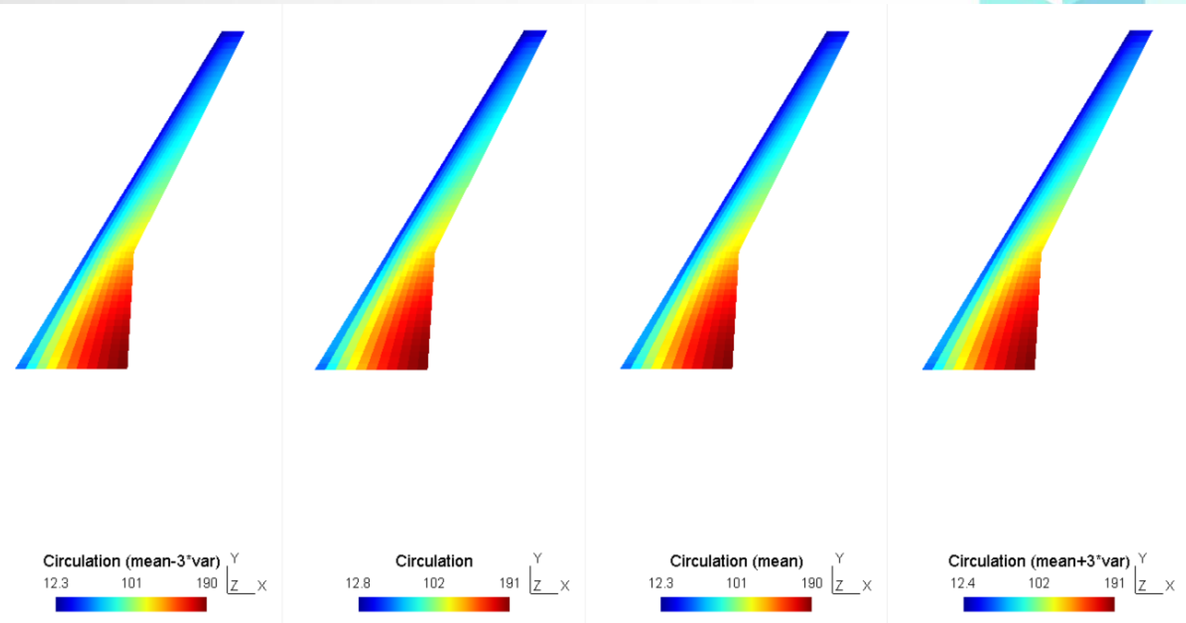
Internship Reporting



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Introduction

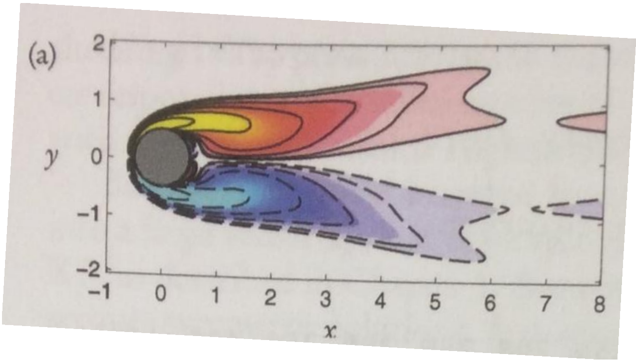
Aeroelasticity

Coupling between aerodynamic forces, inertial forces and elastic forces.

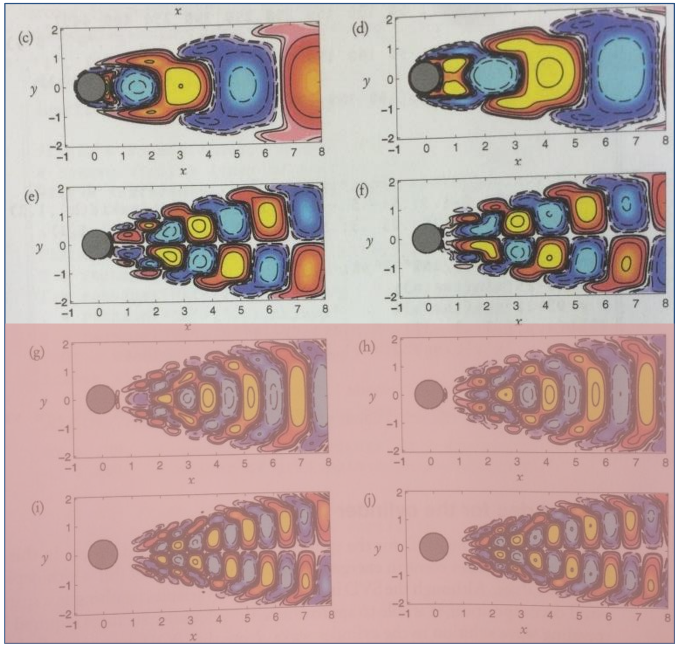
Current issues due to models dimension. Why the ROM?

The calculations have a high cost in time and computational power.

Mean vorticity field



[Kutz et al., Chpt 9, SIAM, 2016]



[Kutz et al., Chpt 9, SIAM, 2016)

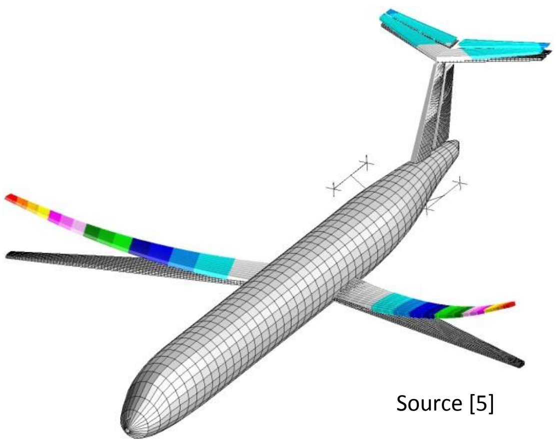
Aim & Objectives

Aim

To implement an approach Greedy+POD to an aeroelastic code in order **to lighten** the further simulations.

Objectives

- To apply the abovementioned technique to the proposed problem.
- To analyse the results.



Source [5]

State of the Art

Principal Component Analysis (PCA) [Kutz, Chpt 15, Oxford University Press, 2013]

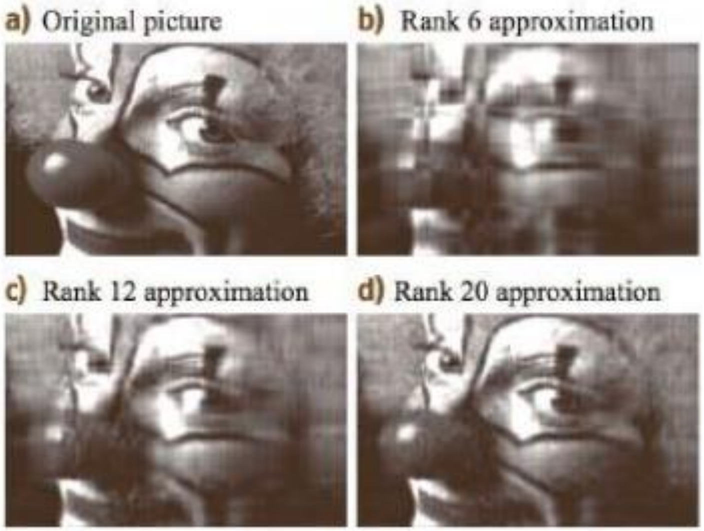
Possible correlated variables (Observations) → Orthogonal Transformation → Values of linearly uncorrelated variables (Principal Components)

- Widely used in the actuality.
- Limitations:
  - The results depend on the scaling variables.
  - The applicability is limited by certain assumptions.
  - No differentiation between classes.
- Used in different disciplines:
  - SVD in Algebra
  - POD in Mechanical Engineering.

SVD approach: $A \approx U\Sigma V^*$

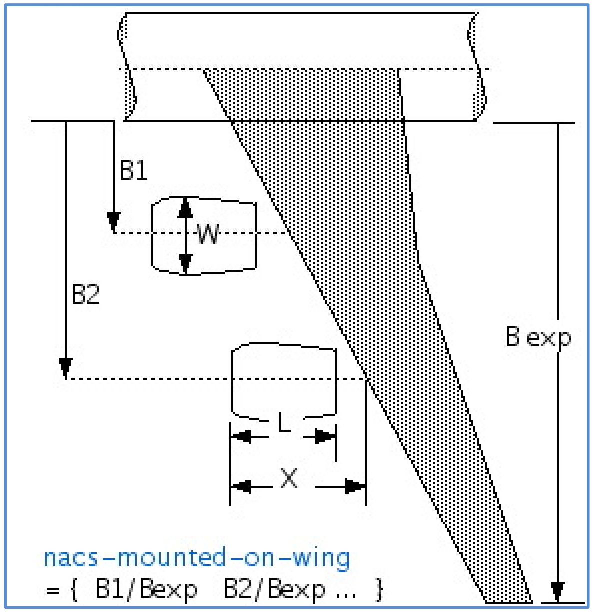
In Matlab:

```
[U,S,V] = svd(A)
[Ar] = U(:,1:k)*S(1:k,1:k)*V(:,1:k)’
```



Results

Aero structure problem



Source [5]

Aerodynamics → VLM Horseshoe

Matrix of Transfer & Mesh deformation

Structures → FEM (DKT for plates)

6 parameters of interest

- $h_{\{skin\}}$
- $h_{\{ribs\}}$
- $h^{LE}_{\{sparse\}}$
- $h^{TE}_{\{sparse\}}$
- Span
- Wing Area

Results

Aero structure problem

Algorithm *Greedy* POD - OFFLINE

[Amsallem, International Journal for Numerical Methods in Engineering, 2014]

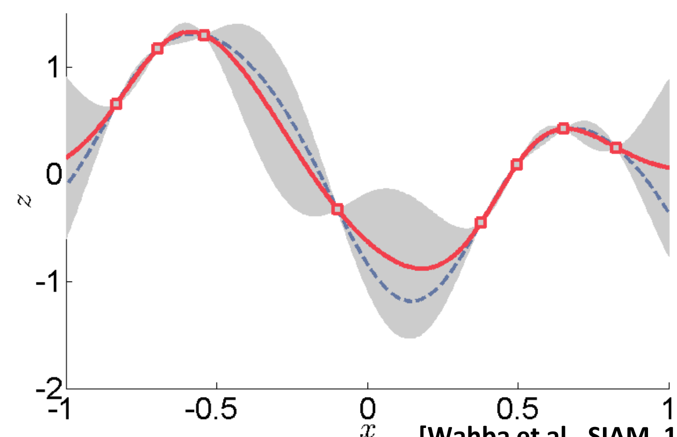
1. Select randomly a first sample: $\boldsymbol{\mu}^{(1)}$
2. Solve the HDM: $f\left(\boldsymbol{w}\left(\boldsymbol{\mu}^{(1)}; \boldsymbol{\mu}^{(1)}\right)\right) = 0$
3. Build a corresponding ROB: $\boldsymbol{V}$
4. For i = 2 ,···, nSample
   a) Solve:
      $\mu^{(1)} = argmax_{\mu \in \{\mu_1, \cdots, \mu_c\}} \|\boldsymbol{r}(\boldsymbol{\mu})\|$
   b) Solve the HDM: $f\left(\boldsymbol{w}\left(\boldsymbol{\mu}^{(i)}; \boldsymbol{\mu}^{(i)}\right)\right) = 0$
   c) Build a ROB $\boldsymbol{V}$ based on the samples
      $\{\boldsymbol{w}(\boldsymbol{\mu}^{(1)}), \cdots, \boldsymbol{w}(\boldsymbol{\mu}^{(i)})\}$

Approximation ONLINE

$$\hat{u}(\xi^{(0)}) = \sum_{i=1}^{nSamples} \hat{\alpha}_i(\xi^{(0)}) \cdot \mathrm{U}_i^V$$

V size:
- Circulation: 627 x **nSamples**
- Displacement: 2988 x **nSamples**

KRIGING: nSamples functions



[Wahba et al., SIAM, 1990]

$$\hat{u} \sim \mathcal{N}(\mu_{\hat{u}}, \sigma_{\hat{u}}^2) \begin{cases} \mu_{\hat{u}} = \sum_{i=1}^{nSamples} \mu_{\hat{\alpha}_i} \cdot \mathrm{U}_i^V \\ \sigma_{\hat{u}}^2 = \sum_{i=1}^{nSamples} \sigma_{\hat{\alpha}_i}^2 \cdot (\mathrm{U}_i^V)^2 \end{cases}$$

Results

Aero structure problem

nSamples = 15 // nCandidates = 30

$h_{\{skin\}}$ = 0,03 m

$h_{\{ribs\}}$ = 0,01 m

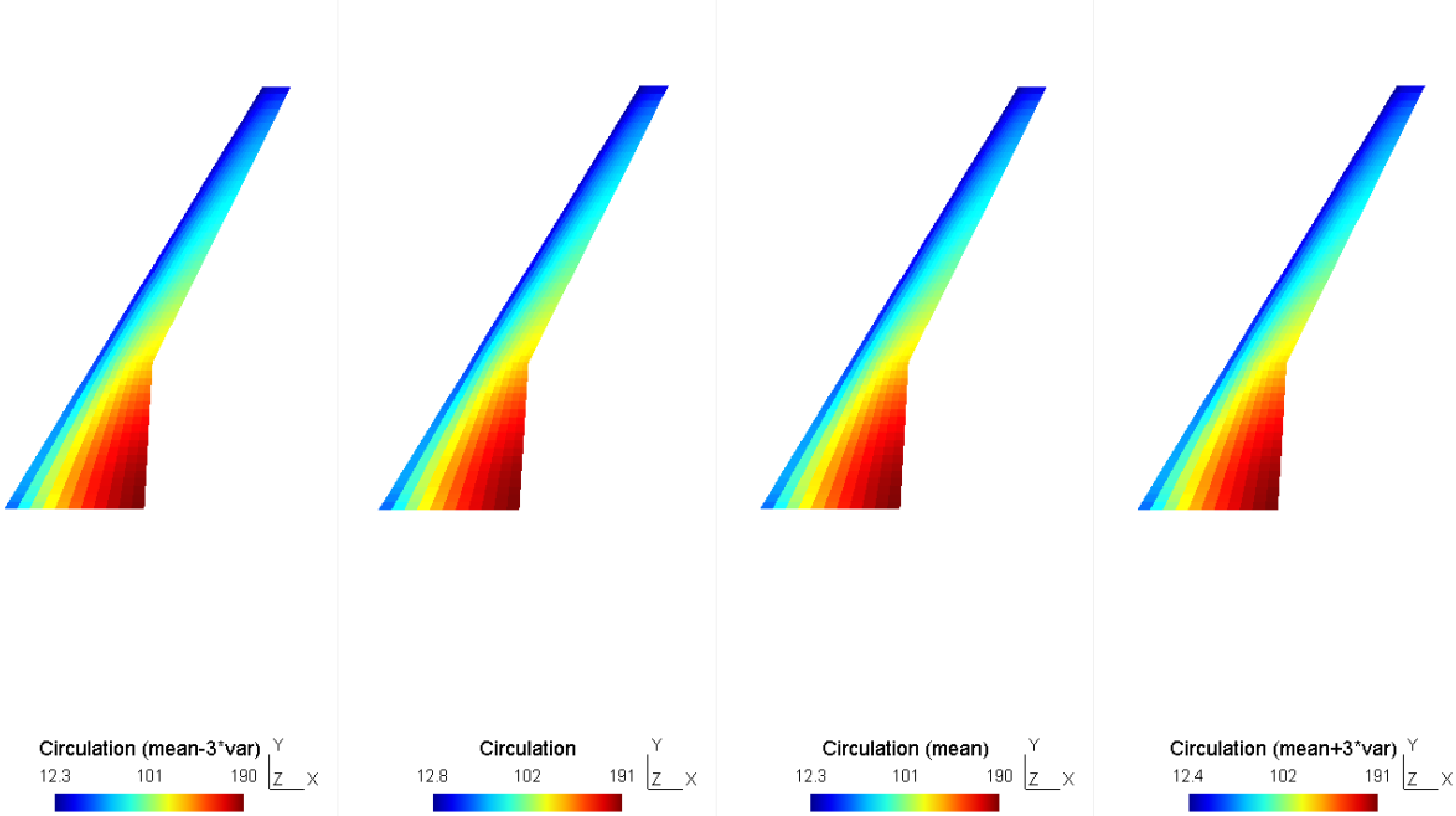
$h^{LE}{}_{\{sparse\}}$= 0,022 m

$h^{TE}{}_{\{sparse\}}$= 0,011 m

b = 73,20 m

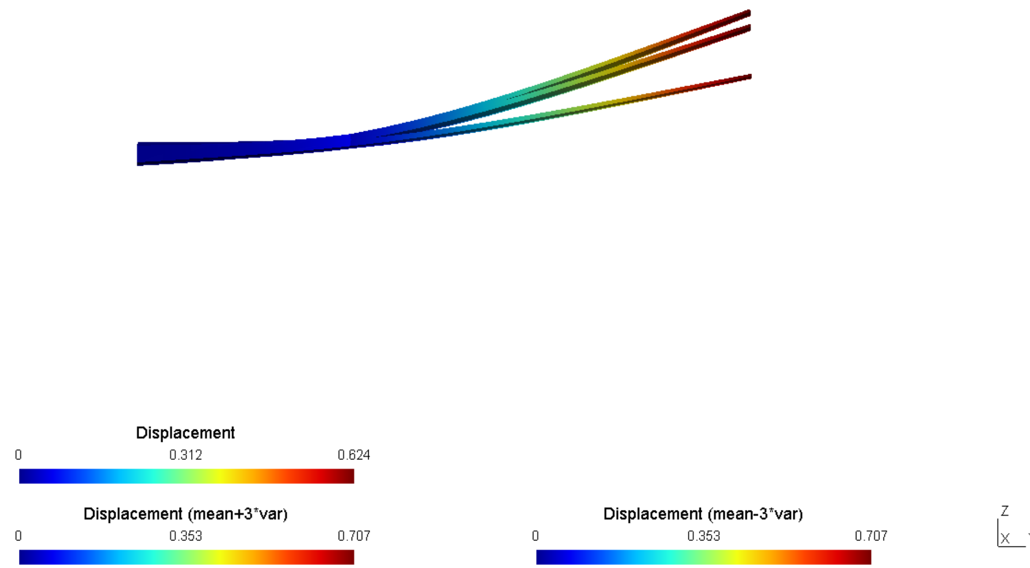
S = 444,44 m^2

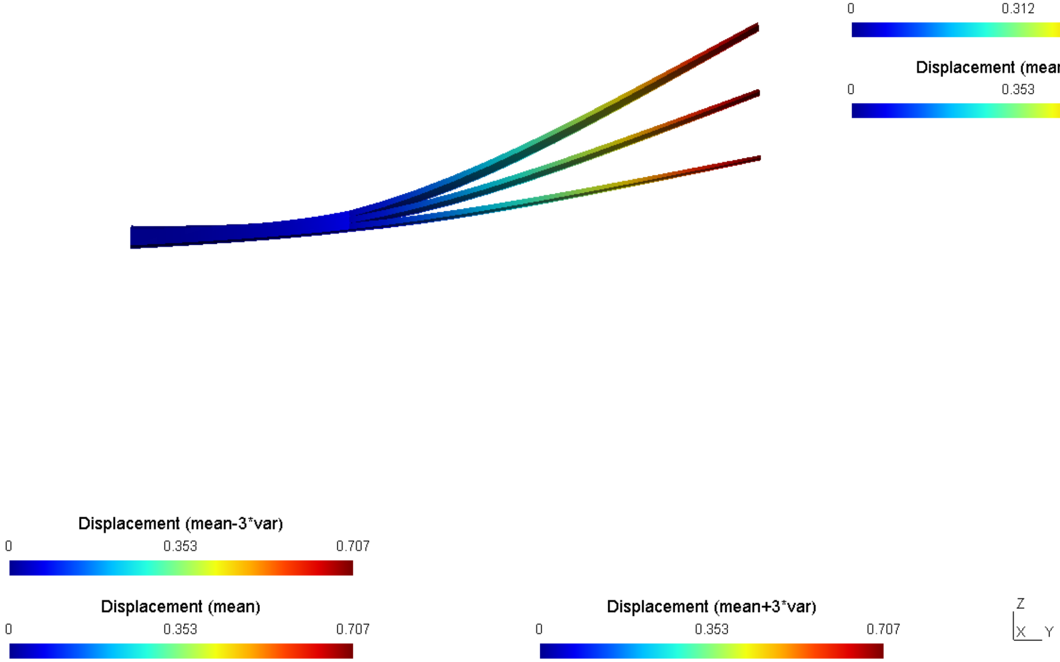
Aero structure problem



Results

Aero structure problem





Conclusions and Future work

CONCLUSIONS

- **Saving** computation time in a **real time** simulation.
- **nSamples** drives the accuracy of the RB.
- Kriging procedure → **acceptable** variance values
- In displacement approximation → real solution is **not localized** between the limits

FUTURE WORK?

- Introduction of the reduced base methodology to the solver *OpenAeroStruct*.

Link to the GitHub Project

References

[1] Kutz, J.N., Brunton, S.L., Brunton, B.W. and Proctor, J.L., "Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems", 1st ed., SIAM, Philadelphia, 2016.

[2] Kutz, J. N., "Data-Driven Modeling & Scientific Computation: Methods for Complex Systems & Big Data", 1st ed., Oxford University Press, Oxford, 2013.

[3] Amsallem, D., “An Adaptive and Efficient Greedy Procedure for the Optimal Training of Parametric Reduced-Order Models”, International Journal for Numerical Methods in Engineering, 2014.

[4] Wahba, G., "Spline Models for Observational Data", Society for Industrial and Applied Mathematics (SIAM), 1990.

[5] http://www.dlr.de/ae/en/desktopdefault.aspx/tabid-1596/

[6] http://www.lissys.demon.co.uk/pug/c03.html