loads using both measurements and HF aerostructures tools ?



https://github.com/ankitchiplunkar/thesis_isae

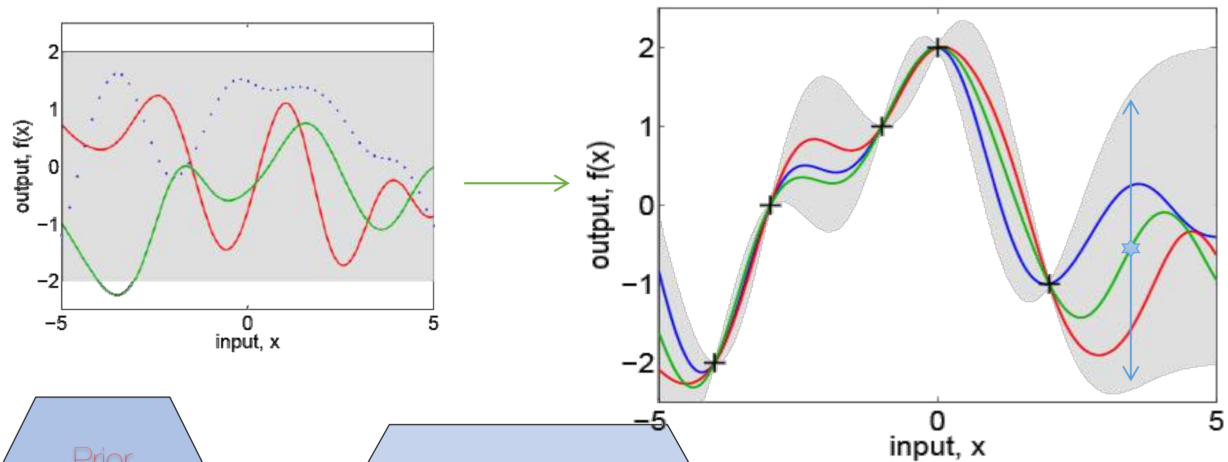
Can we automate the process?

Statistical step

A/C states α, β, M, δ's → Distributed Gaussian Process → Flexible Loads Tz, Mx etc…

Gaussian Process Regression

Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/



Prior
f|x

$GP(0, K)$

X

Y

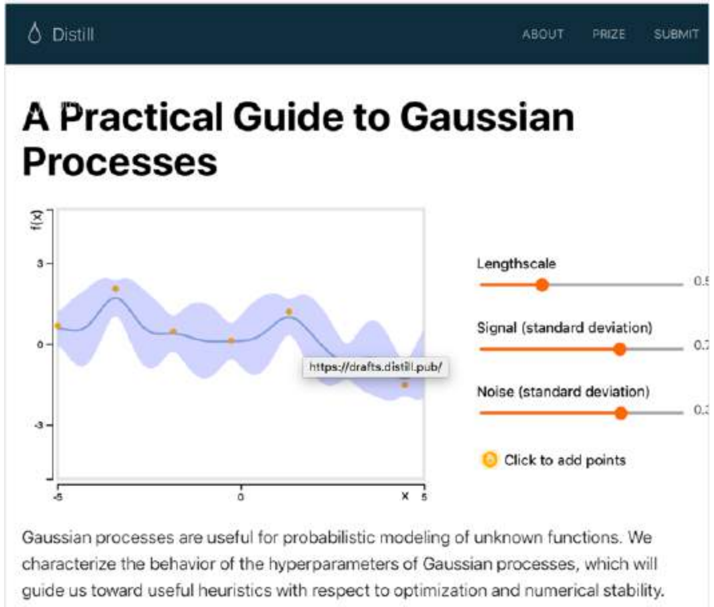
Training
y|x
$GP(0, K + \sigma_n^2 I\ )$
Optimize: for a better hyperparameters and better prior

Posterior on y
$\boldsymbol{y}^* | \boldsymbol{x}^*, \boldsymbol{y}, \boldsymbol{x}$

GP ($K_*[K + {\sigma_n}^2 I]^{-1}$y, $K_{**} + \sigma_n^2 I \ - \ K_*^T [K + {\sigma_n}^2 I]^{-1} K_*$)

A good starting point x_0=Rasmussen's book

- https://drafts.distill.pub/gp/

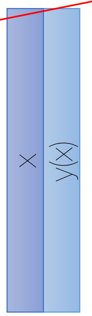


C. E. Rasmussen & C. K. I. Williams, Gaussian Processes for Machine Learning, the MIT Press, 2006, ISBN 026218253X. © 2006 Massachusetts Institute of Technology. www.GaussianProcess.org/gpml

Gaussian Processes for Machine Learning

Matrix view of Gaussian Process

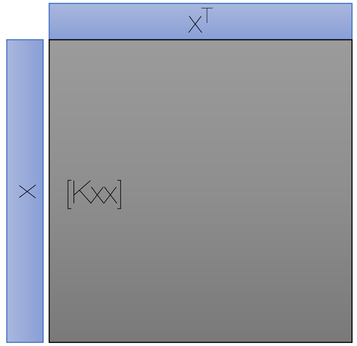
1/ Get your inputs/outputs data

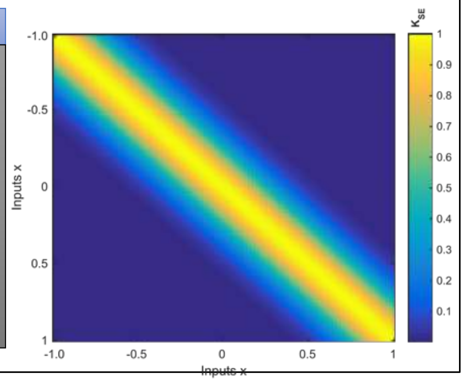


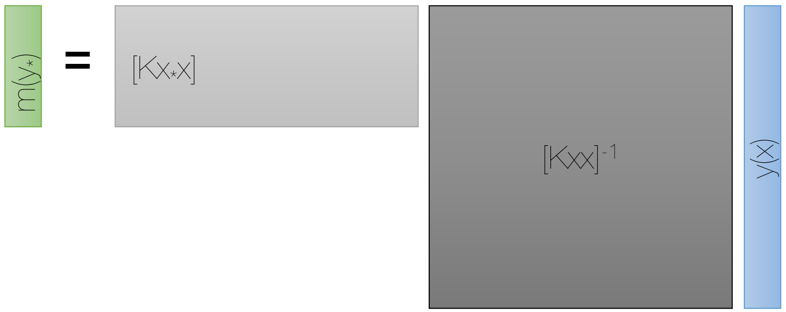
2/ You wan to predict at x*

3/ Choose a Kernel/Construct Kxx and Hyperparameters tuning

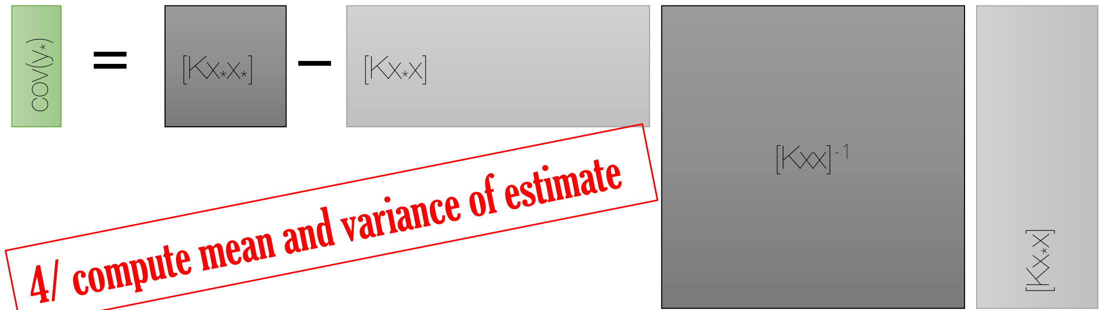
$$k(x,x') = \theta_1{}^2 \exp\left(-\frac{(x-x')^2}{2\theta_2{}^2}\right) =$$







$$m(x_*) = K_*[Kxx]^{-1}y$$



4/ compute mean and variance of estimate

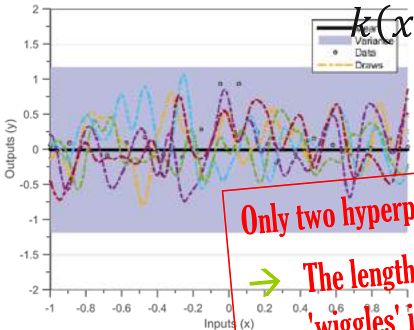
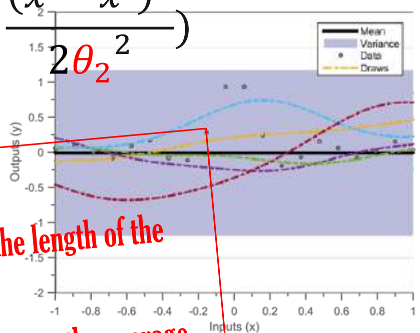
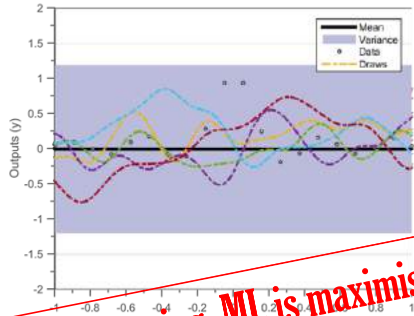
$$var(x_*, x_*') = K_{**} - K_*^T[Kxx]^{-1}K_*$$

Optimizing Marginal Likelihood (ML)

$$\text{ML} = log\left(p(y|X,\theta)\right) = -\frac{1}{2}y^T K^{-1} y - \frac{1}{2}log|K| - \frac{n}{2}log(2\pi)$$

- It is a combination of **data-fit term**, a **complexity penalty** term and a **normalization term**

$$k(x,x') = \theta_1{}^2 \exp\left(-\frac{(x-x')^2}{2\theta_2{}^2}\right)$$

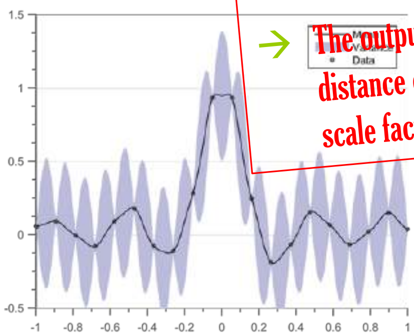
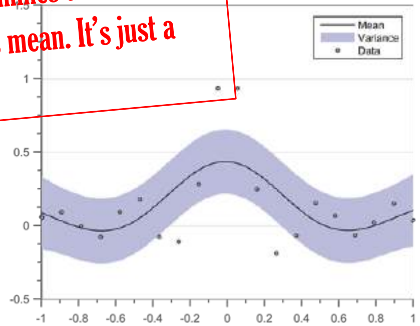
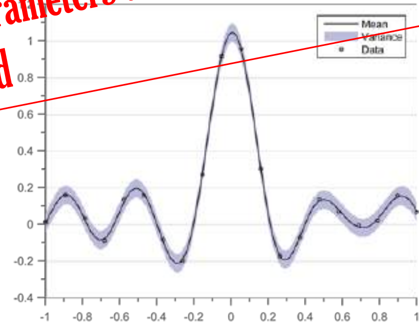




Only two hyperparameters:

→ The lengthscale θ_2 or ℓ determines the length of the 'wiggles' in your function.

→ The output variance $\theta_1{}^2$ or σ^2 determines the average distance of your function away from its mean. It's just a scale factor.

3/ Hyperparameters tuning. ML is maximised, θ^* is found

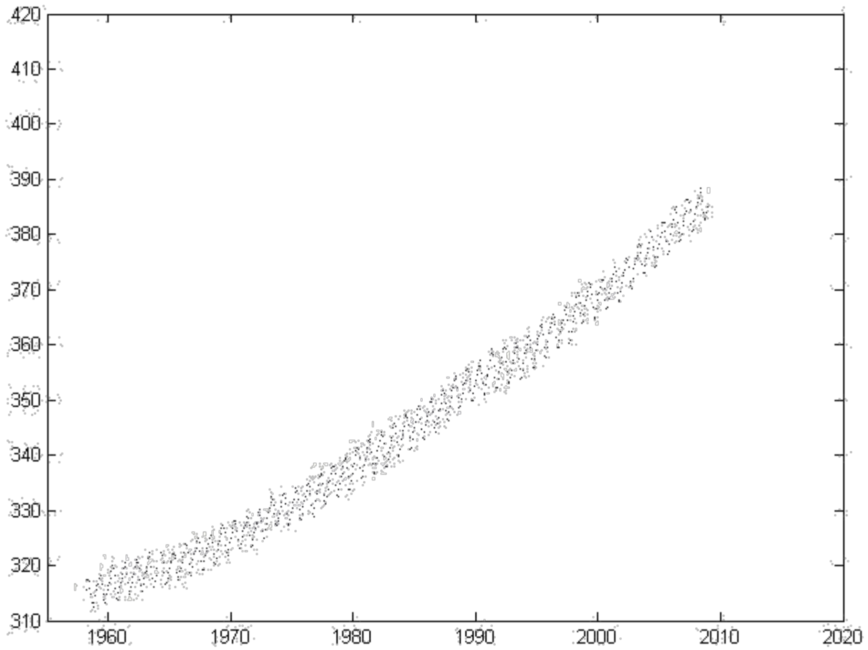




ML = -8.2

ML = -35.3

ML = 6,04

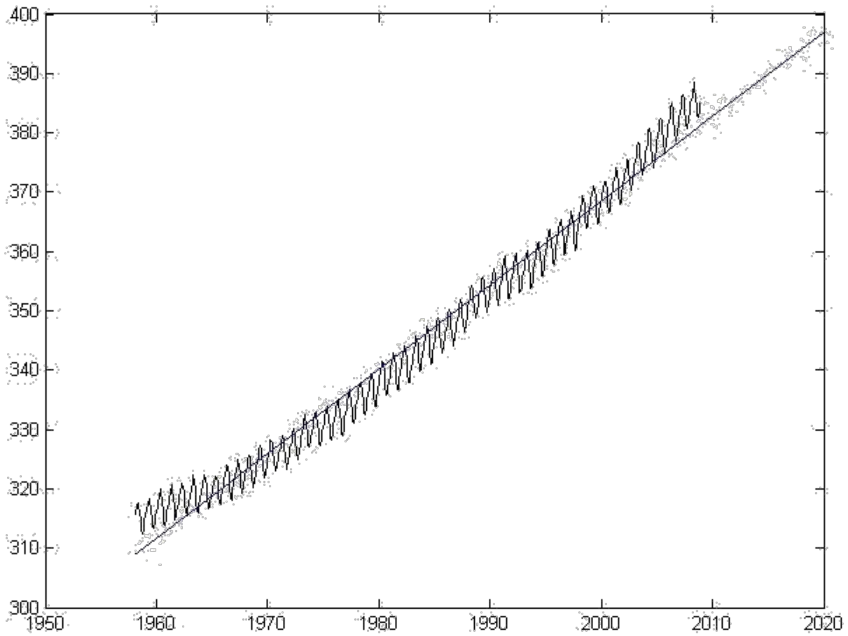
A SIMPle Example



Month-wise data of Co2 concentration in atmosphere at Hawaii

Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/

Example – Linear Regression



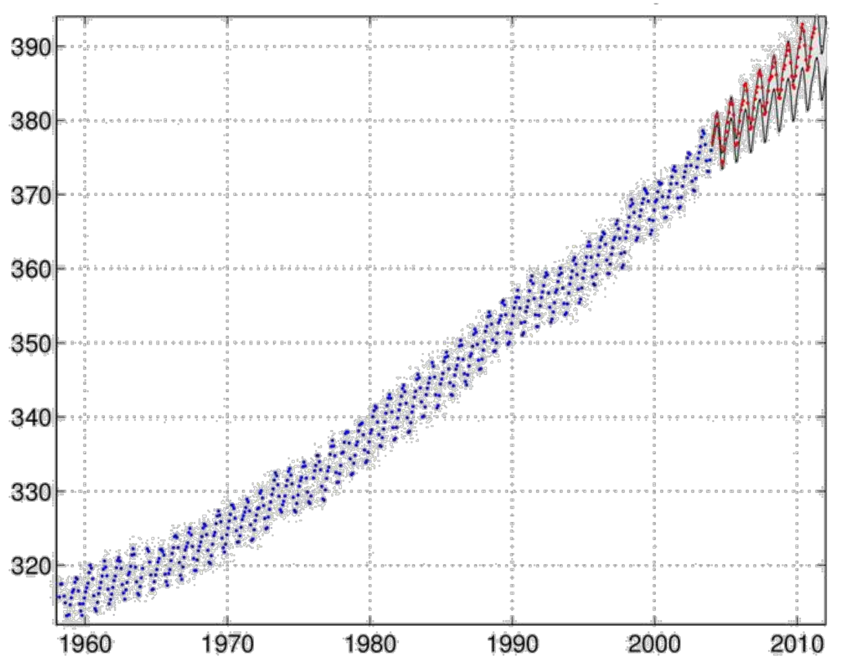
Should we choose a polynomial?

What degree of polynomial should we choose? (overfitting)

For a given degree, what parameters of polynomial should we choose

Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/

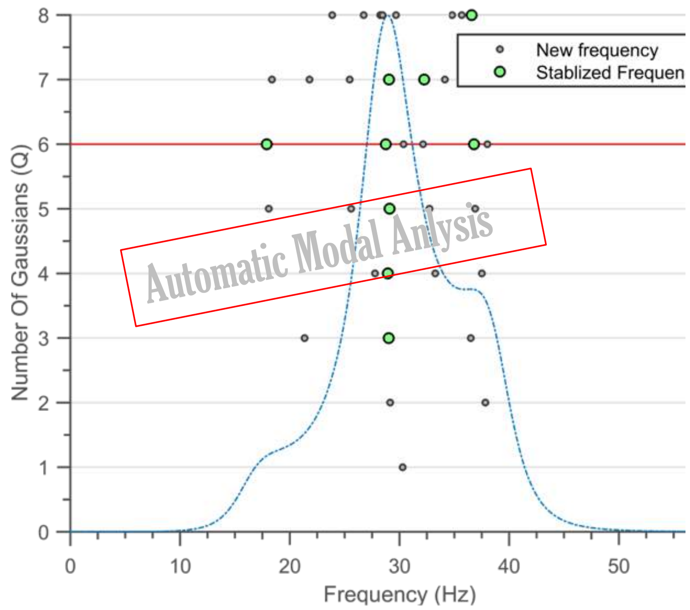
Example – Gaussian Process

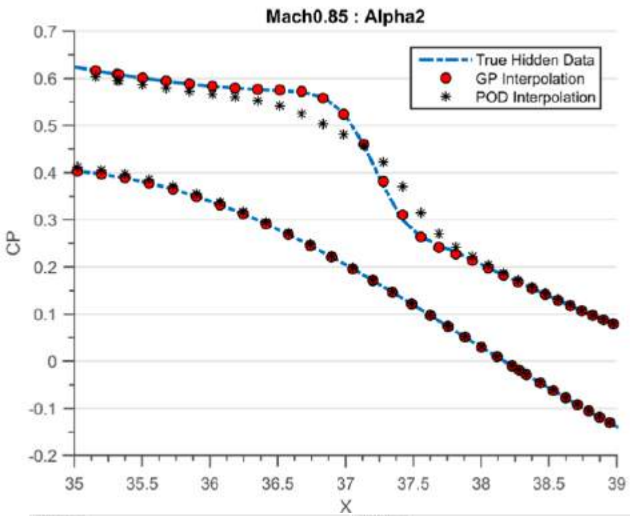


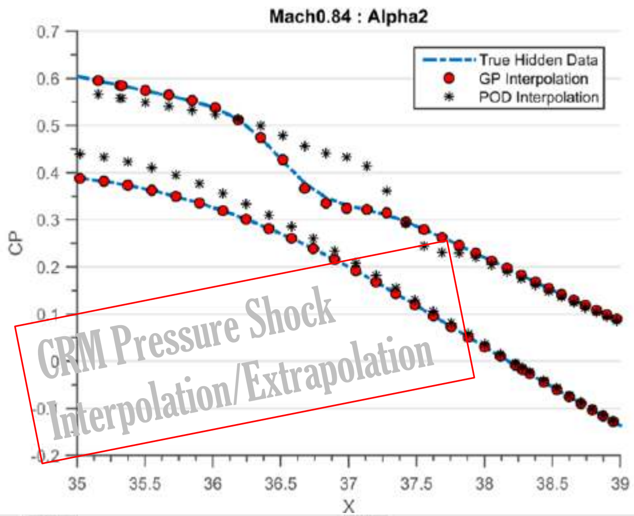
Predicted variance after year 2005 in grey, real data-points in red

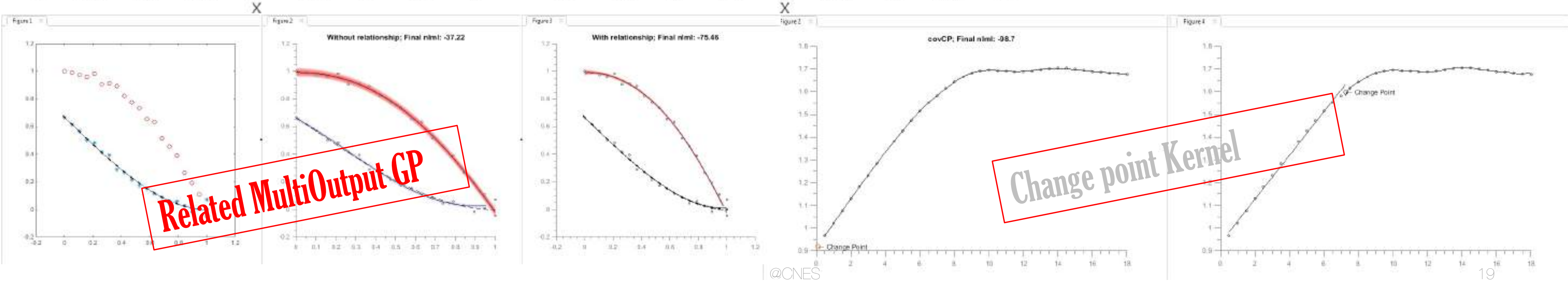
Image Source: http://mlg.eng.cam.ac.uk/teaching/4f13/1314/

Some ML applications









Multi-Output Gaussian Process – Flight Test examples

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Approximate Inference in Related Multi-output Gaussian Process Regression. Lecture Notes in Computer Science. 10163, 88-103. 2017

Given: $f_1 = g(f_2, x)$

- Earlier examples include **Gradient Enhanced Kriging [$]** (GEK) or **Co-kriging ***
- But we want to expand this to integral enhanced kriging, double differential, or any functional relationship between outputs

*** Forrester *et al* (2007) Multi-fidelity optimization via surrogate modelling. Proceedings of the Royal Society A, 463(2088), 3251–3269, (doi:10.1098/rspa.2007.1900).**

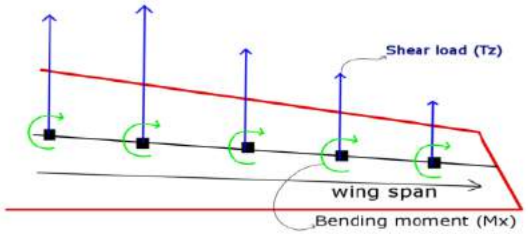
[$] Liu. Development of gradient-enhanced kriging approximations for multidisciplinary design optimization. 2003.

- Add physics[£] as constraints in the mathematical problem

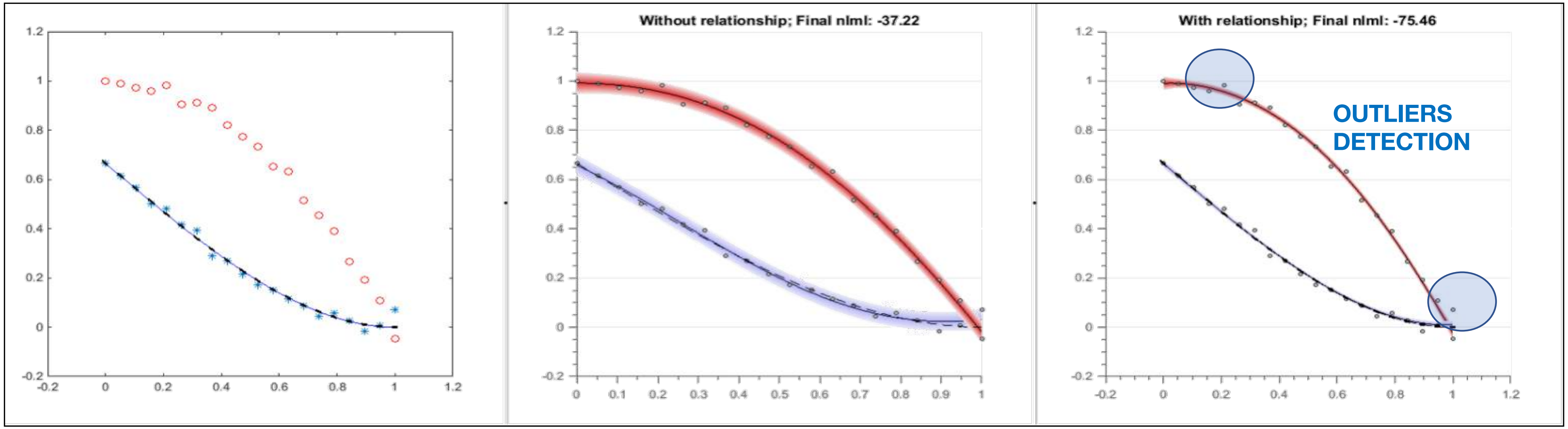
[£]Constantinescu, Physics-based covariance models for Gaussian processes with multiple outputs." International Journal for Unceret *al*tainty Quantification 3.1 (2013).

Example 1 : use the Relationship between Tz and Mx permits to reduce uncertainties

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Sparse Physics-Based Gaussian Process for Multi-output Regression using Variational Inference. Proceedings of ICPRAM 2016 2016



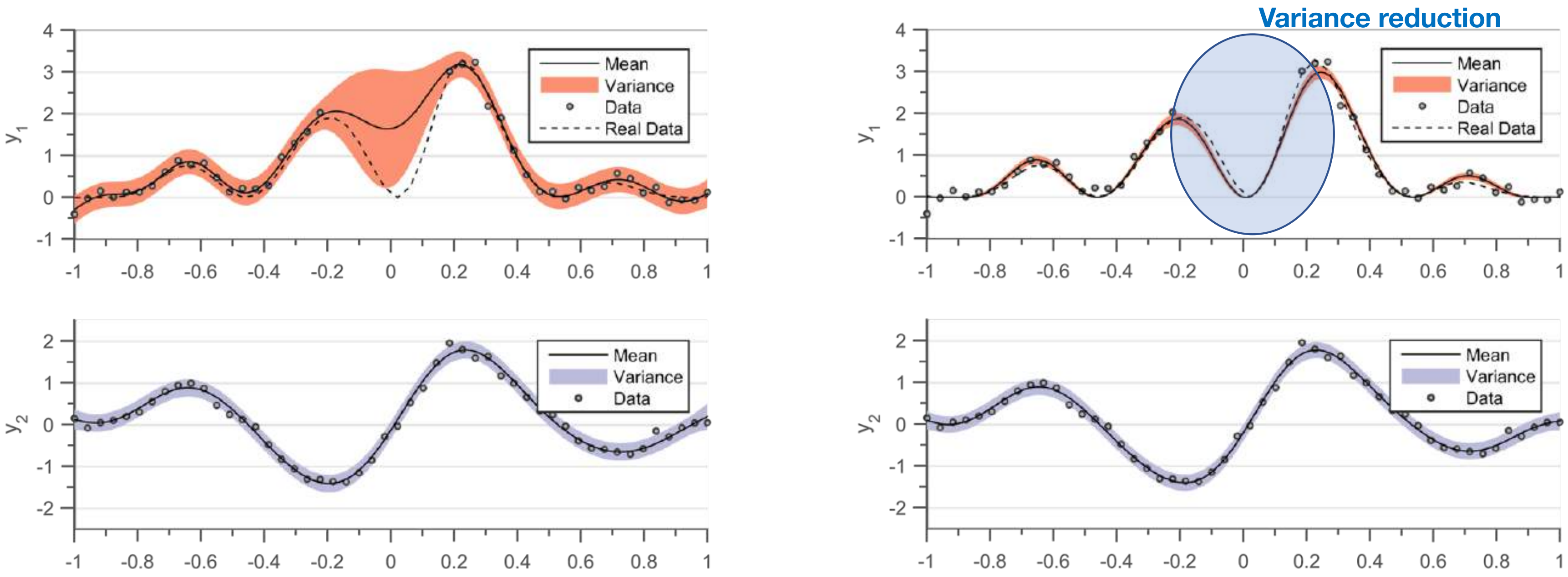
$$Mx = \int_{\eta}^{\eta_{edge}} (x - \eta)\, Tz\, dx$$



Example 2: Faulty sensors (using synthetic data)

$$y_1 = (y_2)^2$$

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Adding Flight Mechanics to Flight Loads Surrogate Model using Multi-Output Gaussian Processes. AIAA AVIATION 2016



Independent GPs

Related GPs

Papers & conf on this topic

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Approximate Inference in Related Multi-output Gaussian Process Regression. Lecture Notes in Computer Science. 10163, 88-103. 2017

Chiplunkar and J. Morlier. Operational Modal Analysis in Frequency Domain using Gaussian Mixture Models . Proceedings of IMAC XXXV, 2017

Chiplunkar, E. Bosco and J. Morlier. Gaussian Process for Aerodynamic Pressures Prediction in Fast Fluid Structure Interaction Simulations. Proceedings of WCSMO12 2017

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Adding Flight Mechanics to Flight Loads Surrogate Model using Multi-Output Gaussian Processes. AIAA AVIATION 2016

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Sparse Physics-Based Gaussian Process for Multi-output Regression using Variational Inference. Proceedings of ICPRAM 2016 2016

Chiplunkar, A., Rachelson, E., Colombo, M., & Morlier, J. (2016, April). Identification of Physical Parameters Using Change-Point Kernels. In Society for Industrial and Applied Mathematics, Uncertainty Quantification, 2016.

Morlier, J., Basile, A., Chiplunkar, A., & Charlotte, M. (2018). An EGO-like optimization framework for sensor placement optimization in modal analysis. Smart Materials and Structures, 27(7), 075004.

Several Papers in preparation

What if

We use Surrogate models to develop Efficient Global Optimization

Algorithms ?

→EGO* (unconstrained problem)
→SEGO$(constrained problem)

*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. Journal of Global optimization, 13(4), 455-492.

$Sasena, M. J., Papalambros, P., & Goovaerts, P. (2002). Exploration of metamodeling sampling criteria for constrained global optimization. Engineering optimization, 34(3), 263-278.

1. GP aka Kriging

2. Kriging for Global Optimization

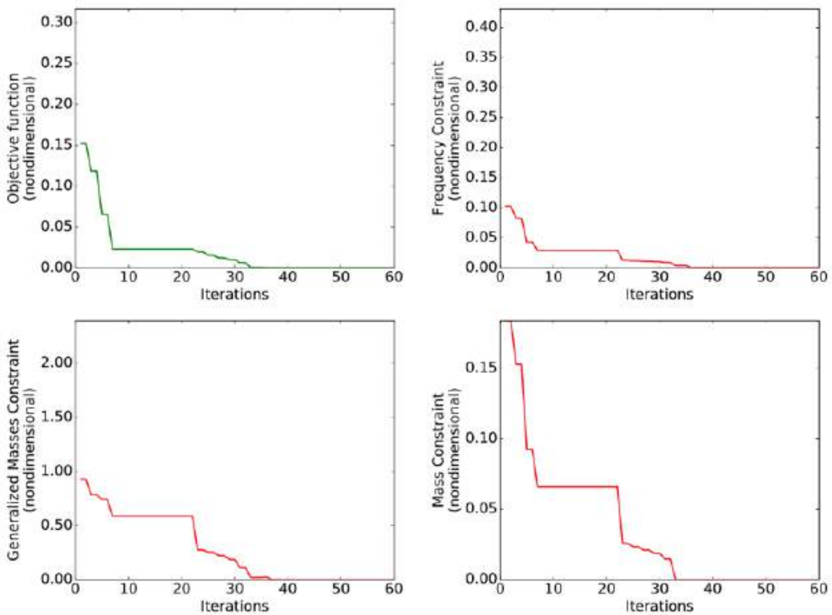
3. New developments in topology optimization

4. Codesign is MDO?

Joint Work since 2007 with N. Bartoli (Onera)

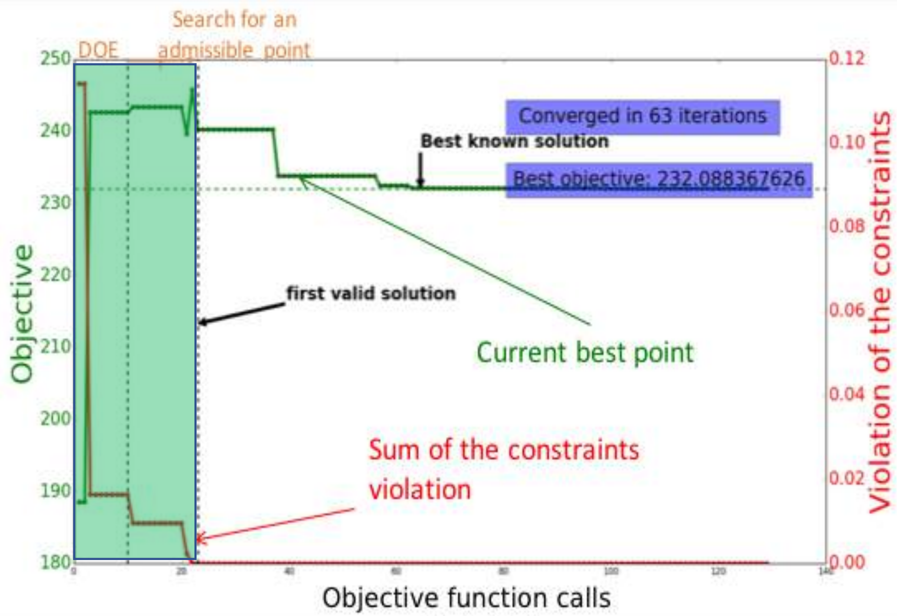
New paradigm for Surrogate Based Optimization (SBO)

Gradient based Optimality, Feasibility



Stopping criteria: tolfun, tolx, maxiter

SBO Exploration, Exploitation



Stopping criteria: Max Budget (Function calls)

A good starting point X_0=Forrester's book

Engineering Design via Surrogate Modelling

A Practical Guide

Alexander I. J. Forrester, András Sóbester and Andy J. Keane

University of Southampton, UK

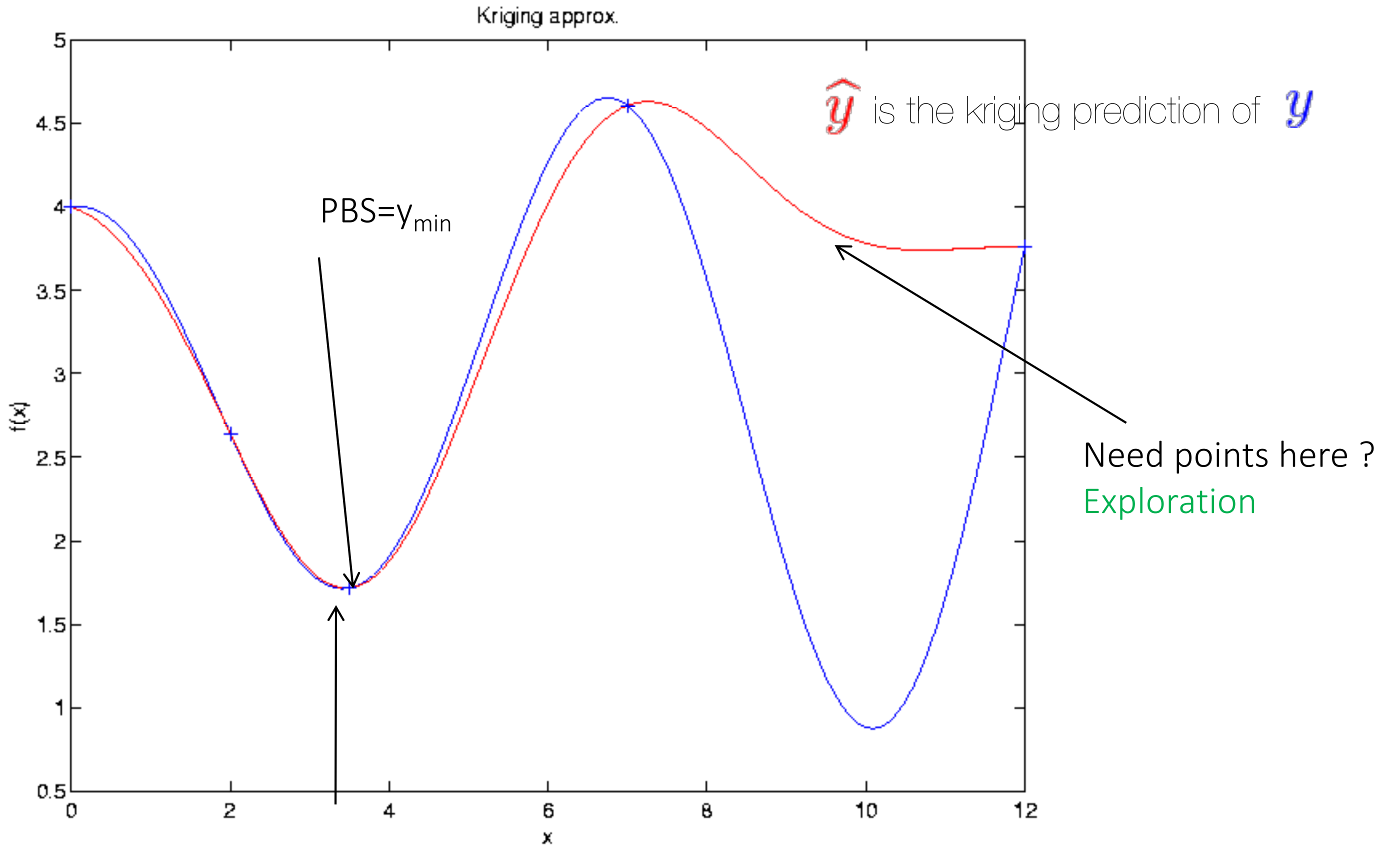
The goal is: find min of f(x) by sampling + and Kriging updating

Where do I need to update my sampling?

We note the present best solution (PBS=y_{min})

At every x there is some chance of improving on the PBS.

Then we ask: Assuming an improvement over the PBS, where is it likely be largest?

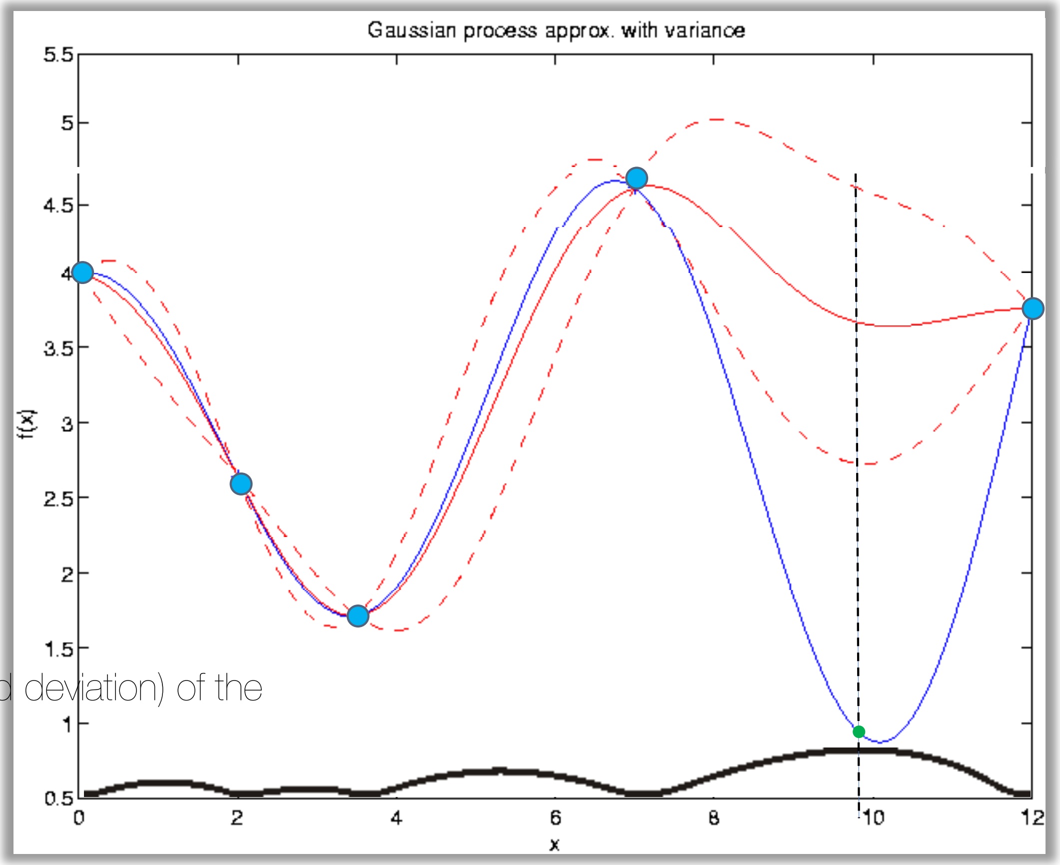


Exploitation may drive the optimization to a local optimum

In supervised mode … have a look to max(RMSE)

Not possible to compute the error: we don't know for each x the true value of the function

But…. Can we use GP properties ?

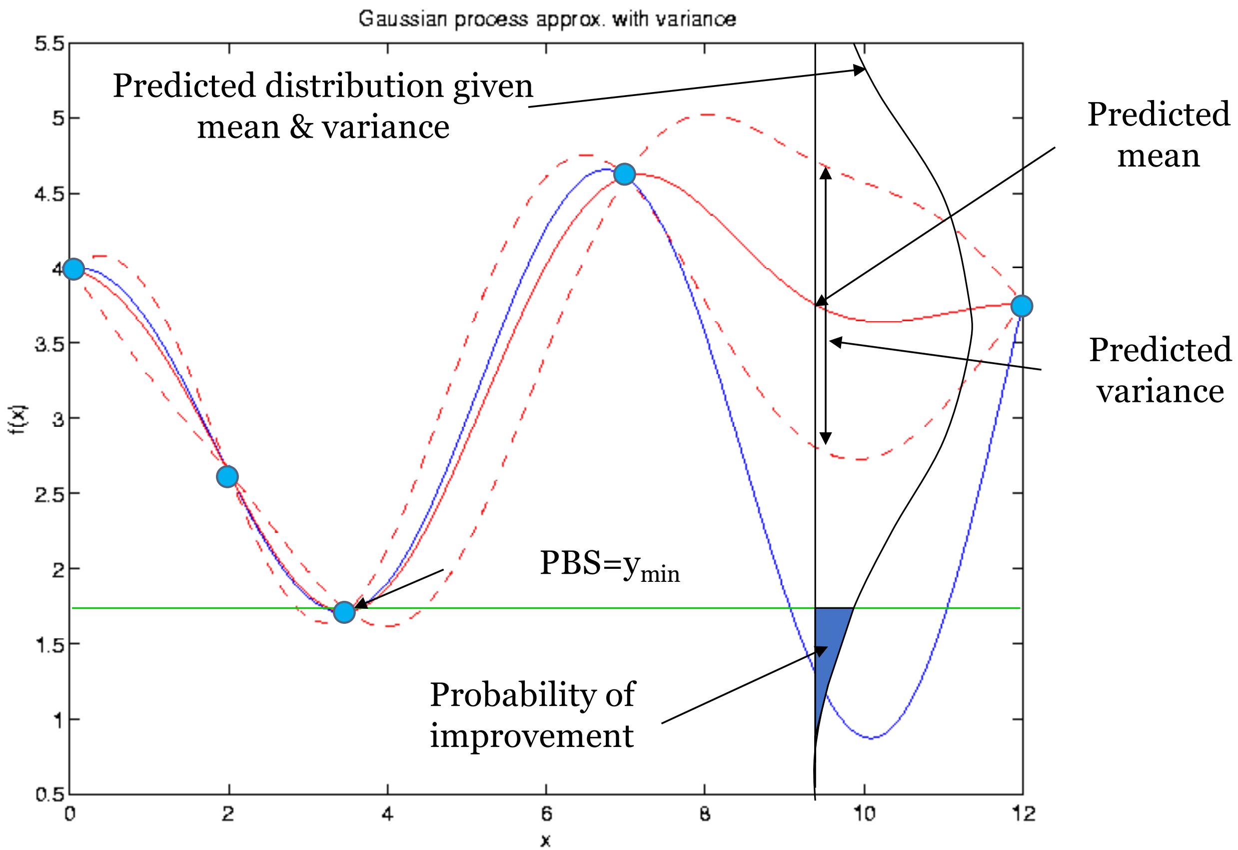


$\hat{y}$ is the kriging prediction of y

$\hat{s}$ is the estimation error (standard deviation) of the prediction (often noted σ_y)

PBS=y_{min}

Probability of improvement



Improvement … explicitely

- *Improvement* : $I(\mathbf{x}) = \max\left(y_{min} - \hat{Y}(\mathbf{x}), 0\right)$
- *Expected Improvement* :

$$\boxed{\mathrm{EI}(x) = \mathrm{E}[\max(0, y_{\min} - \hat{y}(x))]}$$

$$E[I(\mathbf{x})] = \int_{-\infty}^{y_{min}} (y_{min} - \hat{y})\varphi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right) d\hat{y}$$

$$E[I(\mathbf{x})] = (y_{min} - \mu_{\hat{Y}}(\mathbf{x}))\Phi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right) + \sigma_{\hat{Y}}(\mathbf{x})\varphi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right)$$

Exploitation

Exploration

global optimum can be found because *P[I(x)] = 0 when s = 0 so that there is no probability of improvement* at a point which has already been sampled → guarantees global convergence

Φ: cumulative distribution function $\mathcal{N}(0,1)$ ϕ: probability density function $\mathcal{N}(0,1)$

***Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. Journal of Global optimization, 13(4), 455-492.**

Infill Criteria : max(Expected improvement)

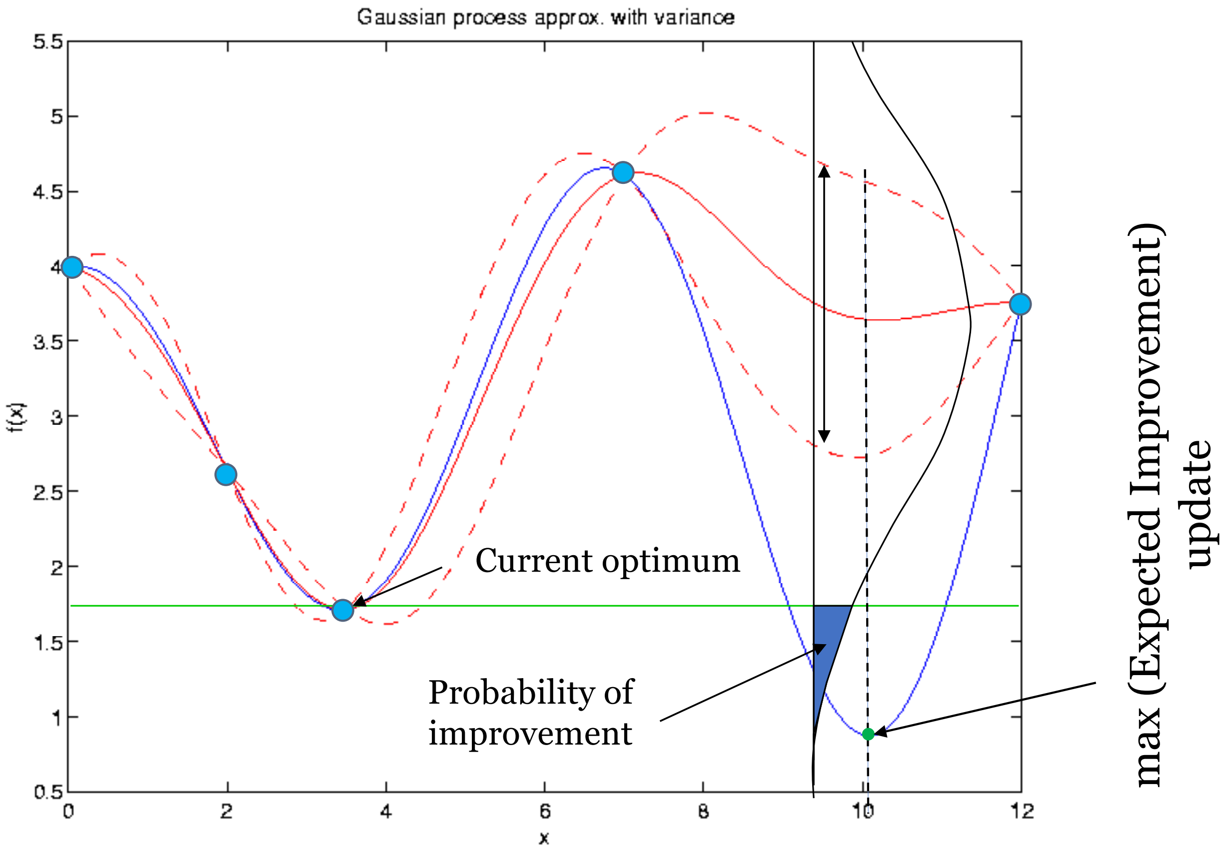
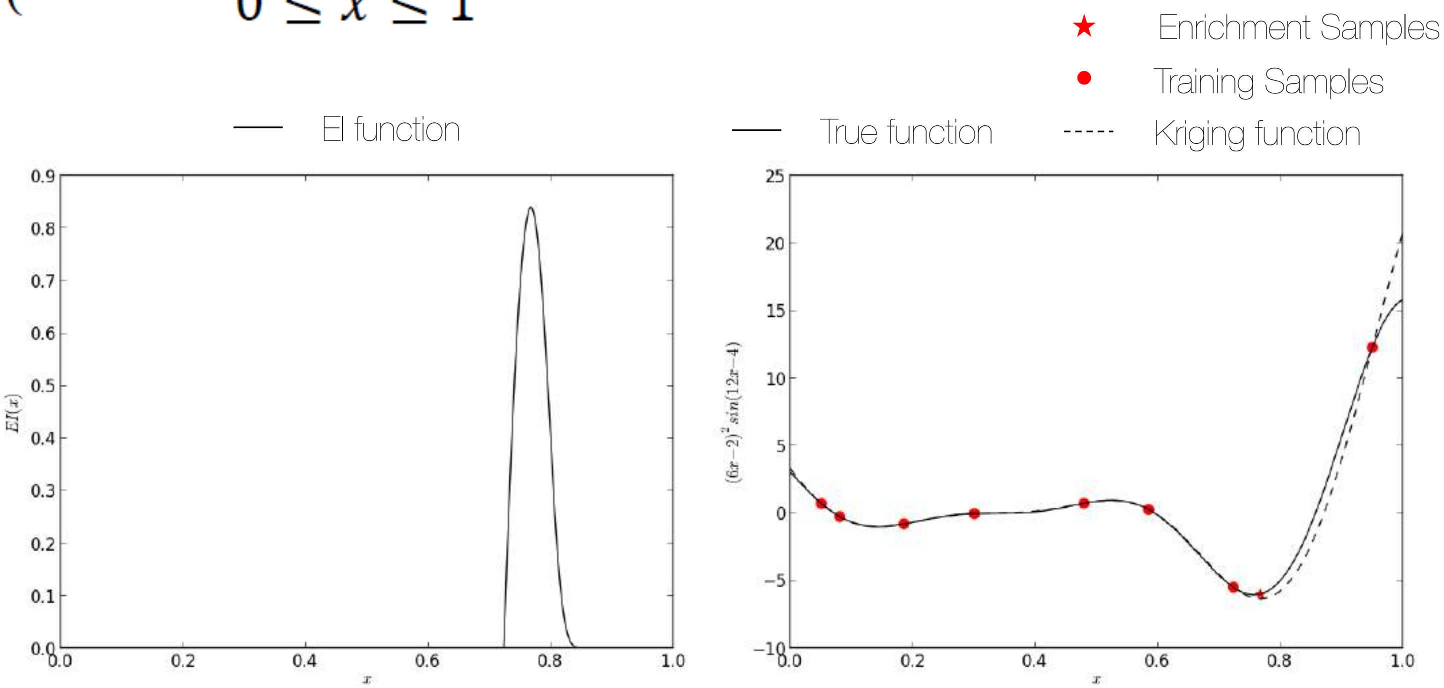


Illustration on 1D example

$$\begin{cases} \min \ (6x-2)^2 \sin(12x-4) \\ s.t. \\ 0 \le x \le 1 \end{cases}$$



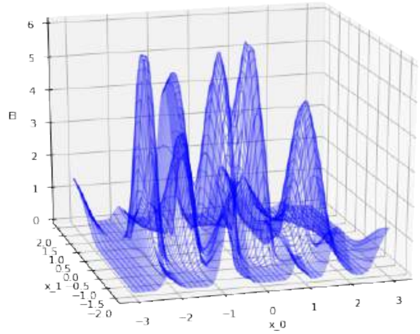
Limits of EI

Expected Improvement criterion (EI) (to maximize)

$$E[I(\mathbf{x})] = (y_{min} - \mu_{\hat{Y}}(\mathbf{x}))\Phi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right) + \sigma_{\hat{Y}}(\mathbf{x})\varphi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right)$$

Exploitation

Exploration



+ Analytical formula ➡ Quick to evaluate
(Criteria and gradient)

- Highly multimodal ➡ Hard to optimize

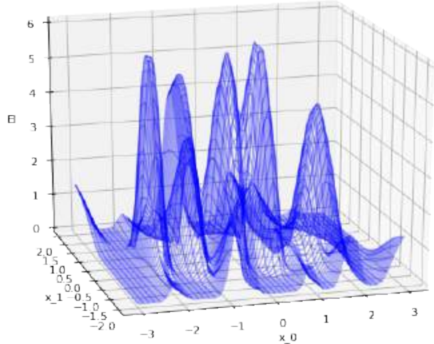
*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. Journal of Global optimization, 13(4), 455-492.

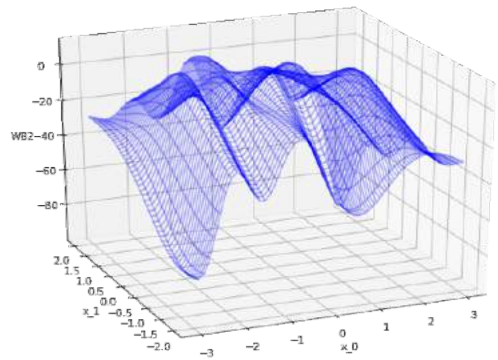
Smoother criteria WB2 (R. Olivanti MsC)

$$\mathrm{WB2}(x) = \mathrm{EI}(x) - \hat{y}(x)$$

Influence of surrogate model prediction

+ 'Smoother' ➡ Easier to optimize
+ Quicker convergence
- Lack of normalization ➡ Reduce the global aspect





EI and WB2 criteria computed on same Kriging surrogate

$Sasena, M. J., Papalambros, P., & Goovaerts, P. (2002). Exploration of metamodeling sampling criteria for constrained global optimization. Engineering optimization, 34(3), 263-278.

New infill sampling criteria

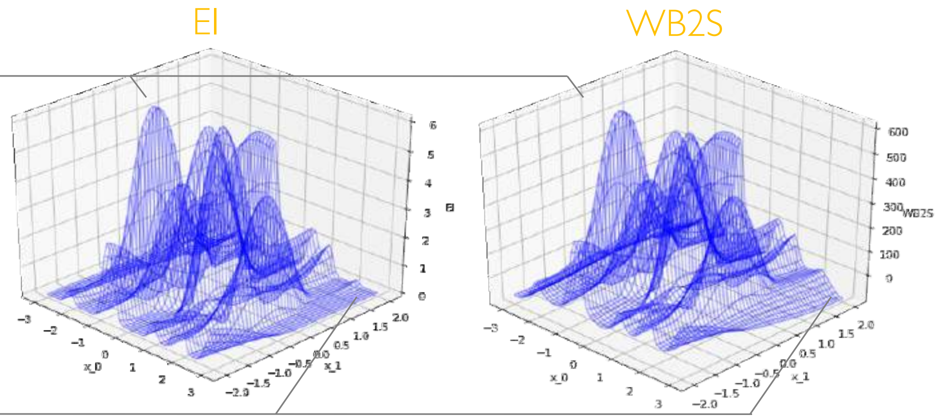
WB2S scaled criterion

$$\mathrm{WB2S}(x) = s\ \mathrm{EI}(x) - \hat{y}(x)$$

- EI numerically improved
- 'dynamical' normalization

+ Exploration similar to EI

+ non negligible value in the area where EI is low

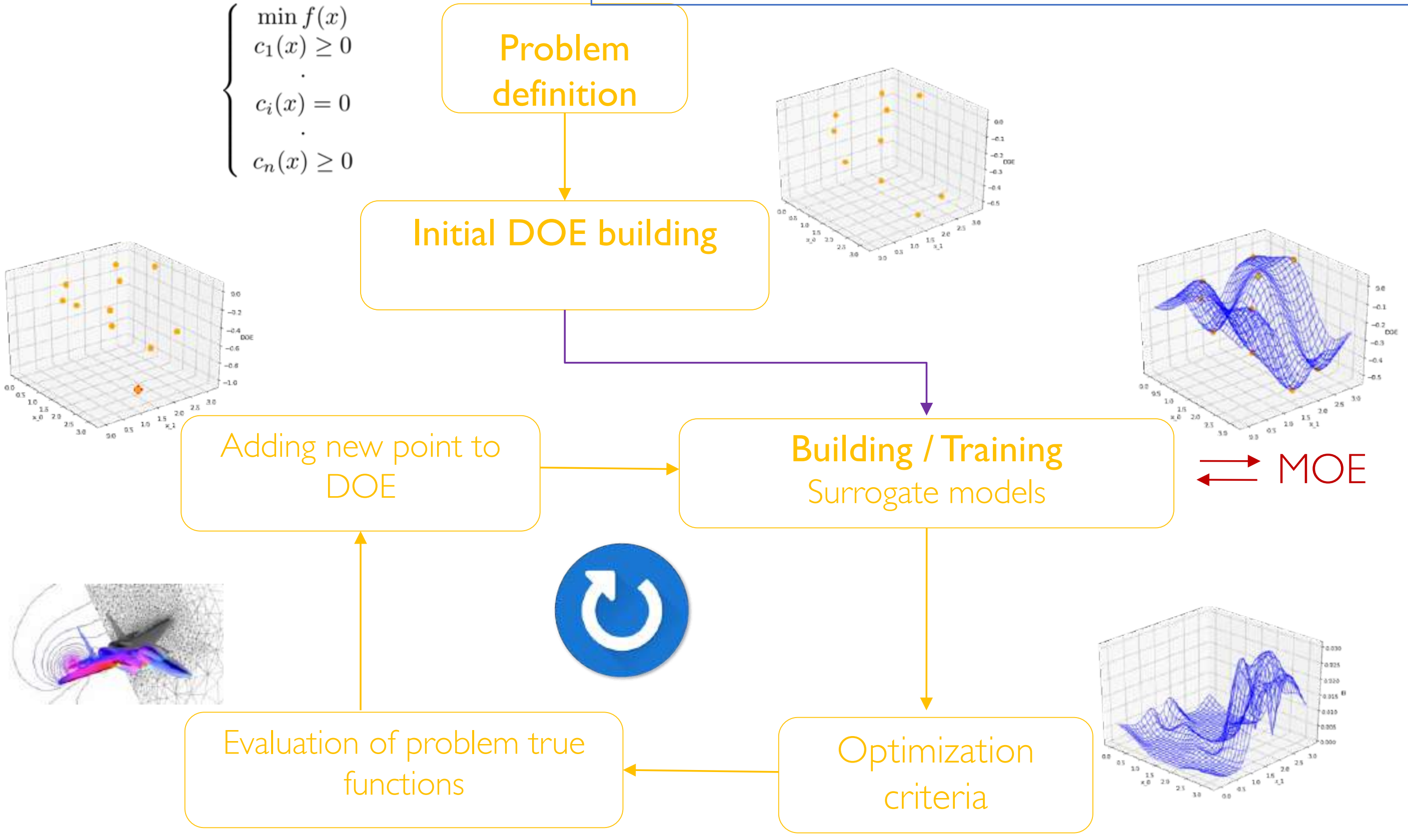


N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, " An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization", 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433,Denver, USA, June 2017

SEGOMOE algorithm

N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, " An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization", 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433,Denver, USA, June 2017

$$\begin{cases} \min f(x) \\ c_1(x) \geq 0 \\ \cdot \\ c_i(x) = 0 \\ \cdot \\ c_n(x) \geq 0 \end{cases}$$



New formulation

N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, " An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization", 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433,Denver, USA, June 2017

Adapted from Super EGO

Costly initial problem

$$\begin{cases} \min f(x) \\ c_1(x) \geq 0 \\ . \\ c_i(x) = 0 \\ . \\ c_n(x) \geq 0 \end{cases}$$

Cheap enrichment problem

$$\begin{cases} \max_{x \in \mathbb{R}^d} \quad \mathrm{EI}(x)/\mathrm{WB2}(x)/\mathrm{WB2s}(x) \\ \text{s.t.} \\ \hat{c_1}(x) \geq 0 \\ . \\ \hat{c_i}(x) = 0 \\ . \\ \hat{c_n}(x) \geq 0 \end{cases}$$

n +1
metamodels

Possibly Multimodal

Multimodal

Global optimization method

ADODG6 * testcase (R. Olivanti, R. Priem MsC)

CFD guys know very well the multimodality of this problem...

Wing drag minimization problem (subsonic, Euler equations)

	Function/variable	Description	Quantity	Range
minimize	C_D	Drag coefficient	1	
with respect to	α	Angle of attack	1	$[-3.0, 6.0]$ (°)
	θ	Twist	8	$[-3.12, 3.12]$ (°)
	δ	Dihedral	8	$[-0.25, 0.25]$ (unit of chord)
		Total variables	17	
subject to	$C_L = 0.2625$	Lift coefficient	1	
		Total constraints	1	

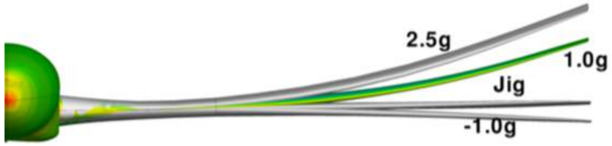
Can SEGOMOE help us to reach the global optimum ?

Is it less dependant on X0 compared to SNOPT £?

*AIAA, Aerodynamic Design Optimization Discussion Group
http://mdolab.engin.umich.edu/content/aerodynamic-design-optimization-workshop

£ https://web.stanford.edu/group/SOL/snopt.htm

ADODG* 6 TOOLS



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UNIVERSITY of MICHIGAN ■ COLLEGE of ENGINEERING

CFD solver
ADflow (adjoint)

pyOptSparse
Optimization
environment

Geometry update
FFD (Volume)

Mesh update
analytic inverse
distance method

UCSD/
Stanford

isae
SUPAERO

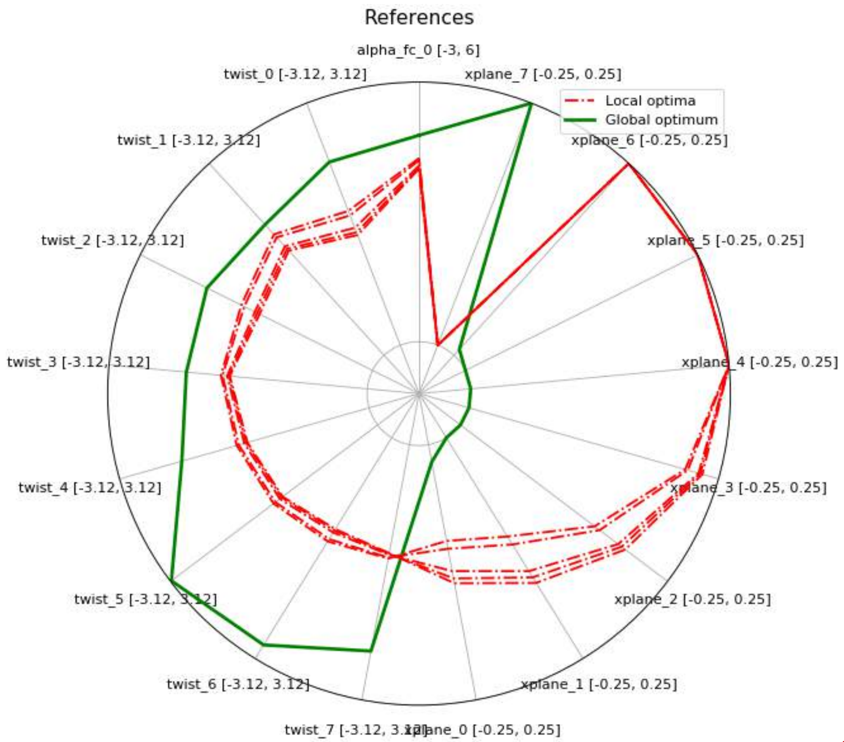
ONERA
THE FRENCH AEROSPACE LAB

SNOPT | SEGOMOE

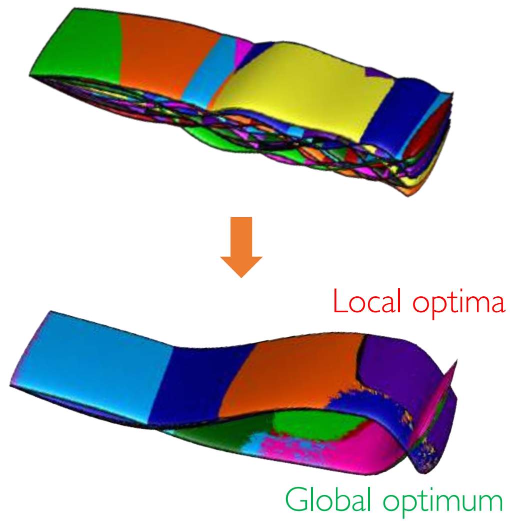
GOAL: Bench our SBO method with a reference SNOPT
→Compare the robustness of the solution with respect to X_0

Multimodal optimization problem (SNOPT Benchmark)

Wing drag minimization problem (subsonic, Euler equations with ADFlow solver) (Mesh 180K cells)



X_0:Multi start with DOE of 15 (LHS)



➔ 5/15 runs found the global optimum

67% Local optimum

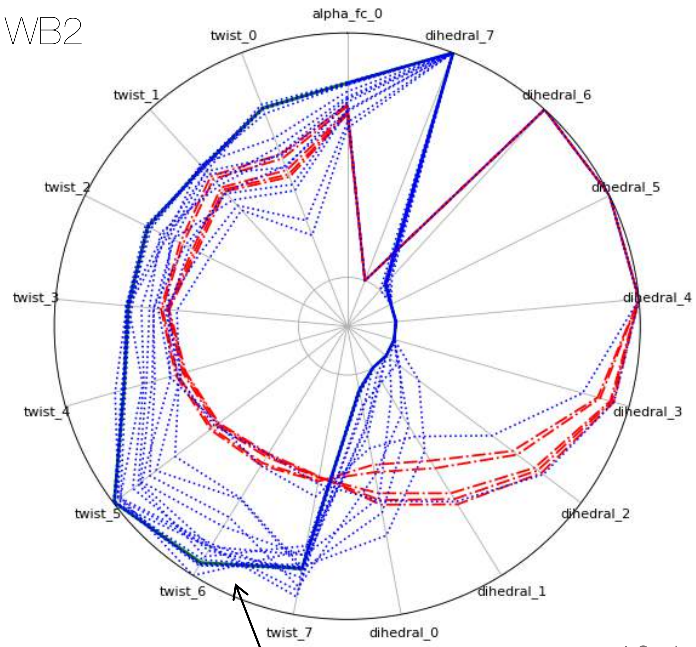
Accuracy Cz ~$1e^{-10}$ – 300 calls to ADflow

Multimodal optimization problem (SEGOMOE 2)

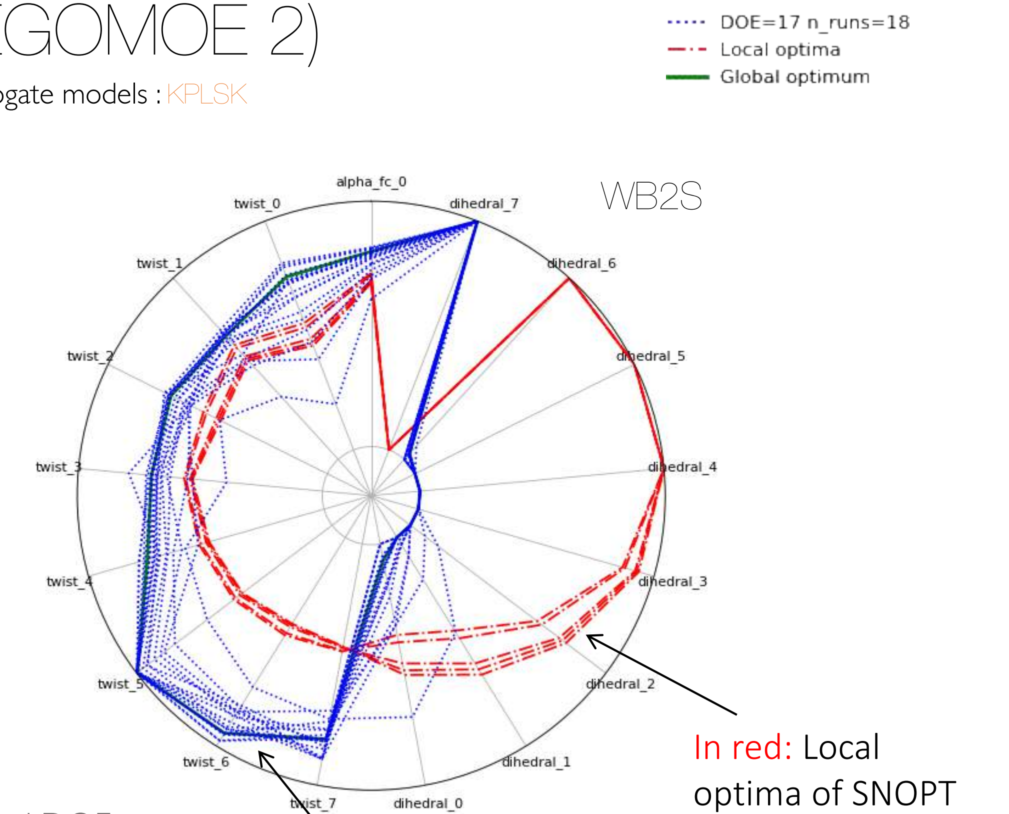
Initial DOE= 68 points (4xd)

Frozen budget: 500 evaluations   Surrogate models : KPLSK

WB2



WB2S



In red: Local optima of SNOPT

18 shared DOE

Global optimum 16 / 18

Best point ~ 409 evaluations   Accuracy Cz ~ $2.97e^{-6}$

Global optimum 18 / 18

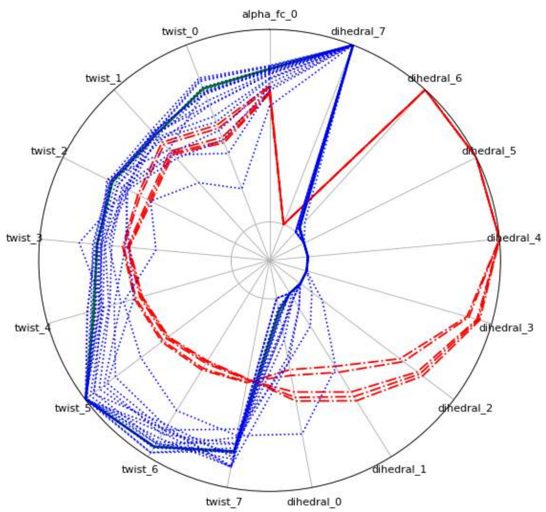
Best point ~ 477 evaluations

Accuracy Cz ~ $3.24e^{-6}$

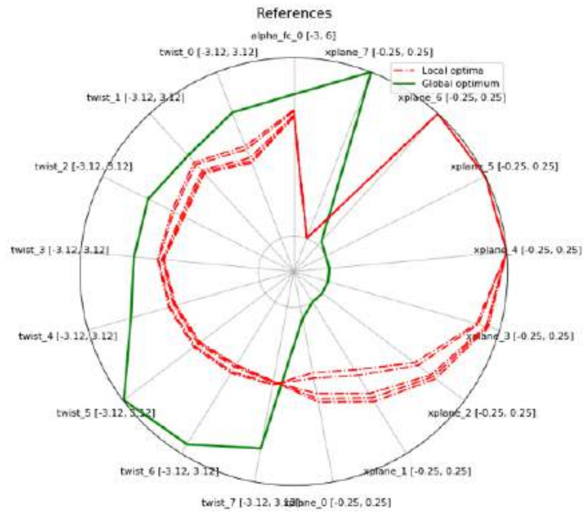
Idea: 2 steps method

1. Start with SEGOMOE, stop with maxiter

x° with high confidence near x^{*} (so we avoid to get stuck in local minima !)



2. Use x° as x_0 in SNOPT to reach rapidly the Global Optimum x^{*}



A surrogate model toolbox: SMT (M. Bouhlel Postdoc)

SMT 0.2 documentation »

An open source python toolbox for surrogate models (since July 2017)

Bouhlel, M. A., Bartoli, N., Otsmane, A., & Morlier, J. (2016). Improving kriging surrogates of high-dimensional design models by Partial Least Squares dimension reduction. Structural and Multidisciplinary Optimization, 53(5), 935-952.

- Including the models KPLS and KPLS-K (for processing HD input variables)
- Focus on derivatives:
  - training derivatives used for gradient-enhanced modeling,
  - prediction derivatives,
  - derivatives with respect to the training data

SMT: A Python Surrogate Model Toolbox

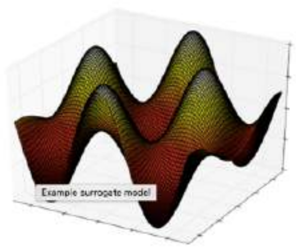
preamble

SMT, surrogate model toolbox, is a Python toolbox that includes four types of surrogate models---the least square (ls), square polynomial (pa2), inverse distance weighting (idw) and kriging-based models.

The toolbox supports Linux and Microsoft Windows, except the idw model which is available only with Linux.

This package is devoted to gathering several types of surrogate models within the same platform, for providing a benchmark to the engineering field. Another purpose of the SMT is for research purposes.

SMT is typically used as a surrogate model for a (time-consuming) computer model.



https://github.com/SMTorg/SMT

Papers & conf on this topic

Bouhlel, M. A., Bartoli, N., Otsmane, A., & Morlier, J. (2016). Improving kriging surrogates of high-dimensional design models by Partial Least Squares dimension reduction. Structural and Multidisciplinary Optimization, 53(5), 935-952.

Bouhlel, M. A., Bartoli, N., Otsmane, A., & Morlier, J. (2016). An improved approach for estimating the hyperparameters of the kriging model for high-dimensional problems through the partial least squares method. Mathematical Problems in Engineering, 2016.

Bouhlel, M., Bartoli, N., Regis, R. G., Otsmane, A., & Morlier, J. (2018). Efficient global optimization for high-dimensional constrained problems by using the Kriging models combined with the partial least squares method. Engineering Optimization, 1-16.

Bartoli, N., Lefebvre, T., Dubreuil, S., Olivanti, R., Bons, N., Martins, J. & Morlier, J. (2017). An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization. In 18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4433).

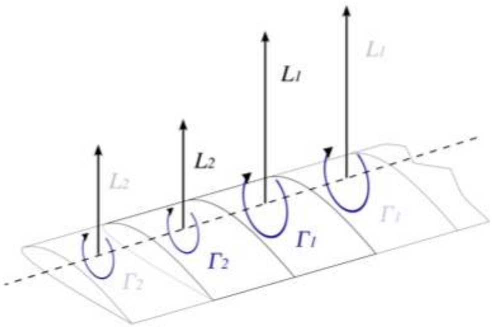
Bartoli, N., Bouhlel, M. A., Kurek, I., Lafage, R., Lefebvre, T., Morlier, J., & Regis, R. (2016). Improvement of efficient global optimization with application to aircraft wing design. In 17th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4001).

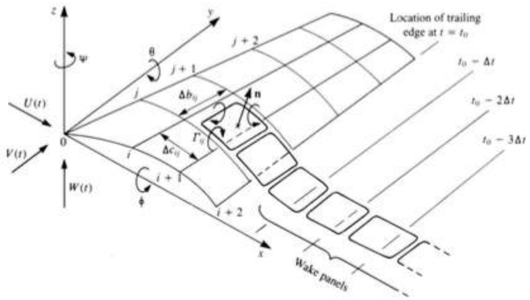
Several Papers in preparation

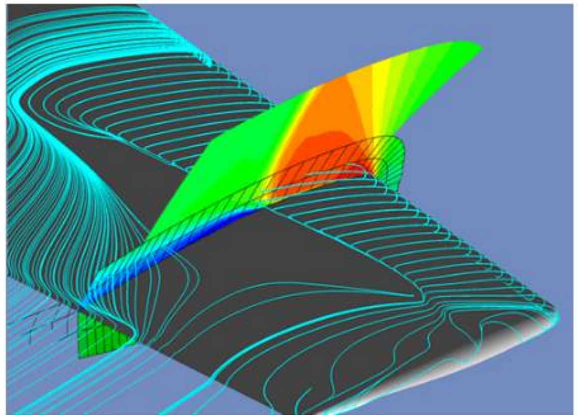
What if ?

- Several levels of fidelity of the same simulation are available

→ For example, in aerodynamics: Lifting line theory, Vortex lattice method, and RANS CFD code

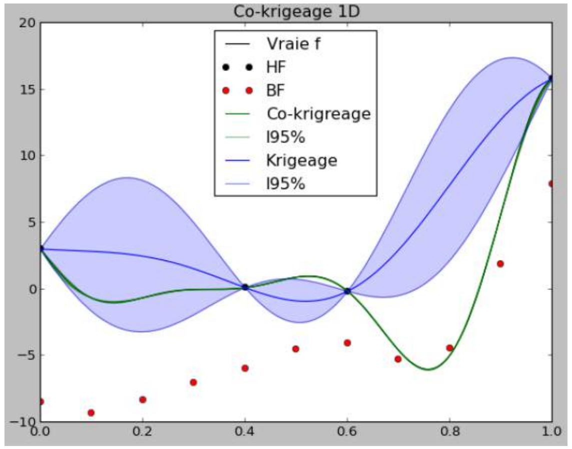


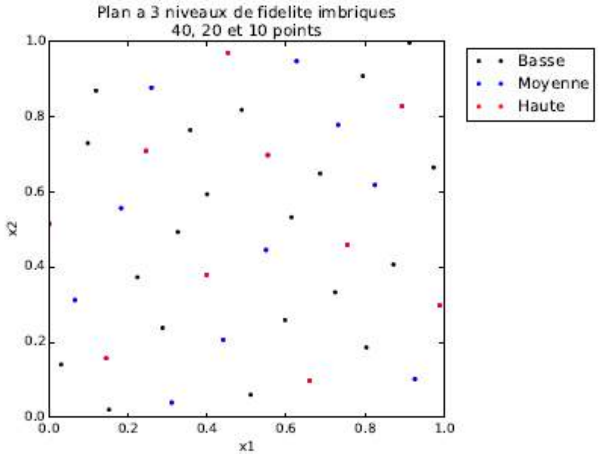




Multi-fidelity kriging (M. Meliani Msc)

- Surrogate adapted for multi-fidelity data (co-kriging with recursive approach)
- Design of experiments adapted to multi-fidelity (nested DOE)
- Developments of modules integrated in the OpenMDAO Framework (since Jan 2015) and now in SMT (since 2018)

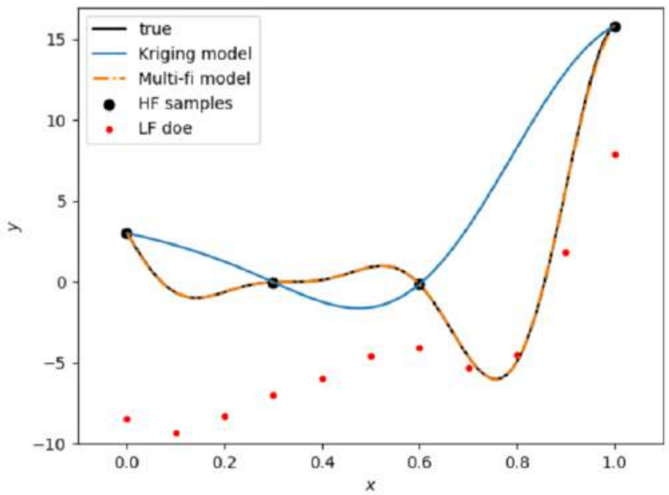


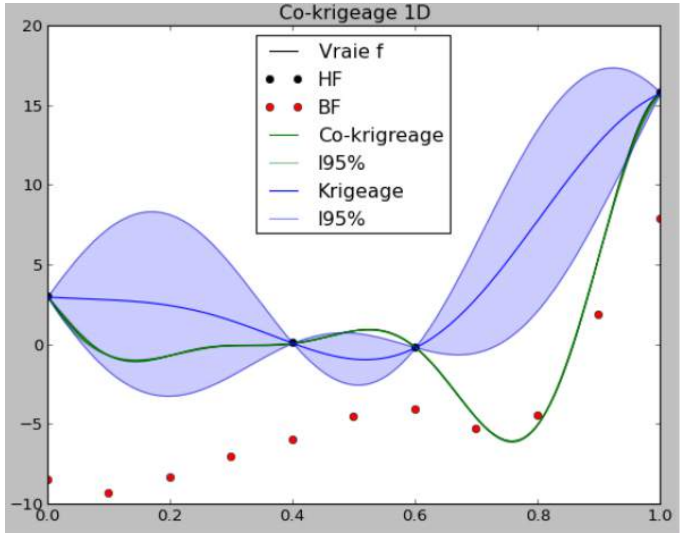


Le Gratiet, L. and Garnier, J., "Recursive co-kriging model for Design of Computer experiments with multiple levels of fidelity," International Journal for Uncertainty Quantification, 2014, pp. 365–386

Vauclin, R., "Développement de modèles réduits multi-fidélité en vue de l'optimisation de structures aéronautiques," Tech. rep., ISAE-SUPAERO, July 2014

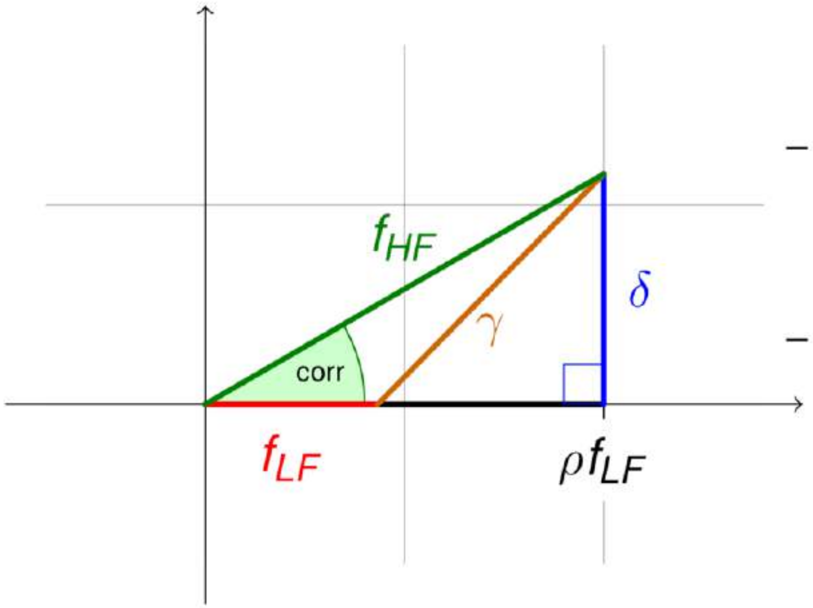
How to use low-fidelity information to speed up the optimization?





Remember Co-Kriging is the way to learn the difference between HF & LF …

Co Kriging



– Additive formulation [Lewis 2000]

$$f_{HF}(x) = f_{LF}(x) + \gamma(x)$$

– Kennedy-O'Hagan [Kennedy 2001]

$$\begin{cases} f_{HF}(x) = \rho f_{LF}(x) + \delta(x) \\ f_{LF}(\cdot) \perp \delta(\cdot) \end{cases}$$

The addition of the term ρ makes the multi-fidelity learning more robust to poor correlation as well as differences in modelization.

$Alexandrov, N., Lewis, R., Gumbert, C., Green, L., & Newman, P. (2000, January). Optimization with variable-fidelity models applied to wing design. In 38th Aerospace Sciences Meeting and Exhibit (p. 841).

Kennedy, M. C., & O'Hagan, A. (2001). Bayesian calibration of computer models. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 63(3), 425-464.

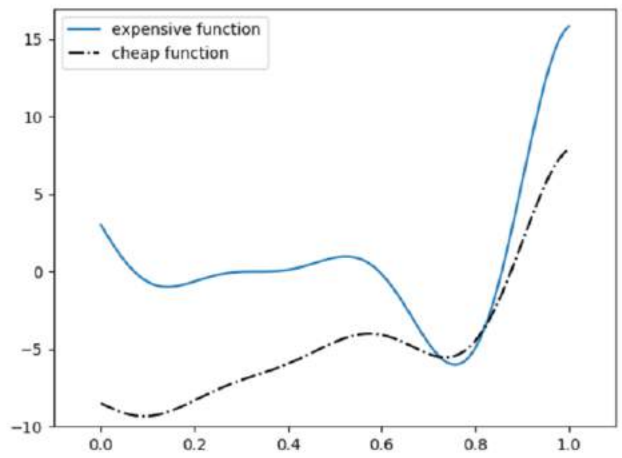
MFEGO

- most promising point: EI criterion

$$x^* = \underset{x}{\text{argmax}}\,(E[I(x)])$$

- choice of levels of enrichment: trade-off information gain/cost

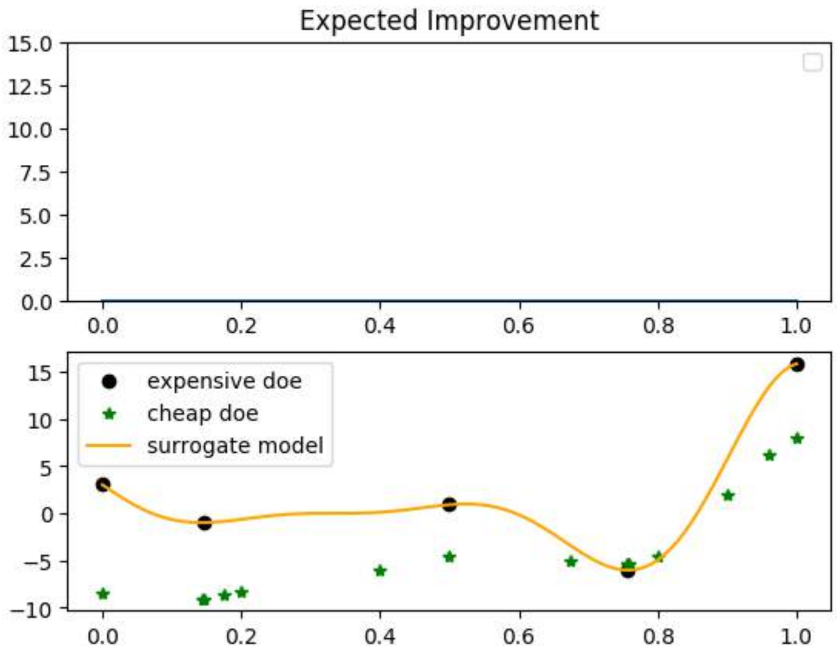
$$k^* = \underset{k \in (0,\dots,l)}{\text{argmax}} \quad \frac{\sigma^2_{red}(k, x^*)}{f(c_k)}$$



$\Rightarrow$ By using low-fidelity to reduce the uncertainty we reduce the Exploration contribution to the EI criterion
$\Rightarrow$ High-fidelity is used for Exploitation and model enhancement

$$f_{HF}(x) = (6x - 2)^2 \times \sin(2(6x - 2))$$
$$f_{LF}(x) = 0.5 f_{HF} + 10(x - 0.5) - 5$$

Results (Toy problem)

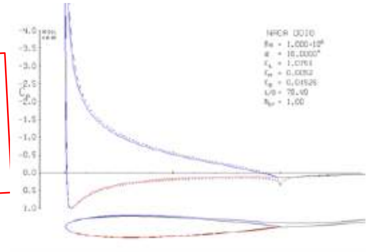


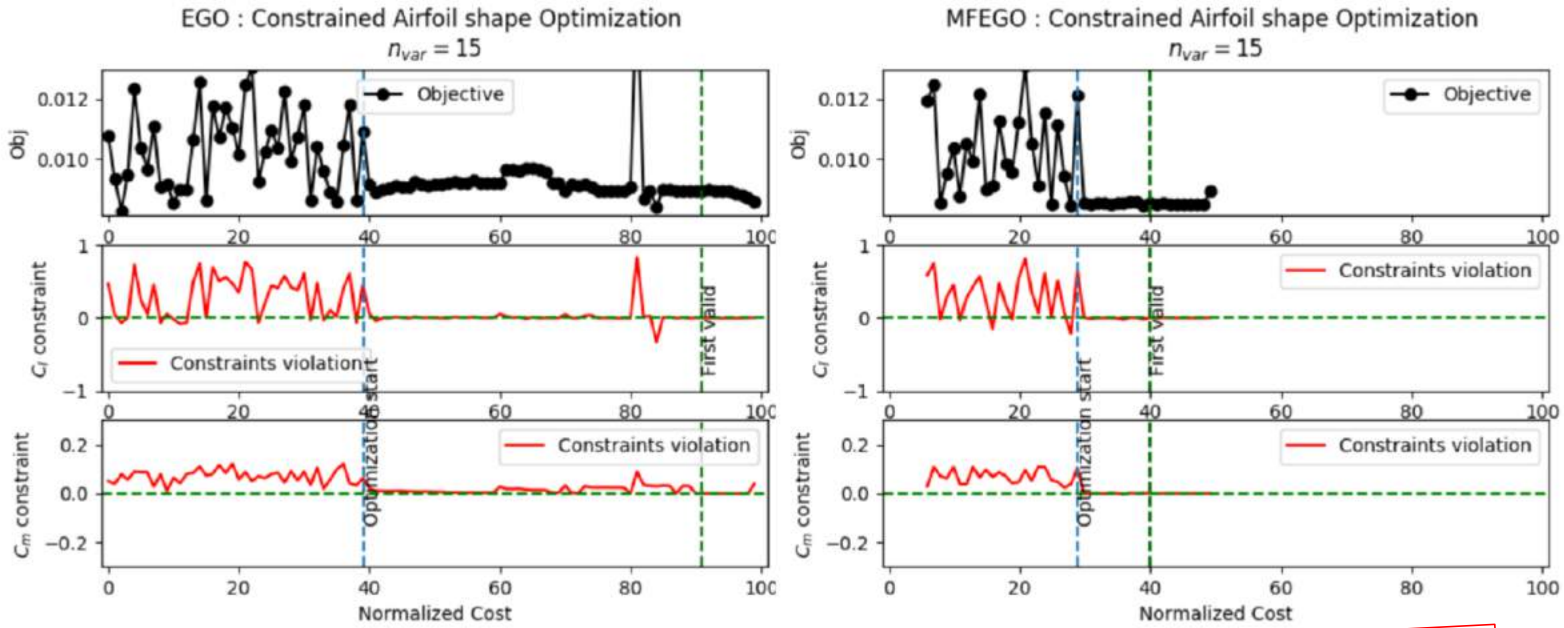
Cost ratio: 1/1000

	HF	LF	Cost
MFEGO	3+2	6+9	5.015
EGO	4+11	-	15

Second application: Constrained Optimization

Estimated COST RATIO: 1/200





[*https://web.mit.edu/drela/Public/web/xfoil/](https://web.mit.edu/drela/Public/web/xfoil/)
[$ http://mdolab.engin.umich.edu](http://mdolab.engin.umich.edu)

MFEGO can speed up the Optimization process by reducing the call to HF expensive code !

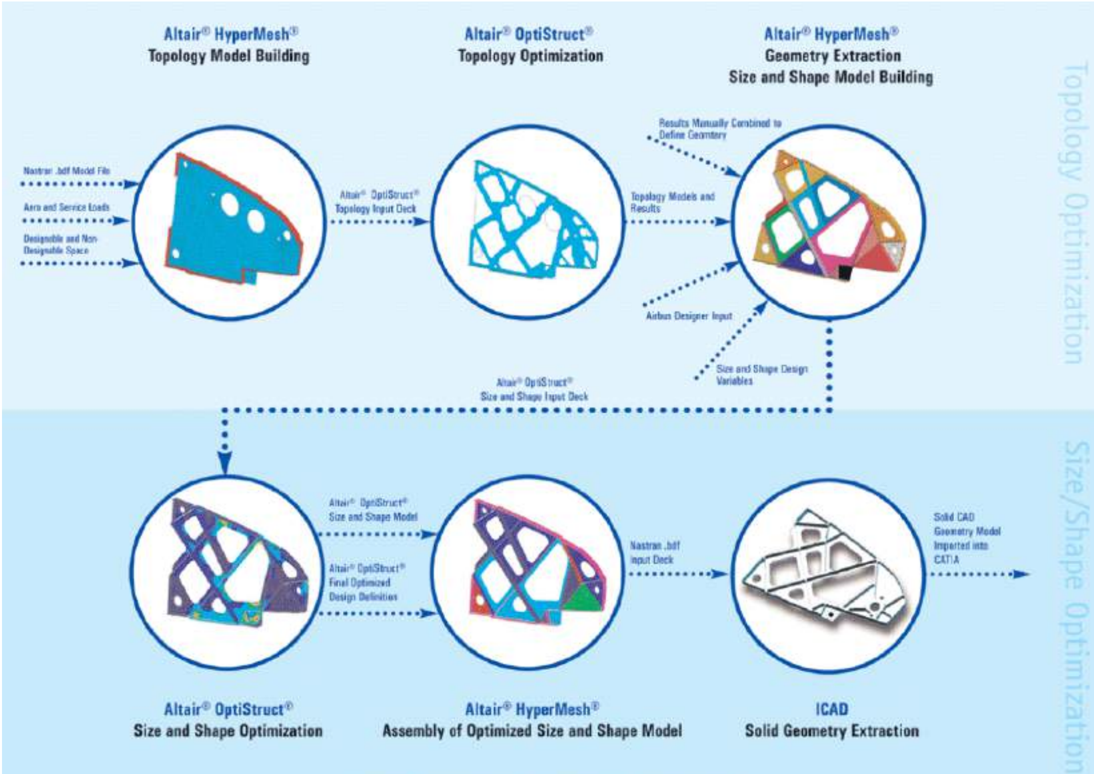
1. GP aka Kriging
2. Kriging for Global Optimization

3. New developments in topology optimization

4. Codesign is MDO ?

INDUSTRIAL PROCESS

"The art of structure is where to put the holes."
~Robert Le Ricolais
(1894-1977)

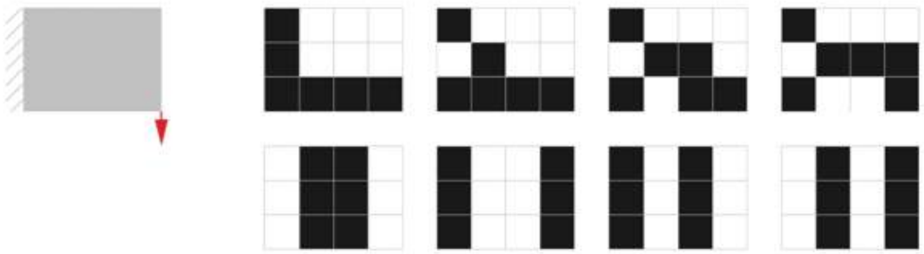


Schematic Representation of Enhanced Rib Design Process

Altair Engineering

Pixels

- Finding a solution by checking all the possible combinations IS impossible since the number of topologies nT increases exponentially with the number of finite elements n
- nT = 2^n,



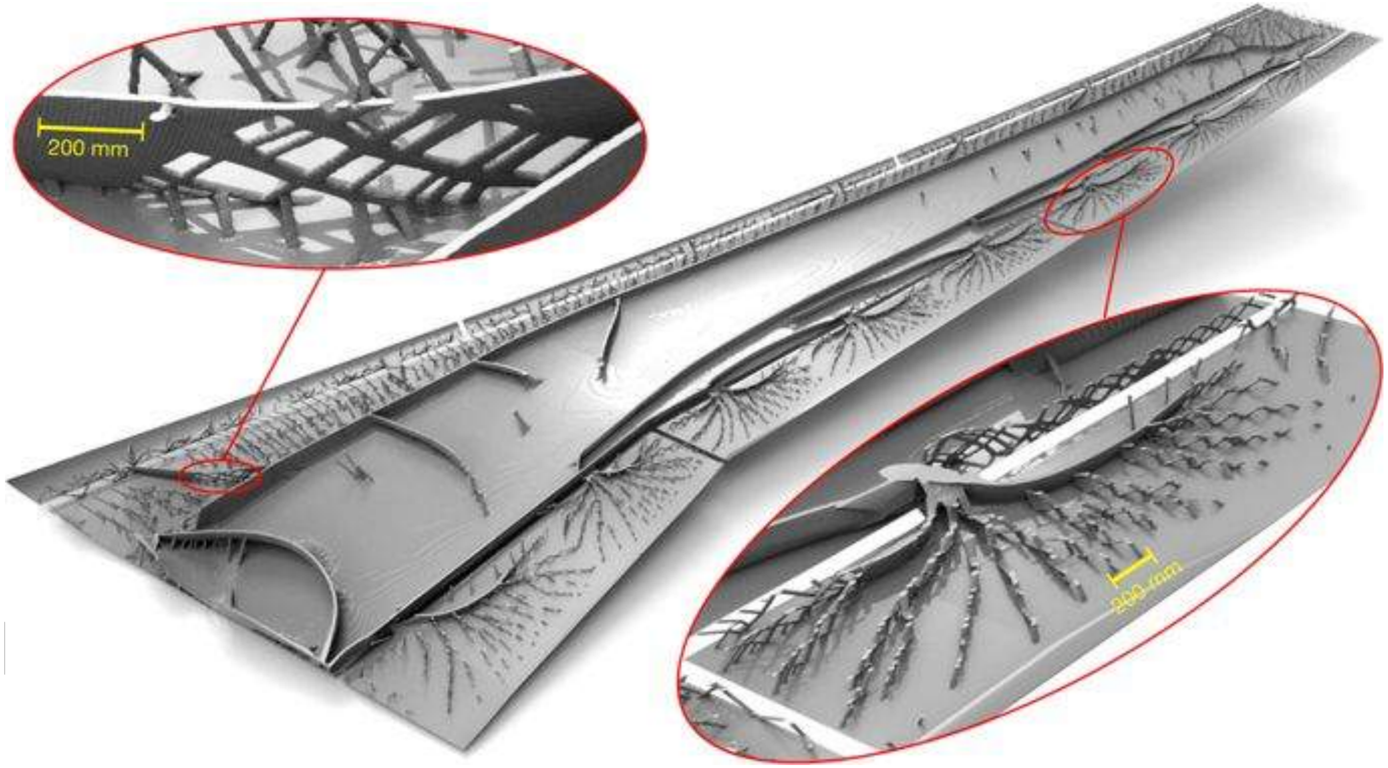
The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

Division into elements (pixels or voxels) and binary decision for each or example 10,000 elements --> 210,000 possible configurations!

Pixels?

When the size of the FE model is increasing, the SIMP optimization problem is … increasing

Niels Aage, Erik Andreassen, Boyan S Lazarov, and Ole Sigmund. Giga-voxel computational morphogenesis for structural design. Nature, 550(7674):84, 2017.



Topmna.m: Visiting scholar at UoM (Thanks Prof MARTINS)

Fondation ISAE-SUPAERO Reconnue d'utilité publique — Aerospace Engineering, University of Michigan

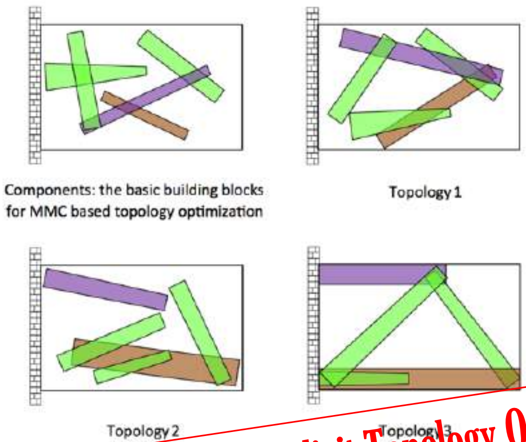
Can we do pattern (structural element) recognition?

Struct Multidisc Optim (2016) 53:1243–1260
DOI 10.1007/s00158-015-1372-3

RESEARCH PAPER

A new topology optimization approach based on Moving Morphable Components (MMC) and the ersatz material model

Weisheng Zhang[1] · Jie Yuan[1] · Jian Zhang[1] · Xu Guo[1]

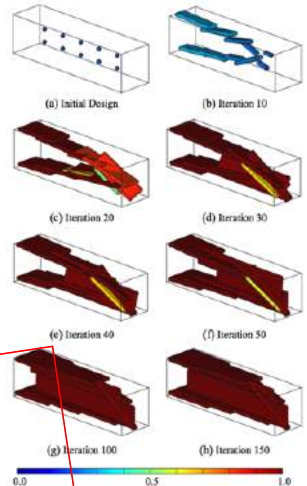


Struct Multidisc Optim
DOI 10.1007/s00158-016-1466-6

RESEARCH PAPER

A geometry projection method for the topology optimization of plate structures

Shanglong Zhang[1] · Julián A. Norato[1] · Arun L. Gain[2] · Naesung Lyu[3]



YES… using Explicit Topology Optimization
And with less design variables !!!! We Speed up the process

Optimization algorithm

Nodal movement

- Optimization algorithm based on decoupling
- Movement of nodes
  - Direction of the movement

- Measure performance of structure by compliance
  - Move mass nodes in the direction of decreasing compliance

$$\min c(\mathbf{x}) = \mathbf{U}^T \mathbf{F} = \mathbf{U}^T \mathbf{K} \mathbf{U}$$

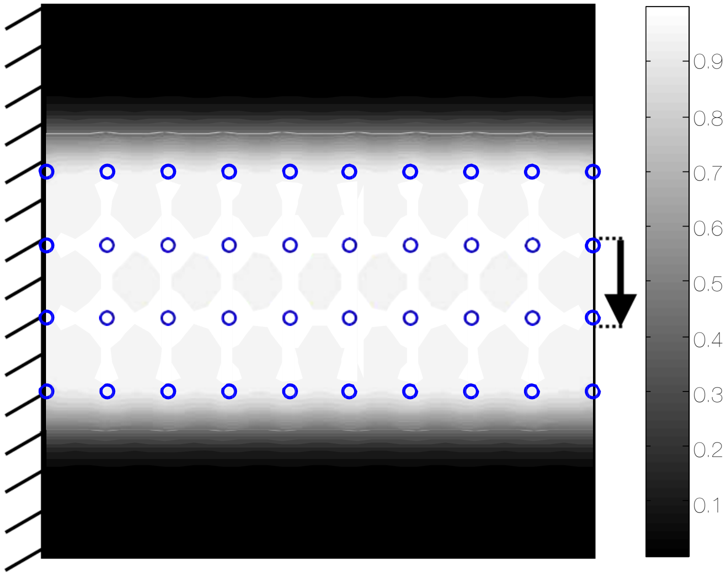
$$\frac{\partial c}{\partial \rho_e} = -p(\rho_e)^{p-1} \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

Original Work →

J. T. Overvelde, "The moving node approach in topology optimization", Master's thesis, TU Delft, Delft University of Technology, 2012.

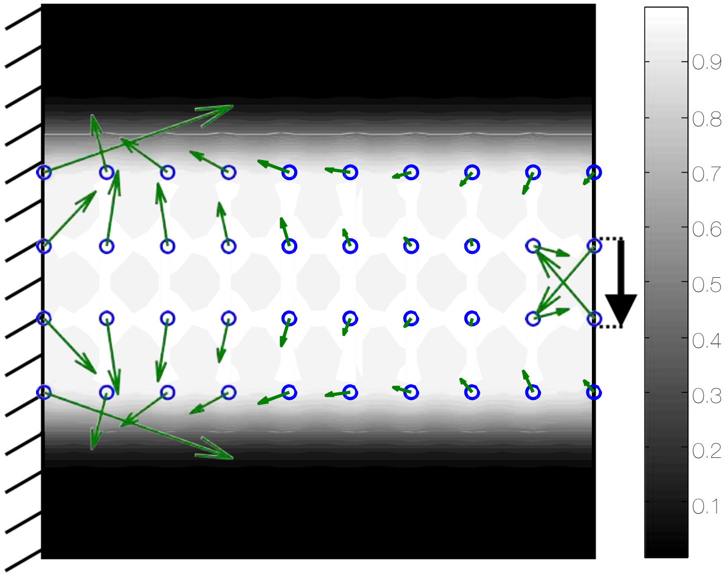
Optimization algorithm

Example of compliance sensitivity



Optimization algorithm

Example of compliance sensitivity



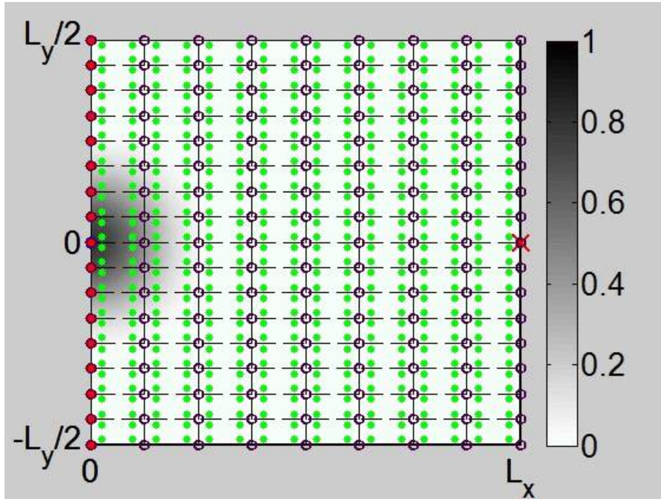
Moving Node Approach (MNA)

- Key idea:

Move material => optimal layout

- Regular discretization = precision

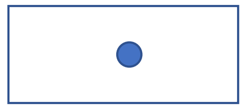
- Material distribution -> mass nodes



The variables (5 per Node)

Optimization variables :

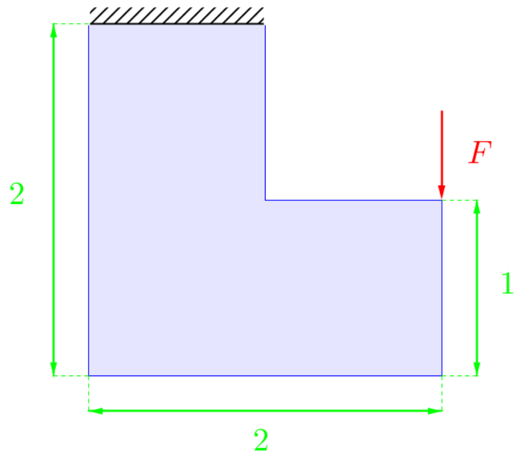
- Positions (x,y)
- Orientation (θ)
- Dimensions (Lx,Ly)

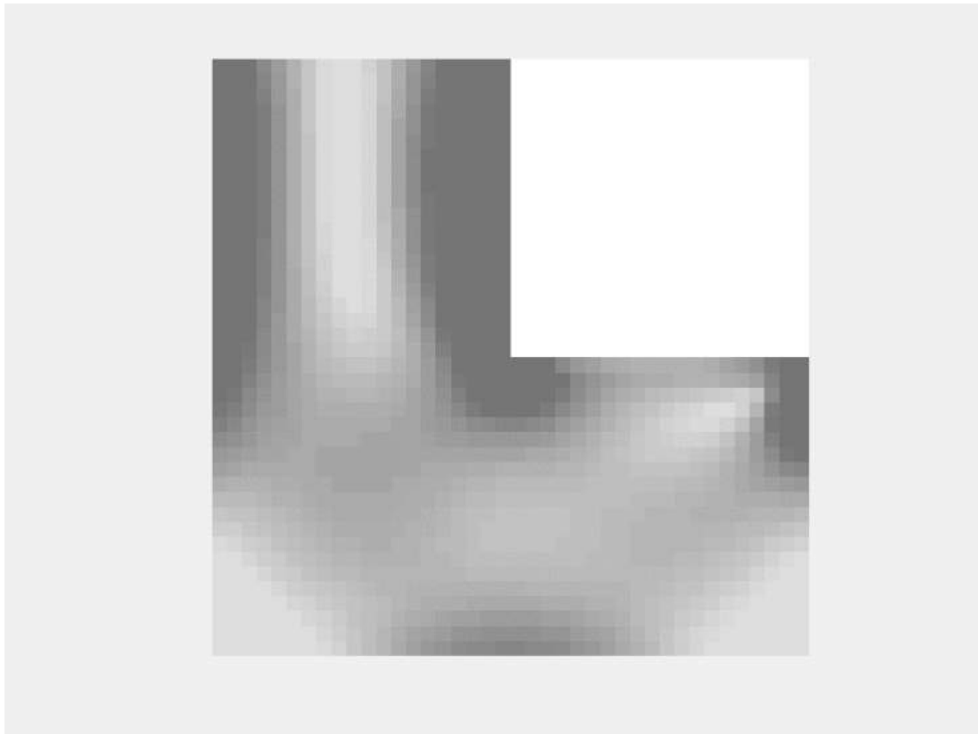


Structural Members: beam's theory

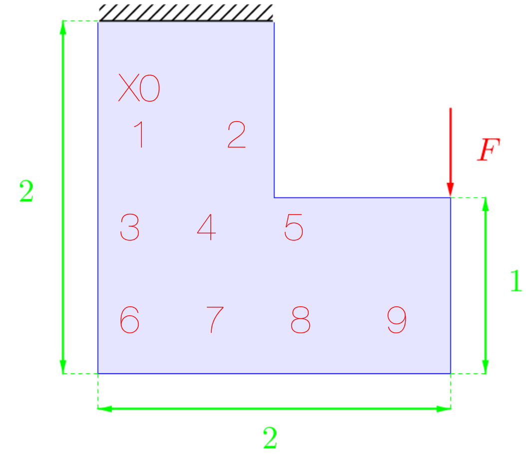
```
disp('SIMP')
top88(nelx,nely,volfrac,3,2,1)

disp('MNA')
topmna(x0,nelx,nely,volfrac,3,[ratio;aspect],tolchange);
```

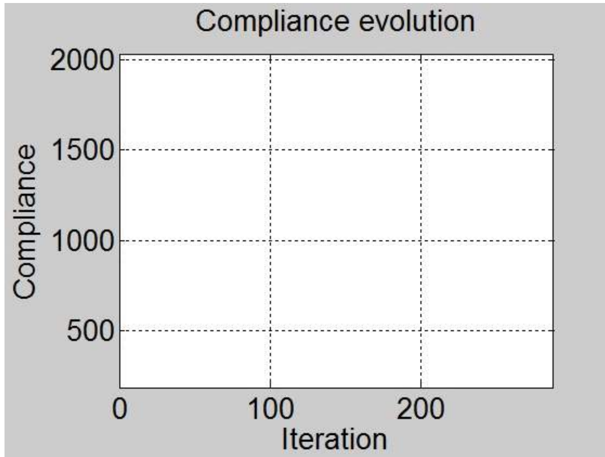
Results SIMP nelx=nely=40→ 1600 design variables
minC st Volfrac=0.25 , Ku=f



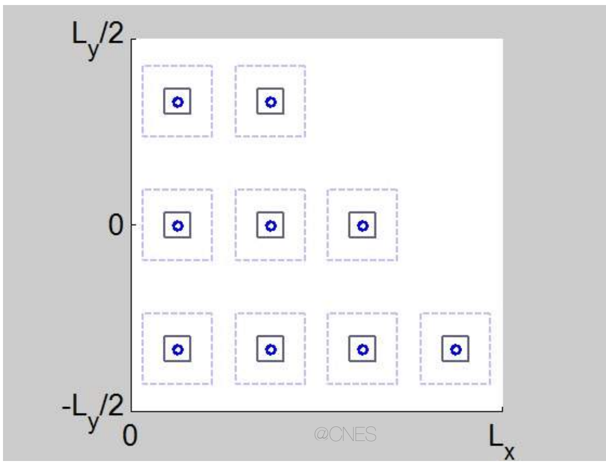


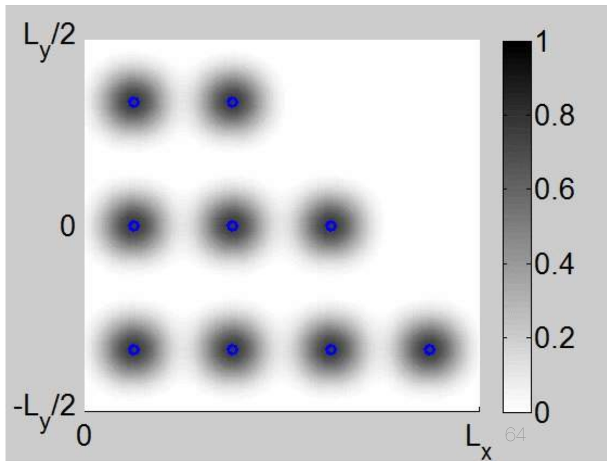
Results MNA, 9*5=45 design variables
minC st Volfrac=0.25 , Ku=f



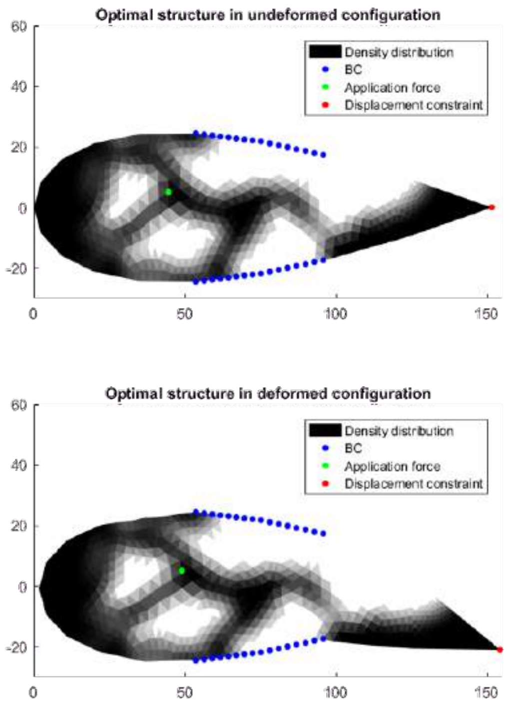


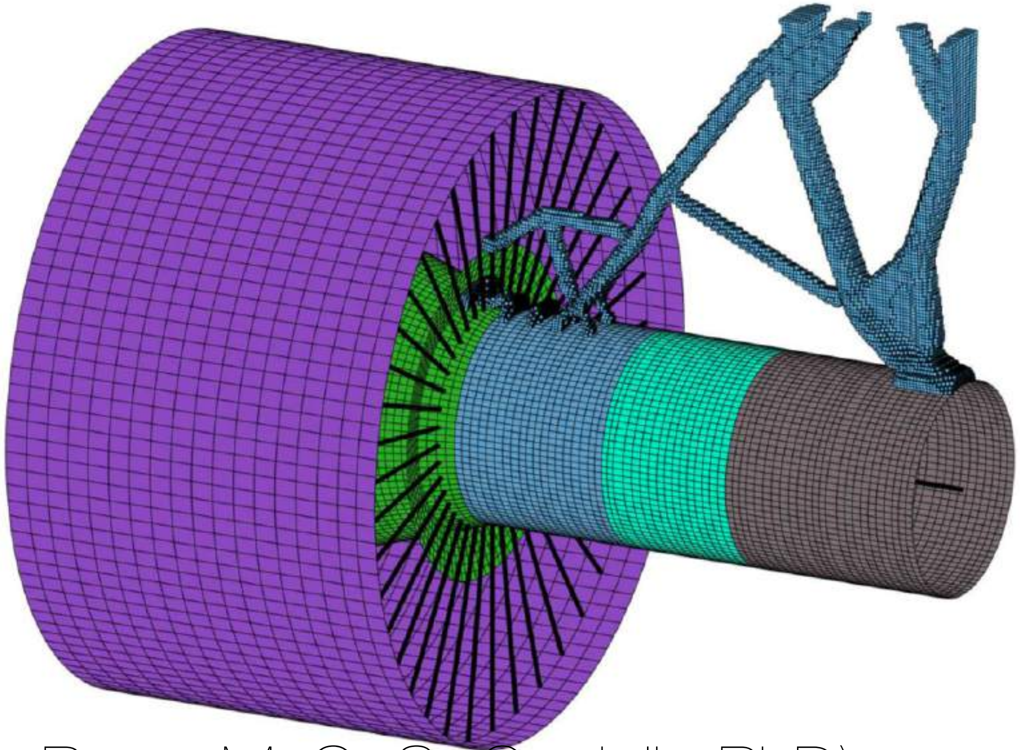
At the end, explicit structural element !





Aeronautical problems → innovative pylon or morphing airfoil





(G. Capasso, M. Herraz, G. Raze, MsC, S. Coniglio PhD)

Papers & conf on this topic

Coniglio, S., Gogu, C., Amargier, R., & Morlier, J. (2017, June). Pylon and engine mounts performance driven structural topology optimization. In World Congress of Structural and Multidisciplinary Optimisation (pp. 1349-1363). Springer, Cham.

Coniglio, S., Gogu, C., & Morlier, J. (2018). Weighted Average Continuity Approach and Moment Correction: New Strategies for Non-consistent Mesh Projection in Structural Mechanics. Archives of Computational Methods in Engineering, 1-29.

Coniglio, S., Morlier, J., Gogu, C., & Amargier, R. (2018). Original Pylon Architecture Design Using 3D HPC Topology Optimization. In 2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference (p. 1388).

Barjhoux, P. J., Diouane, Y., Grihon, S., Bettebghor, D., & Morlier, J. (2017, June). Mixed variable Structural optimization: toward an efficient hybrid algorithm. In World Congress of Structural and Multidisciplinary Optimisation (pp. 1880-1896). Springer, Cham.

Barjhoux, P. J., Diouane, Y., Grihon, S., Bettebghor, D., & Morlier, J. (2018). A Bilevel Methodology for solving a Structural Optimization Problem with both Continuous and Categorical Variables. In 2018 Multidisciplinary Analysis and Optimization Conference (p. 3579).

G. Raze et al, Optimisation topologique sans maillage : vers la reconnaissance d'éléments structuraux, CSMA 2017

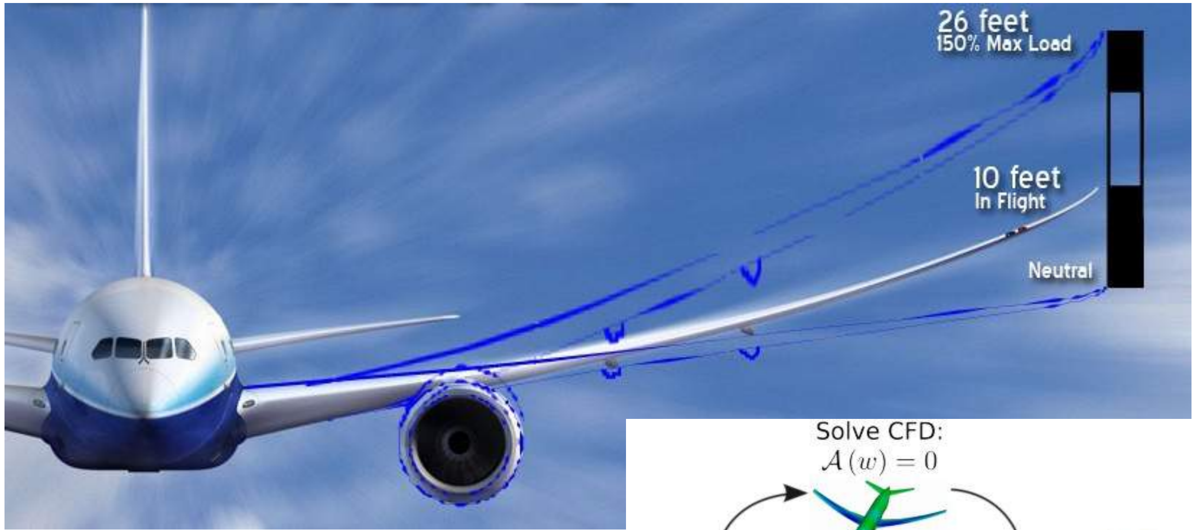
T. Hirshler et al., Analyse Isogéométrique pour les problèmes d'Optimisation de Forme des Structures Coques, , CSMA 2017

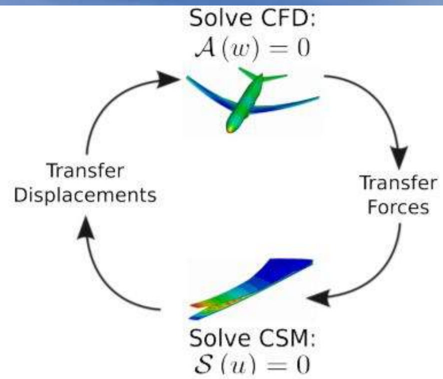
Several Papers in preparation

1. GP aka Kriging
2. Kriging for Global Optimization
3. New developments in topology optimization

4. Codesign is MDO?

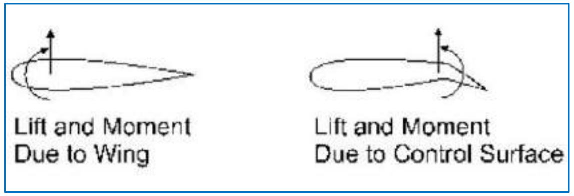
The importance of aerostructural coupling





$\mathcal{A}$: Aerodynamic residuals
w: Aerodynamic states
$\mathcal{S}$: Structural residuals
u: Structural states

+ control law*:

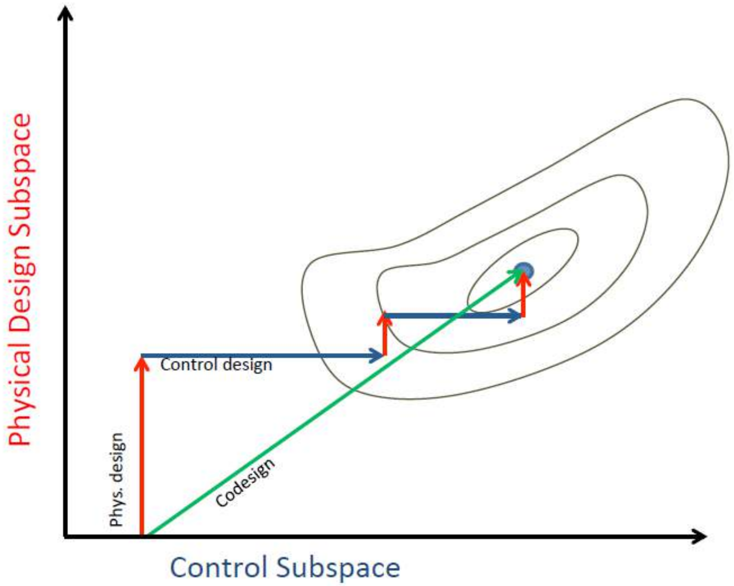


*J. R. Wright, J. E. Cooper. Introduction to Aircraft Aeroelasticity and Loads. 2007.

Co-Design:
Integrated Physical and Control System Design *

Navigate in physical and control design subspaces simultaneously.

➞ Tailor structural/mechanical/control system designs: system optimality

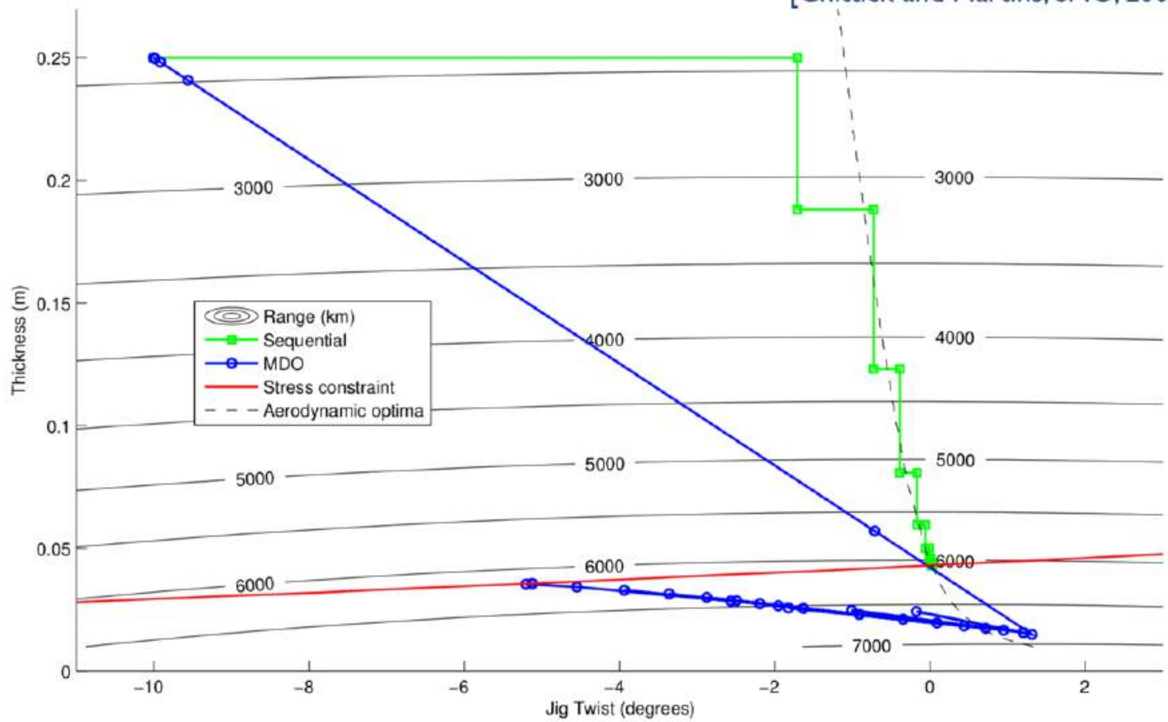


Deshmukh, A. P., & Allison, J. T. (2016). Multidisciplinary dynamic optimization of horizontal axis wind turbine design. Structural and Multidisciplinary Optimization, 53(1), 15-27.

Sequential vs MDO

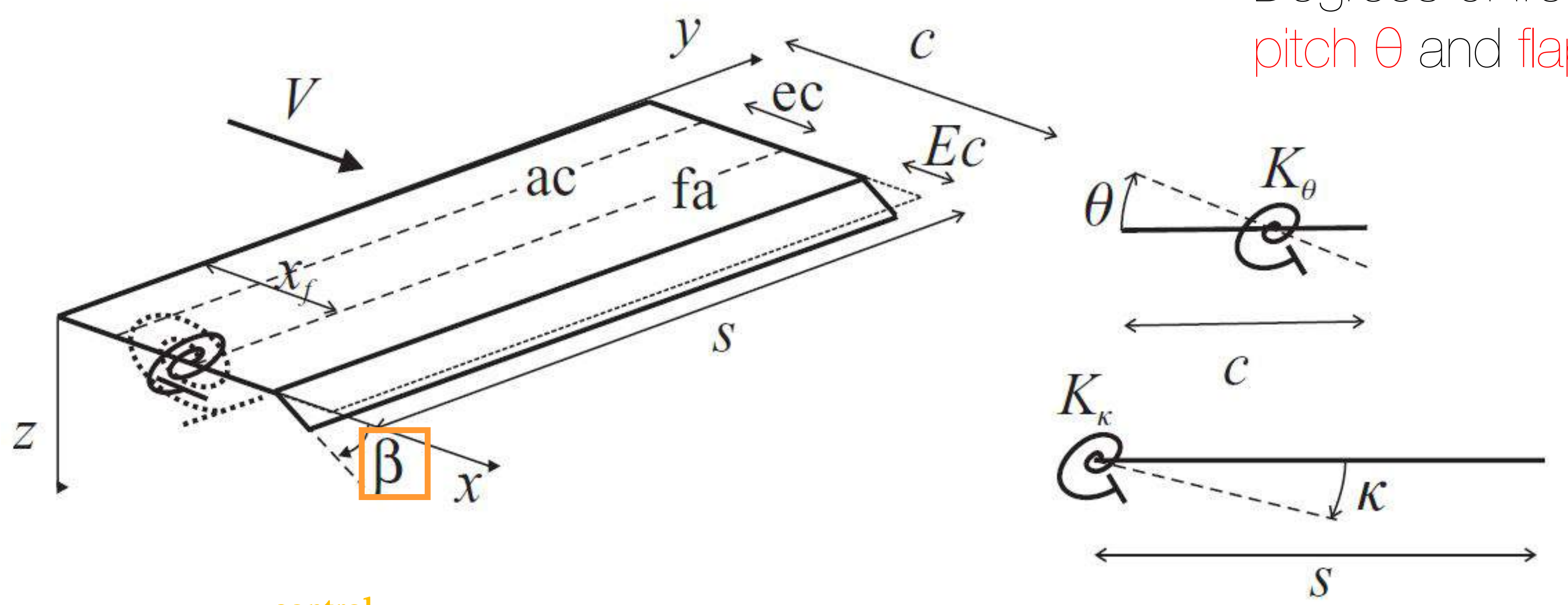
Example: Aerostructural Optimization — Sequential Design vs. MDO 5

[Chittick and Martins, SMO, 2008]



Chittick, I. R., & Martins, J. R. (2008). Aero-structural optimization using adjoint coupled post-optimality sensitivities. Structural and Multidisciplinary Optimization, 36(1), 59-70.

Example: Wing model* (G. Fillipi MsC)



Degrees of freedom:
pitch θ and flap κ

control
surface
angle

*J. R. Wright, J. E. Cooper. Introduction to Aircraft Aeroelasticity and Loads. 2007.

Mathematical modelling → State space modelling

soved with Direct Transcription Method

$$\boxed{\begin{bmatrix} I_k & I_{k\vartheta} \\ I_{k\vartheta} & I_\vartheta \end{bmatrix}}\begin{Bmatrix} \ddot{k} \\ \ddot{\vartheta} \end{Bmatrix} + \rho V \boxed{\begin{bmatrix} \dfrac{cs^3 a_w}{6} & 0 \\ -\dfrac{c^2 s^2 e a_w}{4} & -\dfrac{c^3 s}{8} M_\vartheta \end{bmatrix}}\begin{Bmatrix} \dot{k} \\ \dot{\vartheta} \end{Bmatrix} + \left(\rho V^2 \boxed{\begin{bmatrix} 0 & \dfrac{cs^2 a_w}{4} \\ 0 & -\dfrac{c^2 s e a_w}{2} \end{bmatrix}} + \boxed{\begin{bmatrix} K_k & 0 \\ 0 & K_\vartheta \end{bmatrix}} \right)\begin{Bmatrix} K \\ \vartheta \end{Bmatrix} = \rho V^2 cs \begin{Bmatrix} -\dfrac{s a_c}{4} \\ \dfrac{c b_c}{2} \end{Bmatrix}\boxed{\beta} + \rho V cs \begin{Bmatrix} \dfrac{s}{4} \\ \dfrac{c}{2} \end{Bmatrix}\boxed{w_g}$$

structural inertia

aerodynamic damping

aerodynamic stiffness

structural stiffness

control surface angle

gust term

$$A\ddot{q} + \rho V B \dot{q} + (\rho V^2 C + E) q = g\beta + h w_g$$

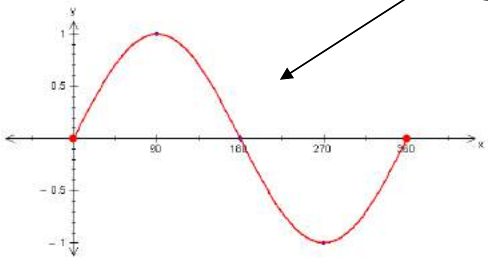
Objective function (multiObj → monoObj to minimize)

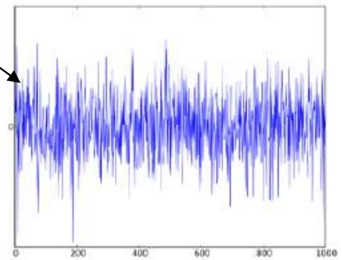
r_i

handling + comfort + control cost

$$J = \int_0^{t_F} (r_1 z^2 + r_2 \ddot{z}^2 + r_3 u^2) dt$$

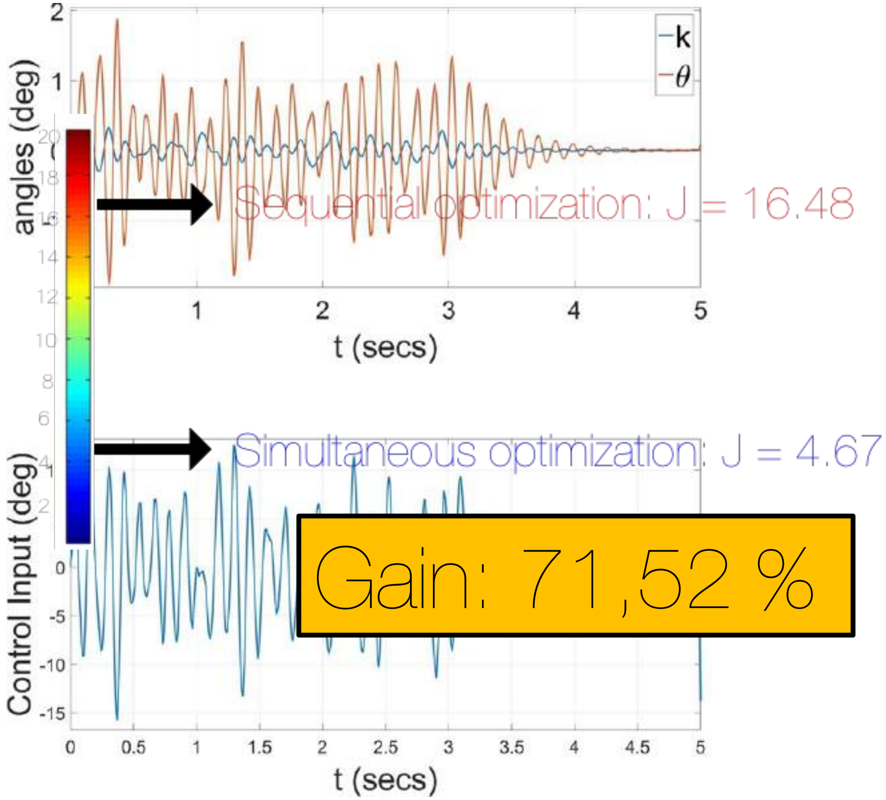
$$J_{tot} = J_{gust} + J_{turb}$$





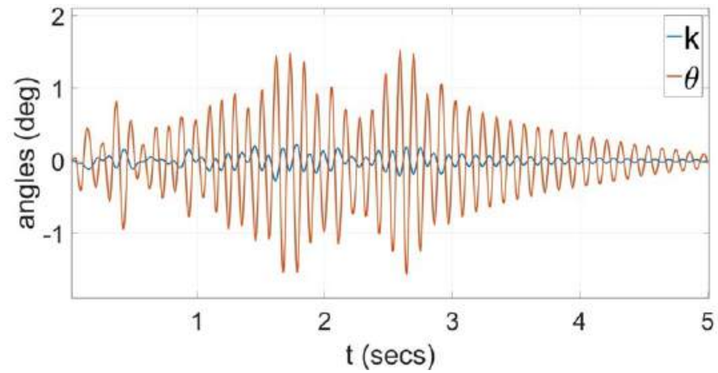
System response (Gust+Random)

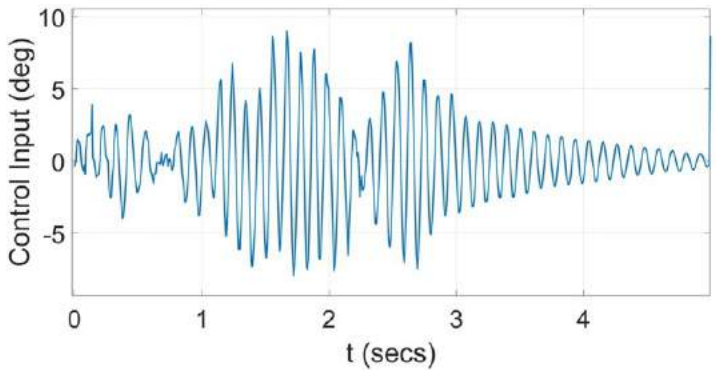
Response to Random turbulence



sequential

Response to Random turbulence





vs simultaneous optimization

Conclusions

- New Surrogates and SBO for an automated optimal design process (industrial constraints)
- **ONERA-SUPAERO** SEGOMOE offers a new solution for global optimization for a constrained problem (up to 100 design variables and several hundred of constraints)
- Opensource solutions due to collaboration with Nasa, University of Michigan and ONERA
- Researches in Structural optimization explore hybrid optimization

→continuous/discrete/categorial variables)

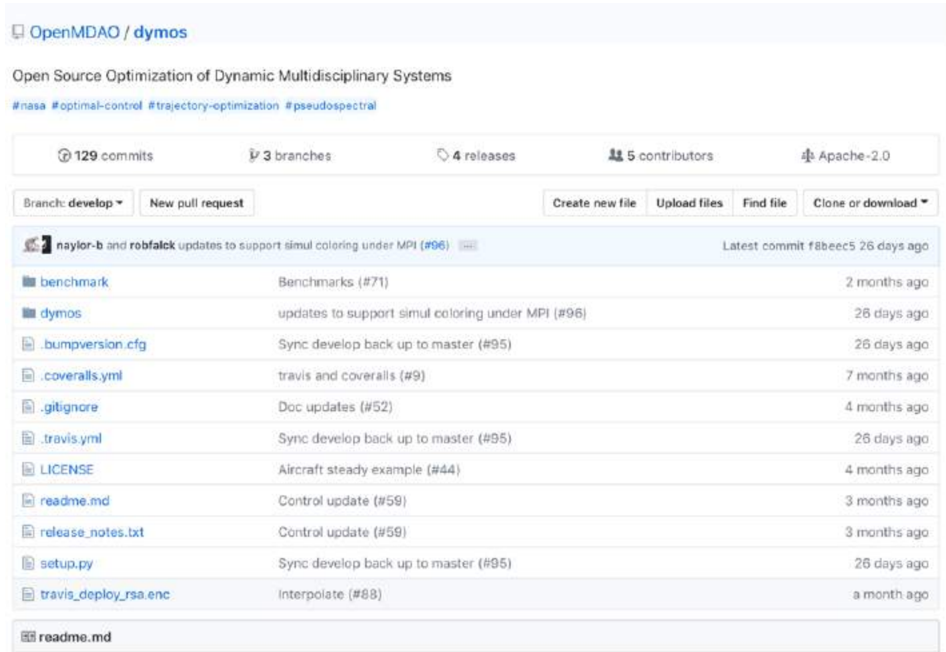
→Stress constraints aggregation (upto Millions FE)

- topmna.m, A new step toward an explicit topology optimization (speed up the CAD/CAE)

Codesign is one of the current trend in MDO

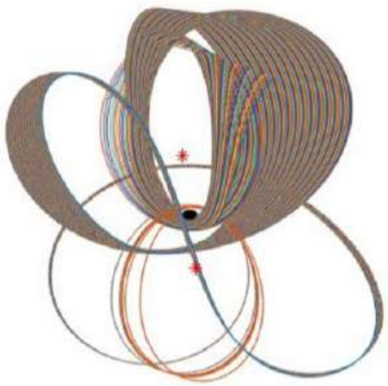
→OpenMDAO/dymos

https://github.com/OpenMDAO/dymos



New collaboration, new PhD (L. Beauregard)

Codesign is also a current trend at SUPAERO for optimal design of reusable launcher (design+ Optimal control of Trajectory)



https://github.com/mid2SUPAERO
Prof J. Morlier's group

- NB: Since 2013 new course at SUPAERO : MDO [Structural&Multidisciplinary Design Optimization, 2*30H] (MsC level] with ONERA/AIRBUS

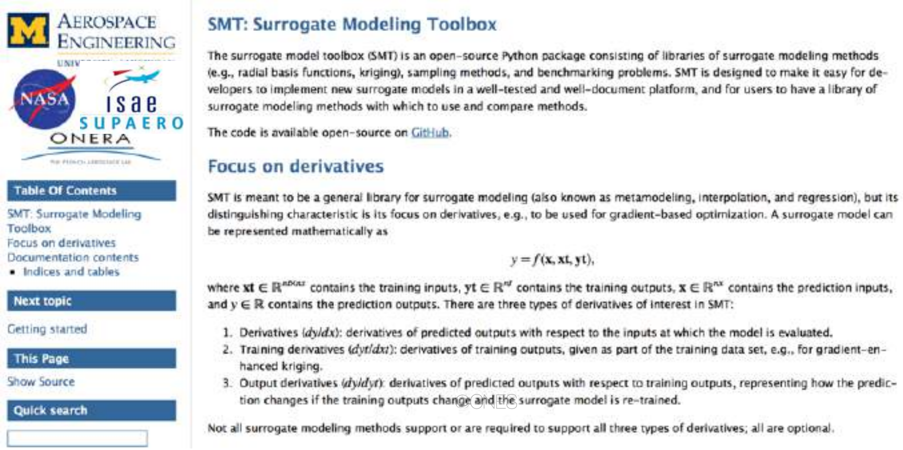
- Since 2017 we offer some fund to students to do research with us in order to be « PhD ready ». Part of this presentation has been made by SUPAERO MsC Students:

Mostafa Meliani, Mahfoud Herraz, Gabriele Capasso, Ghislain Raze, Giovane Filippi etc…

Please Visit :
https://github.com/SMTorg/SMT
https://github.com/mid2SUPAERO for student's project

- Thanks to My co-workers: Joaquim Martins, Nathalie Bartoli, Thierry Lefevbre, Emmanuel Benard, Claudia Bruni, Emmanuel Rachelson, Nicolas Gourdain, John Hwang, Mohamed Bouhlel, Peter Schmolgruber, Youssef Diouane, Sylvain Dubreuil, Christian Gogu, Stephanie Lisy-Destrez and PhDs Pierre-Jean Barjhoux, Simone Coniglio, Elisa Bosco, Joan Mas Colomer, Ankit Chiplunkar, Alessandro Sgueglia, Laurent Beauregard , Romain Olivanti, Remy priem. At Airbus: S. Grihon, A Gazaix, M. Colombo, R. Amargier, S. Trapier, A. Luccheti, F. Vetrano ….

Surrogate modeling in HD, focus on derivatives


SMT: Surrogate Modeling Toolbox

The surrogate model toolbox (SMT) is an open-source Python package consisting of libraries of surrogate modeling methods (e.g., radial basis functions, kriging), sampling methods, and benchmarking problems. SMT is designed to make it easy for developers to implement new surrogate models in a well-tested and well-document platform, and for users to have a library of surrogate modeling methods with which to use and compare methods.

The code is available open-source on GitHub.

Focus on derivatives

SMT is meant to be a general library for surrogate modeling (also known as metamodeling, interpolation, and regression), but its distinguishing characteristic is its focus on derivatives, e.g., to be used for gradient-based optimization. A surrogate model can be represented mathematically as

$$y = f(\mathbf{x}, \mathbf{xt}, \mathbf{yt}),$$

where $\mathbf{xt} \in \mathbb{R}^{nt \times nx}$ contains the training inputs, $\mathbf{yt} \in \mathbb{R}^{nt}$ contains the training outputs, $\mathbf{x} \in \mathbb{R}^{nx}$ contains the prediction inputs, and $y \in \mathbb{R}$ contains the prediction outputs. There are three types of derivatives of interest in SMT:

1. Derivatives (dy/dx): derivatives of predicted outputs with respect to the inputs at which the model is evaluated.
2. Training derivatives (dyt/dxt): derivatives of training outputs, given as part of the training data set, e.g., for gradient-enhanced kriging.
3. Output derivatives (dy/dyt): derivatives of predicted outputs with respect to training outputs, representing how the prediction changes if the training outputs change and the surrogate model is re-trained.

Not all surrogate modeling methods support or are required to support all three types of derivatives; all are optional.

Thanks

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Community of Experts - CYB
Cybersecurity