

# Recent advances in structural and multidisciplinary optimization

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## @SUPAERO

Prof. Joseph Morlier



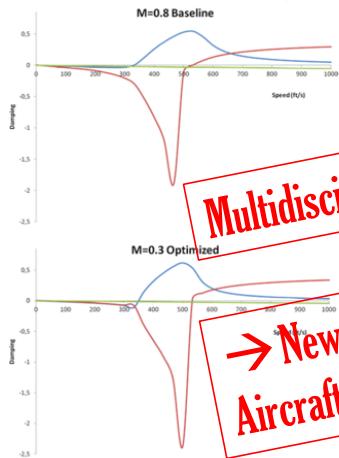
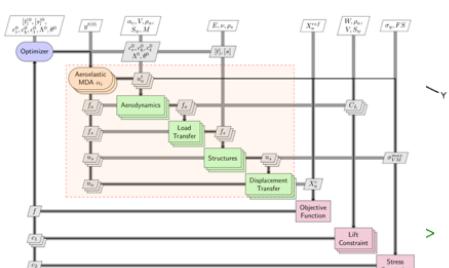
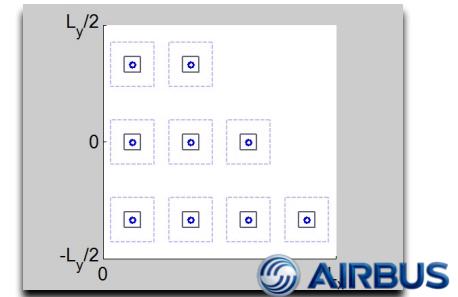
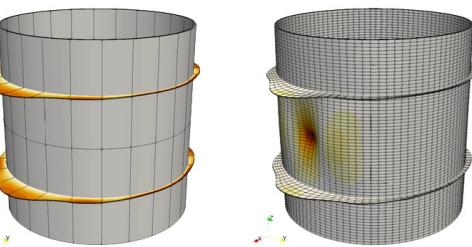
# My Research Group (Joint research with ONERA on MDO)

<http://www.institut-clement-ader.org/pageperso.php?id=jmorlier>

- 4 PhDs, 1 postdoc, 4 MsC

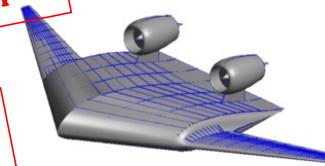
$$\begin{aligned} & \min w(\mathbf{a}, \mathbf{c}) \\ & \mathbf{a} \in \mathbb{R}^{10} \\ & \mathbf{c} \in \Gamma^{10} \\ & \text{s.t. } s(\mathbf{a}, \mathbf{c}) \leq 0 \\ & d(\mathbf{a}, \mathbf{c}) \leq 0 \\ & \underline{\mathbf{a}} \leq \mathbf{a} \leq \bar{\mathbf{a}} \end{aligned}$$

**AIRBUS**



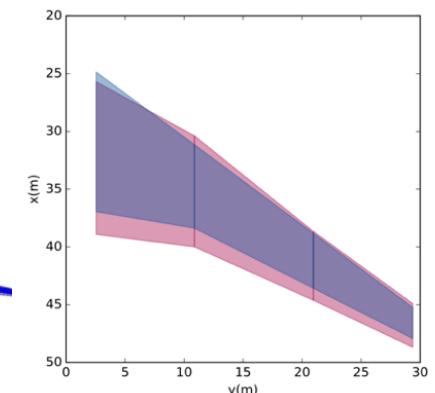
**Multidisciplinary Optimization**

**→ New Aerostructures/  
Aircraft Concept**



**AIRBUS**

**CHAIR FOR ECO DESIGN OF AIRCRAFT**



2

# Our Goals: new optimization process in the design loop of Aerostructures (flexible)

- Reduce in a « smart way » the computation time of optimization for coupled simulations
- Global Optimization using surrogate modeling → fixed budget (enriching process) to deal with INDUSTRIAL problems
- Specialized surrogates for HD (engineering) problems and UQ
- Taking into account different levels of fidelity

**Bartoli, N., Lefebvre, T., Dubreuil, S., Olivanti, R., Bons, N., Martins, J. & Morlier, J. (2017). An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization. In 18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4433).**

→ Methods applied to AD Aircraft Design: Put the aircraft structure / aeroelasticity in the loop at the early stage of MDO process

→ compatible with

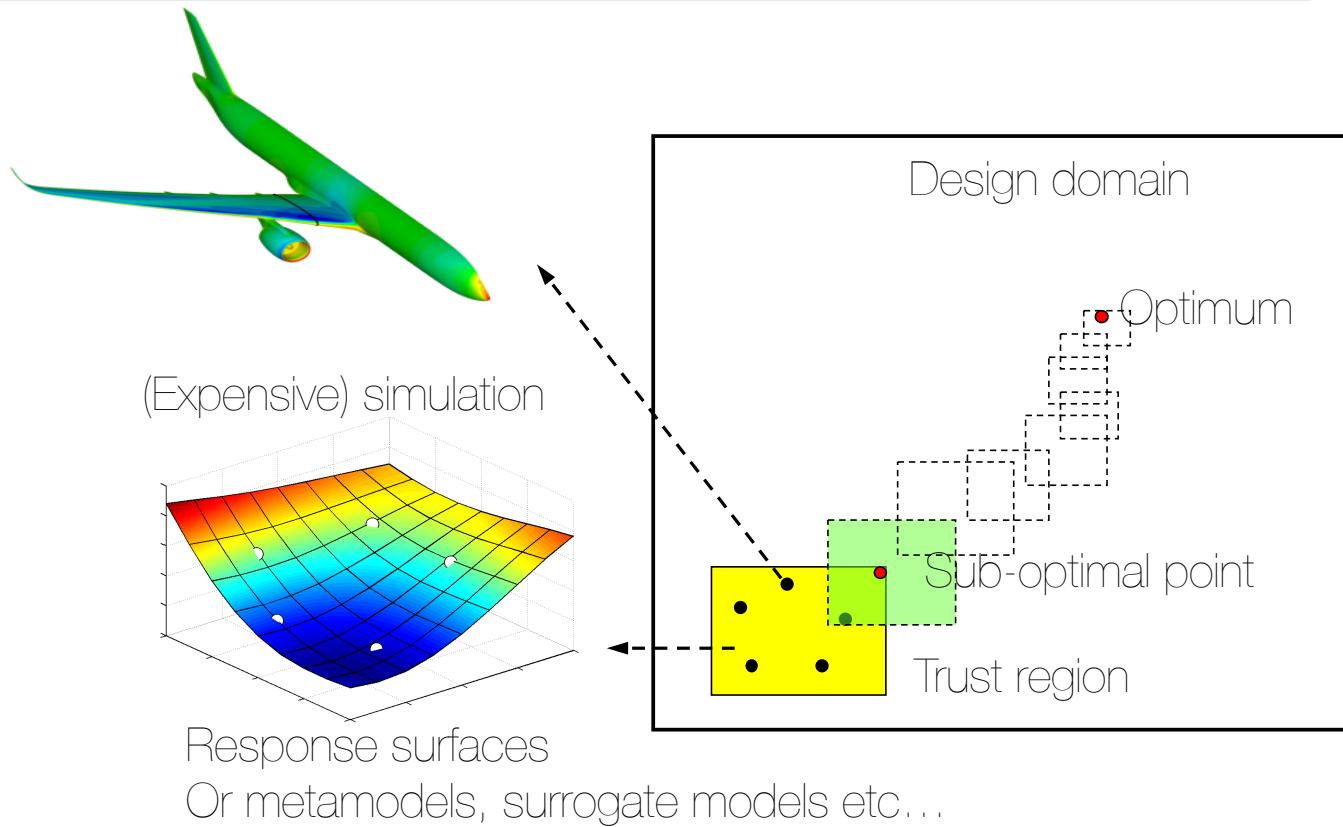


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<http://openmdao.org>

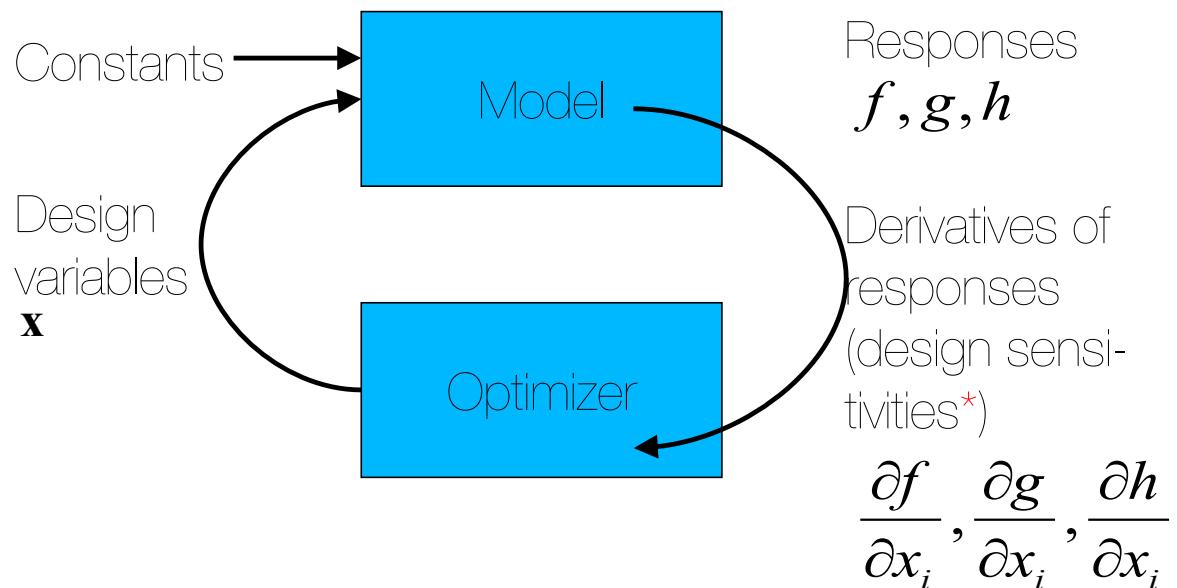
# SURROGATE MODELING (learning for Optimizing)

Jacobs, J. H., Etman, L. F. P., Van Keulen, F., & Rooda, J. E. (2004). Framework for sequential approximate optimization. *Structural and Multidisciplinary Optimization*, 27(5), 384-400.



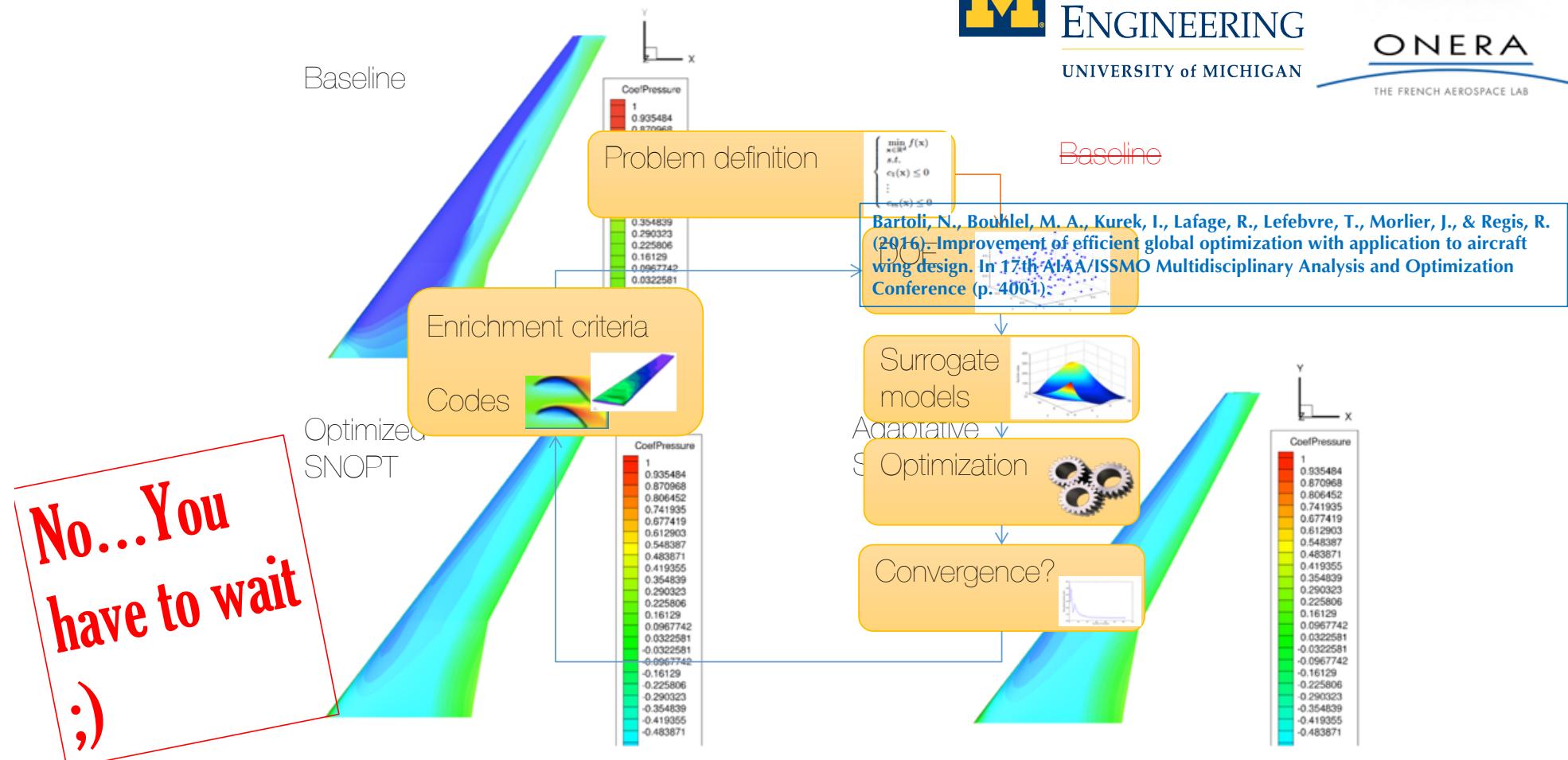
# Gradient Based Optimization

Costly if FD/difficult to implement Adjoint in industrial code  
And also sensitive to discontinuity/sensible to  $X_0$



\*SOL200 in MSC Nastran for example

Is it possible to resume in one slide ?



# Outlines for today

1. GP aka Kriging
2. Kriging for Global Optimization
3. New developments in topology optimization
4. Codesign is MDO?

# 1 .GP aka Kriging

- 2. Kriging for Global Optimization
- 3. New developments in topology optimization
- 4. Codesign is MDO?

# Machine learning for load estimation (A. Chiplunkar, PhD)



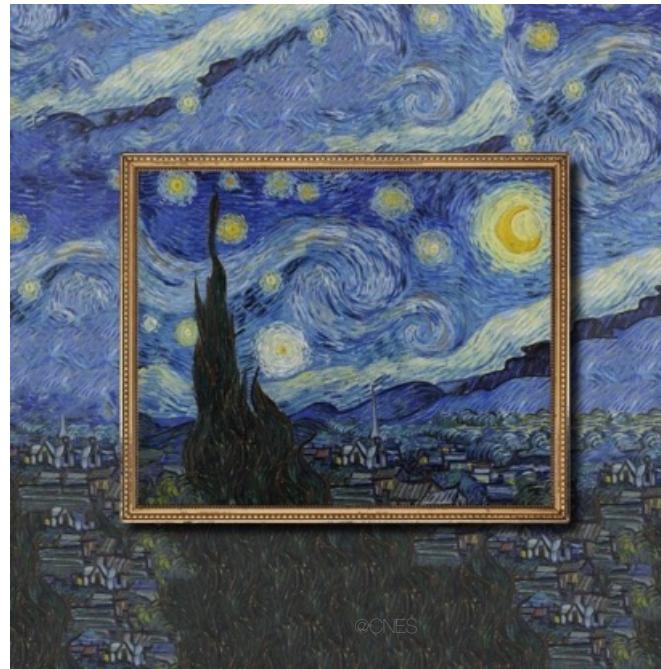
Kriging (Pionneer)	Gaussian Processes (link with AI)
Developed by Daniel Krige – 1951; formalized by Georges Matheron in the 60's (Mines Paris)	Neural network with infinite neurons tend to Gaussian Process 1994
Evaluation: minimize error variance	Evaluation: Marginal Likelihood

Krige, D. G., 1951, A statistical approach to some basic mine valuation problems on the Witwatersrand: J. Chem. Metal. Min. Soc. South Africa, v. 52, p. 119–139.

Matheron, G., 1963b, Principles of geostatistics: Economic Geol., v. 58, p. 1246–1266.

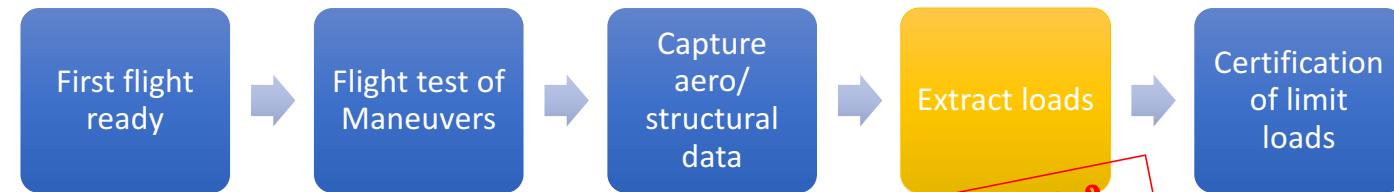
Neal, R. Priors for infinite networks. Tech. rep., University of Toronto, 1994.

Williams, C. K. I., and Rasmussen, C. E. Gaussian processes for regression. *Advances in Neural Information Processing Systems 8* (1996), 514–520.



<http://extrapolated-art.com>

# Loads estimation



Can we extrapolate limit loads using both measurements and HF aerostructures tools ?

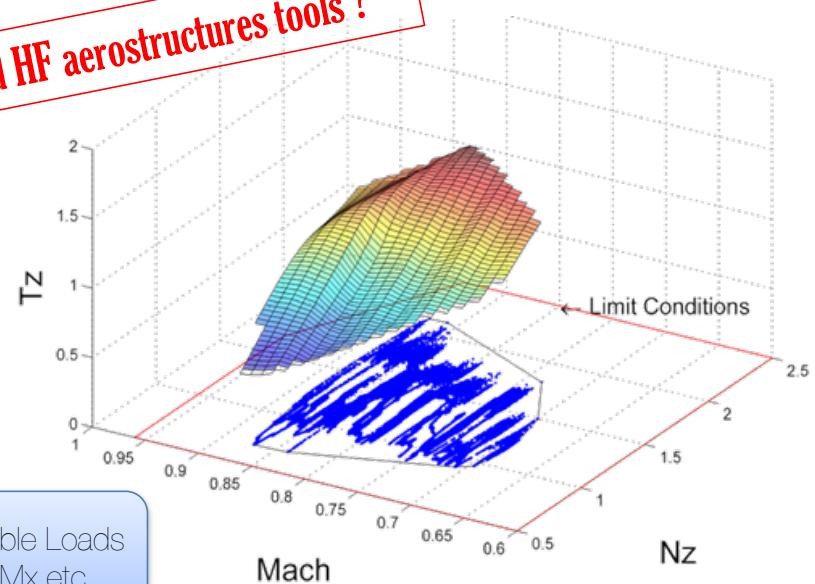
Can we automate the process?

A/C states  
 $\alpha, \beta, M,$   
 $\delta_s$

Distributed Gaussian Process

Flexible Loads  
 $T_z, M_x$  etc...

Statistical step



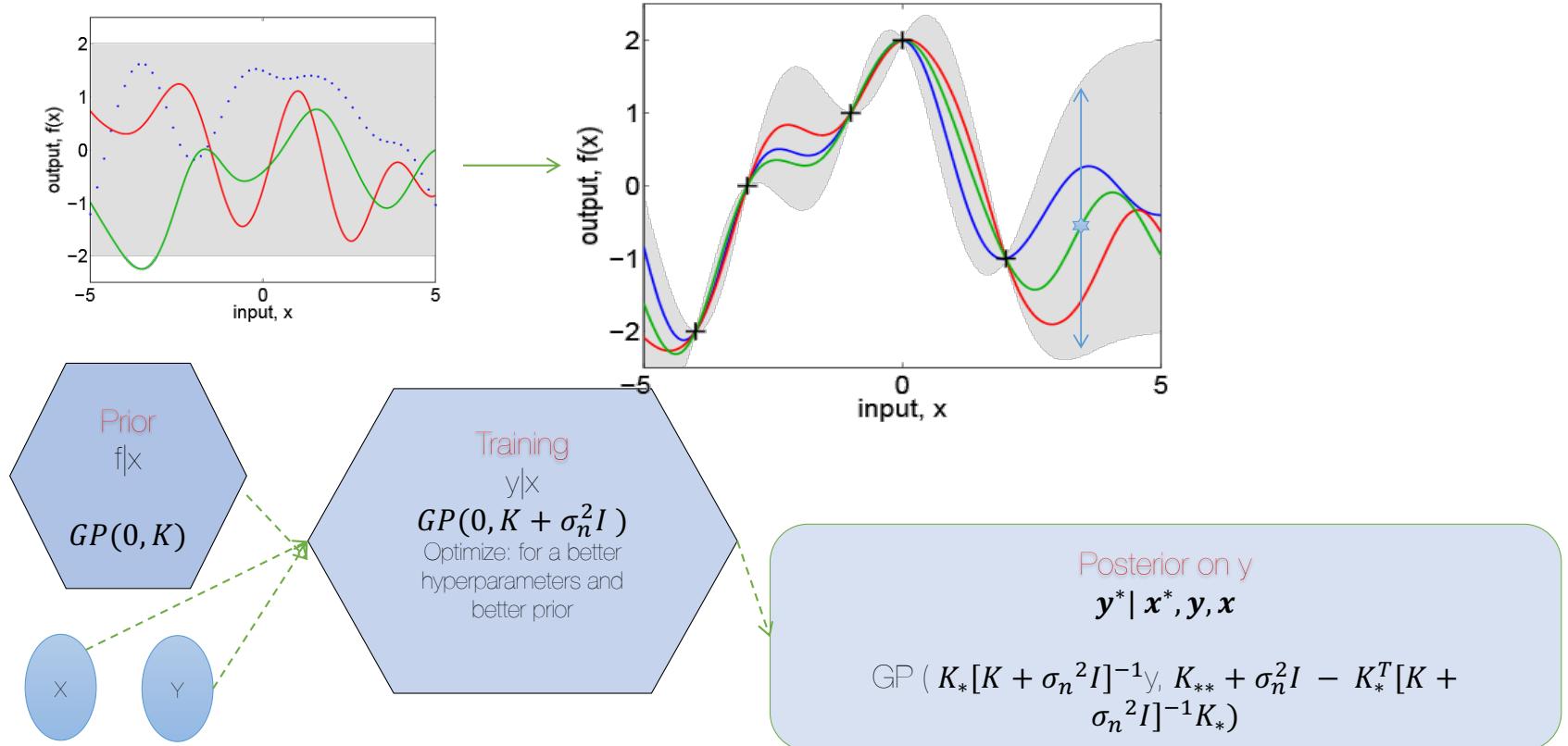
[https://github.com/ankitchiplunkar/thesis\\_isae](https://github.com/ankitchiplunkar/thesis_isae)

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10

# Gaussian Process Regression

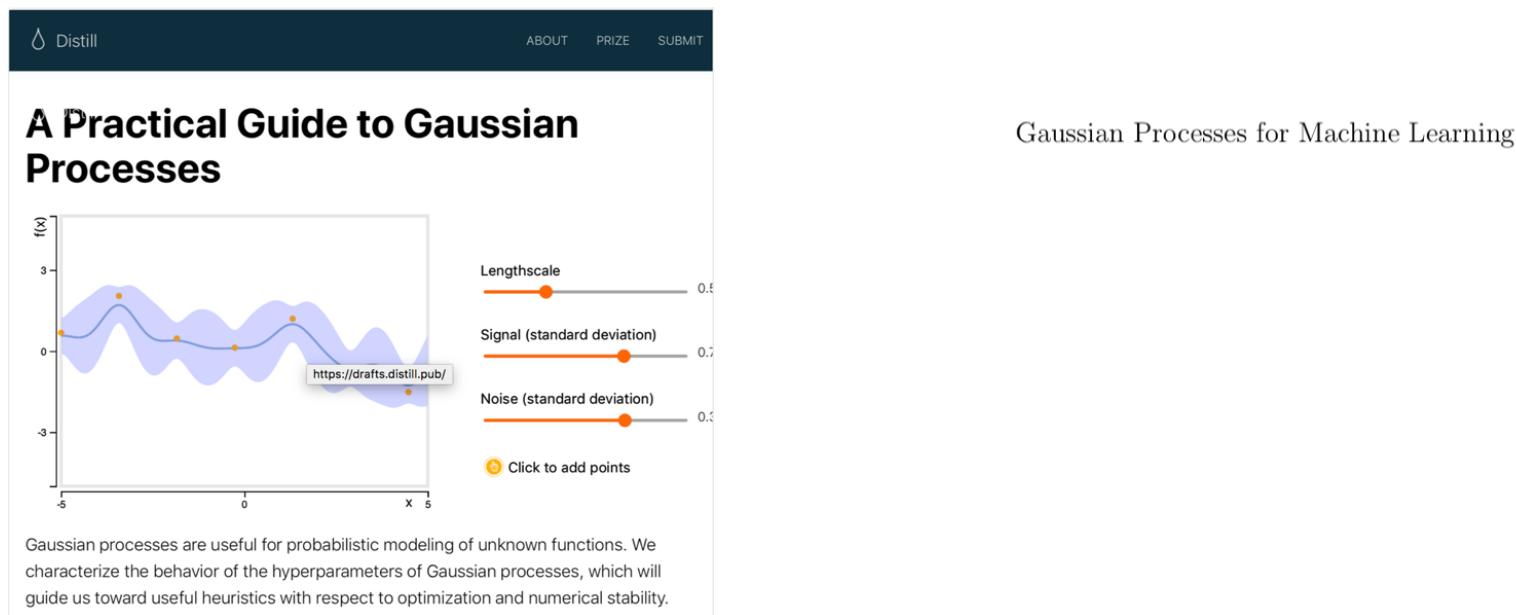
Image Source: <http://mlg.eng.cam.ac.uk/teaching/4f13/1314/>



A good starting point  $x_0$ =Rasmussen's book

- <https://drafts.distill.pub/gp/>

C. E. Rasmussen & C. K. I. Williams, Gaussian Processes for Machine Learning, the MIT Press, 2006,  
ISBN 026218253X. © 2006 Massachusetts Institute of Technology. [www.GaussianProcess.org/gpml](http://www.GaussianProcess.org/gpml)



# Matrix view of Gaussian Process

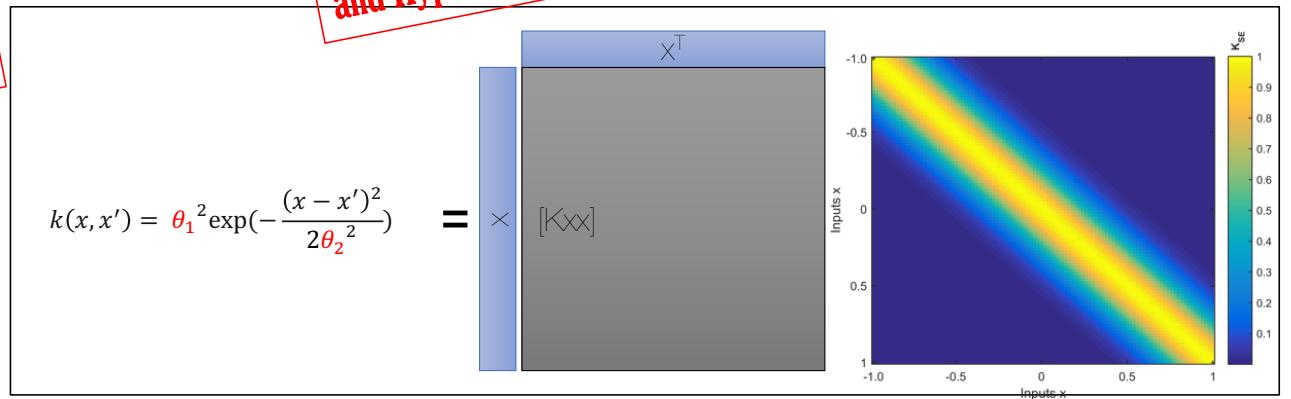
1/ Get your inputs/outputs data

$$\begin{bmatrix} \times \\ \times \end{bmatrix} \quad \begin{bmatrix} y(x) \end{bmatrix}$$

2/ You wan to predict at  $x^*$

$$\begin{bmatrix} \times \\ \times^* \end{bmatrix}$$

3/ Choose a Kernel/Construct  $K_{xx}$   
and Hyperparameters tuning



$$m(y^*) = [K_{xx}x] \cdot [K_{xx}]^{-1} \cdot y(x)$$

$m(x_*) = K_* [K_{xx}]^{-1} y$

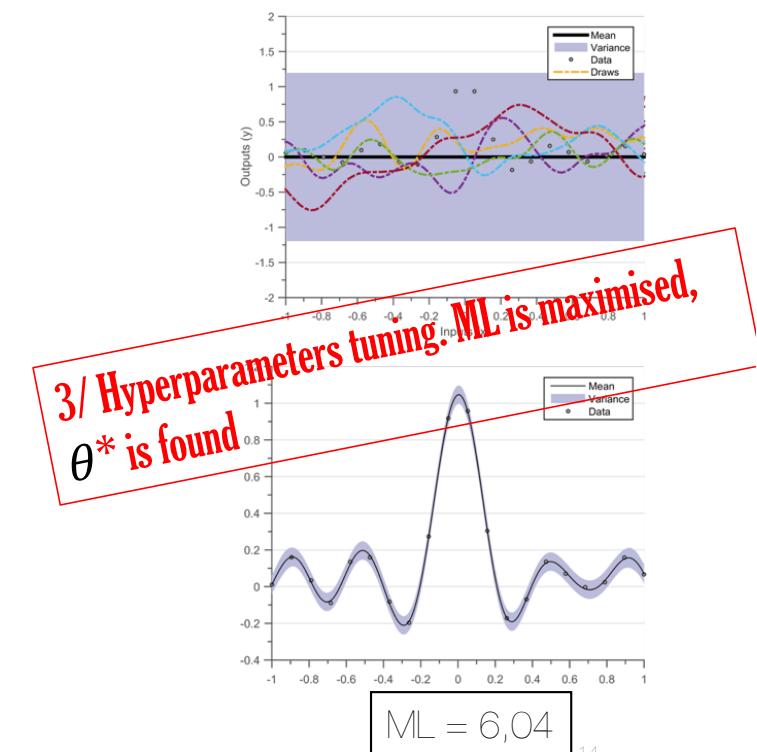
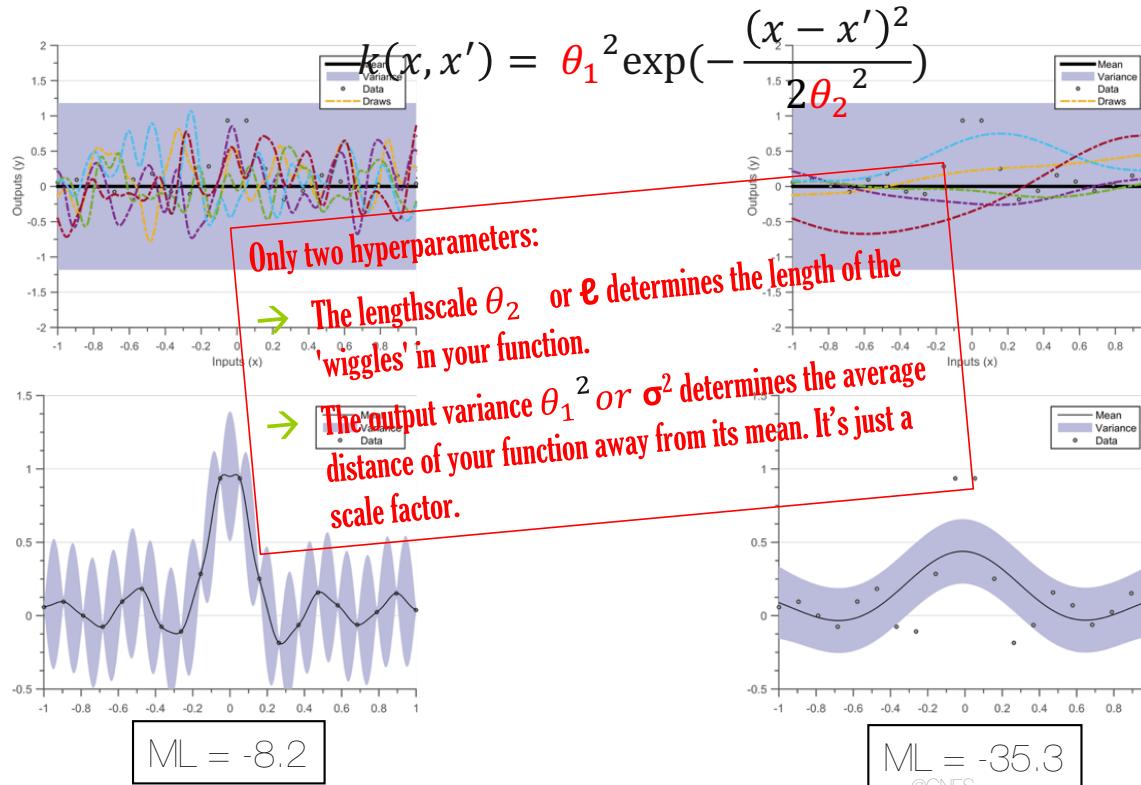
$$\text{cov}(y^*) = [K_{xx^*x^*}] - [K_{xx^*}] \cdot [K_{xx}]^{-1} \cdot [K_{xx^*}]$$

$\text{var}(x_*, x'_*) = K_{**} - K_*^T [K_{xx}]^{-1} K_*$

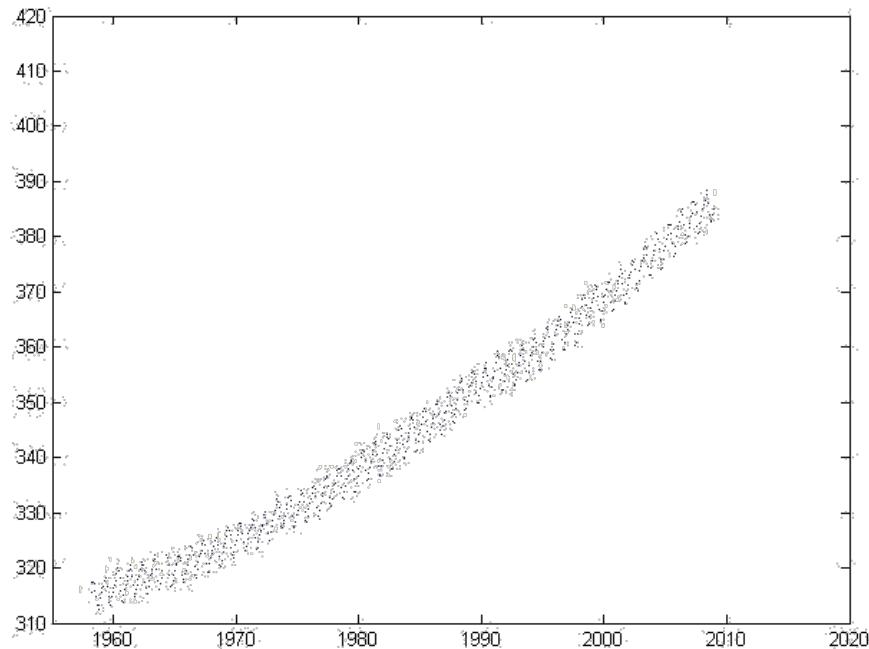
# Optimizing Marginal Likelihood (ML)

$$\text{ML} = \log(p(y|X, \theta)) = -\frac{1}{2}y^T K^{-1}y - \frac{1}{2}\log|K| - \frac{n}{2}\log(2\pi)$$

- It is a combination of **data-fit term**, a **complexity penalty** term and a **normalization term**



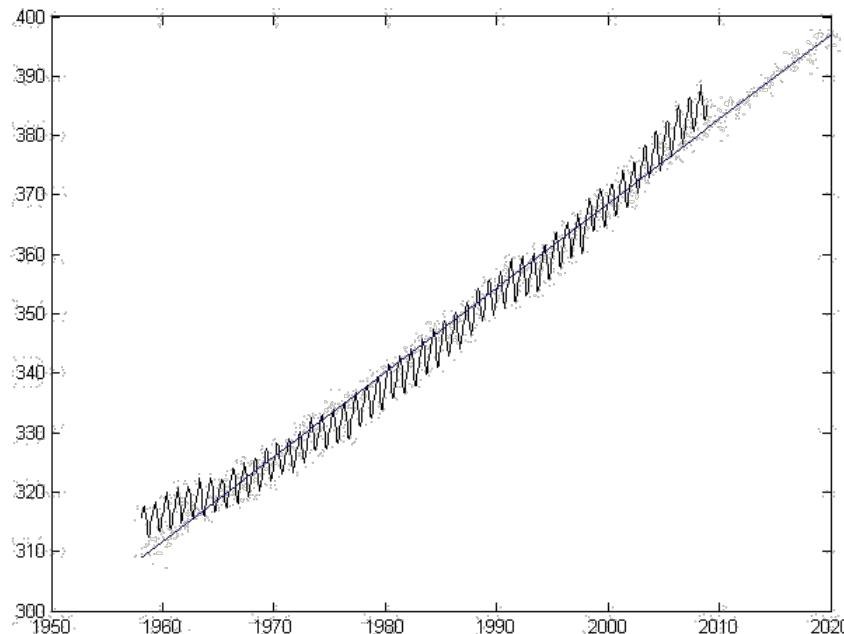
## A SIMPle Example



Month-wise data of Co2 concentration in atmosphere at Hawaii

Image Source: <http://mlg.eng.cam.ac.uk/teaching/4f13/1314/>

## Example – Linear Regression



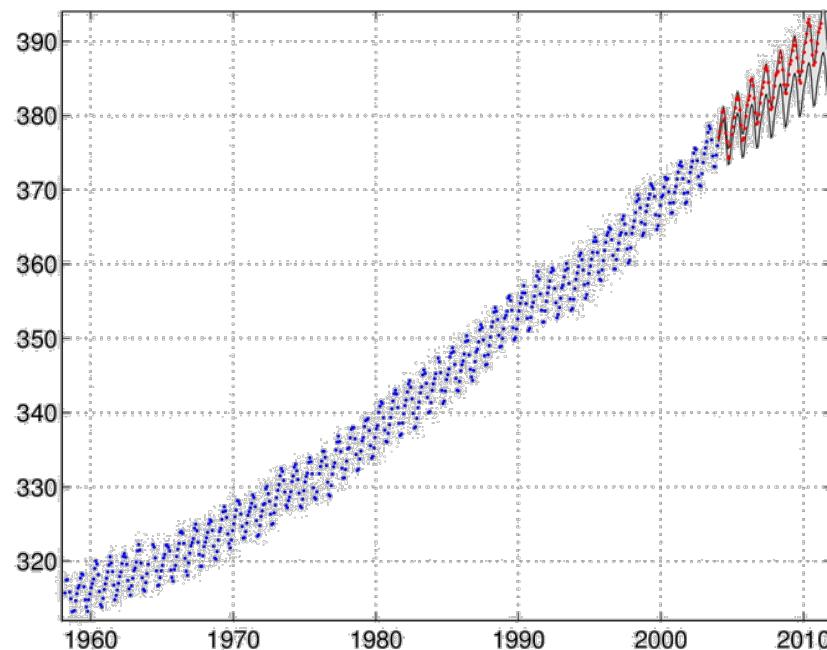
Should we choose a **polynomial**?

What **degree** of polynomial should we choose? (overfitting)

For a given degree, what **parameters** of polynomial should we choose

Image Source: <http://mlg.eng.cam.ac.uk/teaching/4f13/1314/>

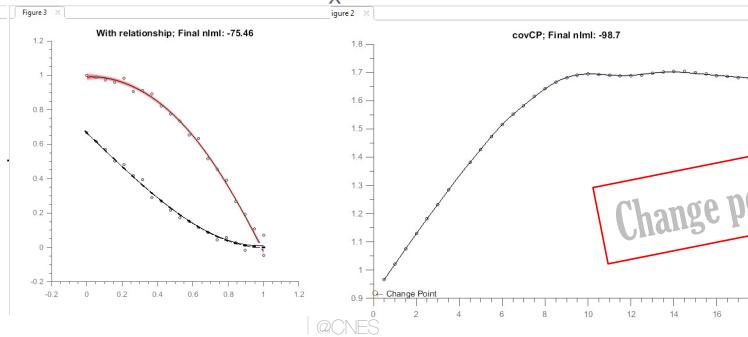
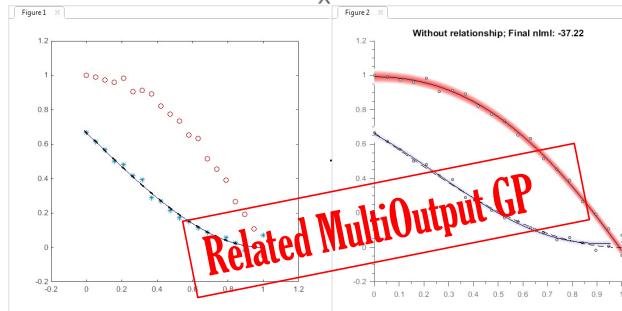
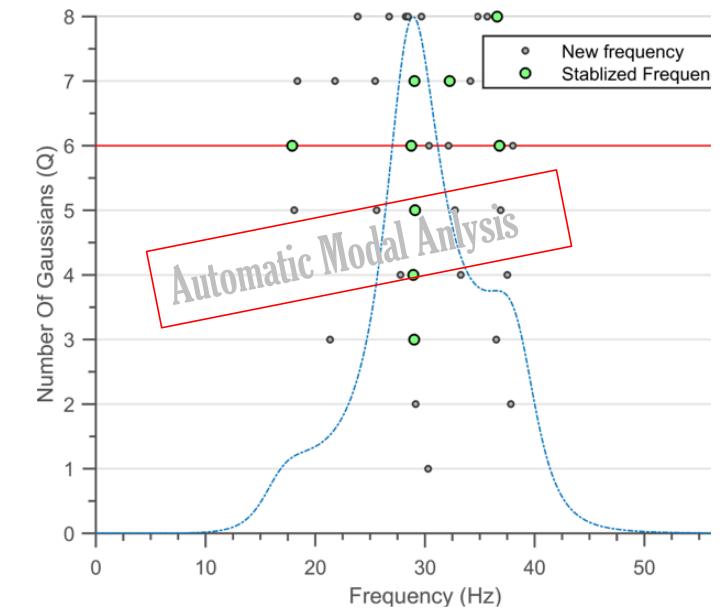
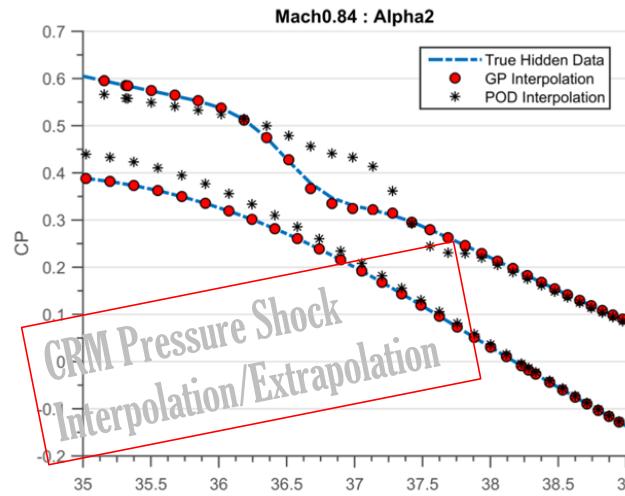
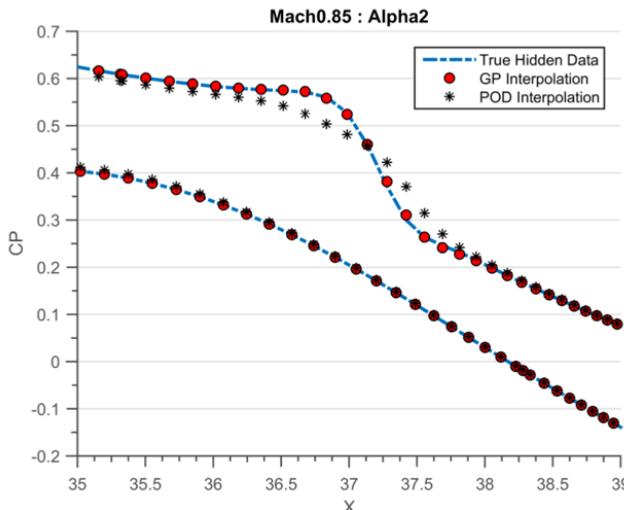
## Example – Gaussian Process



Predicted variance after year 2005 in grey, real data-points in red

Image Source: <http://mlg.eng.cam.ac.uk/teaching/4f13/1314/>

# Some ML applications



# Multi-Output Gaussian Process – Flight Test examples

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Approximate Inference in Related Multi-output Gaussian Process Regression. Lecture Notes in Computer Science. 10163, 88-103. 2017

Given:  $f_1 = g(f_2, x)$

- Earlier examples include **Gradient Enhanced Kriging** <sup>\$</sup> (GEK) or **Co-kriging** \*
- But we want to expand this to integral enhanced kriging, double differential, or any functional relationship between outputs

\* Forrester et al (2007) Multi-fidelity optimization via surrogate modelling. Proceedings of the Royal Society A, 463(2008), 3251–3269, (doi:10.1098/rspa.2007.1900).

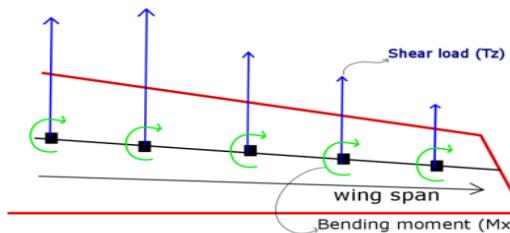
\$ Liu. Development of gradient-enhanced kriging approximations for multidisciplinary design optimization. 2003.

- Add physics<sup>£</sup> as constraints in the mathematical problem

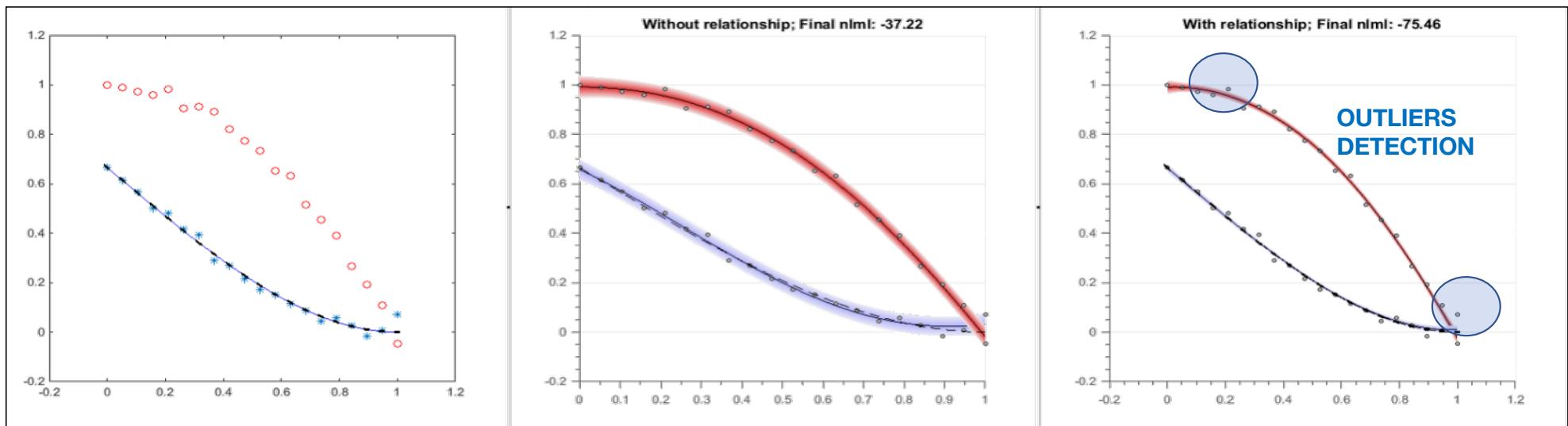
£Constantinescu, Physics-based covariance models for Gaussian processes with multiple outputs." International Journal for Uncertainty Quantification 3.1 (2013).

Example 1: use the Relationship between Tz and Mx permits to reduce uncertainties

Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Sparse Physics-Based Gaussian Process for Multi-output Regression using Variational Inference. Proceedings of ICPRAM 2016 2016



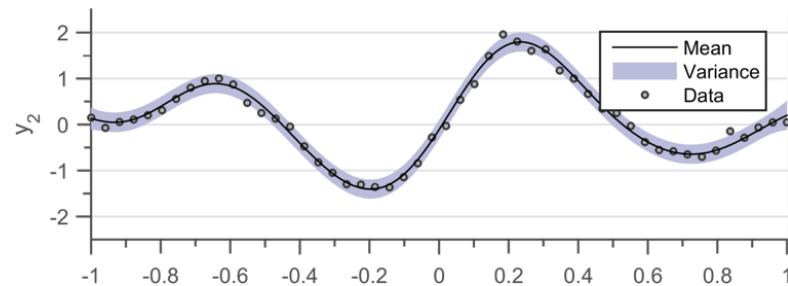
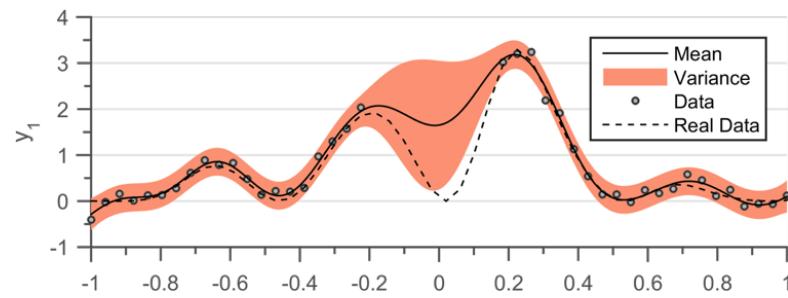
$$Mx = \int_{\eta}^{\eta_{edge}} (x - \eta) Tz \, dx$$



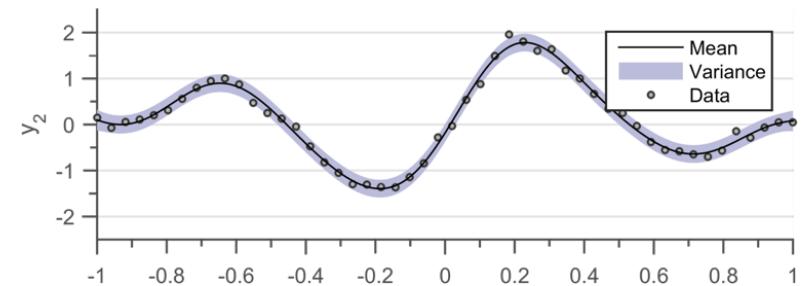
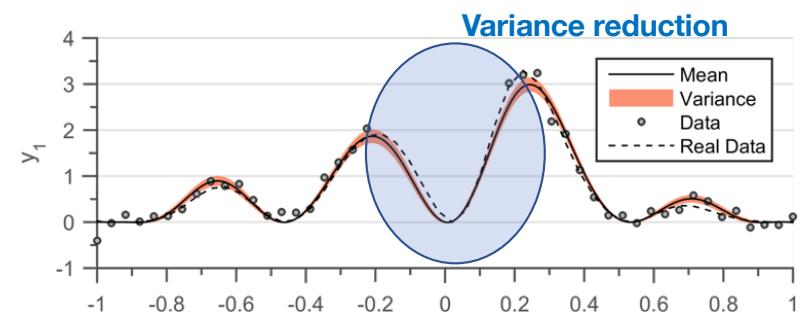
## Example 2: Faulty sensors (using synthetic data)

$$y_1 = (y_2)^2$$

**Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Adding Flight Mechanics to Flight Loads Surrogate Model using Multi-Output Gaussian Processes. AIAA AVIATION 2016**



Independent GPs



Related GPs

## Papers & conf on this topic

*Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Approximate Inference in Related Multi-output Gaussian Process Regression. Lecture Notes in Computer Science. 10163, 88-103. 2017*

*Chiplunkar and J. Morlier. Operational Modal Analysis in Frequency Domain using Gaussian Mixture Models . Proceedings of IMAC XXXV, 2017*

*Chiplunkar, E. Bosco and J. Morlier. Gaussian Process for Aerodynamic Pressures Prediction in Fast Fluid Structure Interaction Simulations. Proceedings of WCSMO12 2017*

*Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Adding Flight Mechanics to Flight Loads Surrogate Model using Multi-Output Gaussian Processes. AIAA AVIATION 2016*

*Chiplunkar, E. Rachelson, M. Colombo and J. Morlier. Sparse Physics-Based Gaussian Process for Multi-output Regression using Variational Inference. Proceedings of ICPRAM 2016 2016*

*Chiplunkar, A., Rachelson, E., Colombo, M., & Morlier, J. (2016, April). Identification of Physical Parameters Using Change-Point Kernels. In Society for Industrial and Applied Mathematics, Uncertainty Quantification, 2016.*

*Morlier, J., Basile, A., Chiplunkar, A., & Charlotte, M. (2018). An EGO-like optimization framework for sensor placement optimization in modal analysis. Smart Materials and Structures, 27(7), 075004.*

*Several Papers in preparation*

# What if

We use Surrogate models to develop Efficient Global Optimization



- EGO\* (unconstrained problem)
- SEGO\$ (constrained problem)

\*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. *Journal of Global optimization*, 13(4), 455-492.

\$Sasena, M. J., Papalambros, P., & Goovaerts, P. (2002). Exploration of metamodeling sampling criteria for constrained global optimization. *Engineering optimization*, 34(3), 263-278.

1. GP aka Kriging

## 2. Kriging for Global Optimization

3. New developments in topology optimization

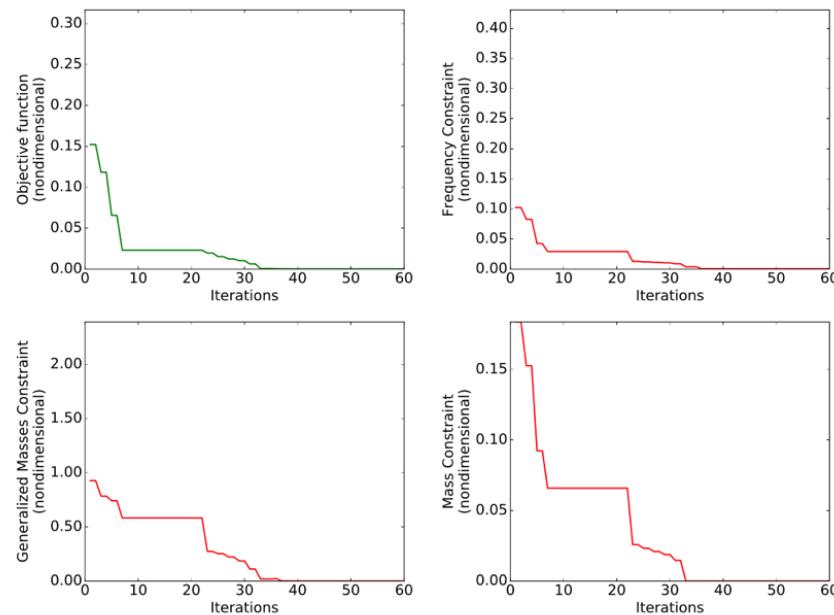
4. Codesign is MDO?

**Joint Work since 2007 with N. Bartoli (Onera)**



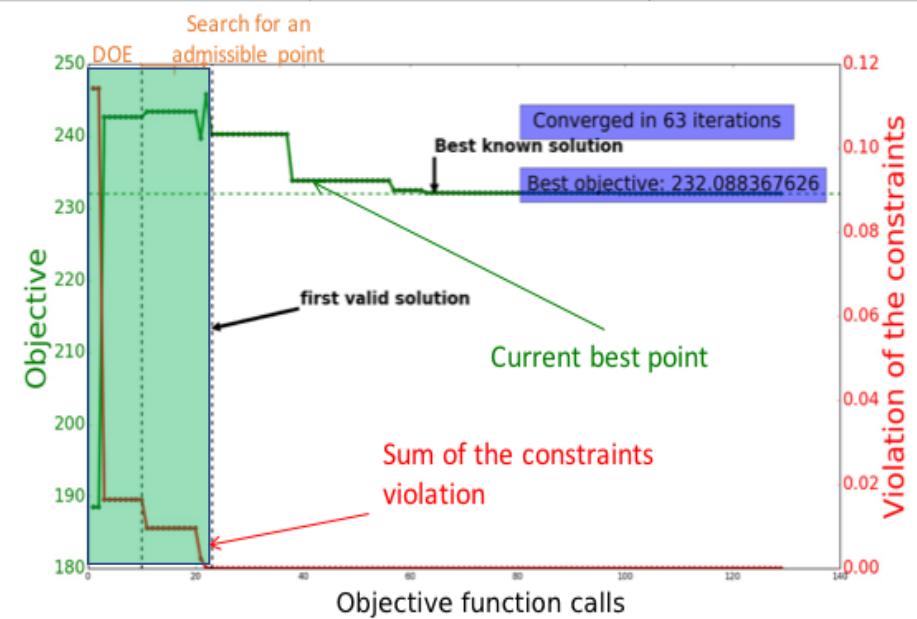
# New paradigm for Surrogate Based Optimization (SBO)

## Gradient based Optimality, Feasibility



Stopping criteria: tolfun, tolx, maxiter

## SBO Exploration, Exploitation



Stopping criteria: Max Budget (Function calls)

A good starting point  $X_0$ =Forrester's book

# **Engineering Design via Surrogate Modelling**

## **A Practical Guide**

**Alexander I. J. Forrester, András Sóbester and Andy J. Keane**

*University of Southampton, UK*

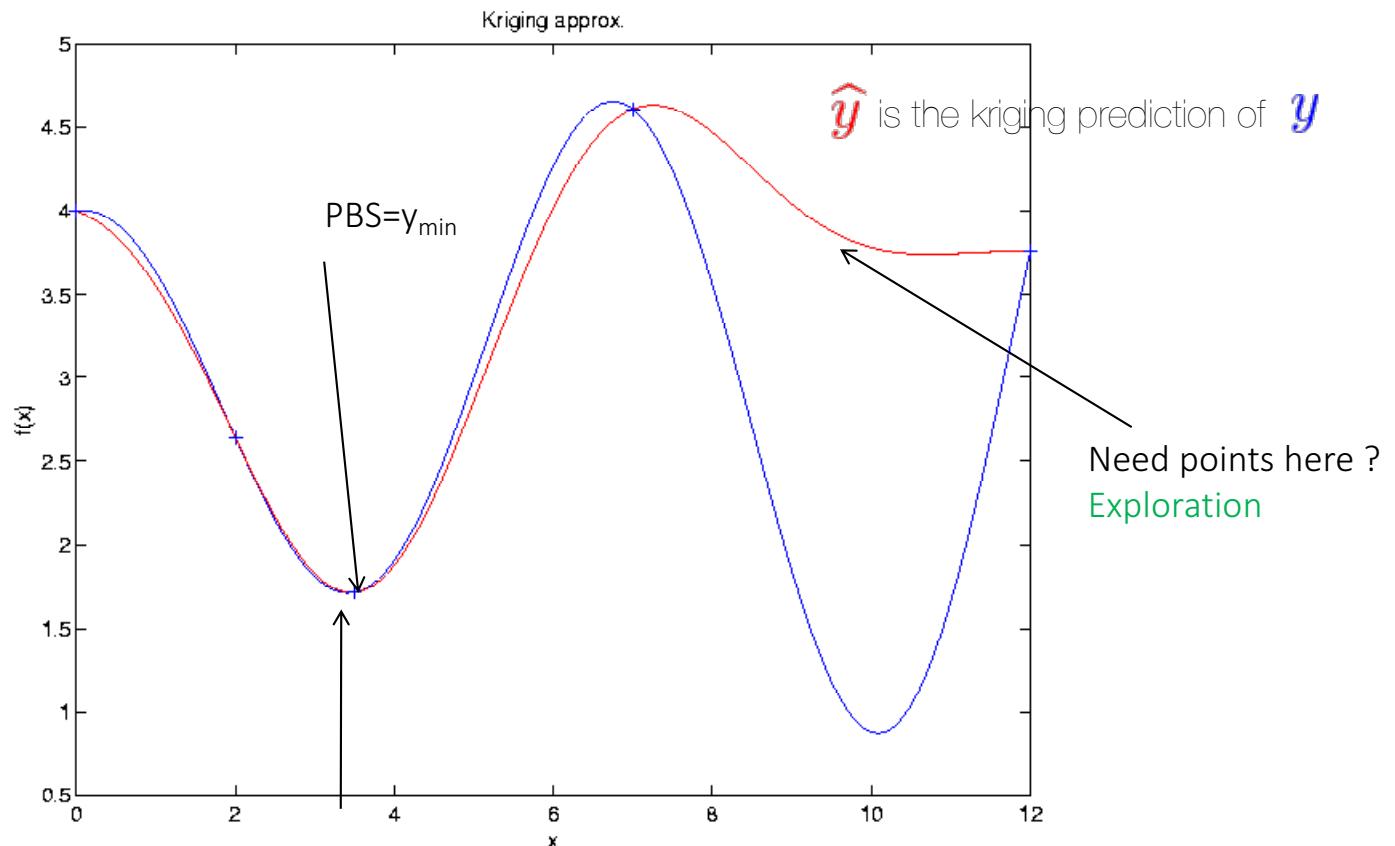
The goal is: find min of  $f(x)$  by sampling + and Kriging updating

Where do I need to update  
my sampling?

We note the present best solution  
(PBS= $y_{\min}$ )

At every  $x$  there is some chance of  
improving on the PBS.

Then we ask: Assuming an  
improvement over the PBS, where is it  
likely to be largest?



Exploitation may drive the optimization to a local optimum

In supervised mode . . . have a look to  $\max(\text{RMSE})$

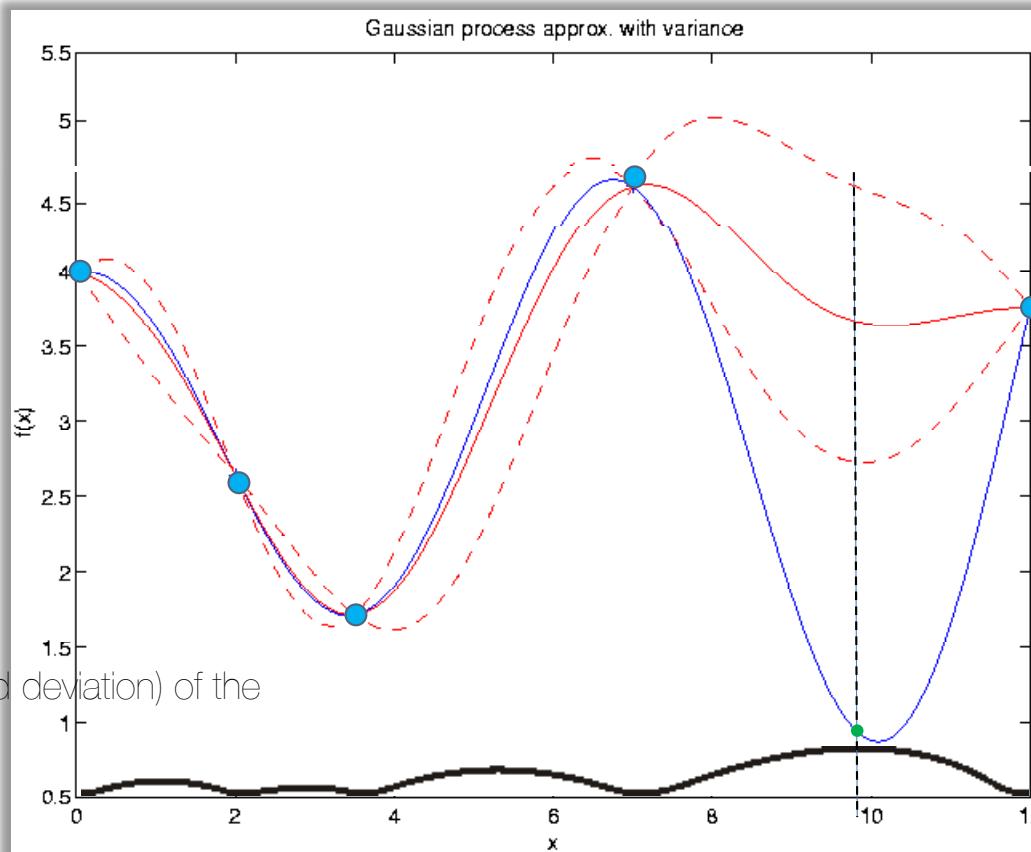
**Not possible to compute  
the error: we don't know  
for each  $x$  the true value of  
the function**

**But.... Can we use GP  
properties ?**

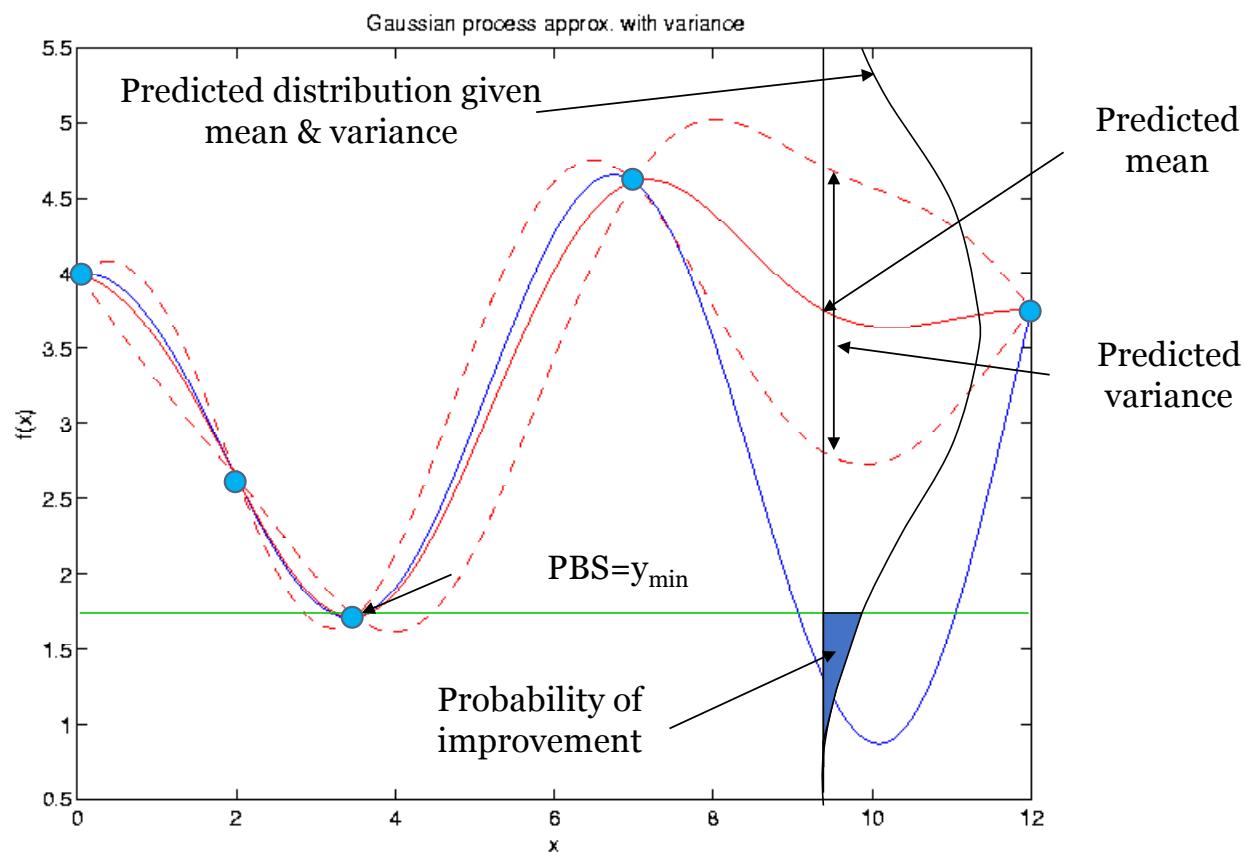
$\hat{y}$  is the kriging prediction of  $y$

$\hat{s}$  is the estimation error (standard deviation) of the  
prediction (often noted  $\sigma_y$ )

PBS= $y_{\min}$



# Probability of improvement



Improvement ... explicitly

- Improvement :  $I(\mathbf{x}) = \max(y_{\min} - \hat{Y}(\mathbf{x}), 0)$
- Expected Improvement :

$$\text{EI}(x) = \mathbb{E}[\max(0, y_{\min} - \hat{y}(x))]$$

$$E[I(\mathbf{x})] = \int_{-\infty}^{y_{\min}} (y_{\min} - \hat{y}) \varphi \left( \frac{y_{\min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})} \right) d\hat{y}$$

$$E[I(\mathbf{x})] = (y_{\min} - \mu_{\hat{Y}}(\mathbf{x})) \Phi \left( \frac{y_{\min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})} \right) + \sigma_{\hat{Y}}(\mathbf{x}) \varphi \left( \frac{y_{\min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})} \right)$$

global optimum can be found because  
 $P[I(x)] = 0$  when  $s = o$  so that there is no  
probability of improvement at a point which  
has already been sampled → guarantees  
global convergence

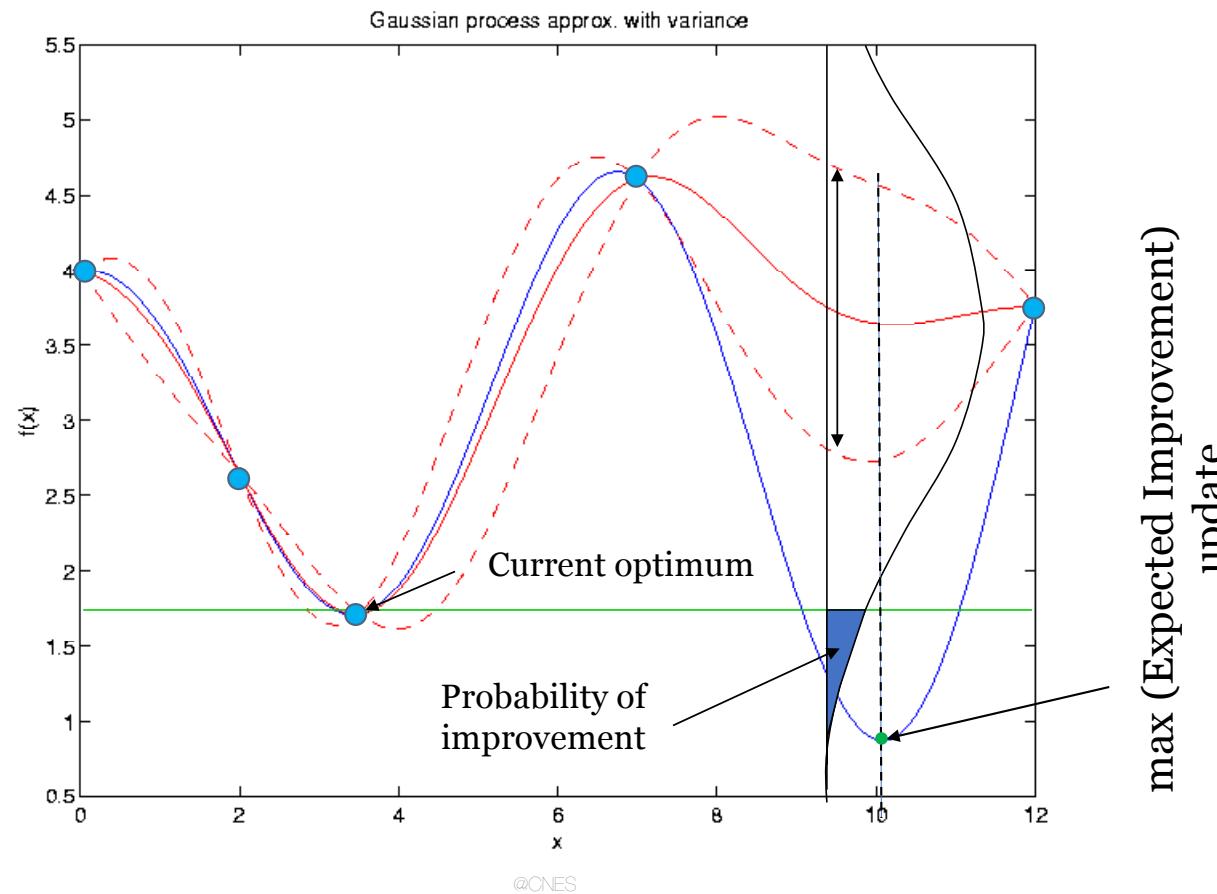
Exploitation

Exploration

$\Phi$ : cumulative distribution function       $\mathcal{N}(0, 1)$      $\phi$ : probability density function       $\mathcal{N}(0, 1)$

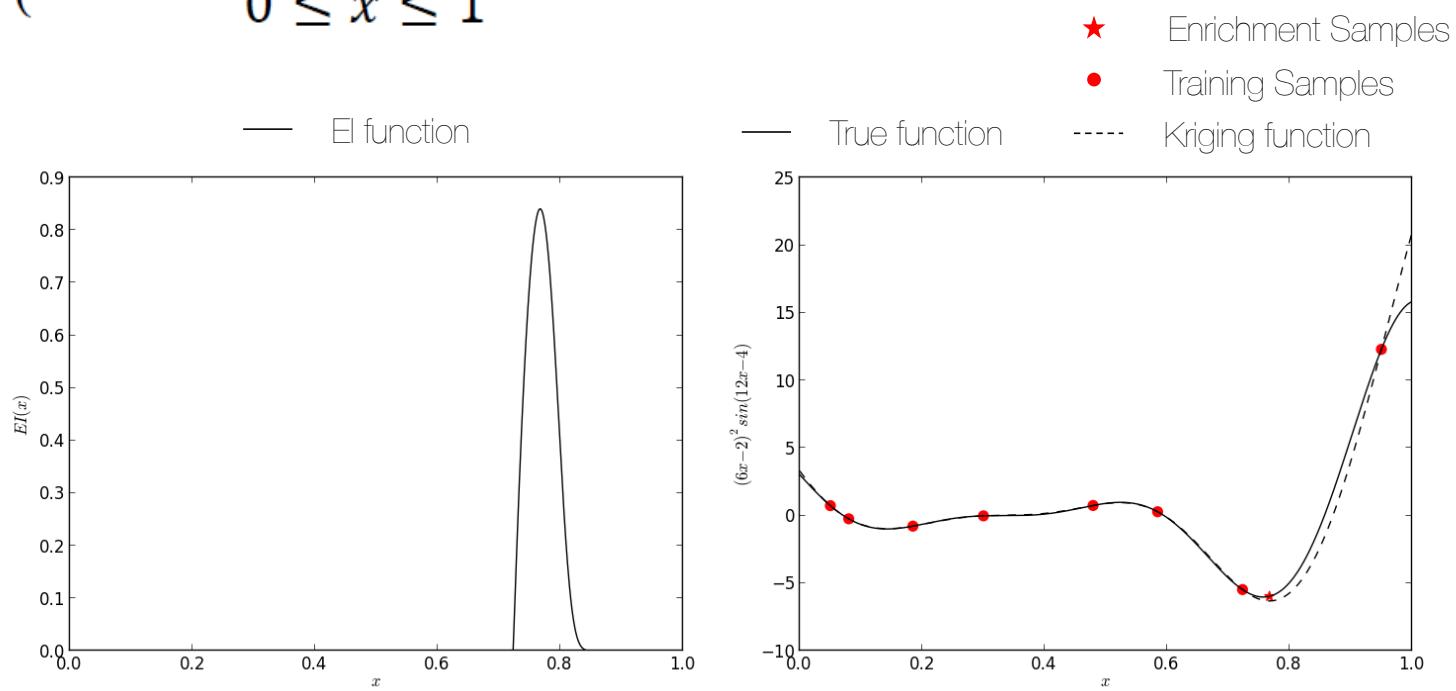
\*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. *Journal of Global optimization*, 13(4), 455-492.

Infill Criteria :  $\max(\text{Expected improvement})$



## Illustration on 1D example

$$\begin{cases} \min & (6x - 2)^2 \sin(12x - 4) \\ & s.t. \\ & 0 \leq x \leq 1 \end{cases}$$



# Limits of EI

Expected Improvement criterion (EI) (to maximize)

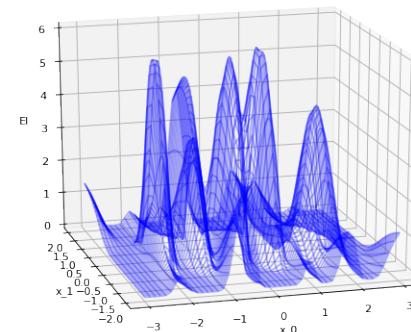
$$E[I(\mathbf{x})] = (y_{min} - \mu_{\hat{Y}}(\mathbf{x}))\Phi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right) + \sigma_{\hat{Y}}(\mathbf{x})\varphi\left(\frac{y_{min} - \mu_{\hat{Y}}(\mathbf{x})}{\sigma_{\hat{Y}}(\mathbf{x})}\right)$$

Exploitation

- + Analytical formula  
(Criteria and gradient)
- Highly multimodal

- ➔ Quick to evaluate
- ➔ Hard to optimize

Exploration



\*Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. *Journal of Global Optimization*, 13(4), 455-492.

# Smoothen criteria WB2 (R. Olivanti MsC)

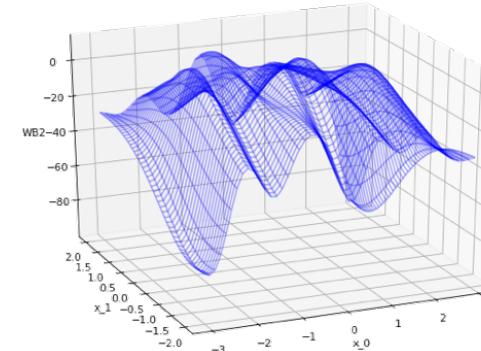
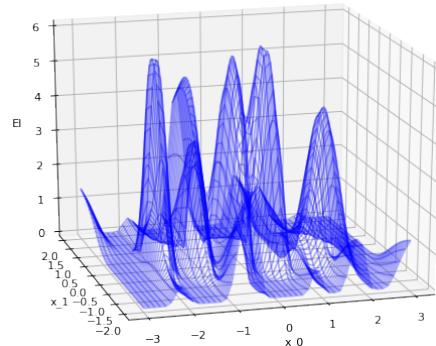
$$\text{WB2}(x) = \text{EI}(x) - \hat{y}(x)$$

Influence of surrogate model prediction

- + 'Smoothen'
- + Quicker convergence
- Lack of normalization

→ Easier to optimize

→ Reduce the global aspect



EI and WB2 criteria computed on same Kriging surrogate

\$Sasena, M. J., Papalambros, P., & Goovaerts, P. (2002). Exploration of metamodeling sampling criteria for constrained global optimization. *Engineering optimization*, 34(3), 263-278.

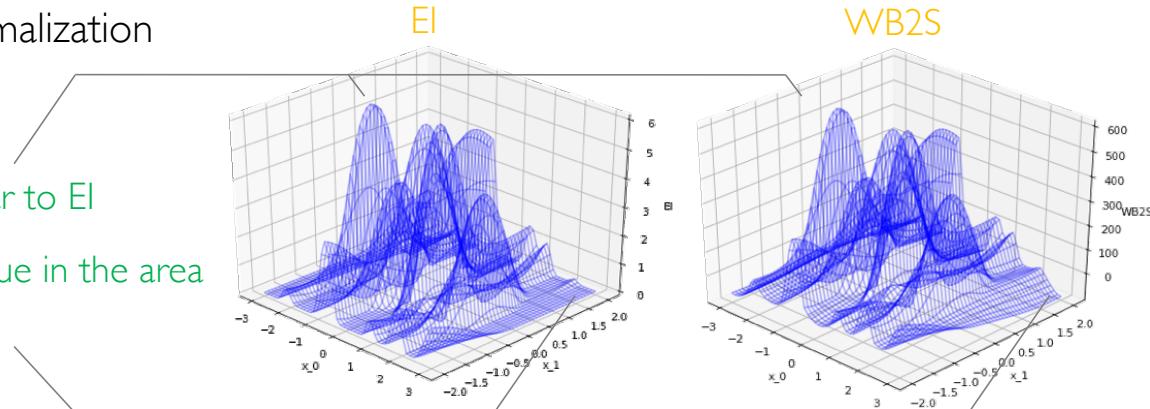
# New infill sampling criteria

WB2S scaled criterion

$$\text{WB2S}(x) = s \text{ EI}(x) - \hat{y}(x)$$

- EI numerically improved
- ‘dynamical’ normalization

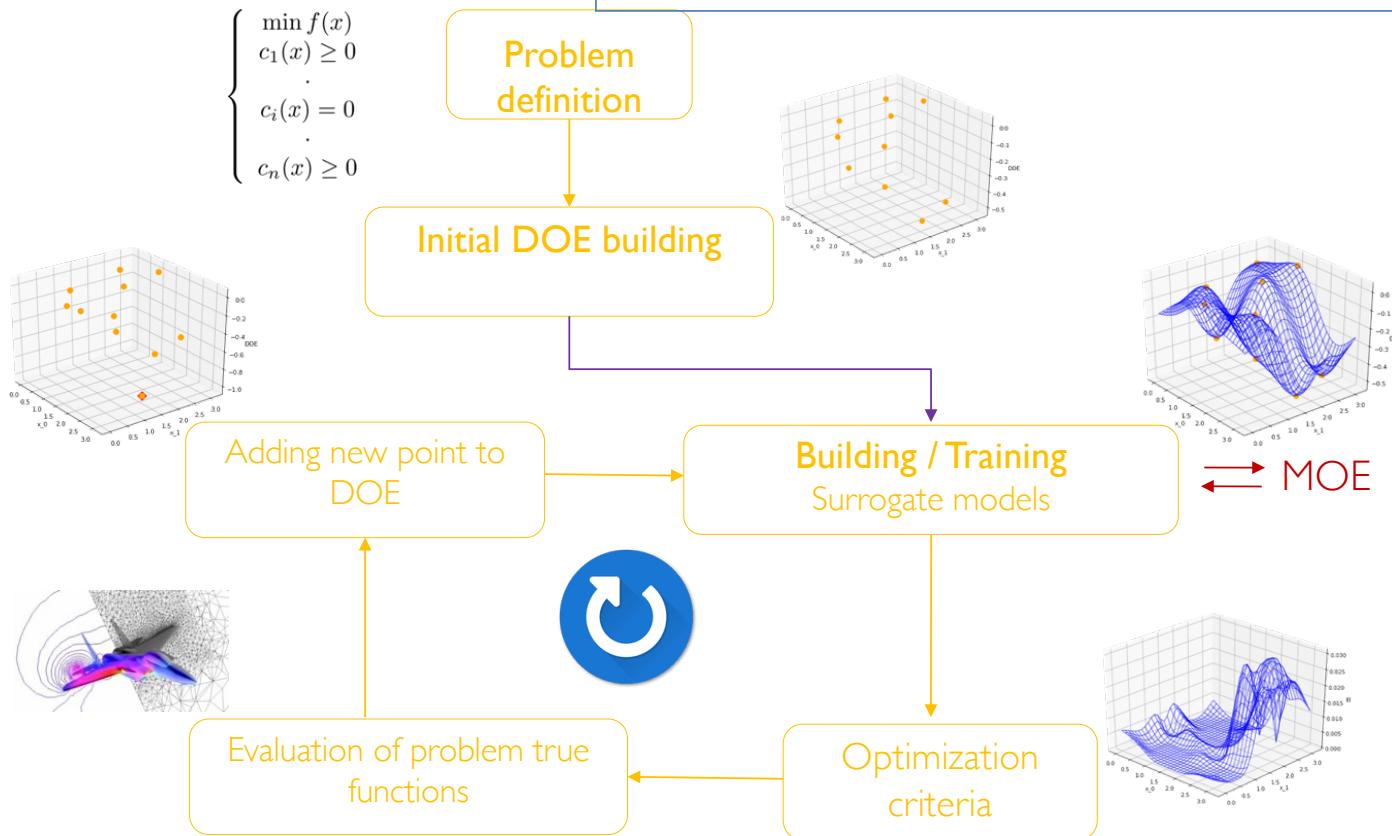
- + Exploration similar to EI
- + non negligible value in the area where EI is low



**N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, “An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization”, 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433, Denver, USA, June 2017**

# SEGOMOE algorithm

**N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, “An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization”, 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433, Denver, USA, June 2017**



# New formulation

Adapted from Super EGO

**N. Bartoli, T. Lefebvre, S. Dubreuil, R. Olivanti, N. Bons, J.R.R.A. Martins, M.-A. Bouhlel, J. Morlier, “An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization”, 18th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference, AIAA-2017-4433, Denver, USA, June 2017**

Costly initial problem

$$\begin{cases} \min f(x) \\ c_1(x) \geq 0 \\ \cdot \\ c_i(x) = 0 \\ \cdot \\ c_n(x) \geq 0 \end{cases}$$

Possibly Multimodal

Global optimization method

Cheap enrichment problem

$$\rightarrow \begin{cases} \max_{x \in \mathbb{R}^d} EI(x)/WB2(x)/WB2s(x) \\ \text{s.t.} \\ \hat{c}_1(x) \geq 0 \\ \cdot \\ \hat{c}_i(x) = 0 \\ \cdot \\ \hat{c}_n(x) \geq 0 \end{cases}$$

*n + 1*  
metamodels

Multimodal



# ADODG6 \* testcase (R. Olivanti, R. Priem MsC)

CFD guys know very well the multimodality of this problem...

Wing drag minimization problem (subsonic, Euler equations)

	Function/variable	Description	Quantity	Range
minimize	$C_D$	Drag coefficient	1	
with respect to	$\alpha$	Angle of attack	1	$[-3.0, 6.0]$ ( $^{\circ}$ )
	$\theta$	Twist	8	$[-3.12, 3.12]$ ( $^{\circ}$ )
	$\delta$	Dihedral	8	$[-0.25, 0.25]$ (unit of chord)
		Total variables	17	
subject to	$C_L = 0.2625$	Lift coefficient	1	
		Total constraints	1	

Can SEGOMOE help us to reach the global optimum ?  
Is it less dependant on X0 compared to SNOPT £?

\*AIAA, Aerodynamic Design Optimization Discussion Group  
<http://mdolab.engin.umich.edu/content/aerodynamic-design-optimization-workshop>

£ <https://web.stanford.edu/group/SOL/snoot.htm>

# ADODG\* 6 TOOLS



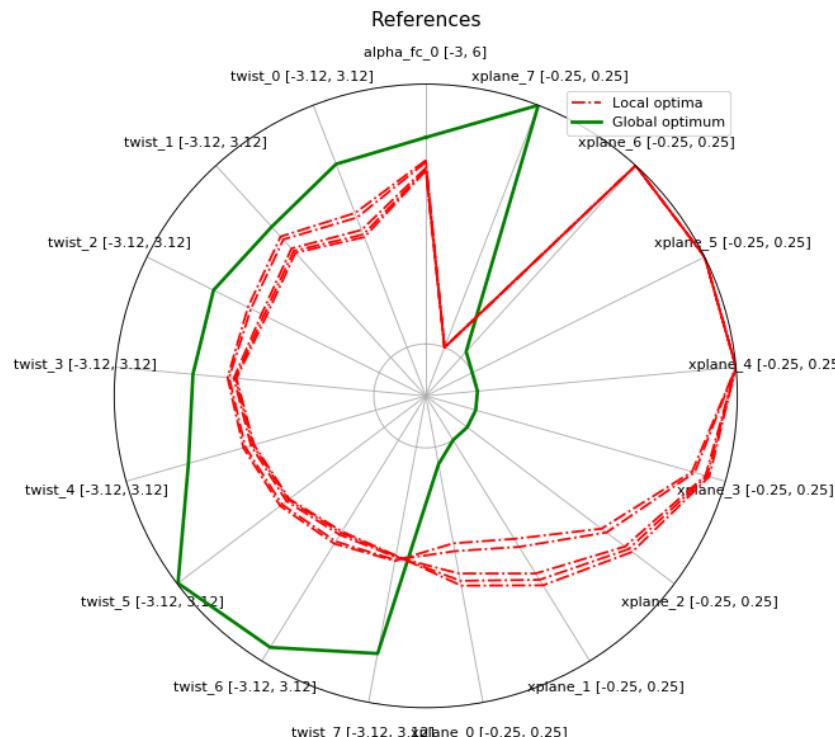
UCSD/  
Stanford



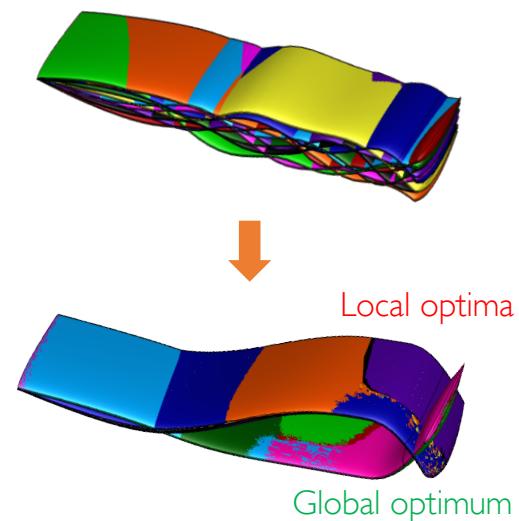
**GOAL:** Bench our SBO method with a reference SNOPT  
→ Compare the robustness of the solution with respect to  $X_0$

# Multimodal optimization problem (SNOPT Benchmark)

Wing drag minimization problem (subsonic, Euler equations with ADFlow solver) (Mesh 180K cells)



$X_0$ : Multi start with DOE of 15 (LHS)



67% Local optimum

Accuracy  $Cz \sim 10^{-10}$  – 300 calls to ADflow

→ 5/15 runs found the global optimum

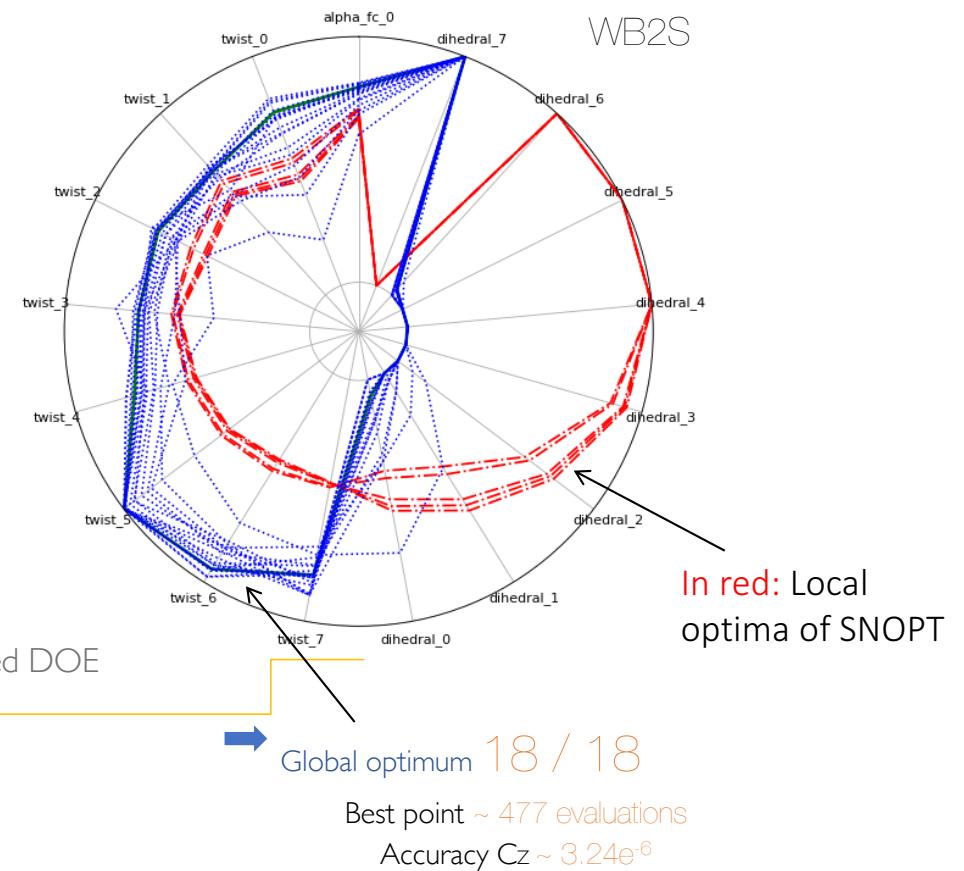
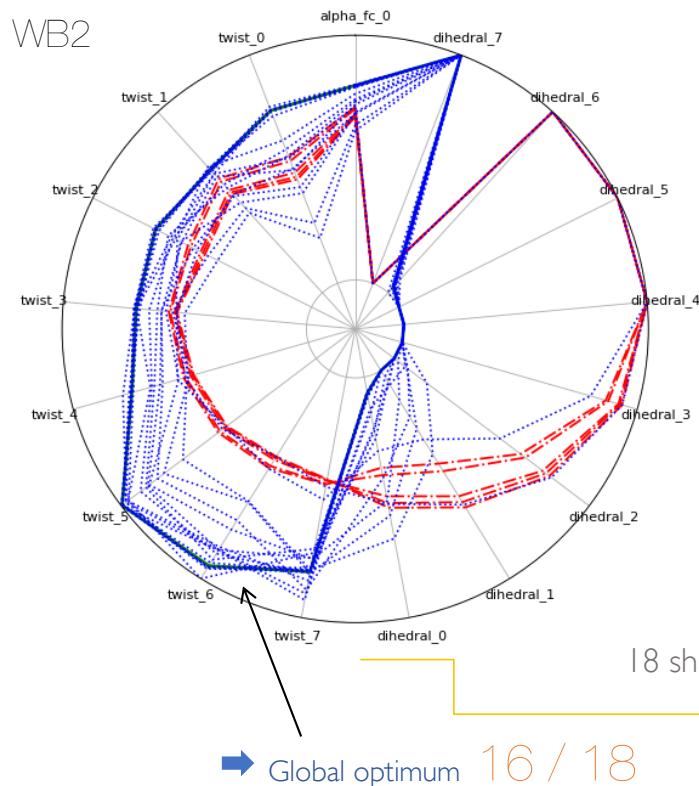
# Multimodal optimization problem (SEGOMOE 2)

Frozen budget: 500 evaluations

Surrogate models : KPLSK

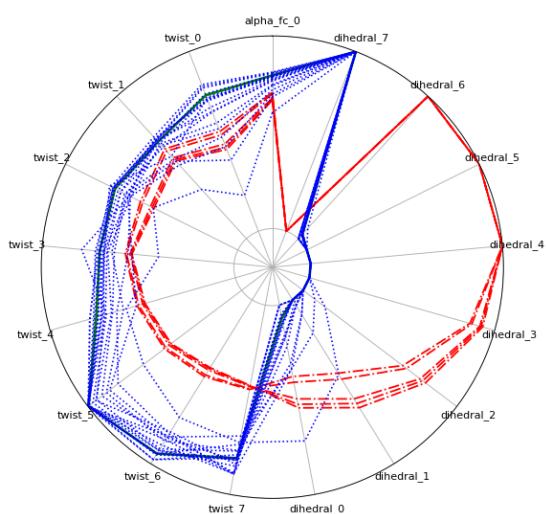
Initial DOE= 68 points (4xd)

- ..... DOE=17 n\_runs=18
- - - Local optima
- Global optimum

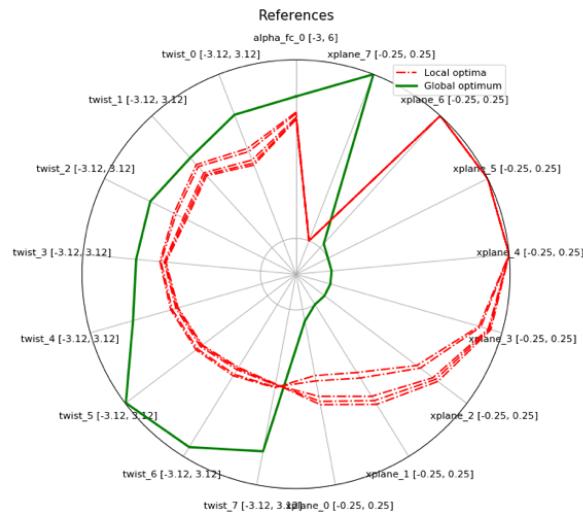


Idea: 2 steps method

1. Start with SEGOMOE, stop with maxiter  
 $x^\circ$  with high confidence near  $x^*$  (so we avoid to get stuck in local minima !)



2. Use  $x^\circ$  as  $x_0$  in SNOPT to reach rapidly the Global Optimum  $x^*$



# A surrogate model toolbox: SMT (M. Bouhlel Postdoc)

[SMT 0.2 documentation »](#)



An open source python toolbox for surrogate models (since July 2017)

**Bouhlel, M. A., Bartoli, N., Otsmane, A., & Morlier, J. (2016). Improving kriging surrogates of high-dimensional design models by Partial Least Squares dimension reduction. Structural and Multidisciplinary Optimization, 53(5), 935-952.**

- Including the models KPLS and KPLS-K (for processing HD input variables)
- Focus on derivatives:
  - training derivatives used for gradient-enhanced modeling,
  - prediction derivatives,
  - derivatives with respect to the training data

## SMT: A Python Surrogate Model Toolbox

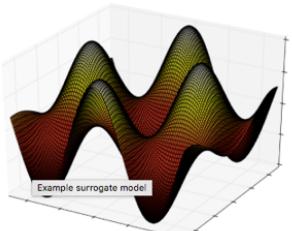
### preamble

SMT, surrogate model toolbox, is a Python toolbox that includes four types of surrogate models---the least square (ls), square polynomial (pa2), inverse distance weighting (idw) and kriging-based models.

The toolbox supports Linux and Microsoft Windows, except the idw model which is available only with Linux.

This package is devoted to gathering several types of surrogate models within the same platform, for providing a benchmark to the engineering field. Another purpose of the SMT is for research purposes.

SMT is typically used as a surrogate model for a (time-consuming) computer model.



<https://github.com/SMTorg/SMT>

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44

## Papers & conf on this topic

Bouhlel, M. A., Bartoli, N., Otsmane, A., & Morlier, J. (2016). *Improving kriging surrogates of high-dimensional design models by Partial Least Squares dimension reduction*. *Structural and Multidisciplinary Optimization*, 53(5), 935-952.

Bouhlel, M. A., Bartoli, N., Otsmane, A., & Morlier, J. (2016). *An improved approach for estimating the hyperparameters of the kriging model for high-dimensional problems through the partial least squares method*. *Mathematical Problems in Engineering*, 2016.

Bouhlel, M., Bartoli, N., Regis, R. G., Otsmane, A., & Morlier, J. (2018). *Efficient global optimization for high-dimensional constrained problems by using the Kriging models combined with the partial least squares method*. *Engineering Optimization*, 1-16.

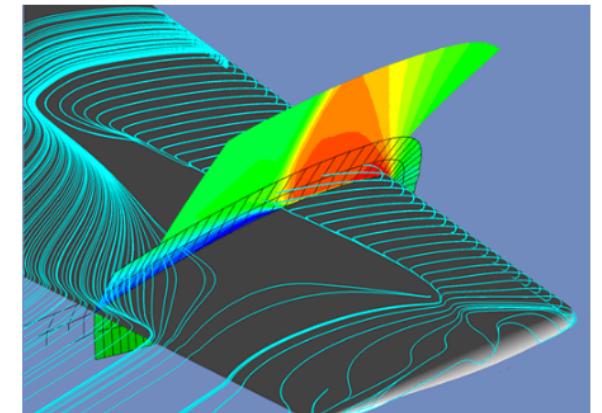
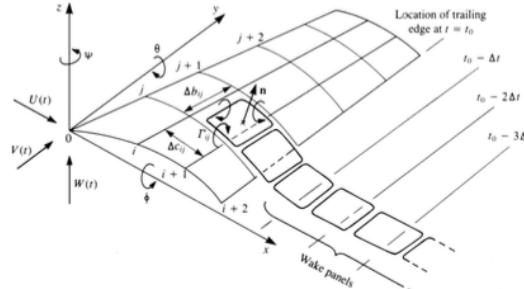
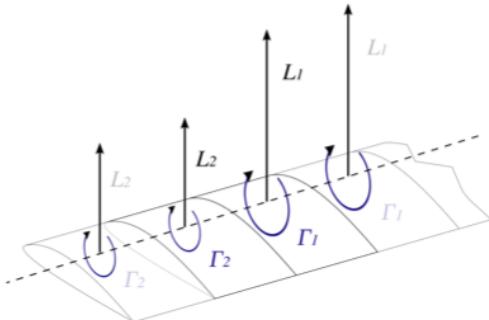
Bartoli, N., Lefebvre, T., Dubreuil, S., Olivanti, R., Bons, N., Martins, J. & Morlier, J. (2017). *An adaptive optimization strategy based on mixture of experts for wing aerodynamic design optimization*. In 18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4433).

Bartoli, N., Bouhlel, M. A., Kurek, I., Lafage, R., Lefebvre, T., Morlier, J., & Regis, R. (2016). *Improvement of efficient global optimization with application to aircraft wing design*. In 17th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4001).

*Several Papers in preparation*

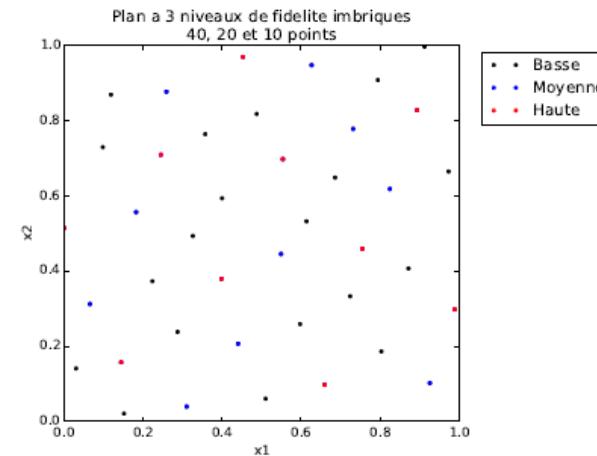
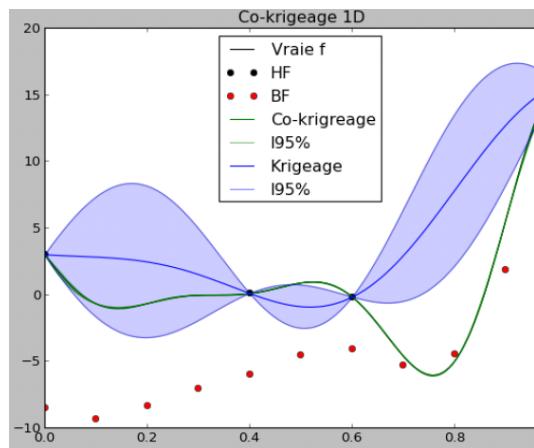
# What if ?

- Several levels of fidelity of the same simulation are available  
→ For example, in aerodynamics: Lifting line theory, Vortex lattice method, and RANS CFD code



# Multi-fidelity kriging (M. Meliani Msc)

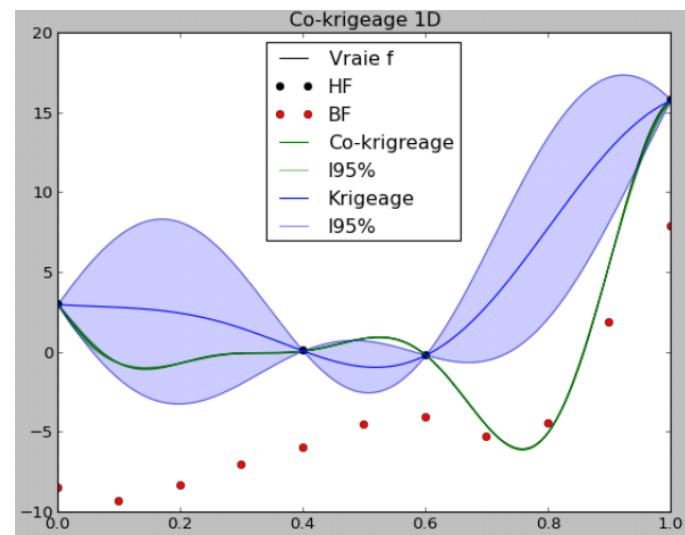
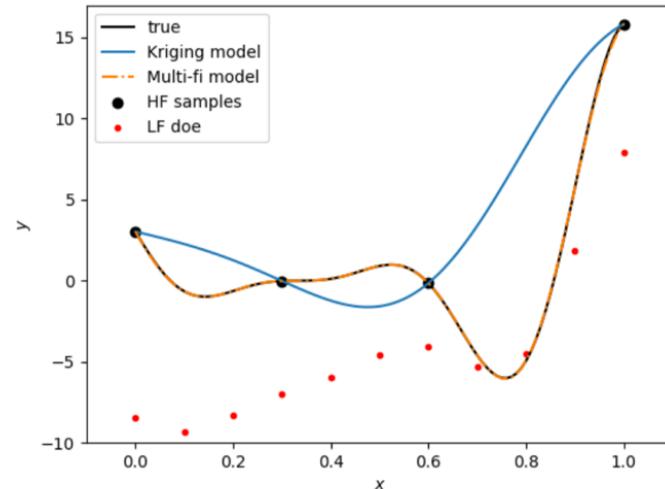
- Surrogate adapted for multi-fidelity data (co-kriging with recursive approach)
- Design of experiments adapted to multi-fidelity (nested DOE)
- Developments of modules integrated in the OpenMDAO Framework (since Jan 2015) and now in SMT (since 2018)



Le Gratiet, L. and Garnier, J., "Recursive co-kriging model for Design of Computer experiments with multiple levels of fidelity," International Journal for Uncertainty Quantification, 2014, pp. 365–386

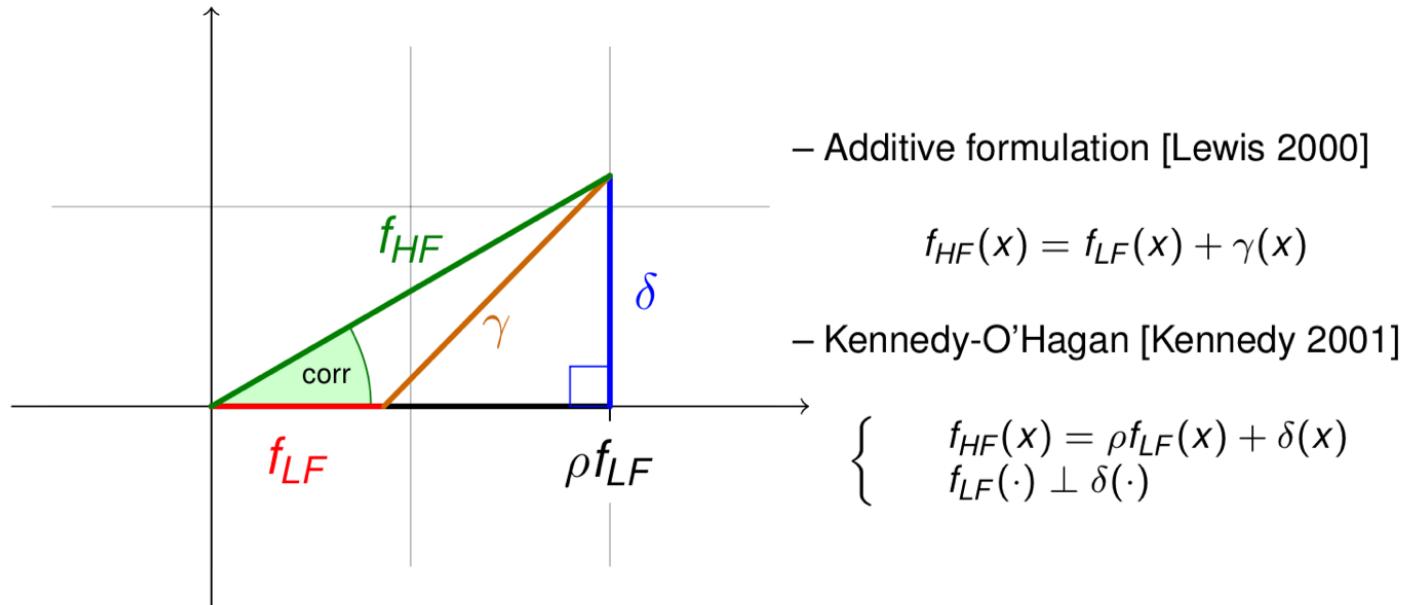
Vauclin, R., "Développement de modèles réduits multi-fidélité en vue de l'optimisation de structures aéronautiques," Tech. rep., ISAE-SUPAERO, July 2014

# How to use low-fidelity information to speed up the optimization?



Remember Co-Kriging is the  
way to learn the difference  
between HF & LF ...

# Co Kriging



The addition of the term  $\rho$  makes the multi-fidelity learning more robust to poor correlation as well as differences in modelization.

<sup>\$</sup>Alexandrov, N., Lewis, R., Gumbert, C., Green, L., & Newman, P. (2000, January). Optimization with variable-fidelity models applied to wing design. In 38th Aerospace Sciences Meeting and Exhibit (p. 841).

Kennedy, M. C., & O'Hagan, A. (2001). Bayesian calibration of computer models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 63(3), 425-464.

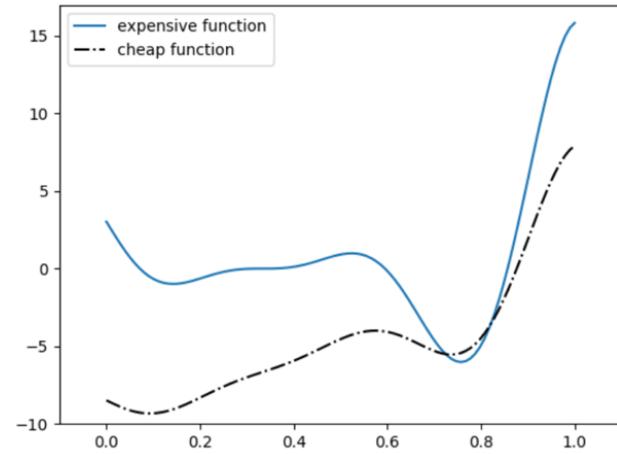
# MFEGO

- ▶ most promising point: EI criterion

$$x^* = \operatorname{argmax}_x (E[I(x)])$$

- ▶ choice of levels of enrichment: trade-off information gain/cost

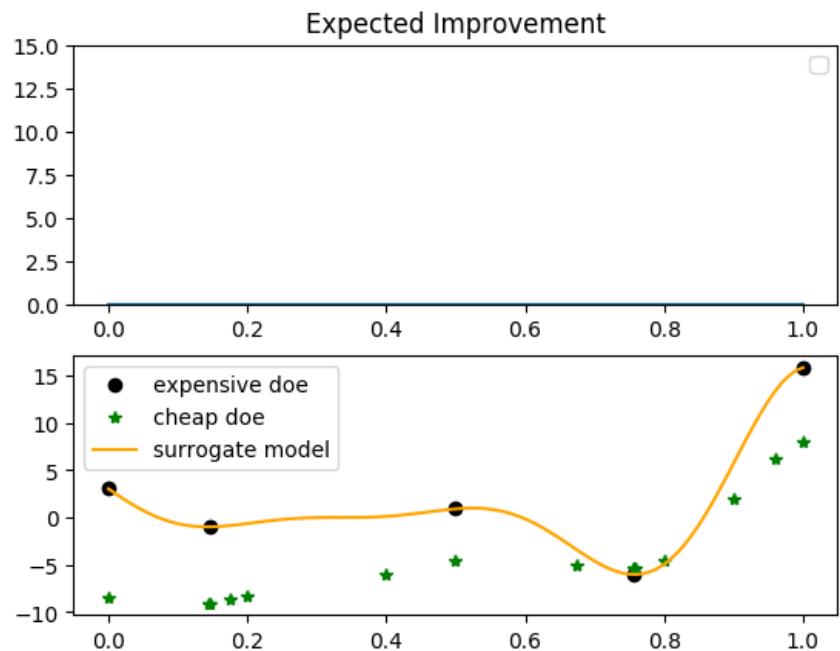
$$k^* = \operatorname{argmax}_{k \in (0, \dots, l)} \frac{\sigma_{red}^2(k, x^*)}{f(c_k)}$$



- ⇒ By using low-fidelity to reduce the uncertainty we reduce the Exploration contribution to the EI criterion  
⇒ High-fidelity is used for Exploitation and model enhancement

$$\begin{aligned} f_{HF}(x) &= (6x - 2)^2 \times \sin(2(6x - 2)) \\ f_{LF}(x) &= 0.5f_{HF} + 10(x - 0.5) - 5 \end{aligned}$$

## Results (Toy problem)

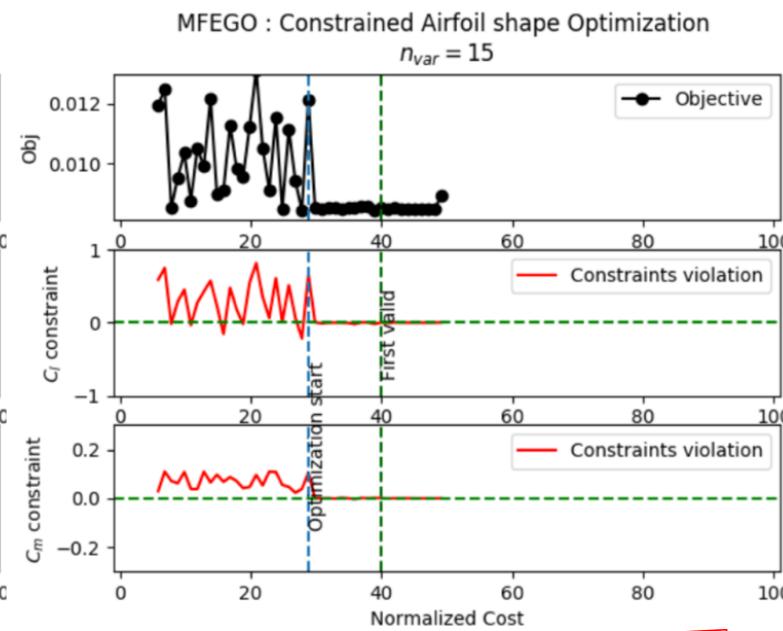
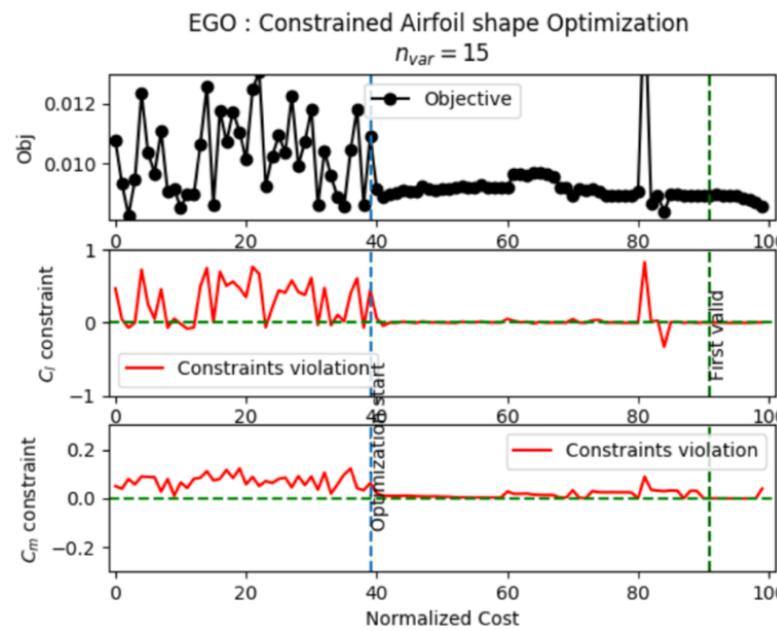
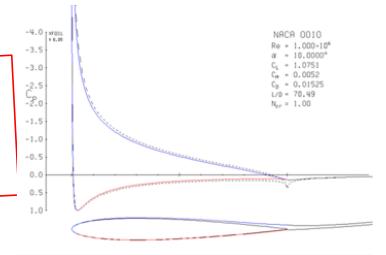


Cost ratio: 1/1000

	HF	LF	Cost
MFEGO	3+2	6+9	5.015
EGO	4+11	-	15

## Second application: Constrained Optimization

**Estimated COST  
RATIO: 1/200**



\*<https://web.mit.edu/drela/Public/web/xfoil/>  
\$ <http://mdolab.engin.umich.edu>

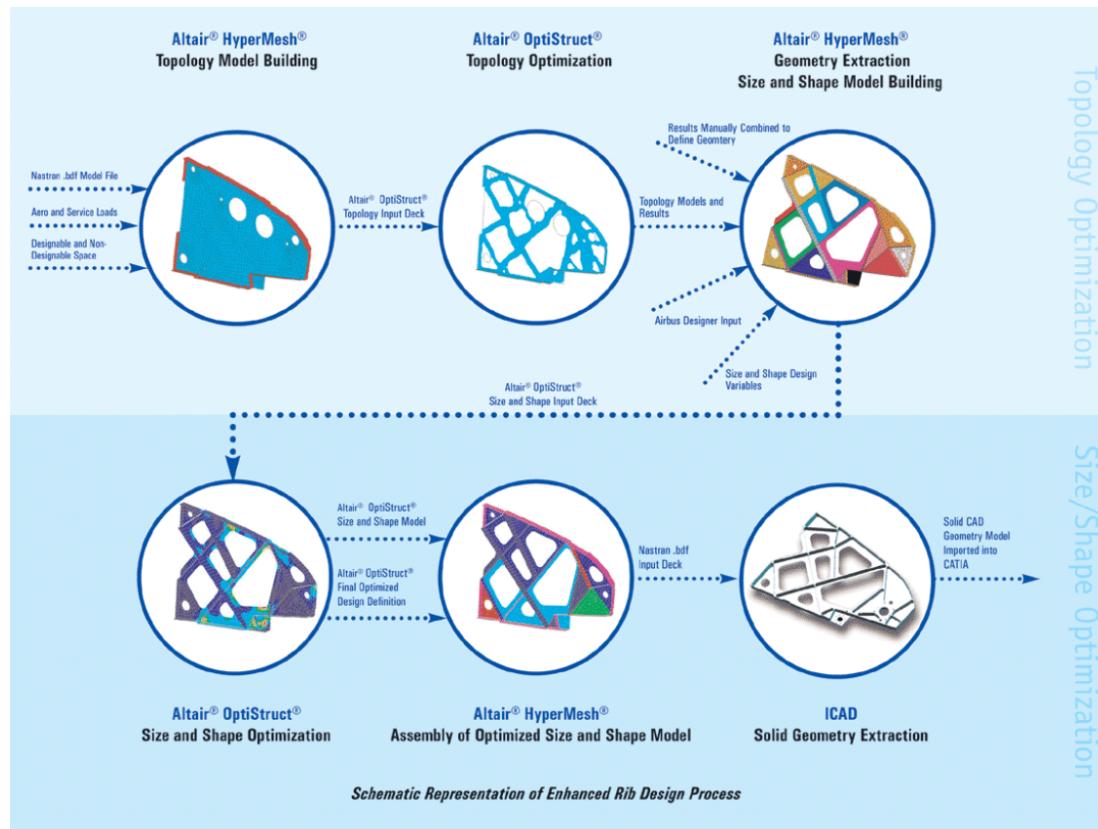
**MFEGO can speed up the Optimization process by reducing the call to HF expensive code !**

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1. GP aka Kriging
2. Kriging for Global Optimization
3. New developments in topology optimization
4. Codesign is MDO ?

# INDUSTRIAL PROCESS

“The art of structure is where to put the holes.”  
~Robert Le Ricolais  
(1894-1977)



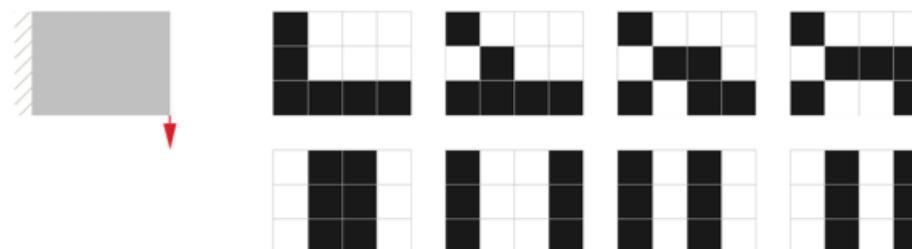
Altair Engineering

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# Pixels

- Finding a solution by checking all the possible combinations IS impossible since the number of topologies  $nT$  increases exponentially with the number of finite elements  $n$
- $nT = 2^n$ ,



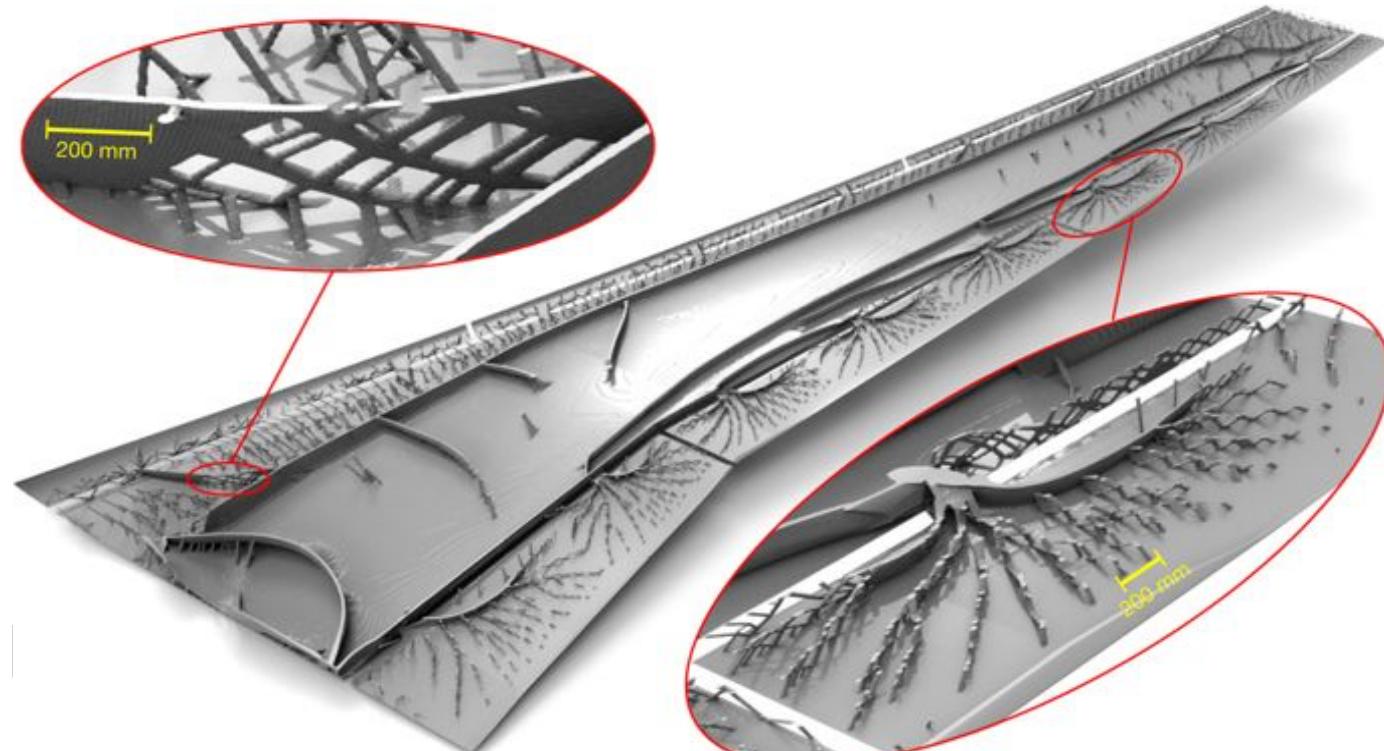
The legal (top) and some illegal (bottom) topologies with 4 by 3 elements

Division into elements (pixels or voxels) and binary decision for each  
or example 10,000 elements --> 210,000 possible configurations!

Pixels?

When the size of the FE model is **increasing**, the SIMP optimization problem is  
... **increasing**

Niels Aage, Erik Andreassen, Boyan S Lazarov, and Ole Sigmund. Giga-voxel computational morphogenesis for structural design. *Nature*, 550(7674):84, 2017.



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56

Topmna.m:  
Visiting scholar at UoM (Thanks Prof MARTINS)

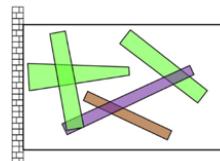
Can we do pattern  
(structural element) recognition?

Struct Multidisc Optim (2016) 53:1243–1260  
DOI 10.1007/s00158-015-1372-3

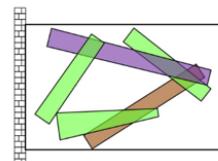
RESEARCH PAPER

**A new topology optimization approach based on Moving Morphable Components (MMC) and the ersatz material model**

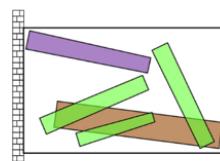
Weisheng Zhang<sup>1</sup> · Jie Yuan<sup>1</sup> · Jian Zhang<sup>1</sup> · Xu Guo<sup>1</sup>



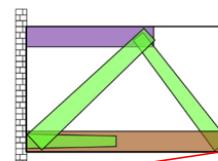
Components: the basic building blocks  
for MMC based topology optimization



Topology 1



Topology 2



Topology 3

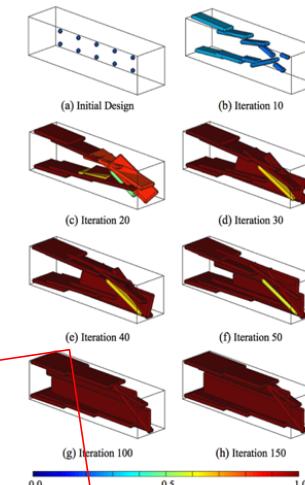
YES... using Explicit Topology Optimization  
And with less design variables !!! We Speed up the process

Struct Multidisc Optim  
DOI 10.1007/s00158-016-1466-6

RESEARCH PAPER

**A geometry projection method for the topology optimization  
of plate structures**

Shanglong Zhang<sup>1</sup> · Julián A. Norato<sup>1</sup> · Arun L. Gain<sup>2</sup> · Naesung Lyu<sup>3</sup>



# Optimization algorithm

## Nodal movement

- Optimization algorithm based on decoupling
- Movement of nodes
  - Direction of the movement
- Measure performance of structure by compliance
  - Move mass nodes in the direction of decreasing compliance

$$\min c(\mathbf{x}) = \mathbf{U}^T \mathbf{F} = \mathbf{U}^T \mathbf{K} \mathbf{U}$$

$$\frac{\partial c}{\partial \rho_e} = -p(\rho_e)^{p-1} \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

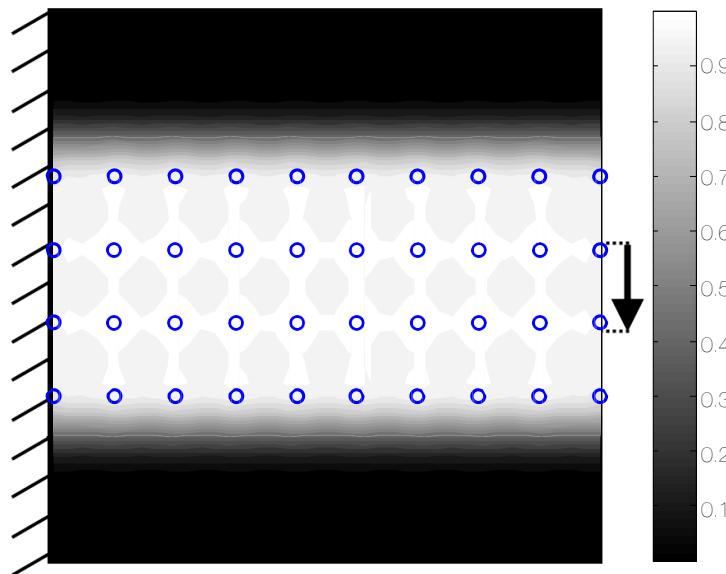
Original Work



J. T. Overvelde, "The moving node approach in topology optimization", Master's thesis, TU Delft, Delft University of Technology, 2012.

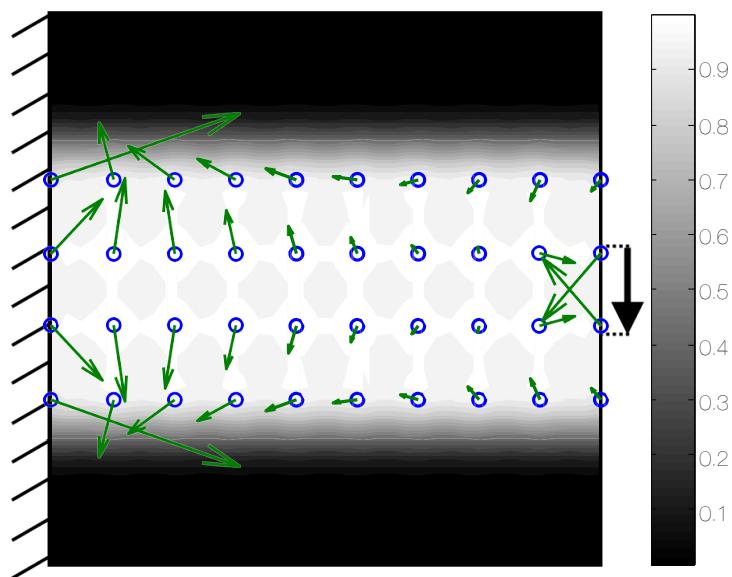
# Optimization algorithm

Example of compliance sensitivity



# Optimization algorithm

Example of compliance sensitivity

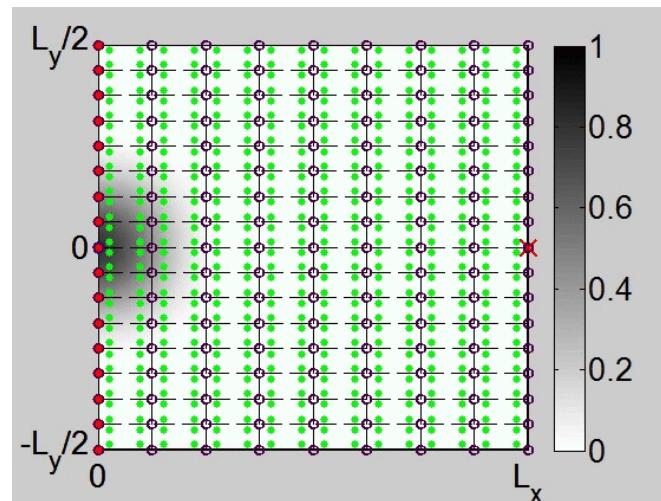


## Moving Node Approach (MNA)

- Key idea:

Move material => optimal layout

- Regular discretization = precision
- Material distribution -> mass nodes



## The variables (5 per Node)

Optimization variables :

- Positions (x,y)
- Orientation ( $\theta$ )
- Dimensions ( $L_x, L_y$ )

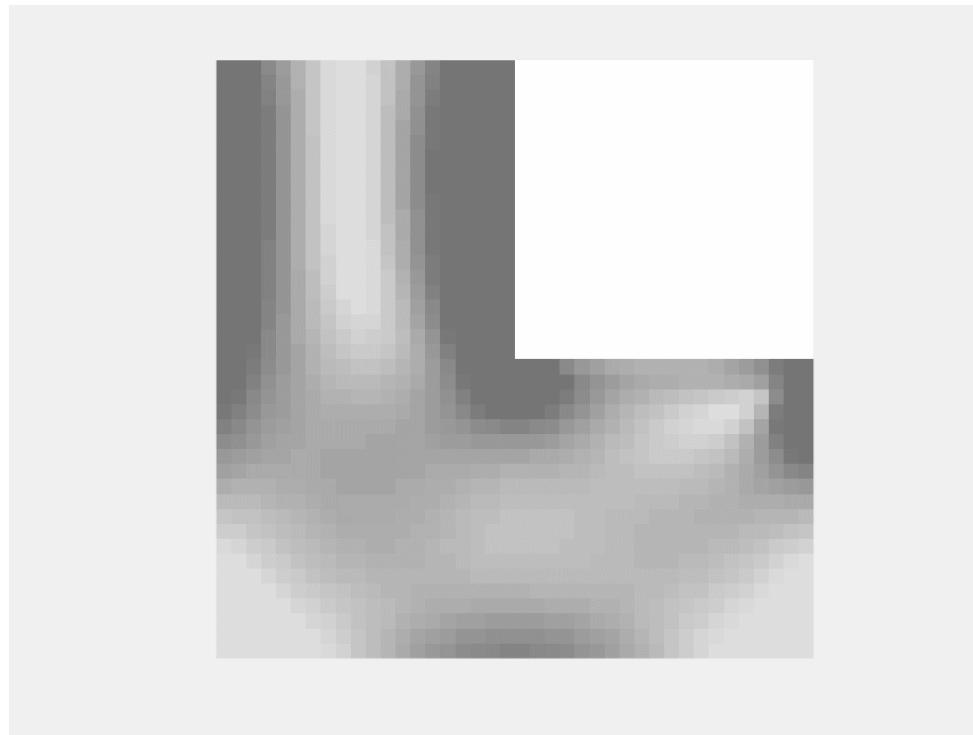
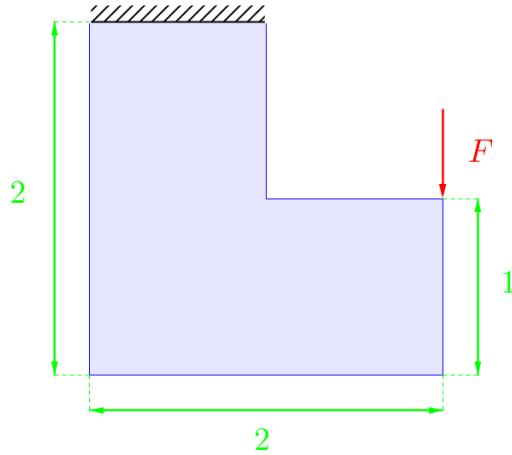


## Structural Members: beam's theory

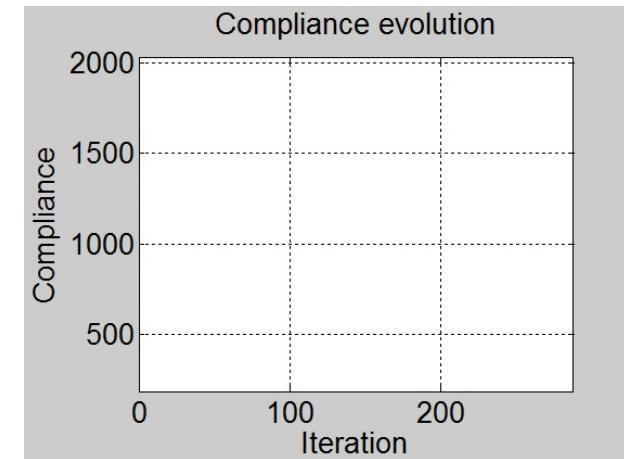
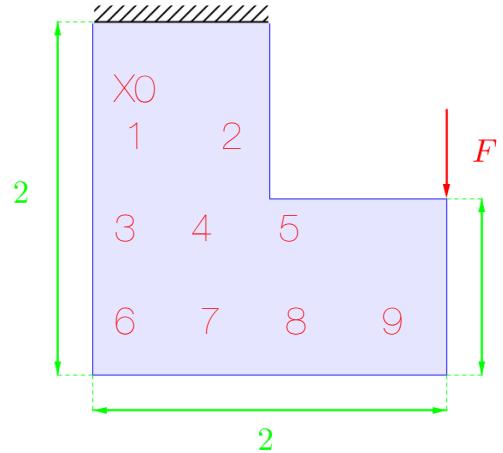
```
disp('SIMP')
top88(nelx,nely,volfrac,3,2,1)

disp('MNA')
topmna(x0,nelx,nely,volfrac,3,[ratio;aspect],tolchange);
```

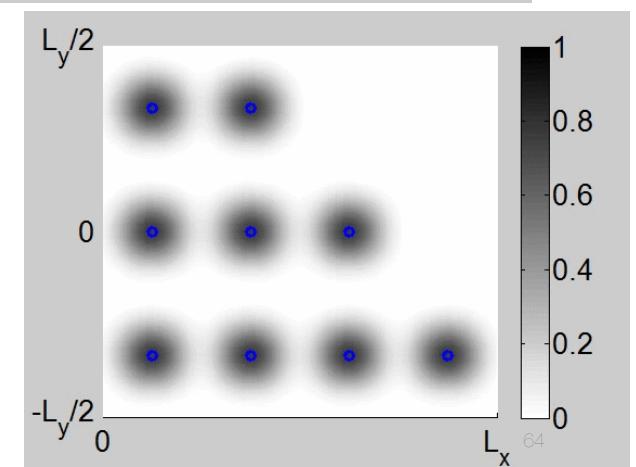
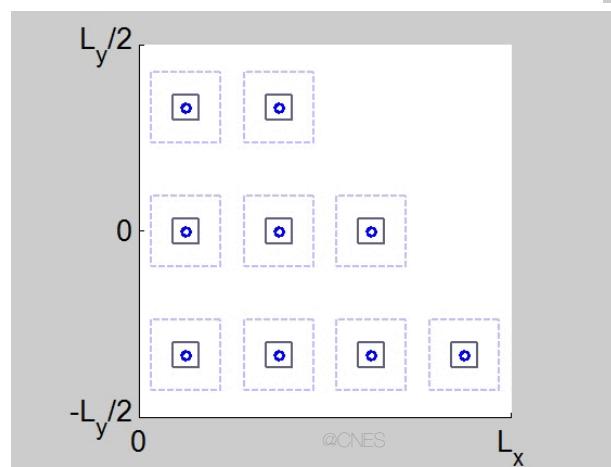
Results SIMP nelx=nely=40 → 1600 design variables  
minC st Volfrac=0.25 , Ku=f



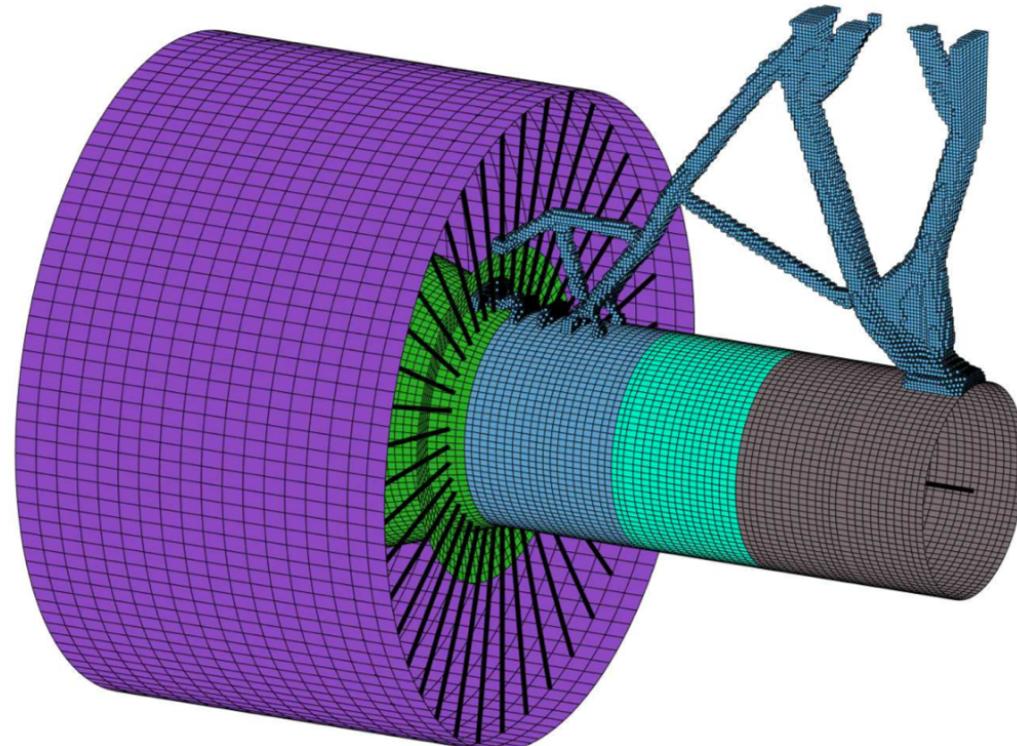
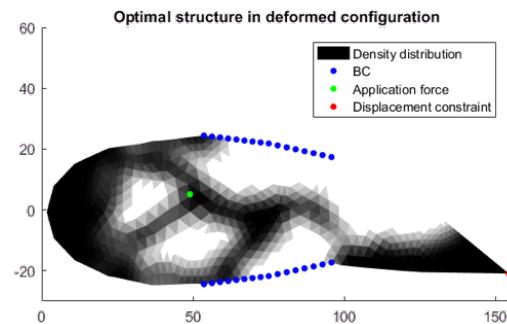
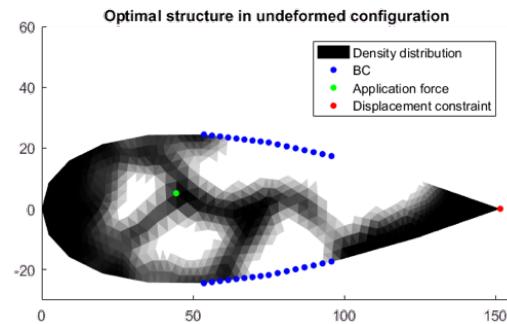
Results MNA,  $9 \times 5 = 45$  design variables  
 $\text{minC st Volfrac}=0.25$ ,  $Ku=f$



At the end, explicit structural element !



Aeronautical problems → innovative pylon or morphing airfoil



(G. Capasso, M. Herraz, G. Raze, MsC, S. Coniglio PhD)

## Papers & conf on this topic

*Coniglio, S., Gogu, C., Amargier, R., & Morlier, J. (2017, June). Pylon and engine mounts performance driven structural topology optimization. In World Congress of Structural and Multidisciplinary Optimisation (pp. 1349-1363). Springer, Cham.*

*Coniglio, S., Gogu, C., & Morlier, J. (2018). Weighted Average Continuity Approach and Moment Correction: New Strategies for Non-consistent Mesh Projection in Structural Mechanics. Archives of Computational Methods in Engineering, 1-29.*

*Coniglio, S., Morlier, J., Gogu, C., & Amargier, R. (2018). Original Pylon Architecture Design Using 3D HPC Topology Optimization. In 2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference (p. 1388).*

*Barjhoux, P. J., Diouane, Y., Grihon, S., Bettebghor, D., & Morlier, J. (2017, June). Mixed variable Structural optimization: toward an efficient hybrid algorithm. In World Congress of Structural and Multidisciplinary Optimisation (pp. 1880-1896). Springer, Cham.*

*Barjhoux, P. J., Diouane, Y., Grihon, S., Bettebghor, D., & Morlier, J. (2018). A Bilevel Methodology for solving a Structural Optimization Problem with both Continuous and Categorical Variables. In 2018 Multidisciplinary Analysis and Optimization Conference (p. 3579).*

*G. Raze et al, Optimisation topologique sans maillage : vers la reconnaissance d'éléments structuraux, CSMA 2017*

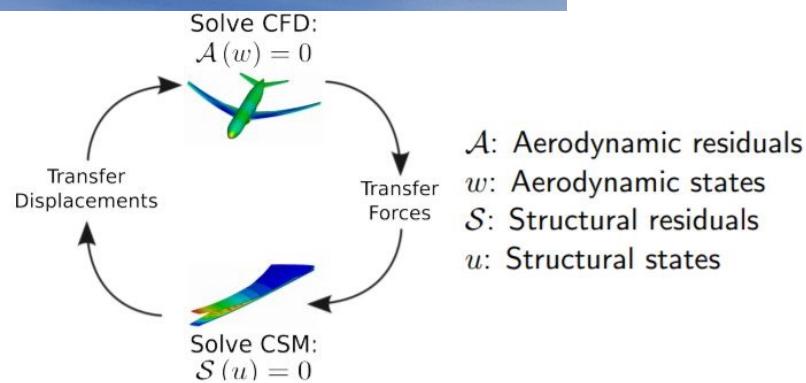
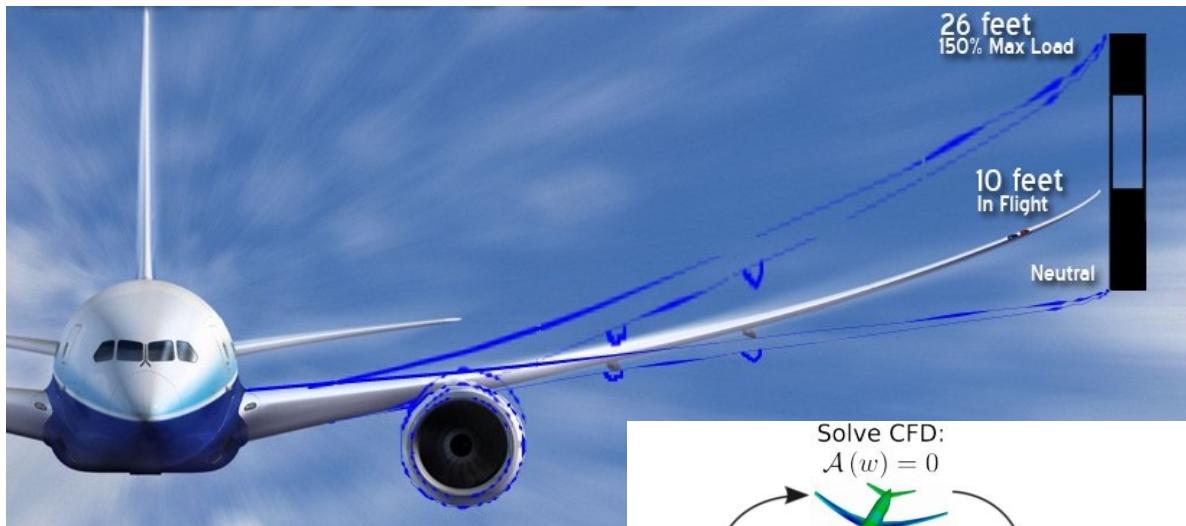
*T. Hirshler et al., Analyse Isogéométrique pour les problèmes d'Optimisation de Forme des Structures Coques, , CSMA 2017*

*Several Papers in preparation*

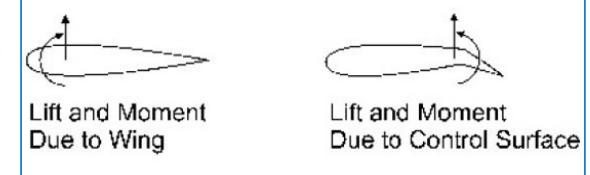
1. GP aka Kriging
2. Kriging for Global Optimization
3. New developments in topology optimization

## 4. Codesign is MDO?

# The importance of aerostructural coupling



+ control law\*:

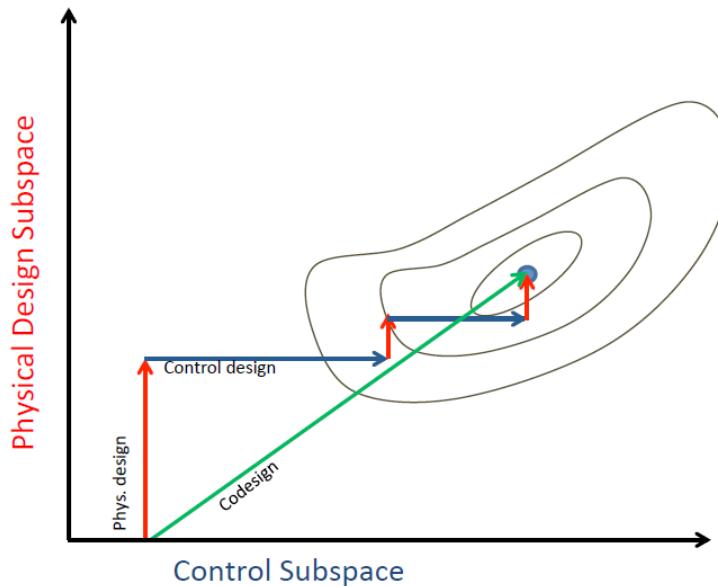


\*J. R. Wright, J. E. Cooper. Introduction to Aircraft Aeroelasticity and Loads. 2007.

# Co-Design: Integrated Physical and Control System Design \*

Navigate in physical and control design subspaces simultaneously.

→ Tailor structural/mechanical/control system designs: system optimality

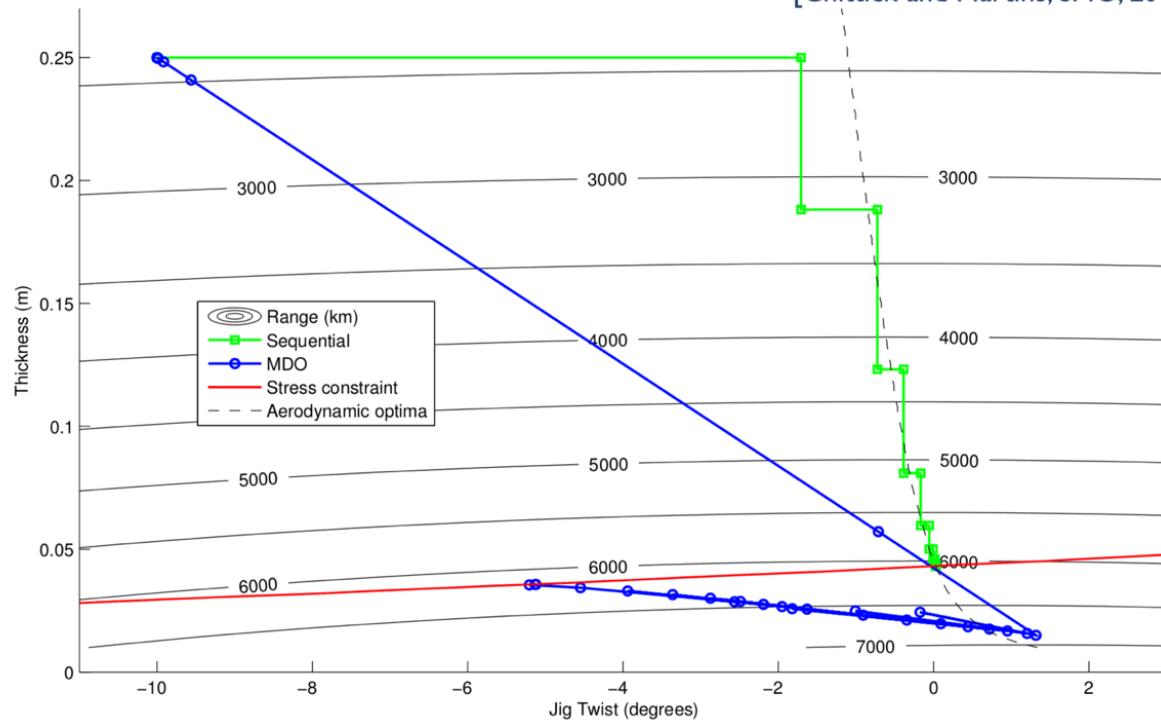


Deshmukh, A. P., & Allison, J. T. (2016). Multidisciplinary dynamic optimization of horizontal axis wind turbine design. *Structural and Multidisciplinary Optimization*, 53(1), 15-27.

# Sequential vs MDO

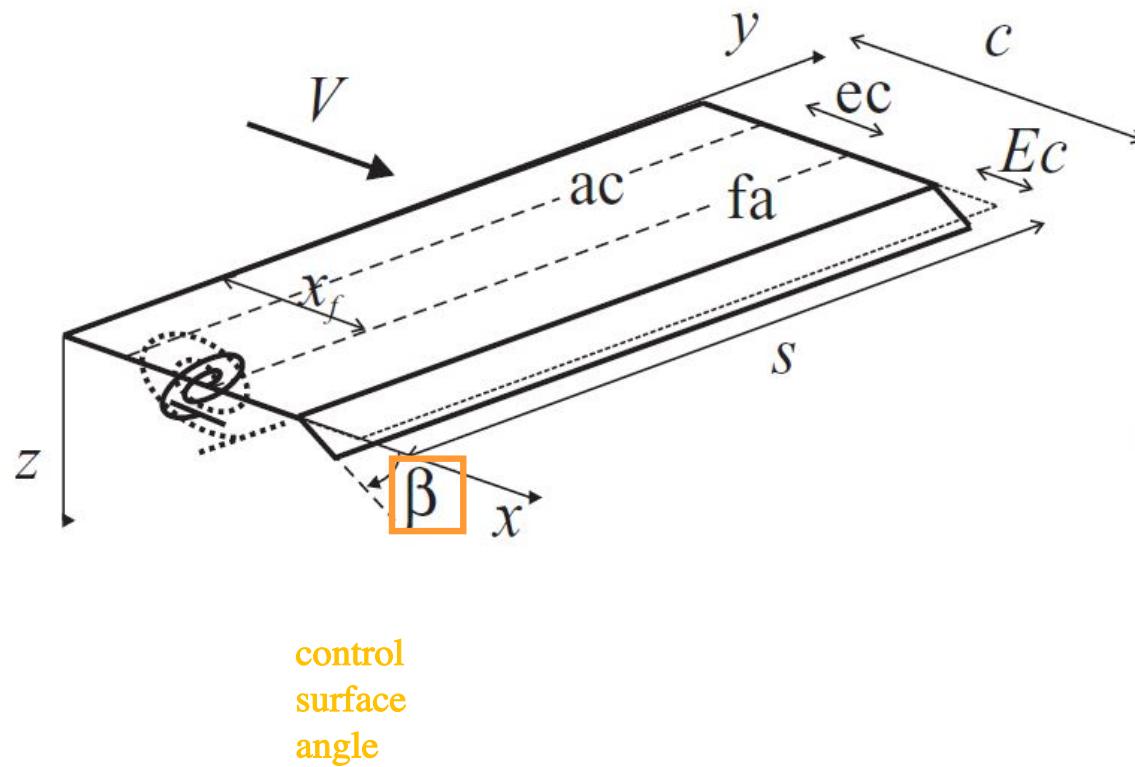
## Example: Aerostructural Optimization — Sequential Design vs. MDO 5

[Chittick and Martins, SMO, 2008]

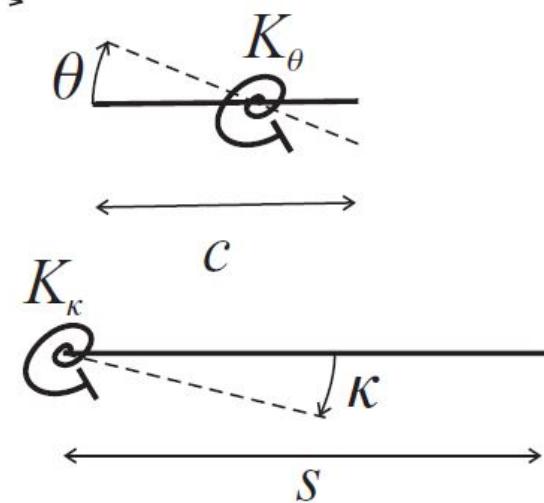


Chittick, I. R., & Martins, J. R. (2008). Aero-structural optimization using adjoint coupled post-optimality sensitivities. *Structural and Multidisciplinary Optimization*, 36(1), 59-70.

## Example: Wing model\* (G. Fillipi MsC)



Degrees of freedom:  
pitch  $\theta$  and flap  $\kappa$



# Mathematical modelling → State space modelling

solved with Direct Transcription Method

$$\begin{bmatrix} I_k & I_{k\theta} \\ I_{k\theta} & I_\theta \end{bmatrix} \begin{Bmatrix} \ddot{k} \\ \dot{\theta} \end{Bmatrix} + \rho V \begin{bmatrix} \frac{cs^3 a_w}{6} & 0 \\ -\frac{c^2 s^2 e a_w}{4} & -\frac{c^3 s}{8} M_\theta \end{bmatrix} \begin{Bmatrix} \dot{k} \\ \dot{\theta} \end{Bmatrix} + \left( \rho V^2 \begin{bmatrix} 0 & \frac{cs^2 a_w}{4} \\ 0 & -\frac{c^2 s e a_w}{2} \end{bmatrix} + \begin{bmatrix} K_k & 0 \\ 0 & K_\theta \end{bmatrix} \right) \begin{Bmatrix} k \\ \theta \end{Bmatrix} = \rho V^2 c s \begin{Bmatrix} -\frac{s a_c}{4} \\ \frac{c b_c}{2} \end{Bmatrix} \beta + \rho V c s \begin{Bmatrix} \frac{s}{4} \\ \frac{c}{2} \end{Bmatrix} w_g$$

structural  
inertia

aerodynamic  
damping

aerodynamic  
stiffness

structural  
stiffness

control  
surface  
angle

gust  
term

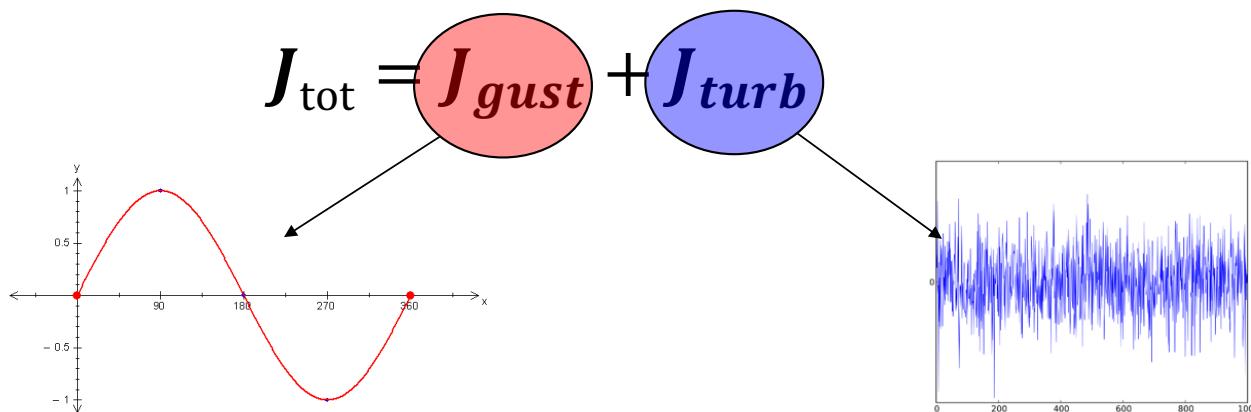
$$A \ddot{q} + \rho V B \dot{q} + (\rho V^2 C + E) q = g \beta + h w_g$$

Objective function (multiObj → monoObj to minimize)

$r_i$

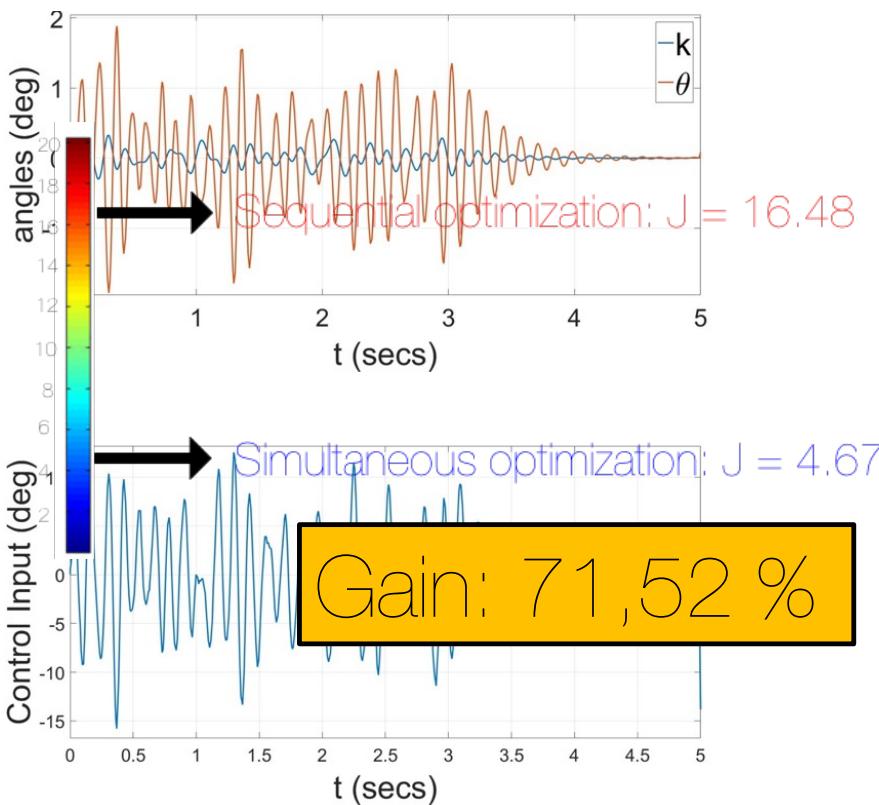
handling + comfort + control cost

$$J = \int_0^{t_F} (r_1 \mathbf{z}^2 + r_2 \ddot{\mathbf{z}}^2 + r_3 \mathbf{u}^2) dt$$

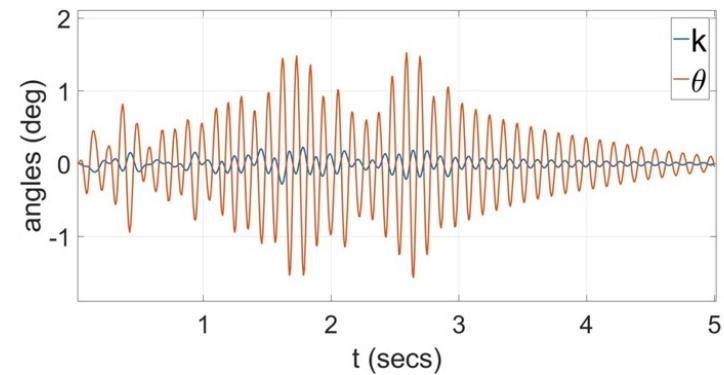


# System response (Gust+Random)

Response to Random turbulence



Response to Random turbulence



# Conclusions

- New Surrogates and SBO for an automated optimal design process (industrial constraints)
- **ONERA-SUPAERO** SEGOMOE offers a new solution for global optimization for a constrained problem (up to 100 design variables and several hundred of constraints)
- Opensource solutions due to collaboration with Nasa, University of Michigan and ONERA
- Researches in Structural optimization explore hybrid optimization
  - continuous/discrete/categorial variables)
  - Stress constraints aggregation (upto Millions FE)



- topmna.m, A new step toward an explicit topology optimization (speed up the CAD/CAE)

@CNES

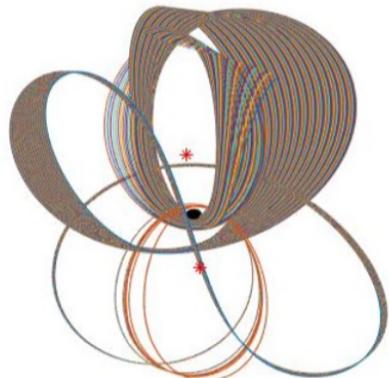
Codesign is one of the current trend in MDO  
→ [OpenMDAO/dymos](#)  
<https://github.com/OpenMDAO/dymos>

The screenshot shows the GitHub repository page for 'OpenMDAO / dymos'. At the top, it displays basic statistics: 129 commits, 3 branches, 4 releases, 5 contributors, and Apache-2.0 license. Below this is a search bar with dropdowns for 'Branch: develop' and 'New pull request', and buttons for 'Create new file', 'Upload files', 'Find file', and 'Clone or download'. The main area shows a list of recent commits by 'naylor-b' and 'robfalck', with details like commit messages, dates, and file changes. A 'readme.md' file is also listed at the bottom.

File	Commit Message	Date
benchmark	Benchmarks (#71)	2 months ago
dymos	updates to support simul coloring under MPI (#96)	26 days ago
.bumpversion.cfg	Sync develop back up to master (#95)	26 days ago
.coveralls.yml	travis and coveralls (#9)	7 months ago
.gitignore	Doc updates (#52)	4 months ago
.travis.yml	Sync develop back up to master (#95)	26 days ago
LICENSE	Aircraft steady example (#44)	4 months ago
readme.md	Control update (#59)	3 months ago
release_notes.txt	Control update (#59)	3 months ago
setup.py	Sync develop back up to master (#95)	26 days ago
travis_deploy_rsa.enc	Interpolate (#88)	a month ago
readme.md		

## New collaboration, new PhD (L. Beauregard)

Codesign is also a current trend at SUPAERO for optimal design of reusable launcher (design+ Optimal control of Trajectory)



<https://github.com/mid2SUPAERO>  
Prof J. Morlier's group

- NB: Since 2013 new course at SUPAERO : MDO [Structural&Multidisciplinary Design Optimization, 2\*30H] (MsC level] with ONERA/AIRBUS
- Since 2017 we offer some fund to students to do research with us in order to be « PhD ready ». Part of this presentation has been made by SUPAERO MsC Students:  
Mostafa Meliani, Mahfoud Herraz, Gabriele Capasso, Ghislain Raze, Giovane Filippi etc...

Please Visit :

<https://github.com/SMTorg/SMT>

<https://github.com/mid2SUPAERO> for student's project

- Thanks to My co-workers: Joaquim Martins, Nathalie Bartoli, Thierry Lefebvre, Emmanuel Benard, Claudia Bruni, Emmanuel Rachelson, Nicolas Gourdain, John Hwang, Mohamed Bouhlel, Peter Schmolgruber, Youssef Diouane, Sylvain Dubreuil, Christian Gogu, Stephanie Lisy-Destrez and PhDs Pierre-Jean Barjhoux, Simone Coniglio, Elisa Bosco, Joan Mas Colomer, Ankit Chiplunkar, Alessandro Sgueglia, Laurent Beauregard , Romain Olivanti, Remy priem. At Airbus: S. Grihon, A Gazaix, M. Colombo, R. Amargier, S. Trapier, A. Luccheti, F. Vetrano ....

Surrogate modeling in HD,  
focus on derivatives



### SMT: Surrogate Modeling Toolbox

The surrogate model toolbox (SMT) is an open-source Python package consisting of libraries of surrogate modeling methods (e.g., radial basis functions, kriging), sampling methods, and benchmarking problems. SMT is designed to make it easy for developers to implement new surrogate models in a well-tested and well-documented platform, and for users to have a library of surrogate modeling methods with which to use and compare methods.

The code is available open-source on [GitHub](#).

### Focus on derivatives

SMT is meant to be a general library for surrogate modeling (also known as metamodeling, interpolation, and regression), but its distinguishing characteristic is its focus on derivatives, e.g., to be used for gradient-based optimization. A surrogate model can be represented mathematically as

$$y = f(\mathbf{x}, \mathbf{xt}, \mathbf{yt}),$$

where  $\mathbf{xt} \in \mathbb{R}^{n_{\text{xt}} \times d_x}$  contains the training inputs,  $\mathbf{yt} \in \mathbb{R}^{n_t \times d_y}$  contains the training outputs,  $\mathbf{x} \in \mathbb{R}^{d_x}$  contains the prediction inputs, and  $y \in \mathbb{R}$  contains the prediction outputs. There are three types of derivatives of interest in SMT:

1. Derivatives ( $dy/dx$ ): derivatives of predicted outputs with respect to the inputs at which the model is evaluated.
2. Training derivatives ( $dy_t/dx_t$ ): derivatives of training outputs, given as part of the training data set, e.g., for gradient-enhanced kriging.
3. Output derivatives ( $dy/dy_t$ ): derivatives of predicted outputs with respect to training outputs, representing how the prediction changes if the training outputs change and the surrogate model is re-trained.

Not all surrogate modeling methods support or are required to support all three types of derivatives; all are optional.

Thanks

