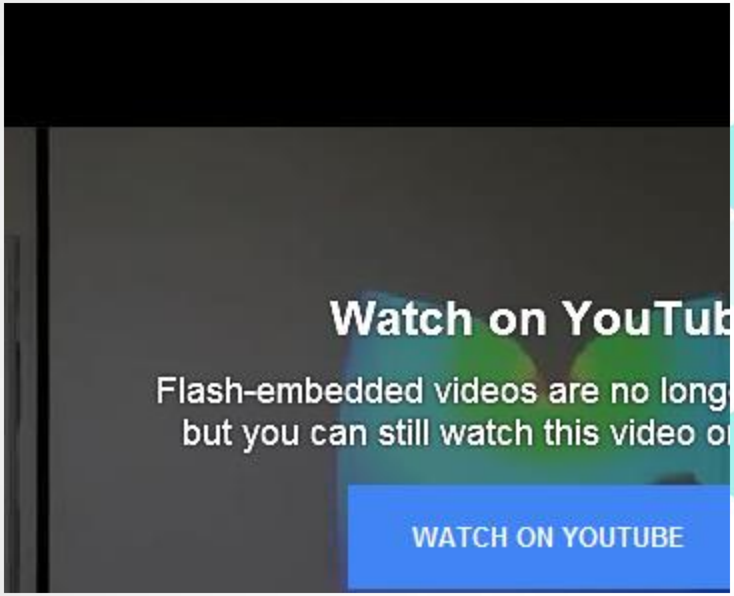
Structural Wing Model Reduction in Fluid-Structure Interactions



[Amsallem, International Journal for Numerical Methods in Engineering, 2014]

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Introduction

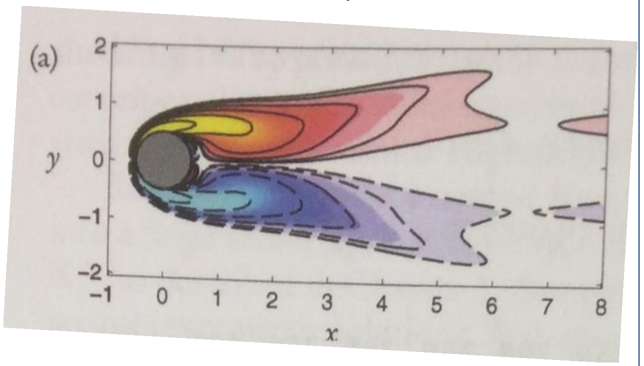
Aeroelasticity

Coupling between aerodynamic forces, inertial forces and elastic forces.

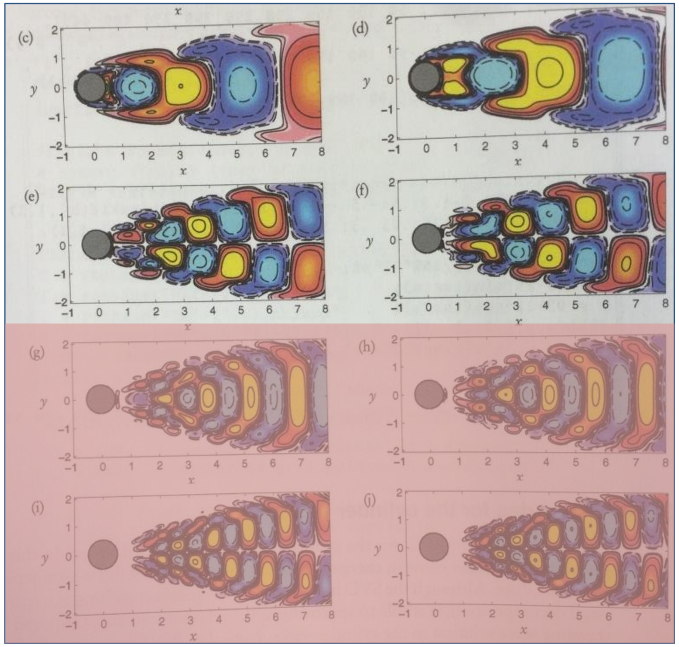
Current issues due to models dimension. Why the ROM?

The calculations have a high cost in time and computational power.

Mean vorticity field



[Kutz et al., Chpt 9, SIAM, 2016]



[Kutz et al., Chpt 9, SIAM, 2016)

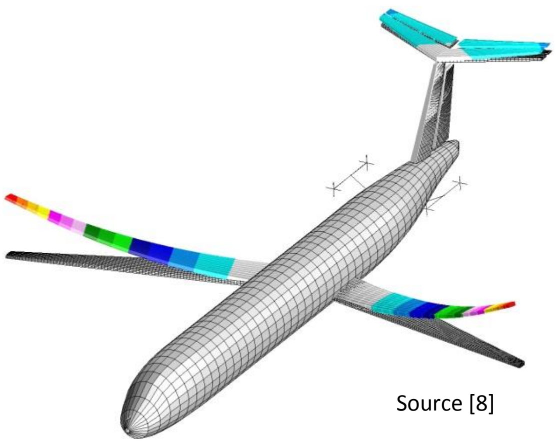
Aim & Objectives

Aim

To study the use of Reduced Order Models (hereafter ROM) in order **to lighten** the computational requirement in fluid-structure problems simulations

Objectives

- To study different ROM methodologies.
- To apply ROM techniques to a simple problem.
- To define a strategy for fluid-structure interactions problems.



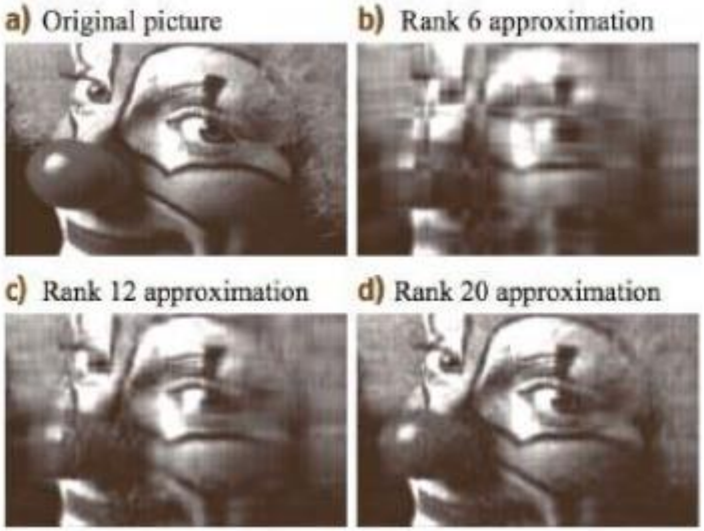
Source [8]

State of the Art: ROM

Principal Component Analysis (PCA) [Kutz, Chpt 15, Oxford University Press, 2013]

Possible correlated variables (Observations) → Orthogonal Transformation → Values of linearly uncorrelated variables (Principal Components)

- Widely used in the actuality.
- Limitations:
  - The results depend on the scaling variables.
  - The applicability is limited by certain assumptions.
  - No differentiation between classes.
- Used in different disciplines:
  - SVD in Algebra
  - POD in Mechanical Engineering.



SVD approach: $A \approx U\Sigma V^*$

In Matlab:

```
[U,S,V] = svd(A)
[Ar] = U(:,1:k)*S(1:k,1:k)*V(:,1:k)'
```

State of the Art: ROM

Dynamic Mode Decomposition (DMD)

[Kutz, SIAM, 2016]
[Kutz, Chpt 20, Oxford University Press, 2013]

AIM:

To take advantage of the low dimensionality in the experimental data.

WHAT DOES DMD PROVIDE?

A decomposition of experimental data into a set of dynamic modes.

HOW DOES IT WORK?

DMD computes the eigenvalues and eigenvectors of the linear model.

ADVANTAGES:

- No equations are needed.
- The future state is known for all time.

WHEN DOES IT FAIL?

- Data matrix is full rank
- No suitable low dimensional structure.

DMD is not a ROM itself, it is more a strategy. However, it uses ROM (usually SVD) to perform a low-rank truncation of the data.

State of the Art: ROM

Proper Generalized Decomposition (PGD) [Chinesta et al., Springer, 2014]

PGD is a numerical method for solving **boundary value** problems.

Its **biggest advantage** is the ability to work in a **Parameterized** approach.

The vectors ***R(x)***, ***S(y)***, ***T(z)***,⋯ , ***Z(n)*** are robustly computed **offline.**

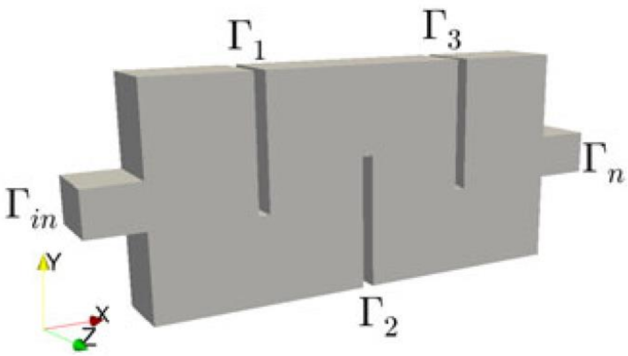
Fast ONLINE solution of a **recurring problem**.

PGD is not a ROM itself, it is more a strategy. However, it can use ROM to economize the vectors solutions ***R***, ***S***, ***T***,⋯, ***Z*** .

State of the Art: ROM

Analysis of Performance of ROM [Quarteroni et al., Springer, 2016]

- Galerkin Reduced Basis (G-RB) + POD or Greedy Algorithm.
- The parametric dependence of the PDE solution is exploited.
- **PROBLEM**: Steady heat conduction-convection problem.



Error bounds :

1) To compute the norm: $\|u_h(\mu) - u_N(\mu)\|_V$
2) To compute the stability factor (β_h) with *Interpolatory Radial Basis function*.
3) To compute the error estimator:

$$\Delta_N = \frac{\|u_h(\mu) - u_N(\mu)\|_V}{\beta_h}$$

Table 2. Computational details for HF and RB models. [Quarteroni et al., Chpt 3, Springer, 2016]

High-fidelity model		Reduced-Order model	
# FE dofs (N_h)	44171	# RB dofs	29
Affine operator components (Q_a)	2	Dofs reduction	1520:1
Affine RHs components (Q_f)	6	Offline CPU time	≈ 5 min
FE solution time	≈ 3,5 s	Online CPU time	1 ms

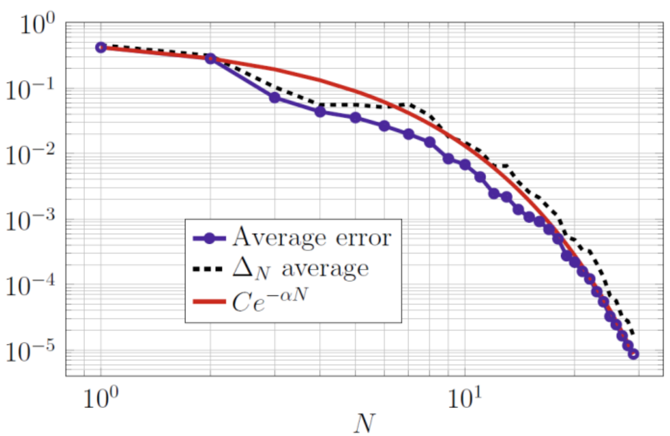


Figure 1. Comparison of the average error and bound error estimator computed on a set of 350 random values. [Quarteroni et al., Chpt 3, Springer, 2016]

State of the Art: ROM

POD approach using:
- ❑ Error estimator $\Delta_N(\mu)$
- ❑ Latin Hypercube Sampling (LHS)

Greedy algorithm critical aspects:
- ❑ Use of $\|u_h(\mu) - u_N(\mu)\|_V$
- ❑ It is not necessarily faster than POD

Greedy algorithm + POD:
- ❑ Error estimator $\Delta_N(\mu)$
- ❑ Latin Hypercube Sampling (LHS)
- ❑ Potentially faster

$n_s = 100$
$N = 26$
$\varepsilon_{POD} = 10^{-5}$

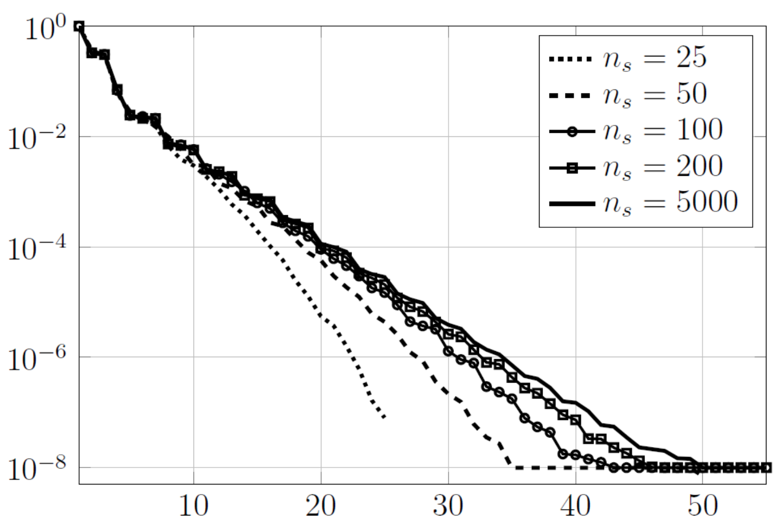


Figure 2. First 55 singular values of the correlation matrix obtained by LHS. [Quarteroni et al., Chpt 6, Springer, 2016]

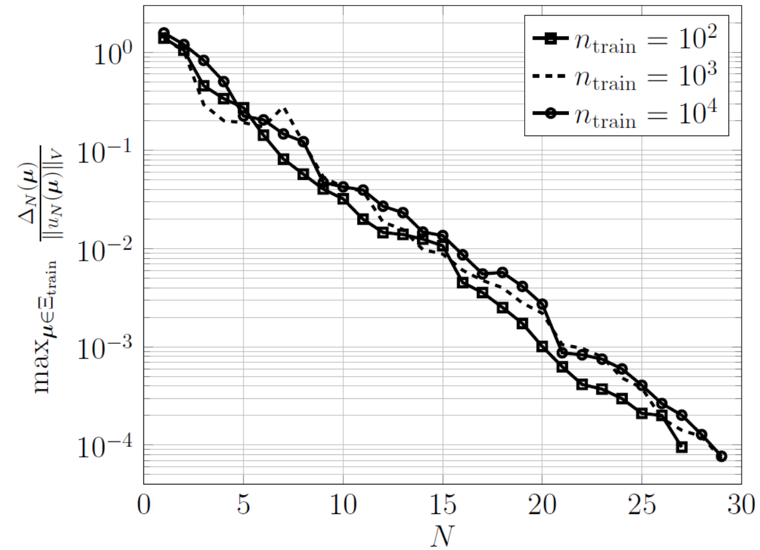


Figure 3. Convergence history of the greedy algorithm. [Quarteroni et al., Chpt 7, Springer, 2016]

Results

Advection-Diffusion problem definition

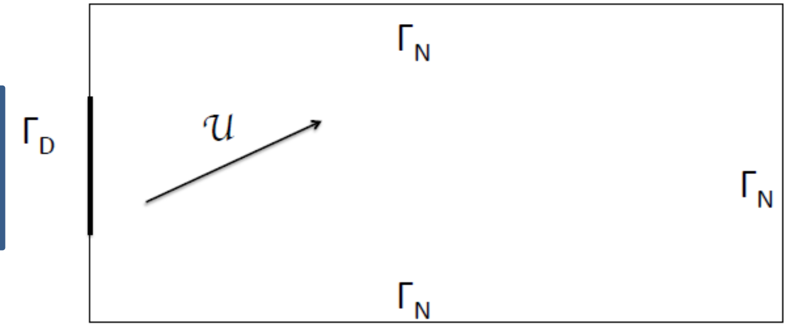
[Amsallem, International Journal for Numerical Methods in Engineering, 2014]

Steady Advection-Diffusion

$$\mathcal{U} \cdot \nabla\mathcal{T} - \kappa \cdot \Delta\mathcal{T} = 0$$

- $\boldsymbol{x} = (x, y) \in [0,1] \times [0,1]$
- $\mathcal{U} = \begin{bmatrix} \mathcal{U}_1 \\ \mathcal{U}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{U}_1 \\ \cancel{0} \end{bmatrix}; \mathcal{U}_1 \epsilon\ [0; 0{,}5]$ → $u_2 \in [0; 0{,}5]$
- $\kappa\ \epsilon\ [0; 0{,}025]$

BOUNDARY CONDITIONS



$$\begin{cases} \mathcal{T}(\boldsymbol{x}; \boldsymbol{\mu}) = \mathcal{T}_D(y; \bar{y}) \\ \nabla\mathcal{T}(\boldsymbol{x}; \boldsymbol{\mu}) \cdot \boldsymbol{n}(\boldsymbol{x}) = 0 \end{cases}$$

$$\mathcal{T}_D(y, t; \bar{y}) = \begin{cases} 300 & \text{if } y \in [0; 1/3] \\ 300 + 325(\sin(3 \cdot \pi \cdot |y - \bar{y}|) + 1) & \text{if } y \in [1/3; 2/3] \\ 300 & \text{if } y \in [2/3; 1] \end{cases}$$

$\bar{y} \in [0\cancel{,1; 0,}6]$ → $\bar{y} = 0{,}4$

Results

Advection-Diffusion problem: POD

Algorithm *Greedy* POD

[Amsallem, International Journal for Numerical Methods in Engineering, 2014]

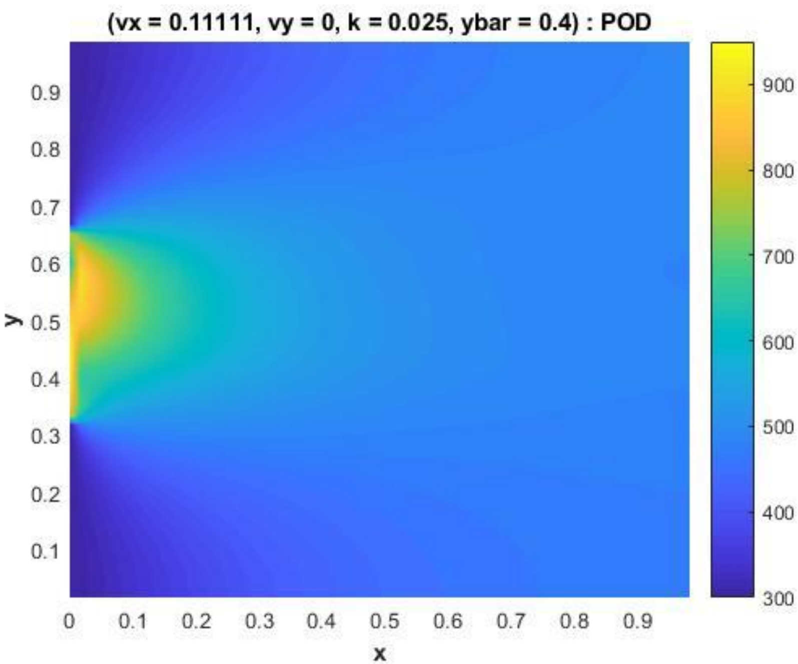
1. Select randomly a first sample: $\boldsymbol{\mu}^{(1)}$
2. Solve the HDM: $f\left(\boldsymbol{w}\left(\boldsymbol{\mu}^{(1)}; \boldsymbol{\mu}^{(1)}\right)\right) = 0$
3. Build a corresponding ROB: $\boldsymbol{V}$
4. For i = 2 ,⋯, s
   a) Solve:
   $\mu^{(1)} = argmax_{\mu \in \{\mu_1, \cdots, \mu_c\}} \|\boldsymbol{r}(\boldsymbol{\mu})\|$
   b) Solve the HDM: $f\left(\boldsymbol{w}\left(\boldsymbol{\mu}^{(i)}; \boldsymbol{\mu}^{(i)}\right)\right) = 0$
   c) Build a ROB $\boldsymbol{V}$ based on the samples $\{\boldsymbol{w}(\boldsymbol{\mu}^{(1)}), \cdots, \boldsymbol{w}(\boldsymbol{\mu}^{(i)})\}$

V size:
3487 x 10 =
34870 terms

$u_1 = 0{,}11 \ {}^{m}/_{s}$
$u_2 = 0 \ {}^{m}/_{s}$
$\kappa = 0{,}025 \ {}^{m^2}/_{s}$

Time computing **performance**: -85%

Solution **accuracy**: $E_{max} = 7\%$ $E_{avg} = 1\%$



Results

Advection-Diffusion problem: PGD

5 coordinates: x, y, u_1, u_2, κ

$$\int_{\Omega} \mathcal{T}^{*}(u\,\nabla\mathcal{T} - \kappa\,\Delta\mathcal{T})\,dx dy du_1 du_2 d\kappa = 0$$

$$\mathcal{T}^{n}(x, y, u_1, u_2, \kappa) = \sum_{i=1}^{n-1} X_i(x) Y_i(y) U_i(u_1) V_i(u_2) K_i(\kappa) + R(x) S(y) T(u_1) W(u_2) Z(\kappa)$$

1. The search of a less intrusive method
2. A non-symmetry induced by the convection terms.

Residual Minimization Technique for PGD

$$\min \; \|A \cdot X - F\|^2$$

[Chinesta et al., Springer, 2014]

AA = cell(5,4) → 2 diffusion eq. + 2 advection eq.

BB = cell(5,1) → RHS: Source terms

N_NT = cell(5,1) → Mass matrices

Dirichlet = cell(5,1) → Where to apply Dirichlet BC: $x = 0$

GG = cell(5,1) → A priori known terms, to enforce the Dirichlet BC

GG {2} = $\mathcal{T}_{\mathrm{D}}\ (y;\ \bar{y})$

Neumann BC

D1x(end,end) = 0;

D1y(1,1) = 0; D1y(end,end) = 0;

Results

Advection-Diffusion problem: PGD

OFFLINE

Loop	Stopping Criterion	Tolerance
Fixed point	$\frac{\lVert FF_n - FF_{n-1}\rVert}{\lVert FF_{n-1}\rVert}$	10^{-8}
PGD enrichment	$\frac{\lVert FF_n\rVert}{\lVert FF_1\rVert}$	10^{-6} **!!!**

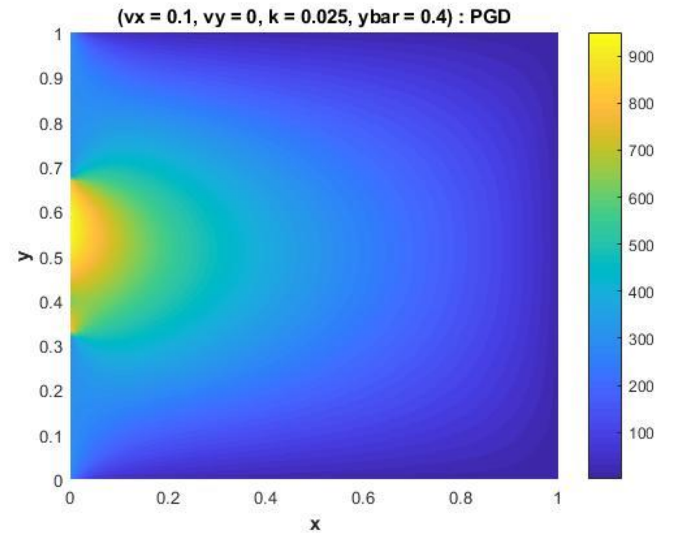
Computing time: 43min 25s

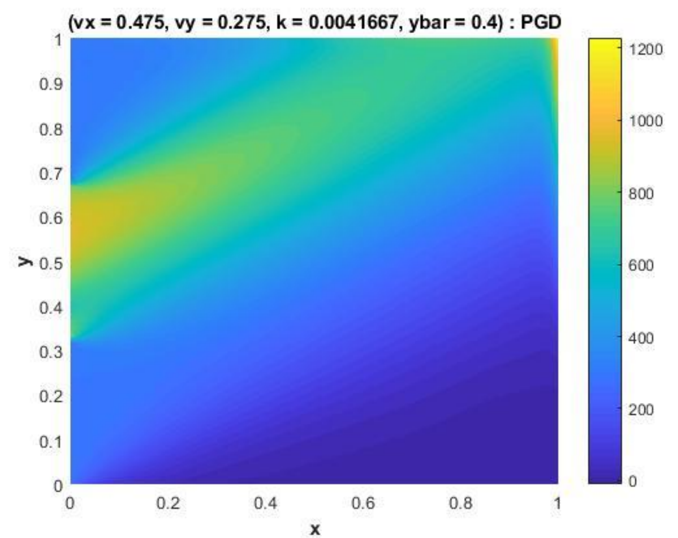
FF size: 292500 terms

ONLINE

```
alph = ones(size(FF{1},2),1);
alpha = FF{3}(5,:) .* FF{4}(1,:) .* FF{5}(15,:)
.* alph';
T = FF{2} * diag(alpha) * FF{1}';
```

Computing time: 0,1 – 0,5 s





Conclusions

- PGD → a **lot of cases** of the same (normally expensive) problem are expected.

⇩

POD

- DMD → simulations in **time domain**. We need a film of snapshots to create the reduced model.

PGD

- The most interesting PGD's approach is the **Parametric PGD**.
- PGD allows to introduce **Boundary Conditions** as extra-coordinates of the problem.
- The **Stopping Criterion** and **Tolerance** effectiveness should be evaluated for each problem.

Future work

Modification of the Stopping Criterion.

SUMMER INTERSHIP IN ONERA

Application of the strategy into *OpenAeroStruct.*

SUMMER INTERSHIP IN ONERA

Link to the GitHub Project

References

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[6] http://www.dlr.de/ae/en/desktopdefault.aspx/tabid-1596/