On some topology optimization recipes

{Stiffer, Lighter, Greener}

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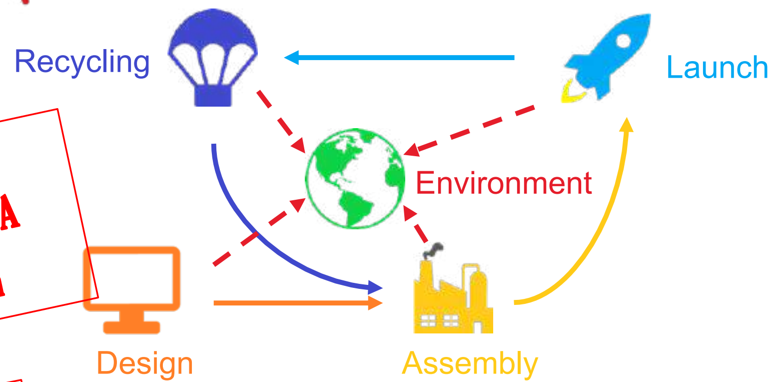
ICA cnrs dépasser les frontières

isae Institut Supérieur de l'Aéronautique et de l'Espace SUPAERO

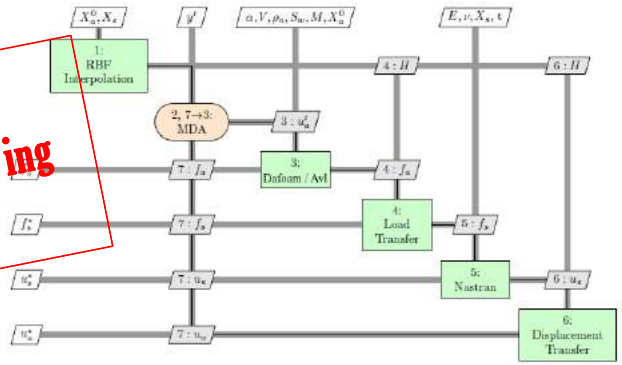
About Me? https://ica.cnrs.fr/en/author/jmorlier/

- 6 PhDs, 3 MsCs

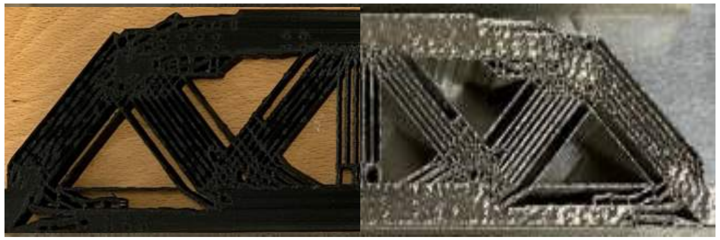
MDO for Aerospace systems Including LCA := EcoOptimization



MDO for Aeroelastic Wing design

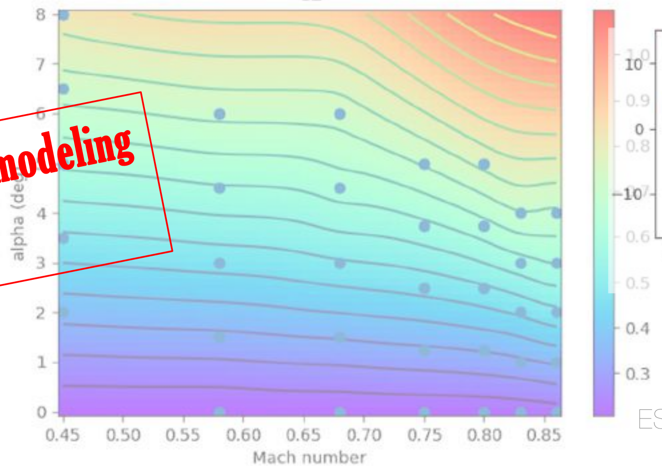


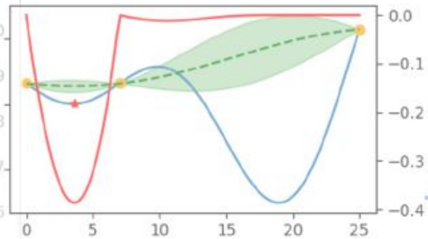
Digital fabrication



https://github.com/SMTorg/SMT

Surrogate modeling AI4E





On the road to design optimization

https://medium.com/daptablog/on-the-road-to-design-optimisation-a3c9867f29b6

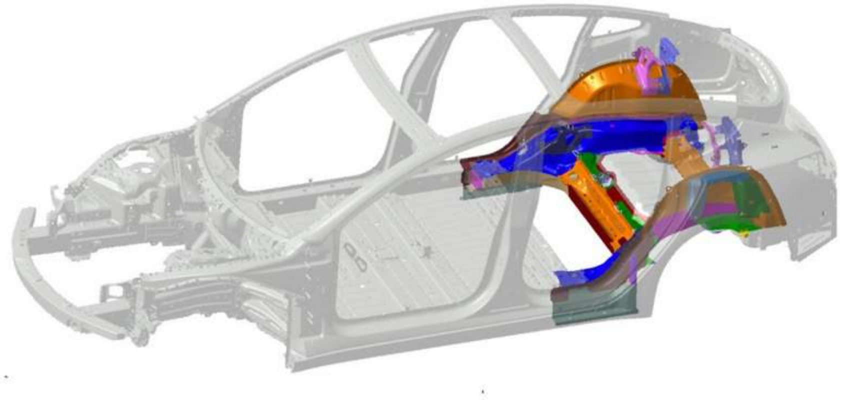
- Optimization
  noun [U] (UK usually **optimisation**)
  the act of making something as good as possible
  (Cambridge Dictionary)

Do Better with Less

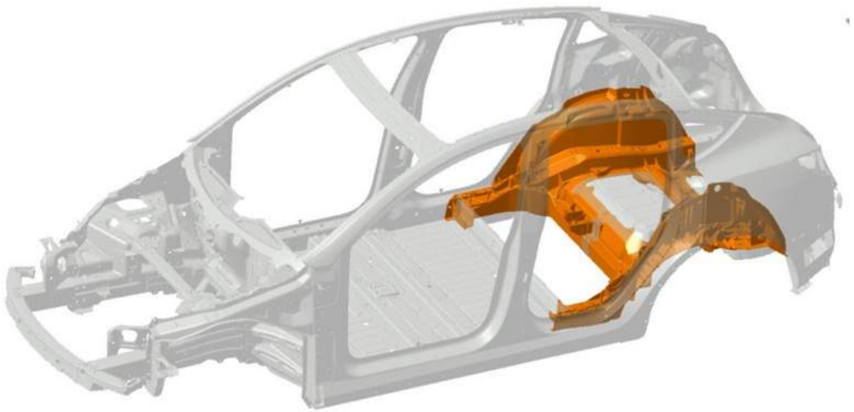
- Design optimization is an engineering design methodology **using a mathematical formulation of a design problem** to support selection of the optimal design among many alternatives. (Wikipedia)

Think different!

Do Better with Less



Model 3 rear underbody
70 pieces of metal

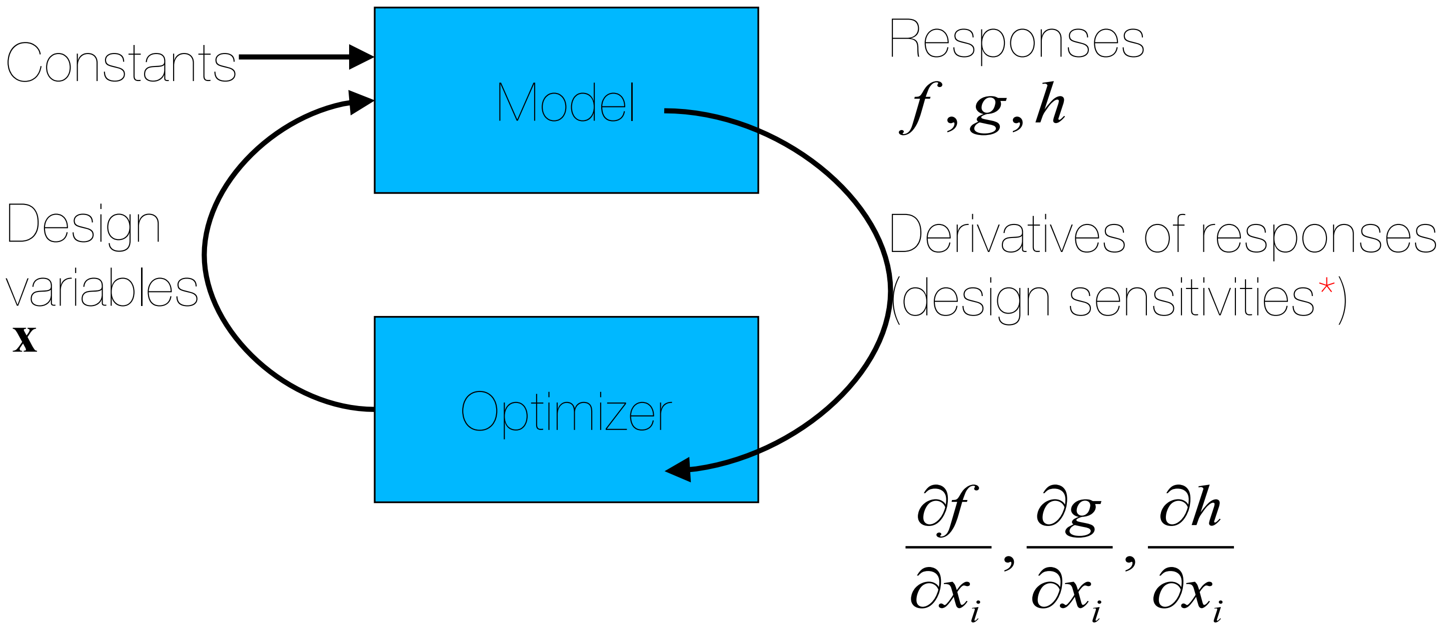


Model Y rear underbody
2 pieces of metal (eventually a single piece)

The use of 3D printing → enable the reduction of subassemblies (form 70 to 1)
Tesla talk about digital casting technology but this is a Design / Topology Optimization problem !!!

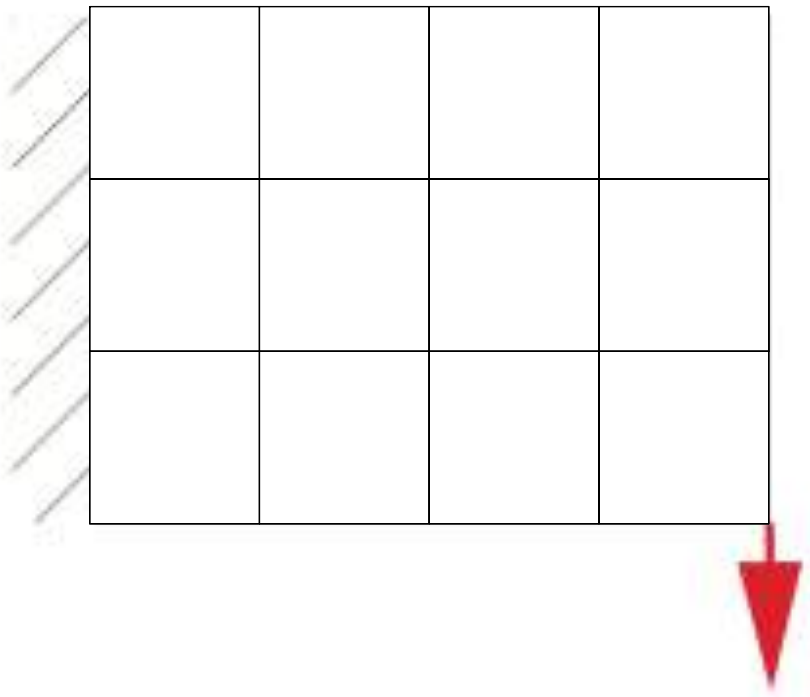
Gradient Based Optimization

- Costly if Finite Differences is used for sensitivities
- Difficult to implement Adjoint in industrial code
- Sensitive to discontinuity
- Sensitive to X_0



*SOL200 in MSC Nastran for example

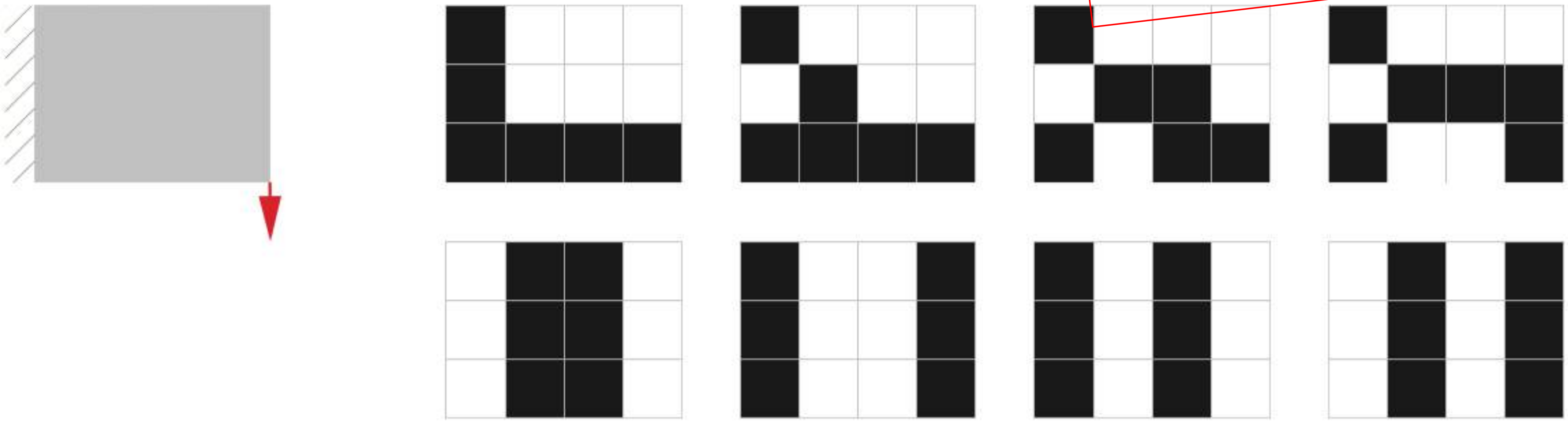
Quiz ! draw material (black) or void (white)



Goal: 50%
gain in mass

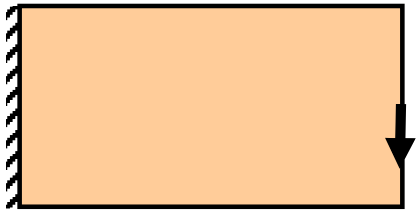
Results

discrete or continuous variables ?



The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

TopOpt



$Ku = f$

1. Objective?
2. Constraints?
3. Method?

Minimize Compliance

Volume Constraint

$$\boxed{\begin{array}{l}\text{Minimize } J \\ \text{Vol.Frac} \leq 0.5\end{array}}$$

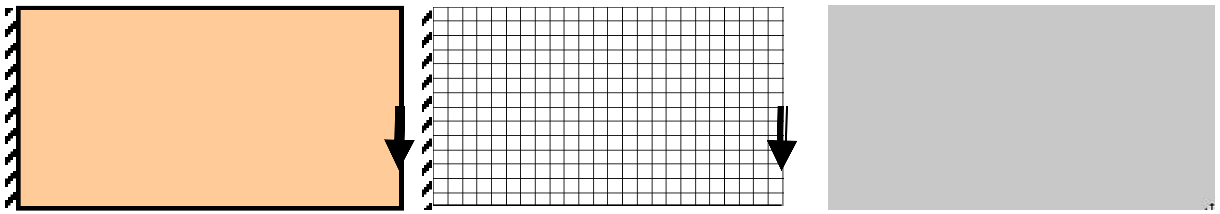
Compliance $J = f^T u$

Method: Gradient based:
Need sensitivities…

Compliance = 1/Stiffness

SIMP

SIMP: Solid Isotropic Material with Penalization



Min Compliance

$v = 0.5v_0$

Where do we add holes?

$0 < \rho_e \leq 1$: 'PseudoDensity'

$$\underset{\rho_e}{Min} \text{ Compliance}$$

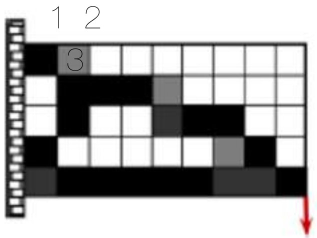
$$\sum \rho_e v_e = 0.5v_0$$

Pixels?

When the size of the FE model is increasing, the SIMP optimization problem is … increasing

Chris Columbus et al, Pixels, movie 2015

Intuitive Problem? Quadratic Form



$x_1 = 1$
$x_2 = 0.5$
$x_3 = 0$
...

- Objective function; Strain energy

$$\min c(\mathbf{x}) = \mathbf{U}^T\mathbf{F} = \mathbf{U}^T\mathbf{K}\mathbf{U}$$

with $x_e = \frac{\rho_e}{\rho_0}$ (4)

with $\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^{N} x_e^p$ one can write:

$$\min c(\mathbf{x}) = \sum_{e=1}^{N} (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

Scalar

(5)

- Contraints: mass target

$$\frac{V(\mathbf{x})}{V_0} = f = const \Leftrightarrow \sum_{e=1}^{N} V_e x_e - V_0 f = 0 = h(\mathbf{x})$$

Scalar

$$0 < \rho_{\min} \leq \rho_e \leq 1$$

K is linked through E and xe

Rozvany, G.I.N. , Zhou, M., and Gollub, M. (1989). Continuum Type Optimality Criteria Methods for Large Finite Element Systems with a Displacement Connstraint, Part 1. Structural Optimization 1:47-72.

$$\mathbf{K} = \mathbf{K}_0 \sum_{e=1}^{N} x_e^p \qquad x_e = \frac{\rho_e}{\rho_0}$$

What is p, (simP)?????????

- But HOW ??

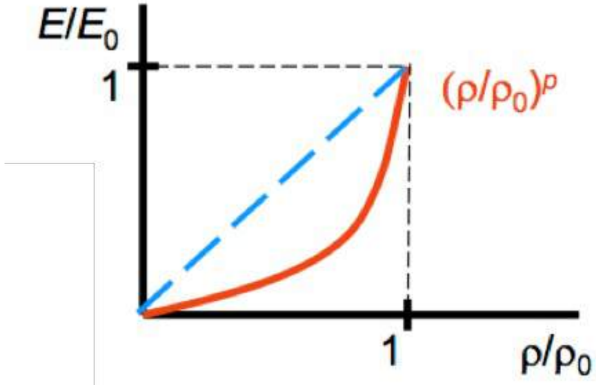
Avoid intermediate densities !

Solid Isotropic Material with Penalization (SIMP)

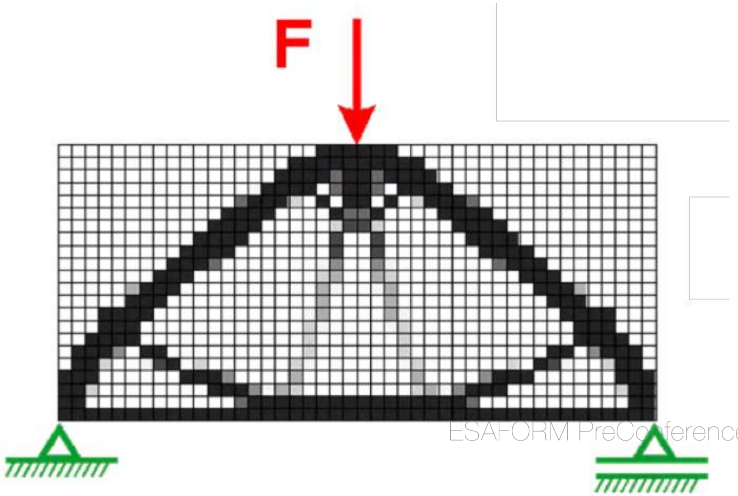
$$E(x) = E_{min} + (E_0 - E_{min})x^p$$

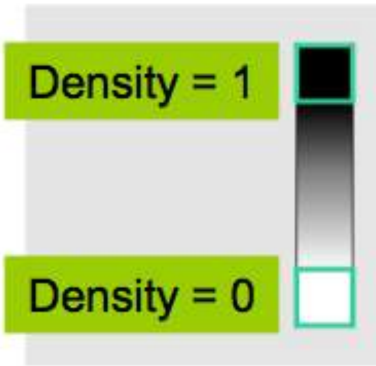
p is the penalty parameter to push densities to black (1) and white (0).

E_{min} is a small value that avoid stiffness matrix singularity



Penalization for altering stiffness localy

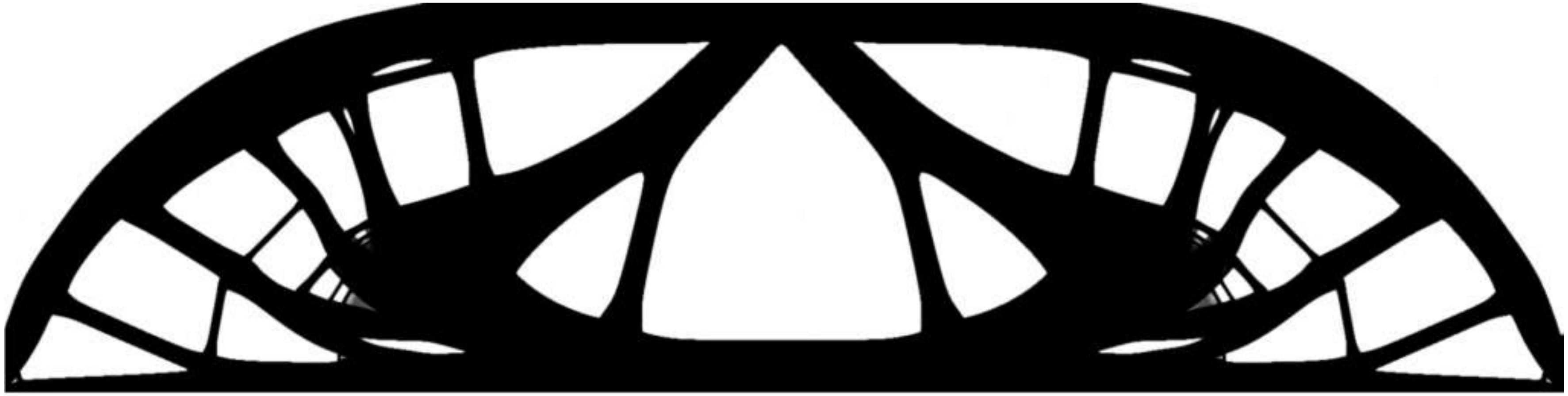




Nice idea !

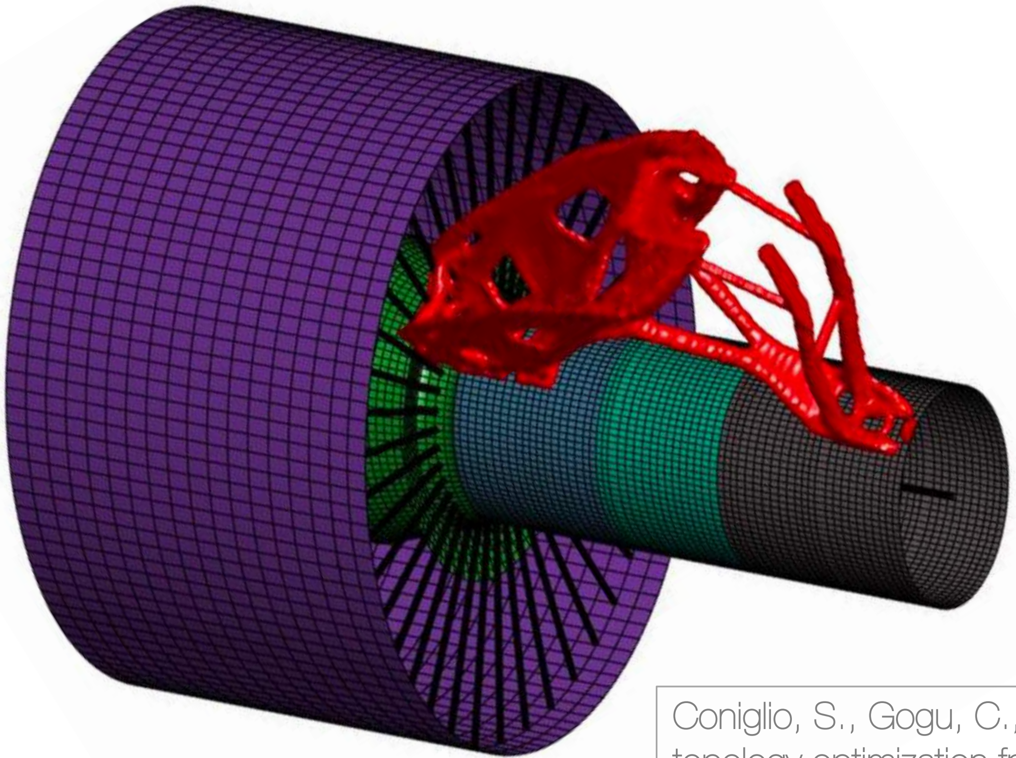
1. Transform discrete variables in continuous ones (→TO USE gradient-based algorithms)
2. Find an objective function with "cheap" derivatives (It is !!! Check TOPOPT textbooks)

BUT PROF… IN PRACTICE… HOW CAN I DO THAT ??



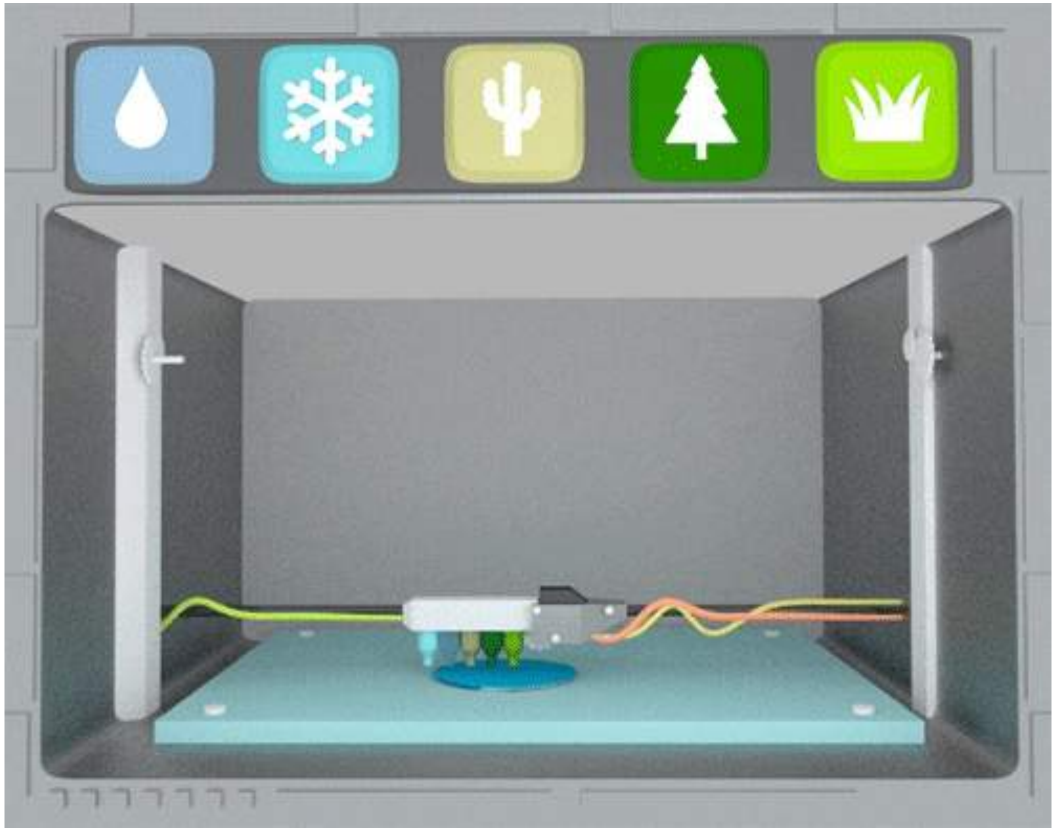
https://www.topopt.mek.dtu.dk/apps-and-software/topology-optimization-codes-written-in-python

Pylon Design



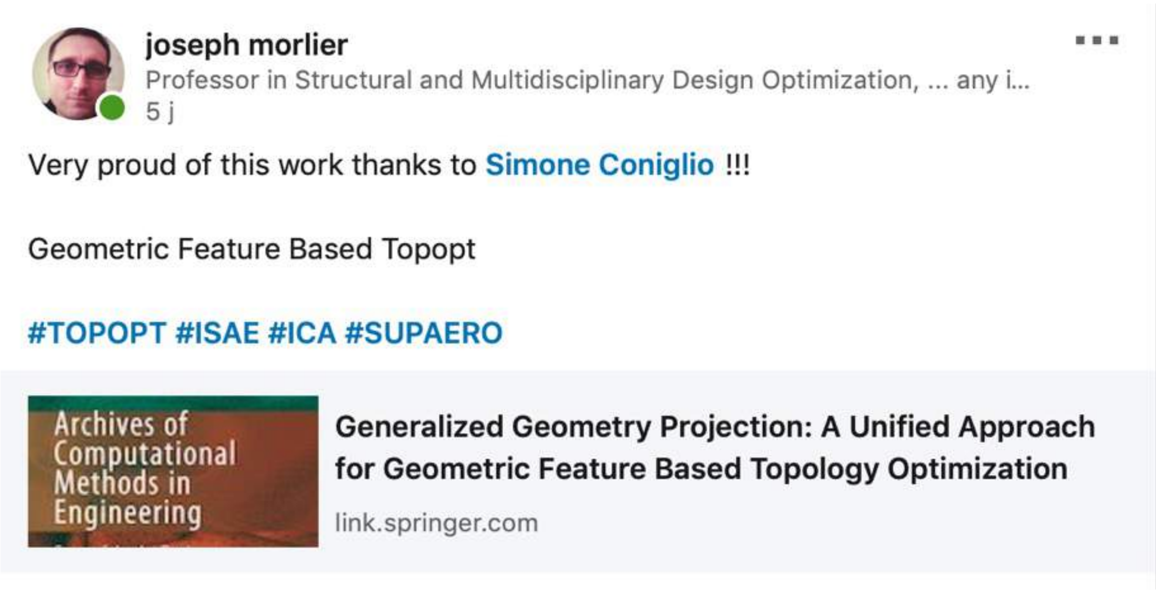
Coniglio, S., Gogu, C., Amargier, R., & Morlier, J. (2019). Engine pylon topology optimization framework based on performance and stress criteria. AIAA Journal, 57(12), 5514-5526.

Can we do **{Stiffer, Lighter, Greener}??**



Source: 3D Printing World Environment Day GIF By General Electric

GGP for AM


joseph morlier
Professor in Structural and Multidisciplinary Design Optimization, ... any i...
5 j

Very proud of this work thanks to Simone Coniglio !!!

Geometric Feature Based Topopt

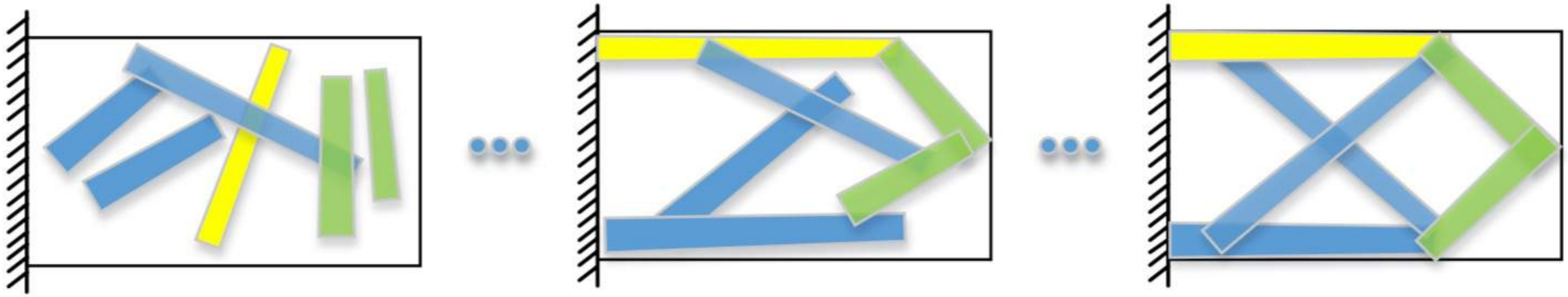
#TOPOPT #ISAE #ICA #SUPAERO

Archives of Computational Methods in Engineering

Generalized Geometry Projection: A Unified Approach for Geometric Feature Based Topology Optimization

link.springer.com

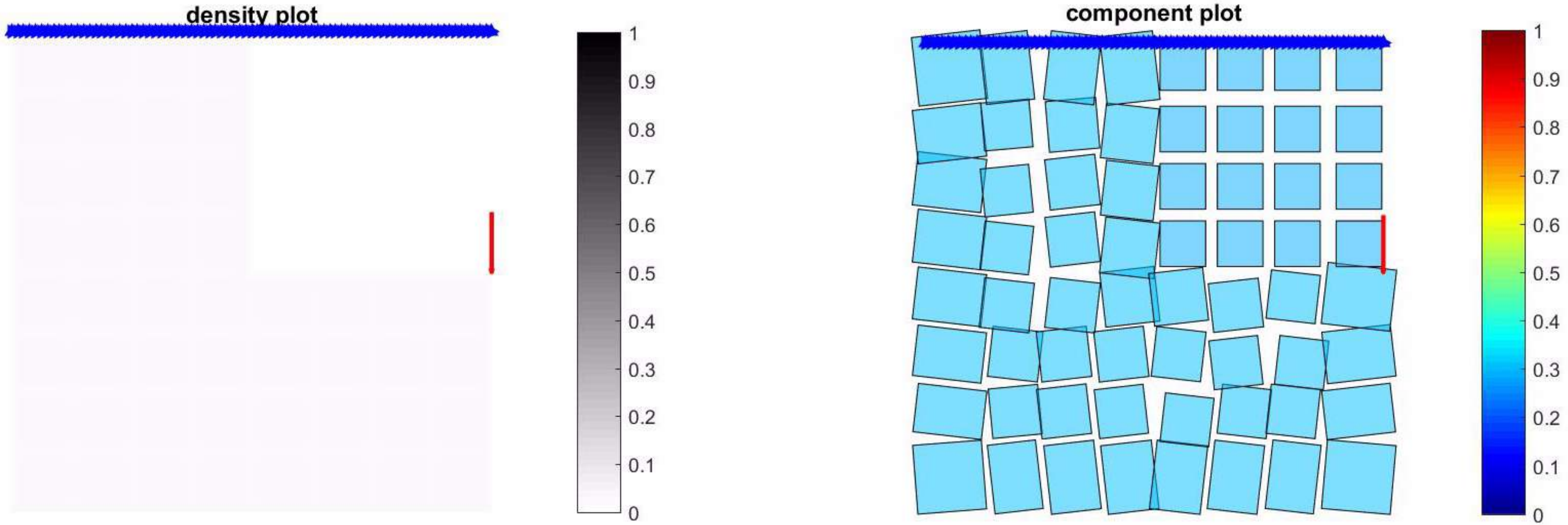
https://github.com/topggp/blog



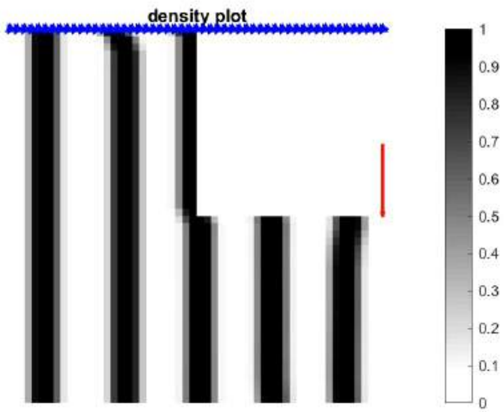
GGP

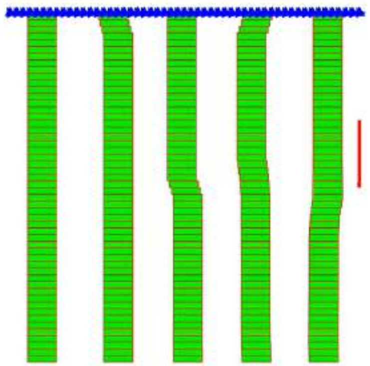
8*8*6=384 design variables
minC st Volfrac=0,4

LEGO style!



Optimizing printing path





CSMA 2019
14ème Colloque National en Calcul des Structures
13-17 Mai 2019, Presqu'île de Giens (Var)

Une approche par projection pour l'optimisation topologique
:tructures imprimées par fabrication additive

ilasraj Bhat[1], S. Coniglio[2], J. Morlier[3], M. Charlotte[4]

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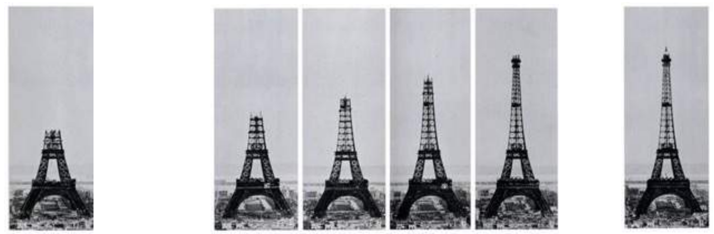
UMR5312, Institut Clément Ader

5 Toulouse Cedex 04, France,

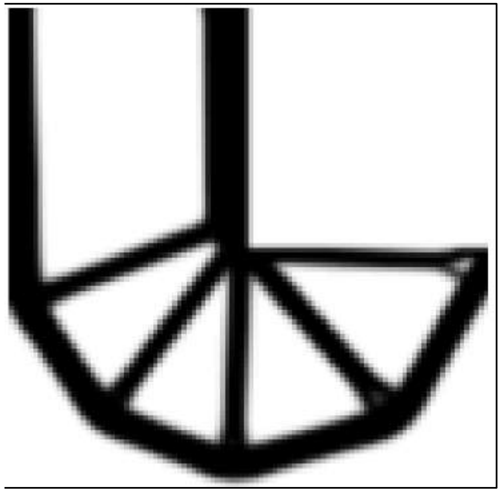
ımé — Ce papier présente une exploration et l'application de méthodes visant à intégrer la fabri-
n additive (FA) à l'optimisation topologique. Les contraintes classiques dites d'overhang sont ap-
ées sans traitement supplémentaire (post processing). Les techniques courantes de post-traitement
ent souvent l'interprétation de la solution (lissage) et des éléments structuraux (poutre, plaque etc...)
: logiciel de post traitement. La méthodologie proposée fournit une expression explicite de la so-
ı, contenant notamment pour les procédés de FA par déposition des informations sur les largeurs
pression, les positions et le nombre de couches de matériaux déposés.

; **clés** — Optimisation topologique, fabrication additive, méthode par dépôt en fusion

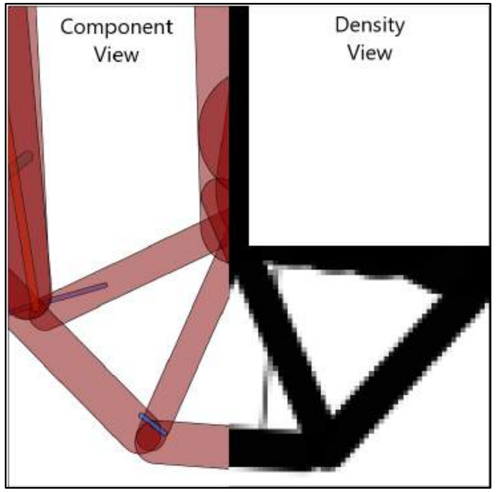
$N_x = N_y = 52$
$v_f = 0.4$
$5\ printing\ components$
$18\ printing\ intervals$
$5{\times}18{\times}2\ design\ variables$



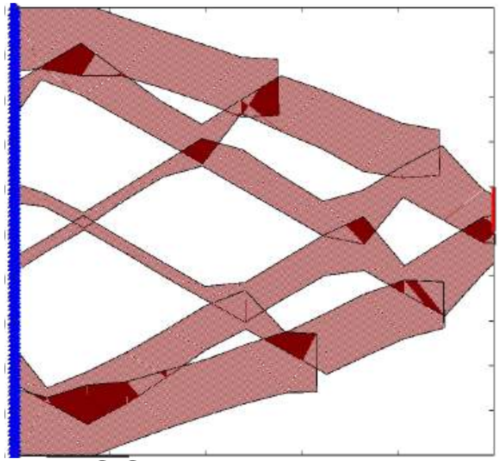
GGP For ALM (DfAM as constraints) ?



SIMP



GGP



GGP for 3D printing

3D Printed part

Krishnaraj Vilasraj Bhat, Gabriele Capasso, Simone Coniglio, Joseph Morlier, Christian Gogu, On some applications of Generalized Geometric Projection to optimal 3D printing, Computers & Graphics,2021,

Au programme

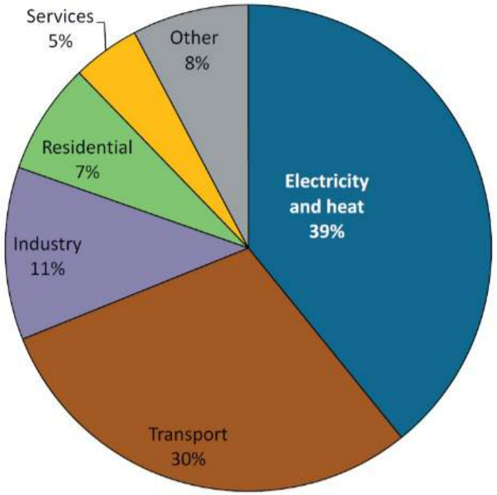
- Part1 :Unit Cell/Material/Process as design variables
- Part2: Ecodesign of 3D volumetric structures with fiber/resin topology optimization
- Conclusions

Au programme

Thanks to Edouard Duriez
+ Miguel Charlotte=
Catherine Azzaro Pantel

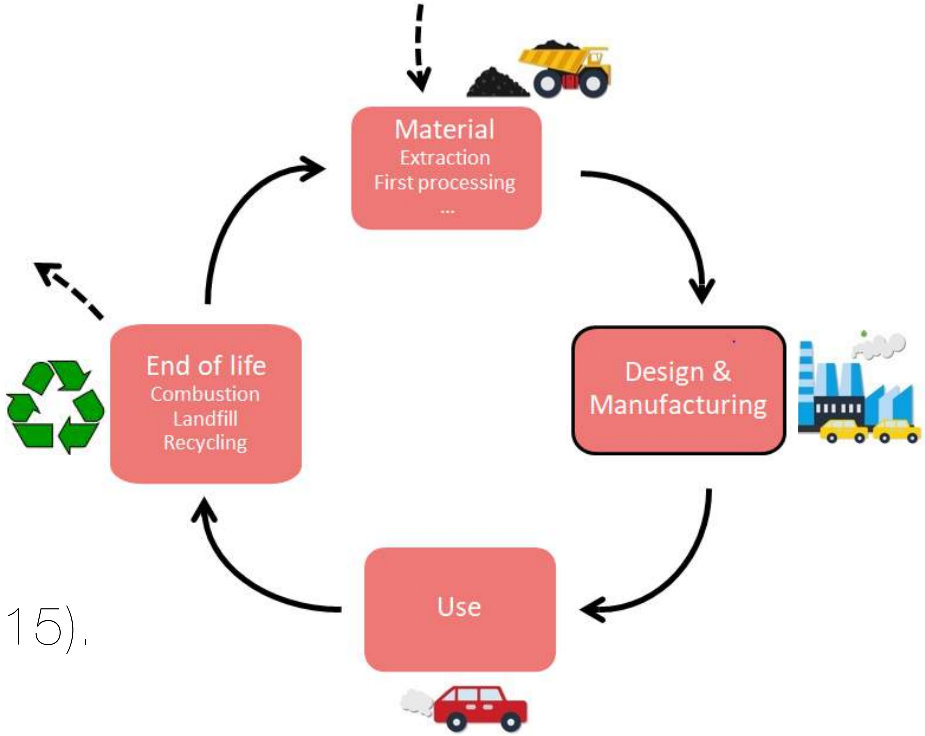
- Part1 :Unit Cell/Material/Process as design variables
- Part2: Ecodesign of 3D volumetric structures with fiber/resin topology optimization
- Conclusions

Main question



CO_2 emissions of the OECD (2015).

International Energy Agency IEA. Energy and CO2 emissions in the OECD. 2017

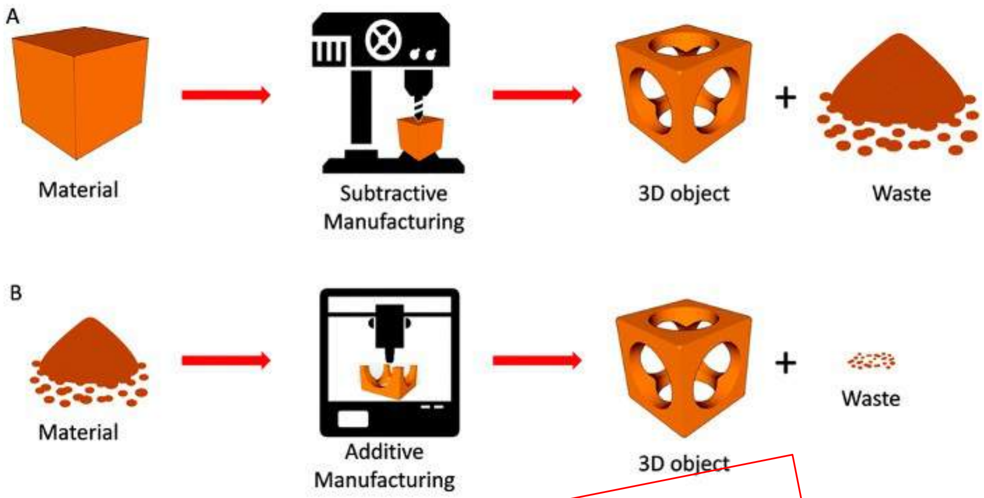


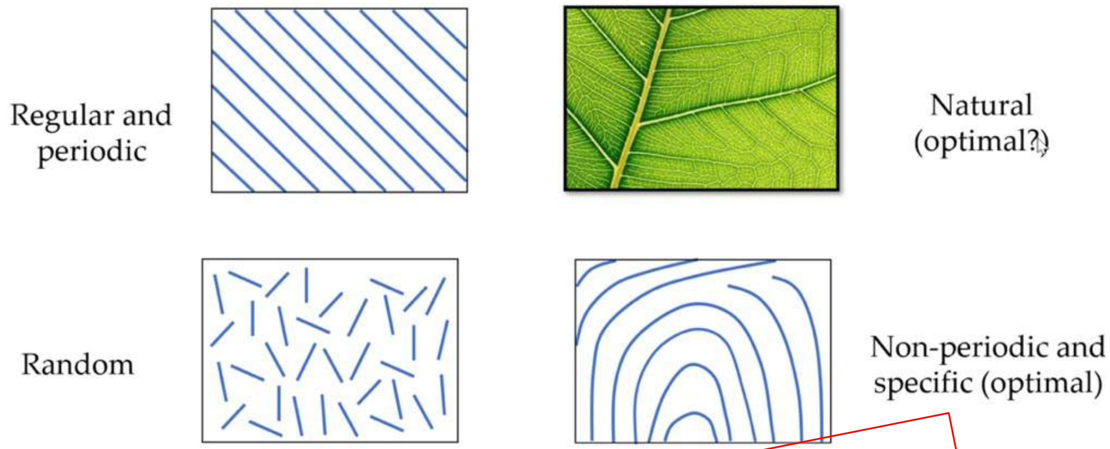
Vehicle life cycle

How to find structural designs, materials and additive manufacturing processes with the lowest life-cycle CO_2 footprint?

Process is AM, but WHY?

https://dfam.substack.com/p/dfam-education-in-2022





+Near 100% material utilization
+Recyclability, Buy to fly ratio
+LCA of 3D printing machine
+Monitoring

+ Automatic Fiber Placement +
eco-fiber/resin selection
+Tailoring / high strength-to-weight ratio

Ecodesign and Additive Manufacturing

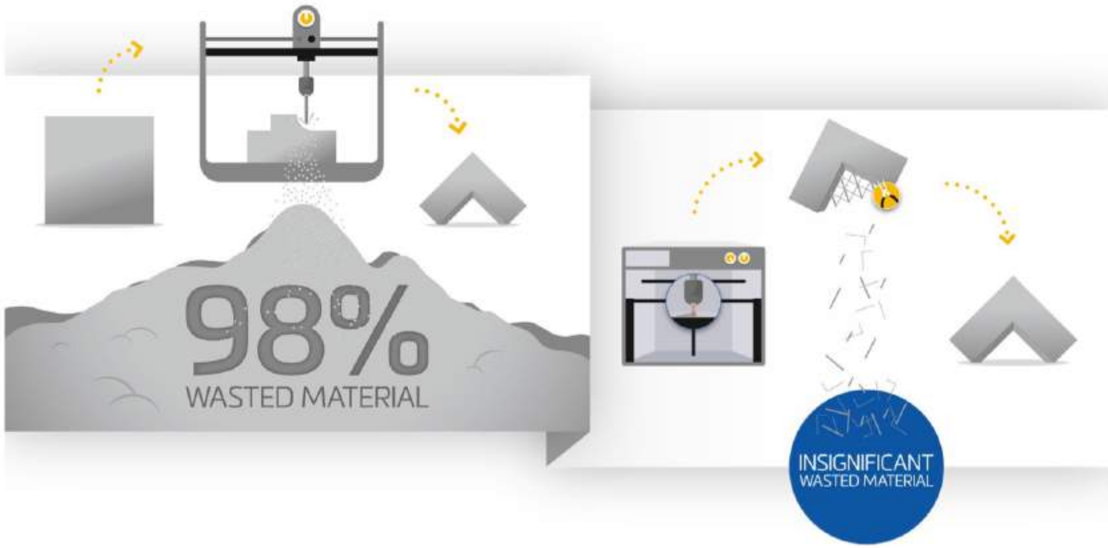
AM environmental opportunities*	AM environmental risks**
Lower mass	Higher specific energy demand
Improved resource efficiency	Longer manufacturing times
Durability (repair/replacement)	Quality issues
Reduced transport	New process (tooling / choices)

*S. Ford and M. Despeisse. « Additive manufacturing and sustainability : an exploratory study of the advantages and challenges ». en. In : J. of Cleaner Production 137 (nov. 2016),
*R. Huang, M. Riddle, D. Graziano, Joshua Warren, Sujit Das, Sachin Nimbalkar, Joe Cresko and Eric Masanet. « Energy and emissions saving potential of additive manufacturing : the case of lightweight aircraft components ». en. In : J. of Cleaner Production 135 (nov. 2016)
**C. Herrmann, W. Dewulf, M. Hauschild, A. Kaluza, S. Kara and S. Skerlos. « Life cycle engineering of lightweight structures ». en. In : CIRP Annals 67.2 (jan. 2018)
**D. Chen, S. Heyer, S. Ibbotson, K. Salonitis, J. G. Steingrímsson and S. Thiede. « Direct digital manufacturing : definition, evolution, and sustainability implications ». en. In : J. of Cleaner Production 107 (nov. 2015)

'Buy to-Fly' Ratio

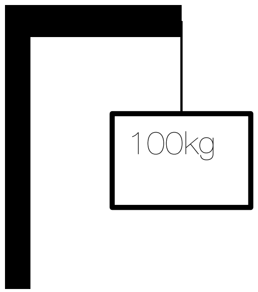
- Traditional subtractive manufacturing machining techniques often result in a costly imbalance between the weight of raw material required to make a specific component, and the weight of the component itself — a relationship more commonly referred to (from its aerospace heritage) as the 'Buy-to-Fly' ratio.

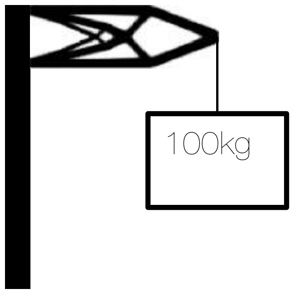
https://www.materialise.com/sites/default/files/resources/Whitepaper_Buy-to-Fly-Ratio_E.pdf



The answer

- CO_2 emissions minimization of parts
  - If material choice is fixed => mass minimization





Other answer ;)

- CO_2 emissions minimization of parts
  - If material choice is free => more complicated
  - → scope of the part of these 2 papers

ELSEVIER

Procedia CIRP
Volume 109, 2022, Pages 454-459

Ecodesign with topology optimization

Edouard Duriez [a], Joseph Morlier [a], Catherine Azzaro-Pantel [b], Miguel Charlotte [a]

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https://doi.org/10.1016/j.procir.2022.05.278

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Open access

ELSEVIER

Cleaner Environmental Systems
Volume 9, June 2023, 100114

A fast method of material, design and process eco-selection via topology optimization, for additive manufactured structures

Edouard Duriez [a], Catherine Azzaro-Pantel [b], Joseph Morlier [a], Miguel Charlotte [a]

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https://doi.org/10.1016/j.cesys.2023.100114

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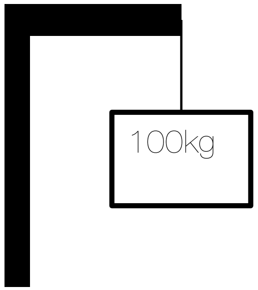
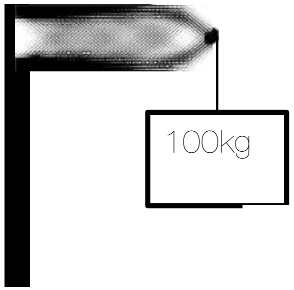
Ecodesign/Manufacturing

- Mass minimization of parts
  - Redesign through topology optimization => same performance but lower mass

multiscale topology optimization

- Reparability
- Fail-safe design
- Reusability and robot for assembly (see NASA MADCAT)

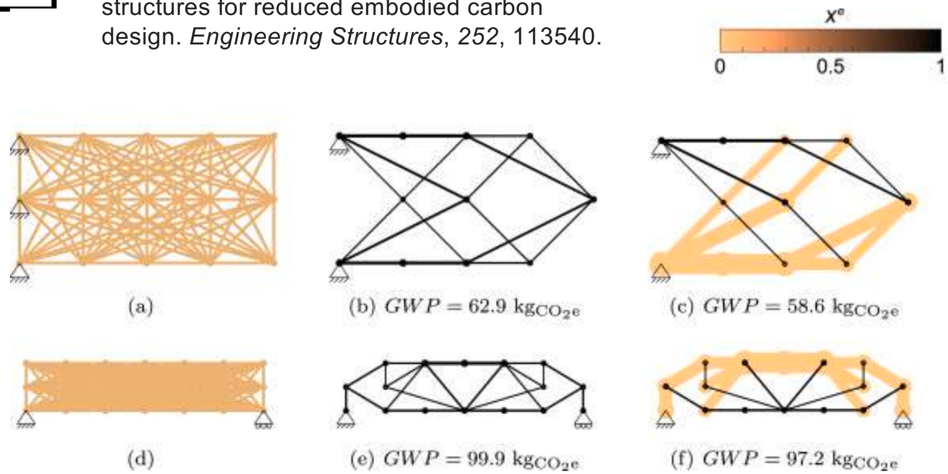


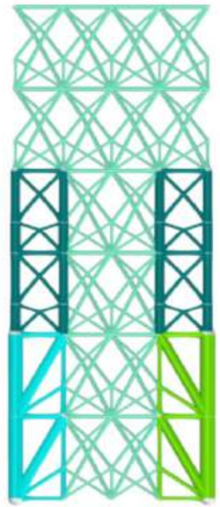


- Multimaterial
- GWP under stress

Ching, E., & Carstensen, J. V. (2022). Truss topology optimization of timber–steel structures for reduced embodied carbon design. *Engineering Structures*, *252*, 113540.

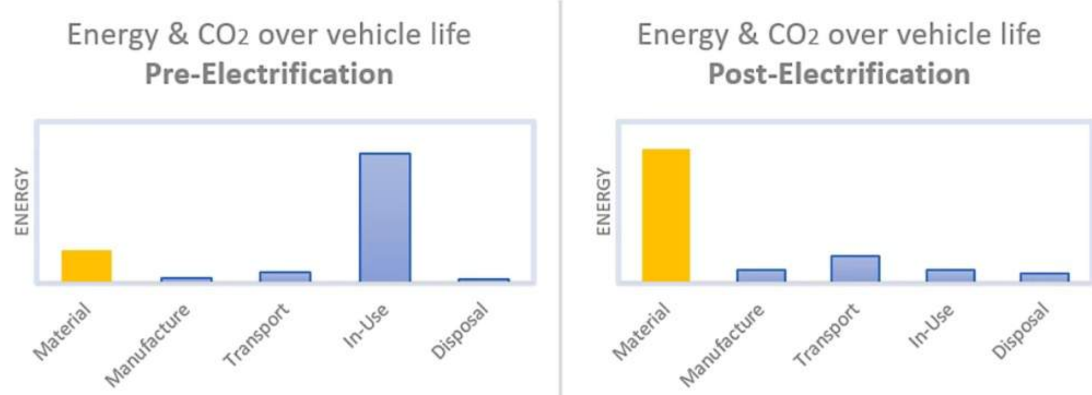
Liu, Y., Wang, Z., Lu, H., Ye, J., Zhao, Y., & Xie, Y. M. (2023, September). Layout optimization of truss structures with modular constraints. In *Structures* (Vol. 55, pp. 1460-1469). Elsevier.



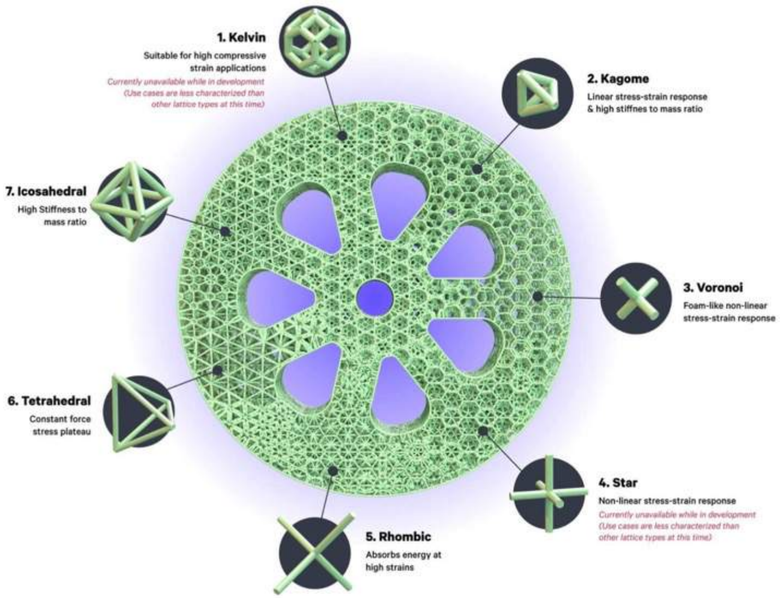


Material/Process as new design variables in MDO

Eco Material selection
Eco Process selection



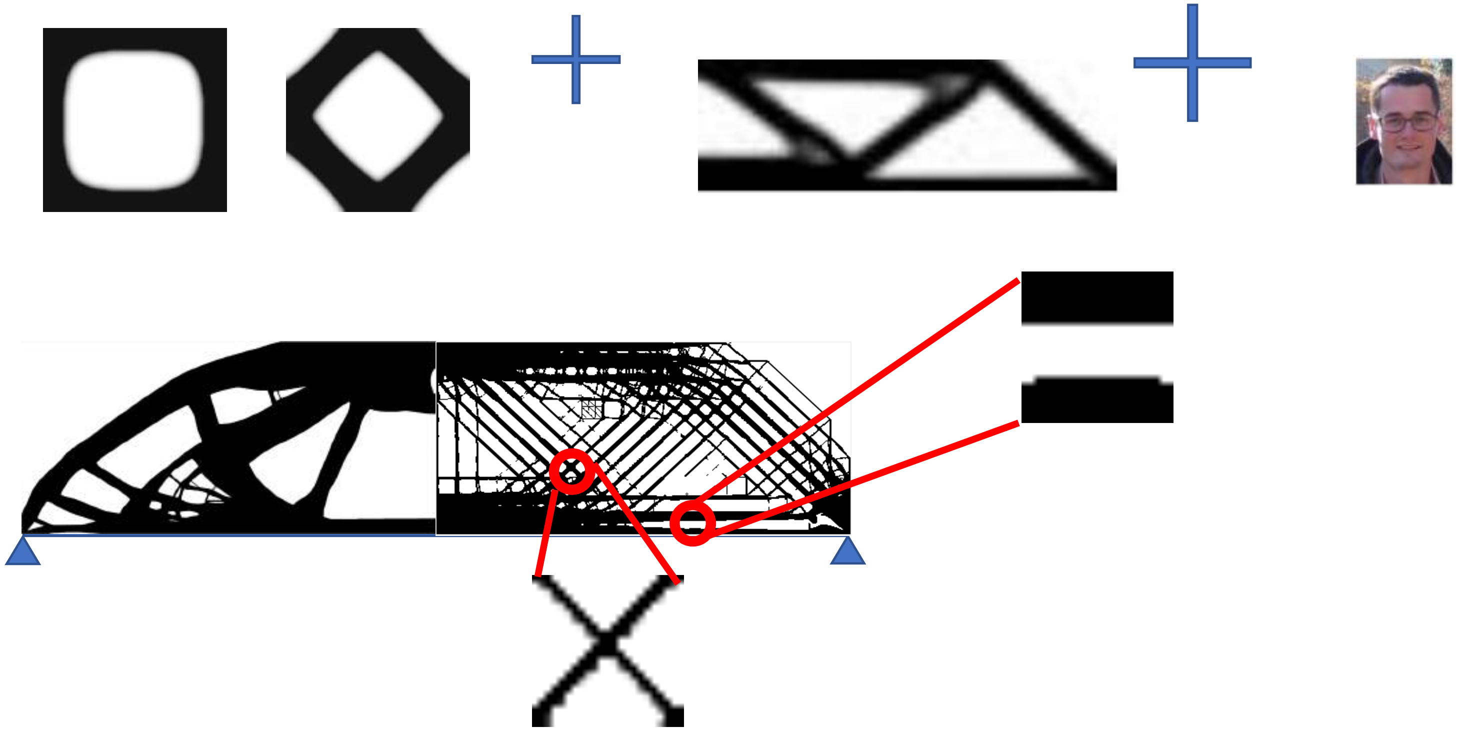
https://www.ansys.com/blog/the-impact-of-materials-on-sustainability-part-2



Unit cell design (anisotropy)
Digital materials

Multi-scale TO (well connected+ locally-oriented)

A two level optimization that combines Unit cell design & Topology Optimization

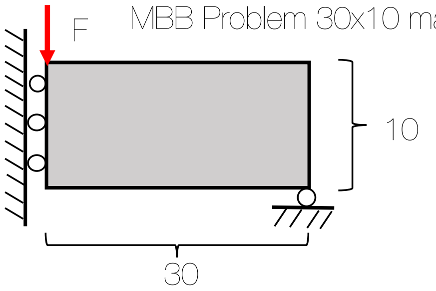


Xia L, Breitkopf P (2015) Design of materials using topology optimization and energy-based homogenization approach in Matlab. Struct Multidisc Optim 52(6):1229–1241.
Wu, Jun, Ole Sigmund, and Jeroen P. Groen. "Topology optimization of multi-scale structures: a review." Structural and Multidisciplinary Optimization 63.3 (2021): 1455-1480.

EMTO results

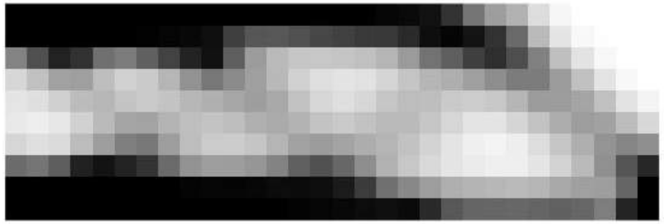
Duriez, E., Morlier, J., Charlotte, M., & Azzaro-Pantel, C. (2021). A well connected, locally-oriented and efficient multi-scale topology optimization (EMTO) strategy. Structural and Multidisciplinary Optimization, 64(6), 3705-3728.

MBB Problem 30x10 macro elements

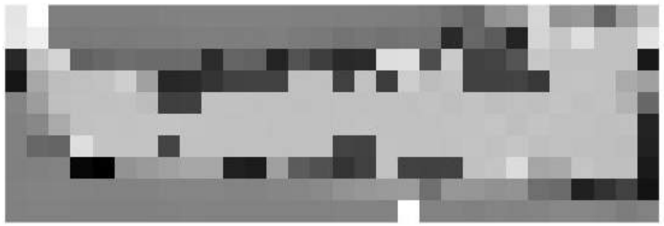


Theoretical compliance 192.7

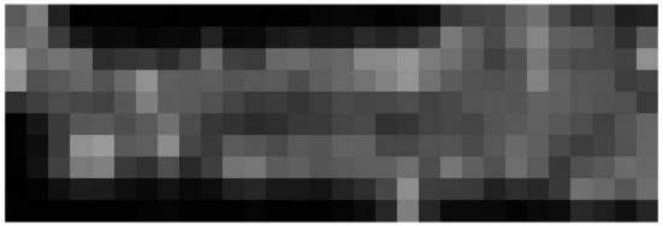
Density of the cells



Orientation of the cells



Cubicity of the cells

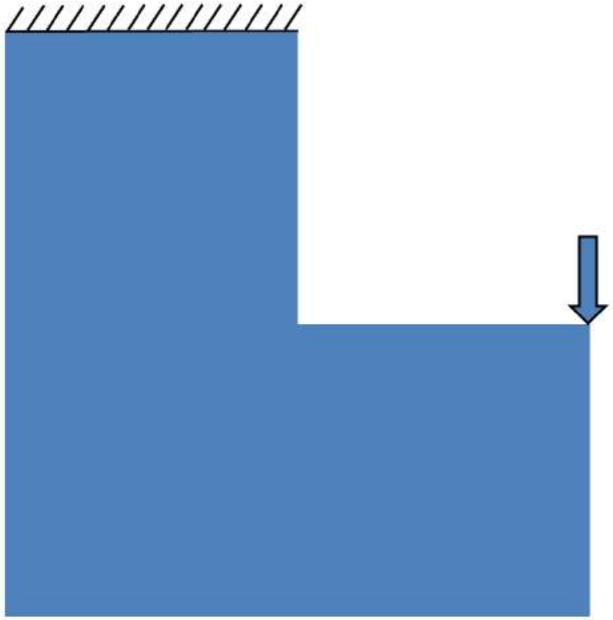


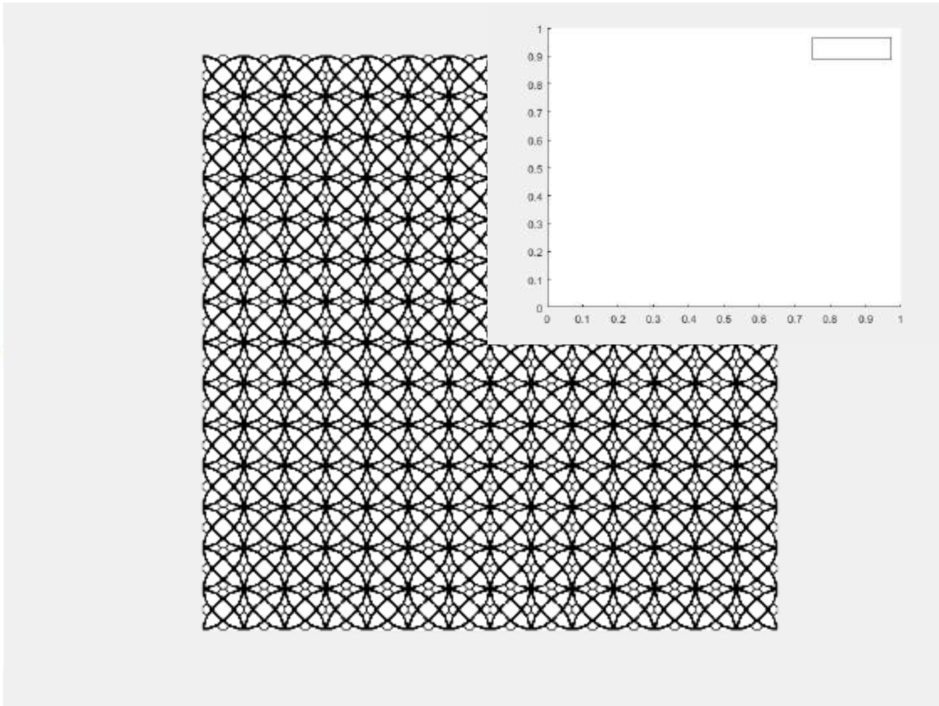
Structure Database and post-treatment

EMTO MBB solution

top88 MBB solution

EMTO on L-shape (cellular /digital materials)





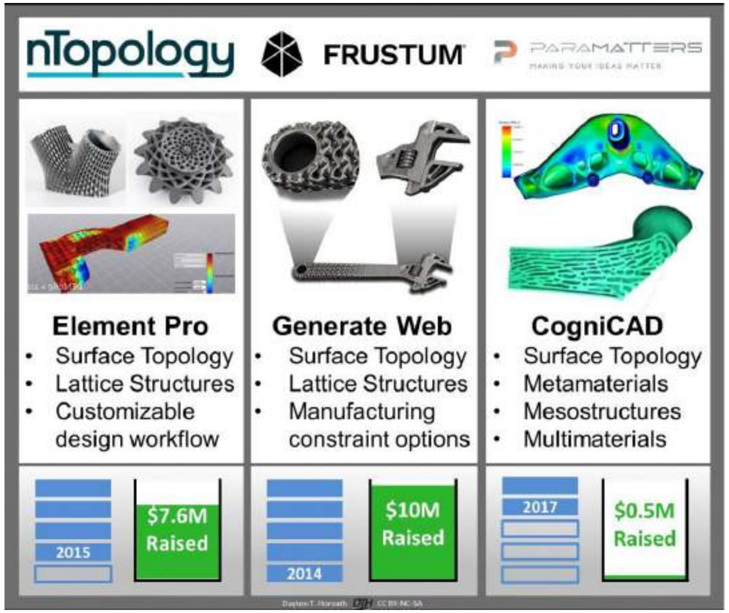
https://github.com/mid2SUPAERO/EMTO

Softwares for hierarchical design

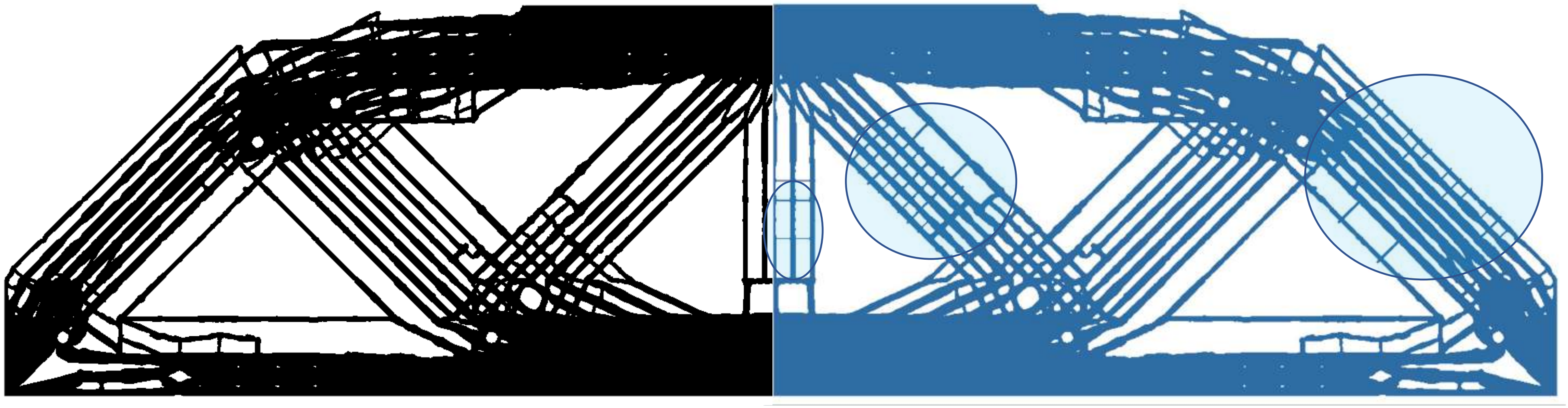
Conventional CAD programs do not work well
New players are emerging

Examples:
- ntopology (see case studies): https://ntopology.com/
- additiveflow: https://www.additiveflow.com/
- Hyperganic
- ParaMatters: https://paramatters.com/
- Fusion 360 (Autodesk)

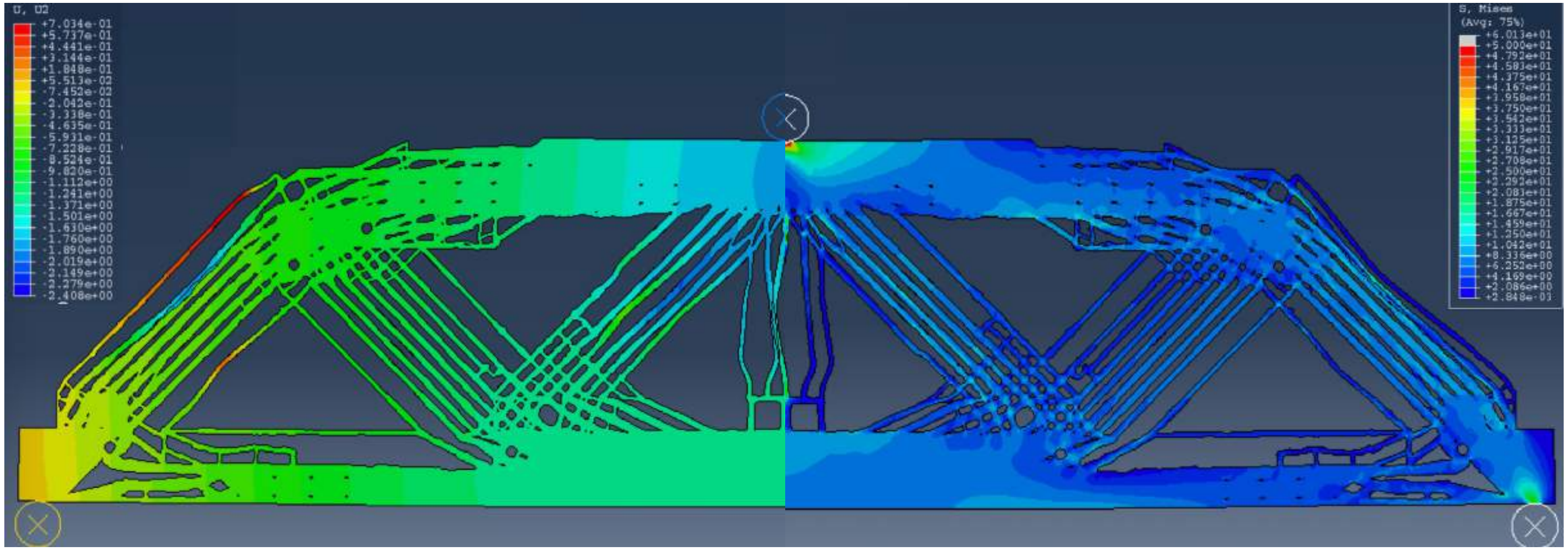
French one: cognitive design



Do you see a difference (Left2Right)?



EMTO 3pts bending (disp vs stress)



Selective Laser Melting (**SLM**)

ABAQUS REANALYSE

Universiteitsfonds

Au programme

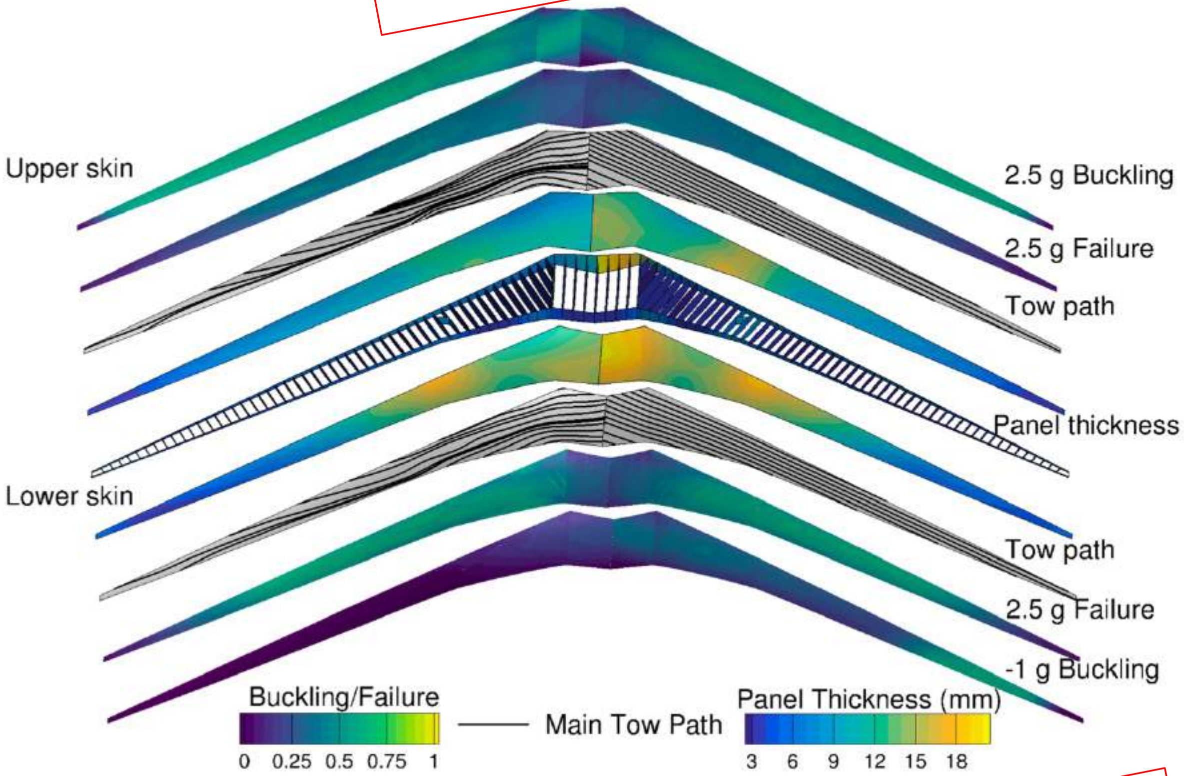
Thanks to Enrico Stragiotti, Alexandre Coehlo and Gustavo Asai+ Frederic Lachaud+Kunal Masania

- Part1 :Unit Cell/Material/Process as design variables
- **Part2: Ecodesign of 3D volumetric structures with fiber/resin topology optimization**
- Conclusions

Composites Fiber Placement as DVs

https://www.compositesworld.com/articles/tow-steering-part-2-the-next-generation

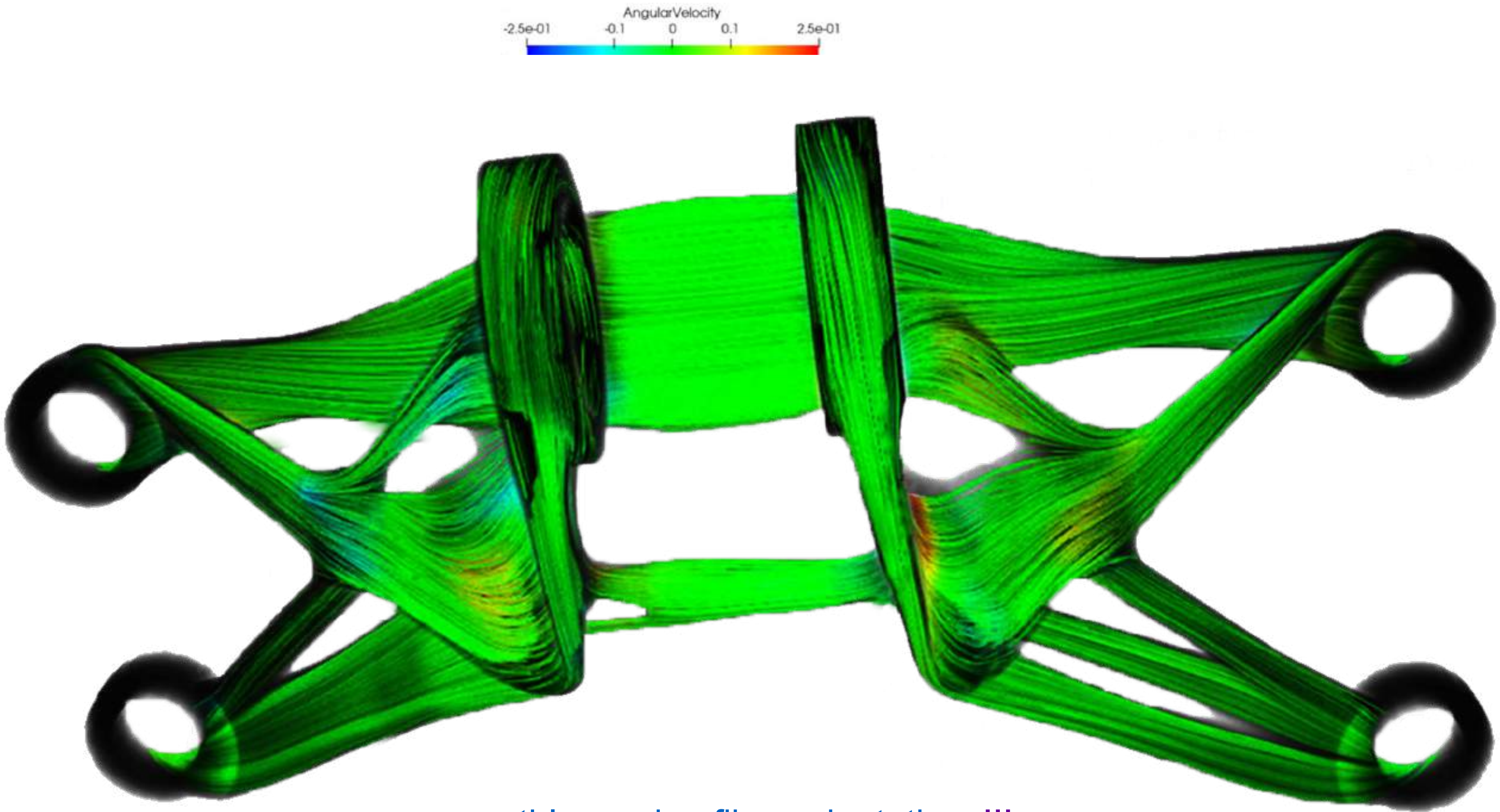
X* is a UHAR Wing ;)



Brooks, T. R., Martins, J. R., & Kennedy, G. J. (2019). High-fidelity aerostructural optimization of tow-steered composite wings. Journal of Fluids and Structures, 88, 122-147.
Brooks, T. R., Martins, J. R., & Kennedy, G. J. (2020). Aerostructural tradeoffs for tow-steered composite wings. Journal of Aircraft, 57(5), 787-799.

Lower wing mass, Less fuel burn

GE Bracket by Schmidt et al., Struct. Multidiscip. Optim. (2020)



smoothly varying fiber orientations!!!

First in 2D:

In-plane fibre orientations

- Optimisation problem formulation

$$\min_{\boldsymbol{\rho},\boldsymbol{\theta}} c(\boldsymbol{\rho},\boldsymbol{\theta}) = \sum_e \rho_e^p\, \boldsymbol{u_e^T}\, \boldsymbol{k_0}(\theta_e)\, \boldsymbol{u_e^T}$$

The finite element analysis step calls the Ansys solver via the PyMAPDL interface

$$\text{s.t.} \begin{cases} \frac{V(\boldsymbol{\rho})}{V_0} \leq f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \leq \boldsymbol{\rho} \leq 1 \\ -\pi \leq \boldsymbol{\theta} \leq \pi \end{cases}$$

solved with initial random point

- Filters

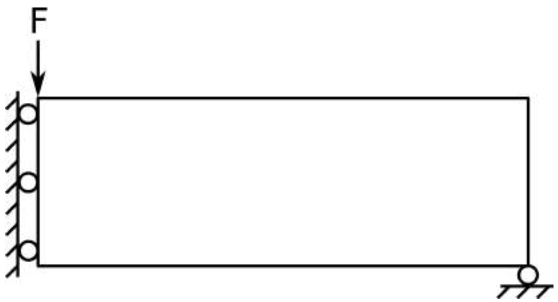
$$\rho_e \widetilde{\frac{\partial c}{\partial \rho_e}} = \frac{1}{\sum_i H_{ei}^\rho} \sum_i H_{ei}^\rho\, \rho_i \frac{\partial c}{\partial \rho_i} \qquad \tilde{\theta}_e = \frac{1}{\sum_i H_{ei}^\theta\, \rho_i} \sum_i H_{ei}^\theta\, \rho_i \theta_i$$

$$H_{ei}^\rho = \max(0, r_\rho - \Delta(e,i)) \qquad H_{ei}^\theta = \max(0, r_\theta - \Delta(e,i))$$

AFP?

Problem 1 - MBB beam

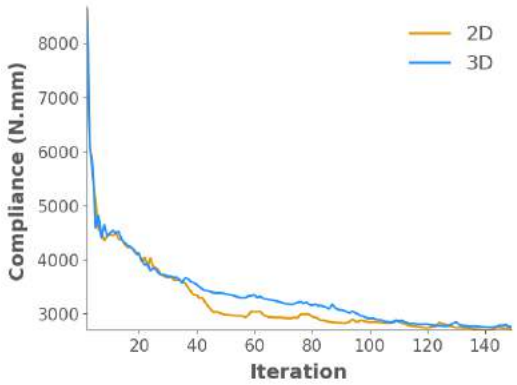
2D and in-plane 3D solutions were compared to verify the sensitivity calculations for 3D elements



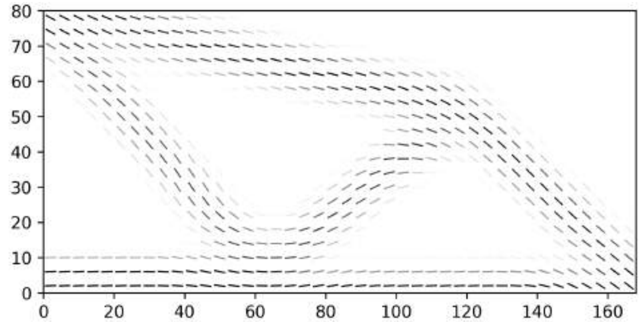
The composite material: longitudinal Young modulus E_x, transversal Young modulus E_y, in-plane Poisson's ratio ν_{xy}, out-of-plane Poisson's ratio ν_{yz}, and in-plane shear modulus G_{xy}. These elastic constants are obtained from the application of the rule of mixtures in a 2-phase (fiber+resin) micromechanical model.

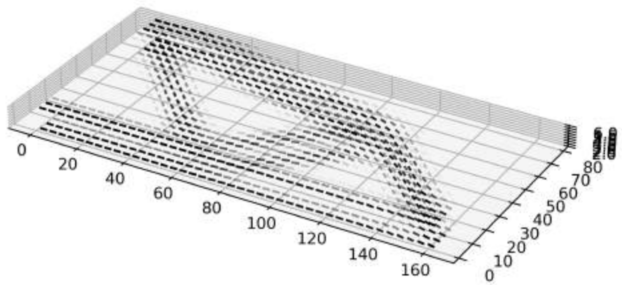
- Half MBB beam, 186 mm $\times$ 80 mm $\times$ 8 mm
- Element size: 4 mm
- Volume fraction constraint: 0.3
- Density filter radius: 8 mm $\Rightarrow$ 3D layers behave similar to 2D
- Orientation filter radius: 20 mm
- Same initial orientation: 50°

Problem 1 - MBB beam



- 2D - Comp. = 2691 N.mm
- 3D - Comp. = 2733 N.mm





CO2?

In this work, the environmental impact of the structure is measured in terms of the mass of CO_2 emitted during material production $CO_{2,mat}$ and during its use in a long distance aircraft $CO_{2,use}$, following the methodology from [1], adapted to composite materials. The value used to compare different designs is the total footprint $CO_{2,tot} = CO_{2,mat} + CO_{2,use}$.

The impact of the material production depends on the total mass M and the CO_2 intensity of the material $CO^i_{2,mat}$ (mass of CO_2 emitted per mass of material). Its expression is given by (8), where ρ_f is the fiber density, $CO^i_{2,f}$ is the fiber CO_2 intensity, ρ_m is the matrix density, $CO^i_{2,m}$ is the matrix CO_2 intensity, and V_f is the fiber volume fraction in the composite material.

$$CO_{2,mat} = M \cdot CO^i_{2,mat} = M \cdot \frac{\rho_f V_f CO^i_{2,f} + \rho_m (1 - V_f) CO^i_{2,m}}{\rho_f V_f + \rho_m (1 - V_f)}$$

The impact of the use phase is calculated as the emissions that would be saved if the component was lighter. Reducing the mass by 1 kg in a long distance aircraft leads to a reduction of 98.8 tCO_2 during its lifetime [1], i.e., $CO_{2,use} = M \cdot 98.8$ tCO_2/kg.

CO2?

	Material	ρ (kg/m^3)	E (GPa)	ν	CO_2^i (kg CO_2/kg)
Fibers	Bamboo	700	17.5	0.39	1.0565
	Flax	1470	53.5	0.355	0.44
	Hemp	1490	62.5	0.275	1.6
	HM Carbon	2105	760	0.105	68.1
	S-Glass	2495	89.5	0.22	2.905
Resins	PLA	1255	3.45	0.39	2.28
	PETG	1270	2.06	0.403	4.375
	Epoxy	1255	2.41	0.399	5.94

CO2?

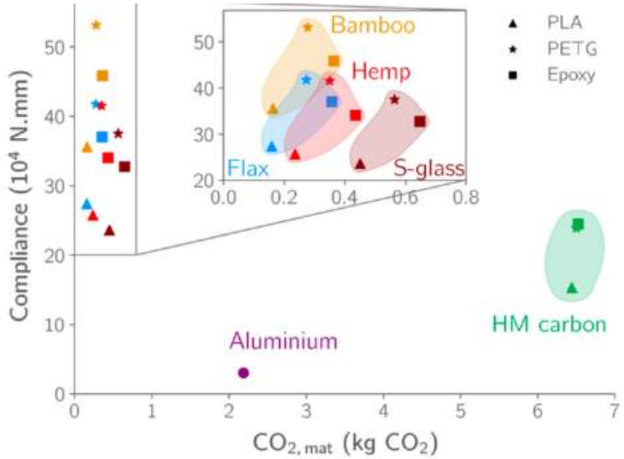


Figure 7: Compliance versus material production footprint of the optimal designs, grouped by fiber.

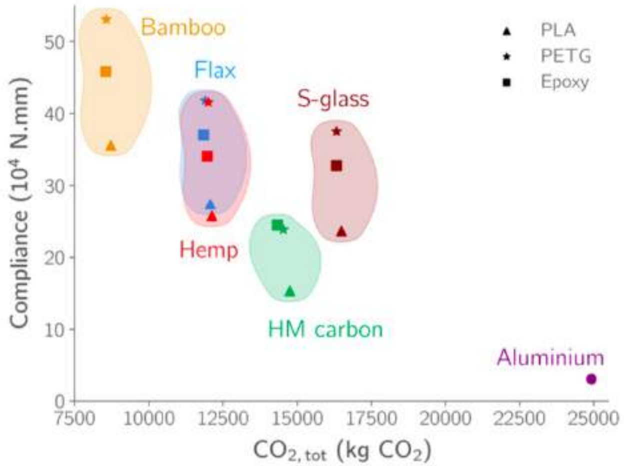


Figure 8: Compliance versus total footprint of the optimal designs, grouped by fiber.

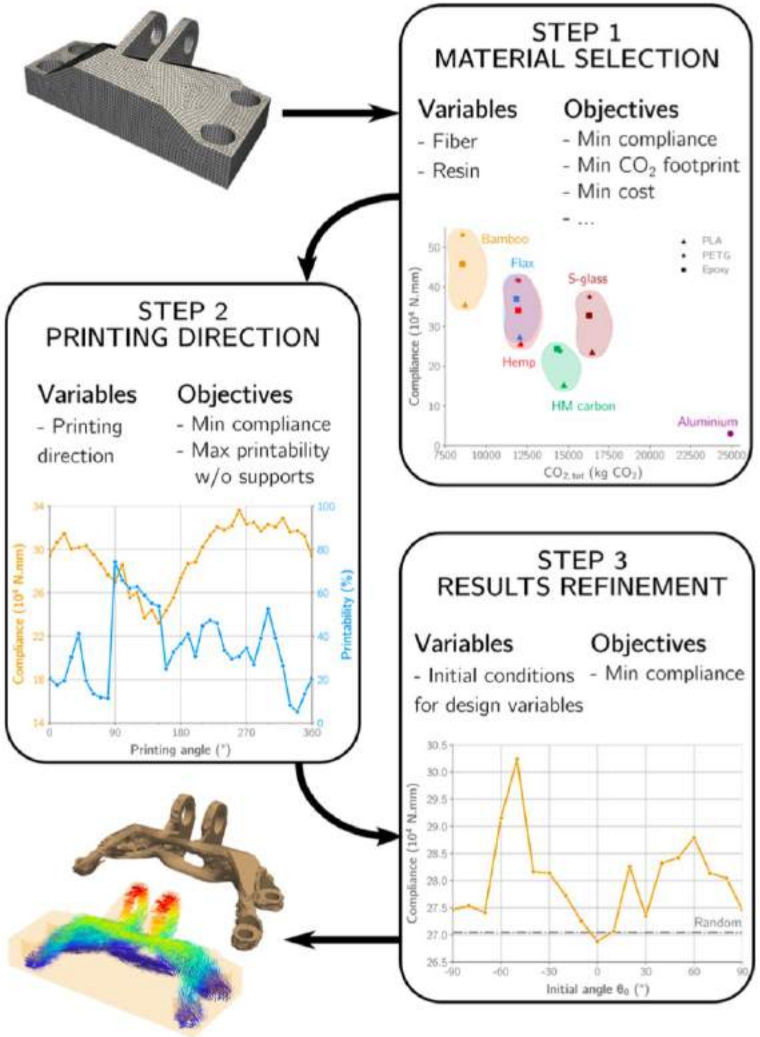
On going works 3D part

$$\min_{\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}} C(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}) = \left(\sum_{i\in LC} c_i(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha})^n\right)^{\frac{1}{n}}$$

$$= \left(\sum_{i\in LC}\left(\sum_e \rho_e^p \boldsymbol{u}_{e,i}^T \boldsymbol{k_0}(\theta_e,\alpha_e)\boldsymbol{u_{e,i}}\right)^n\right)^{\frac{1}{n}}$$

$$\text{s.t.} \begin{cases} \frac{V(\boldsymbol{\rho})}{V_0} \leq f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \leq \boldsymbol{\rho} \leq 1 \\ -\frac{\pi}{2} \leq \boldsymbol{\theta} \leq \frac{\pi}{2} \\ -\frac{\pi}{2} \leq \boldsymbol{\alpha} \leq \frac{\pi}{2} \end{cases}$$

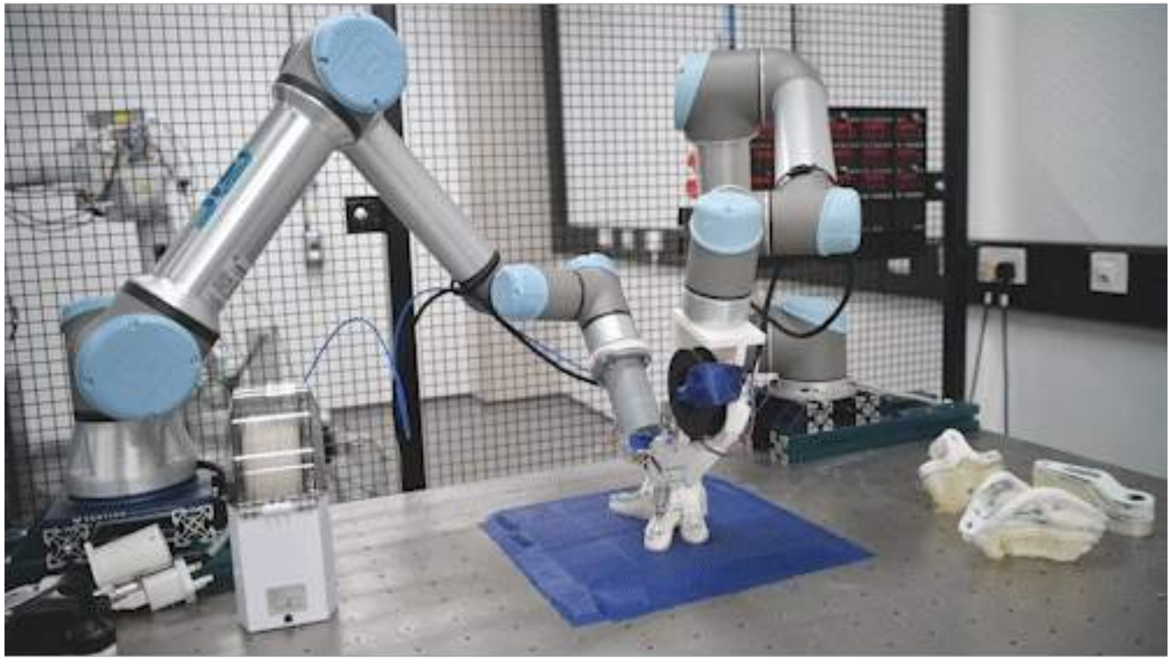
$$\frac{\partial C}{\partial \cdot} = \sum_{i\in LC} c_i^{n-1} C^{1-n} \frac{\partial c_i}{\partial \cdot}$$

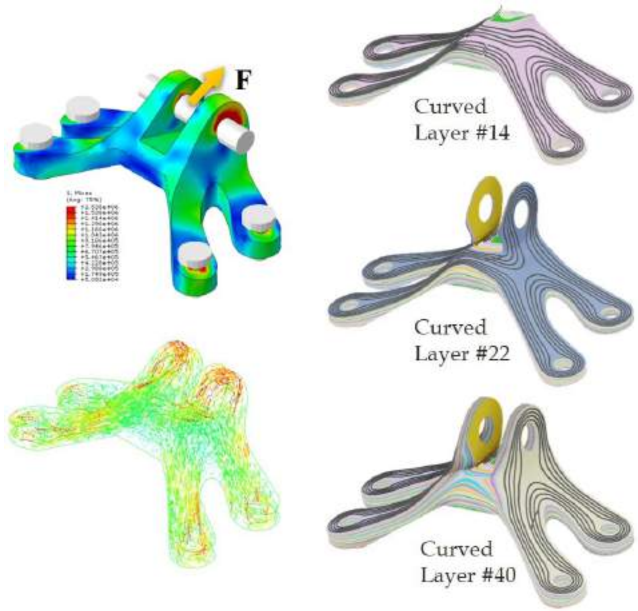


[UNPUBLISHED] From Manufacturable to EcoOptimized part
https://github.com/mid2SUPAERO/SOMP_Ansys

It Can be printed !

https://www.youtube.com/watch?v=7Jxyu9uRMLo





Fang, G., Zhang, T., Huang, Y., Zhang, Z., Masania, K., & Wang, C. C. (2024). Exceptional mechanical performance by spatial printing with continuous fiber: Curved slicing, toolpath generation and physical verification. *Additive Manufacturing*, 104048.

GE bracket Test case

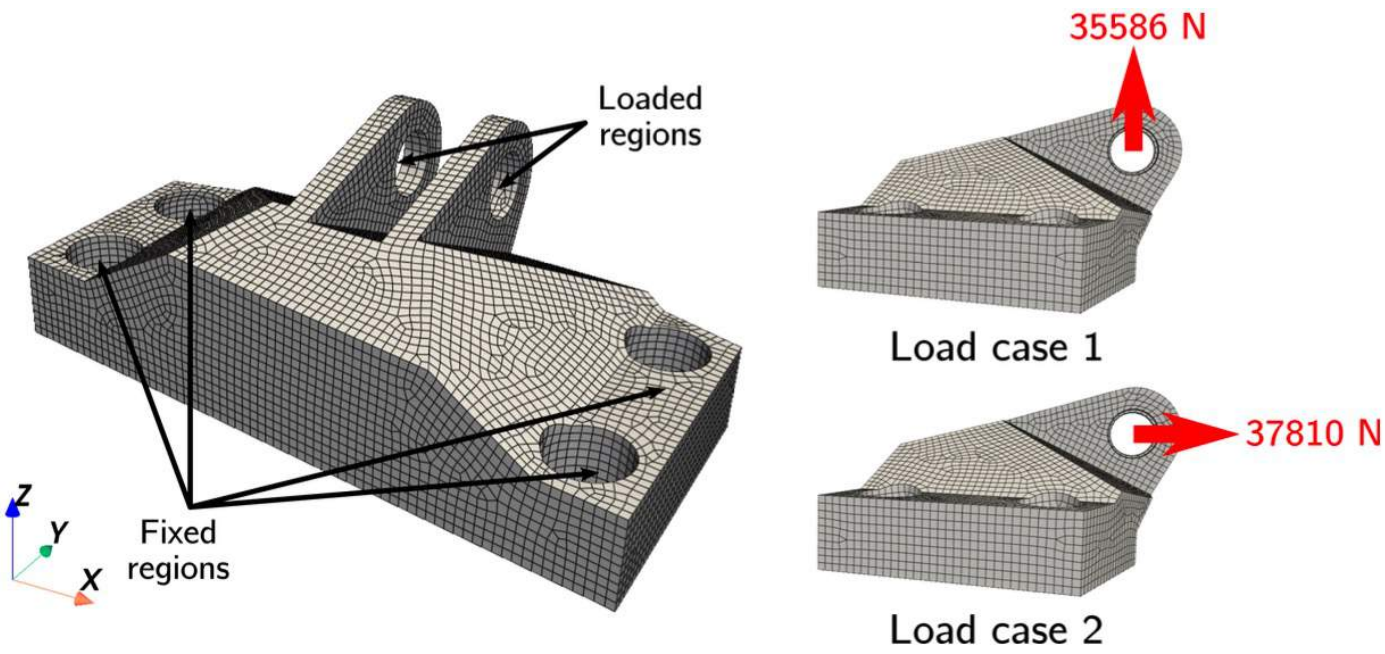
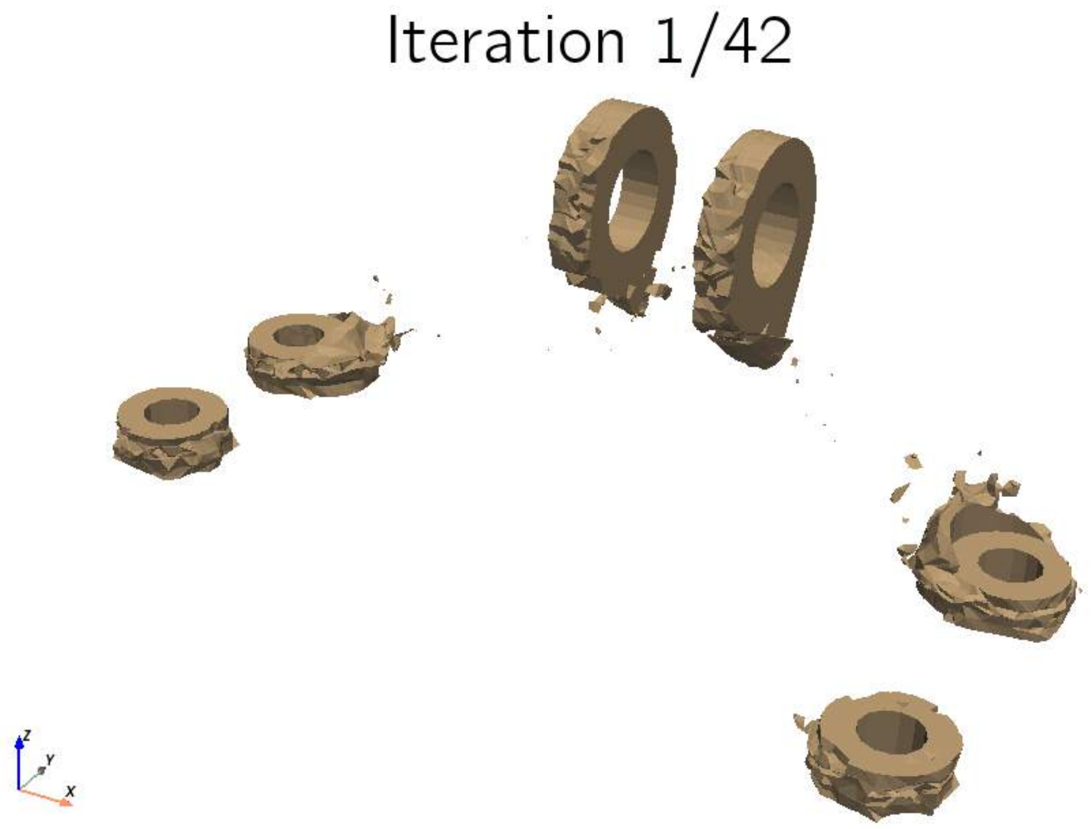
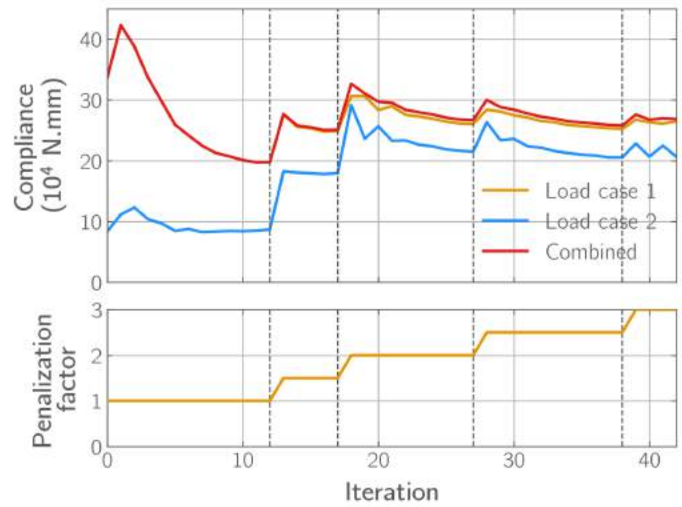
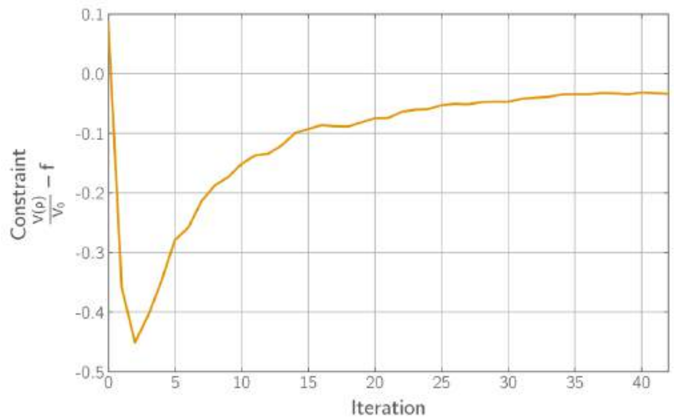


Figure 5: Bracket finite element model.

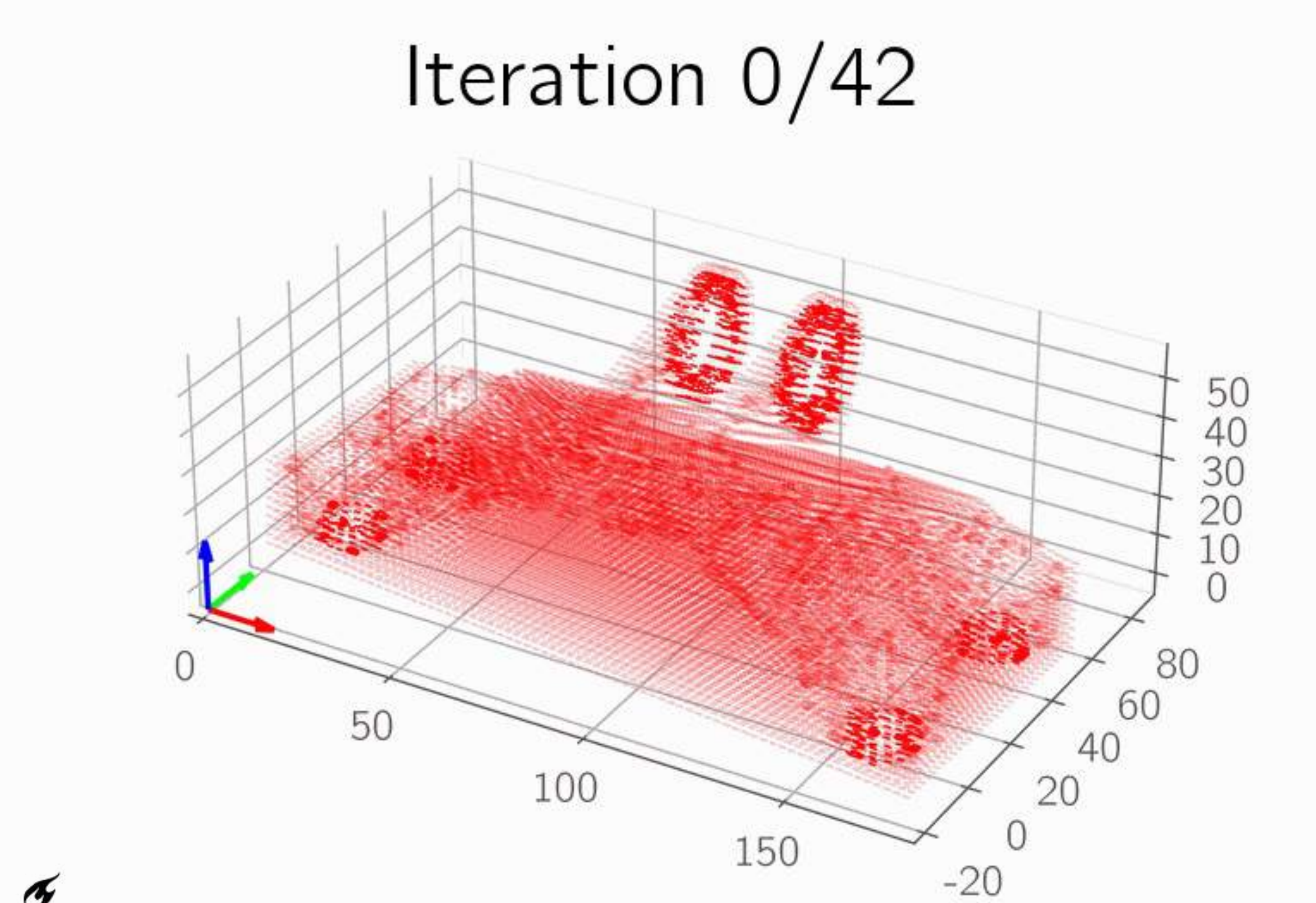
RESULTS


Iteration 1/42
Z
Y
X

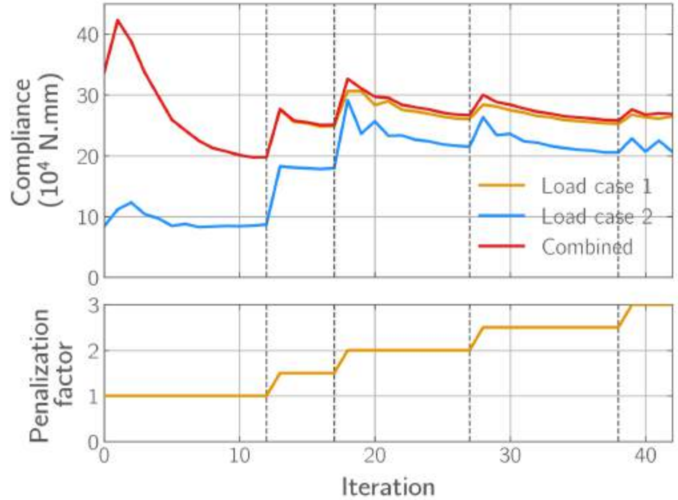

Compliance
(10^4 N.mm)
Load case 1
Load case 2
Combined
Penalization factor
Iteration


Constraint
$\frac{V(\rho)}{V_0} - f$
Iteration

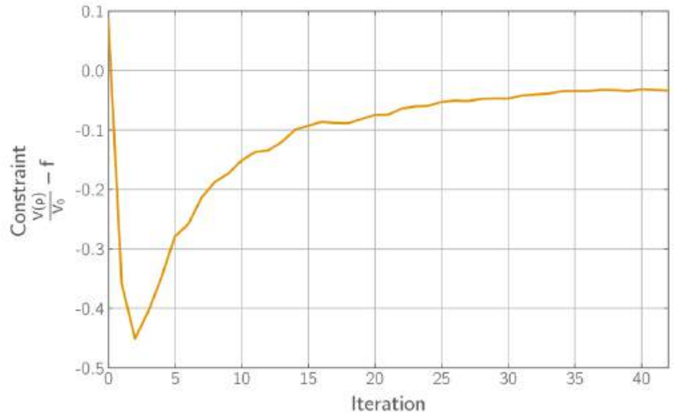
RESULTS



Iteration 0/42
0
50
100
150
-20
0
20
40
60
80
0
10
20
30
40
50



Compliance (10^4 N.mm)
Load case 1
Load case 2
Combined
Penalization factor
Iteration



Constraint $\frac{V(\rho)}{V_0} - f$
Iteration

Au programme

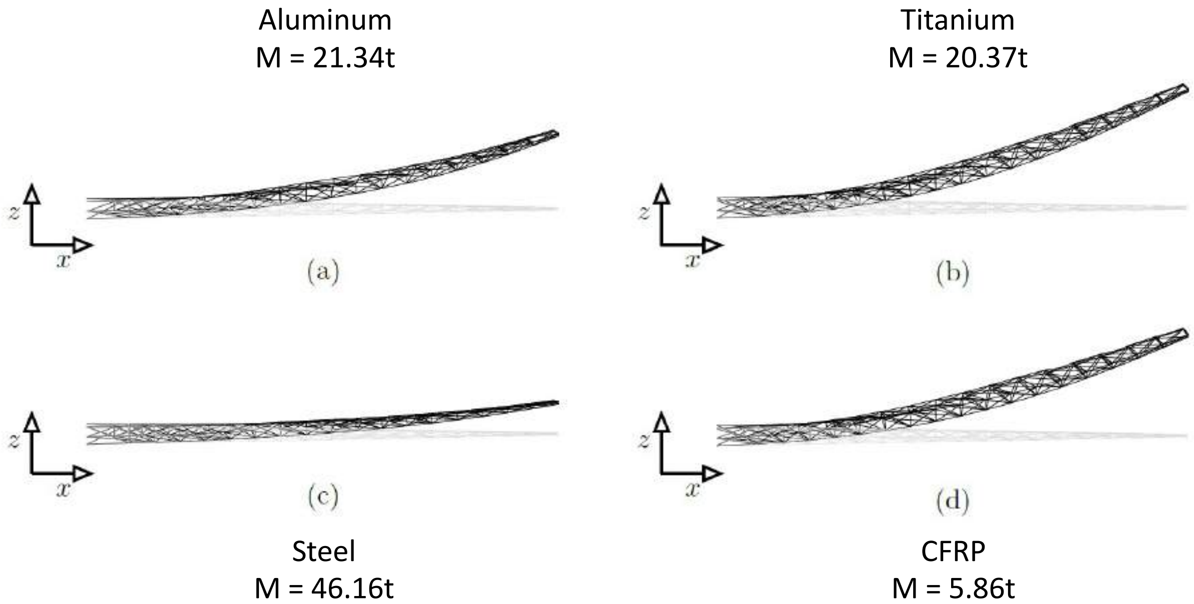
- Part1 :Unit Cell/Material/Process as design variables
- Part2: Ecodesign of 3D volumetric structures with fiber/resin topology optimization
- **Conclusions**

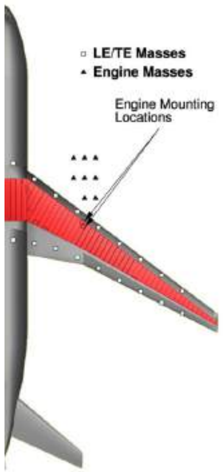
Conclusions

- *Proof of concept of greener aerostructures*
- *Our « open source » solutions can design Metallic and composites 3D printing parts*

Conclusions

- *Proof of concept of greener aerostructures*
- *Our « open source » solutions can design Metallic and composites 3D printing parts*
- *Material as Design variable open new solutions*





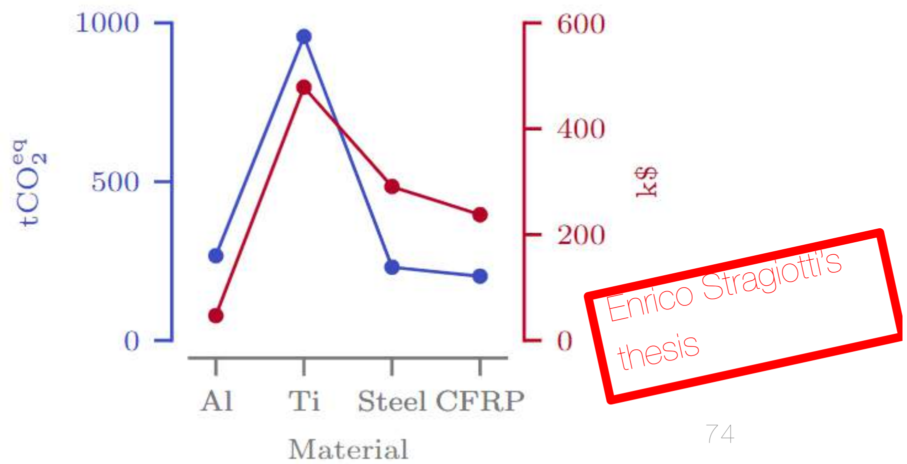
Wingspan, m : 58.76
MTOW, t : 297,55

3 load cases:

- +2.5 g manouver
- -1 g manouver
- Cruise with gust (+1.3 g)

$$
\begin{aligned}
\min_{\substack{a, q^0, \dots, q^{N_p},\\ U^0, \dots, U^{N_p}}} \quad & V = \boldsymbol{\ell}^T \boldsymbol{a} \\
\text{s.t.} \quad & \boldsymbol{B}\boldsymbol{q}^p = \boldsymbol{f}^p && \forall p \in [0, \dots, N_p] \\
& \boldsymbol{q}^p = \frac{\boldsymbol{a}E}{\boldsymbol{\ell}} \boldsymbol{b}^T \boldsymbol{U}^p && \forall p \in [0, \dots, N_p] \\
& \boldsymbol{q}^p \geq -\frac{s\boldsymbol{a}^2}{\boldsymbol{\ell}^{*2}} && \forall p \in [0, \dots, N_p] \\
& -\sigma_c \boldsymbol{a} \leq \boldsymbol{q}^p \leq \sigma_t \boldsymbol{a} && \forall p \in [0, \dots, N_p] \\
& 0 \leq \boldsymbol{a} \leq \frac{4\pi \boldsymbol{\ell}^2}{\lambda_{\max}}.
\end{aligned}
$$

Material	Aluminium	Titanium	Steel	Pultruted CFRP
E	69 GPa	120 GPa	210 GPa	150 GPa
σ_c, σ_t	±270 MPa	±880 MPa	±355 MPa	+1200, −880 MPa
ρ	2.7 g cm^{-3}	4.5 g cm^{-3}	7.8 g cm^{-3}	1.6 g cm^{-3}



THANK YOU

https://www.linkedin.com/pulse/possible-build-aircraft-wing-lego-joseph-morlier/?articleId=6627240732975480832

https://www.tripadvisor.fr/LocationPhotoDirectLink-g187529-d574612-i349532022-Museum_of_Natural_Science_Museo_de_Ciencias_Naturales-Valencia_Province_o.html

Is it possible to build an aircraft wing in LEGO® ?

Publié le 17 février 2020 Modifier l'article | Voir les stats

joseph morlier

Professor in Structural and Multidisciplinary Design Optimization, ... any idea?

5 articles