$$\begin{aligned}
\text{minimize} \quad & f_0\left(x,y\right) + \sum_{i=1}^{N} f_i\left(x_0, x_i, y_i\right) \\
\text{with respect to} \quad & x, y^t, y, \bar{y} \\
\text{subject to} \quad & c_0\left(x,y\right) \geq 0 \\
& c_i\left(x_0, x_i, y_i\right) \geq 0 \qquad\qquad \text{for} \quad i = 1, \ldots, N \\
& c_i^c = y_i^t - y_i = 0 \qquad\qquad \text{for} \quad i = 1, \ldots, N \\
& \mathcal{R}_i\left(x_0, x_i, y_{j\neq i}^t, \bar{y}_i, y_i\right) = 0 \quad \text{for} \quad i = 1, \ldots, N
\end{aligned}$$

Monolithic

AAO — Remove c^c, y^t → **SAND**

AAO — Remove $\mathcal{R}, y, \bar{y}$ → **IDF**

SAND — Remove $\mathcal{R}, y, \bar{y}$ → **MDF**

IDF — Remove c^c, y^t → **MDF**

Distributed IDF

Multilevel

CO: Copies of the shared variables are created for each discipline, together with corresponding consistency constraints. Discipline subproblems minimize difference between the copies of shared and local variables subject to local constraints. System subproblem minimizes objective subject to shared constraints subject to consistency constraints.

BLISS-2000: Discipline subproblems minimize the objective with respect to local variables subject to local constraints. A surrogate model of the local optima with respect to the shared variables is maintained. Then, system subproblem minimizes objective with respect to shared design and coupling variables subject to shared design and consistency constraints, considering the disciplinary preferences.

QSD: Each discipline is assigned a "budget" for a local objective and the discipline problems maximize the margin in their local constraints and the budgeted objective. System subproblem minimizes a shared objective and the budgets of each discipline subject to shared design constraints and positivity of the margin in each discipline.

Penalty

ATC: Copies of the shared variables are used in discipline subproblems together with the corresponding consistency constraints. These consistency constraints are relaxed using a penalty function. System and discipline subproblems solve their respective relaxed problem independently. Penalty weights are increased until the desired consistency is achieved.

IPD/EPD: Applicable to MDO problems with no shared objectives or constraints. Like ATC, copies of shared variables are used for every discipline subproblem and the consistency constraints are relaxed with a penalty function. Unlike ATC, the simple structure of the disciplinary subproblems is exploited to compute post-optimality sensitivities to guide the system subproblem.

ECO: As in CO, copies of the shared design variables are used. Disciplinary subproblems minimize quadratic approximations of the objective subject to local constraints and linear models of nonlocal constraints. Shared variables are determined by the system subproblem, which minimizes the total violation of all consistency constraints.

Distributed MDF

CSSO: In system subproblem, disciplinary analyses are replaced by surrogate models. Discipline subproblems are solved using surrogates for the other disciplines, and the solutions from these discipline subproblems are used to update the surrogate models.

BLISS: Coupled derivatives of the multidisciplinary analysis are used to construct linear subproblems for each discipline with respect to local design variables. Post-optimality derivatives from the solutions of these subproblems are computed to form the system linear subproblem, which is solved with respect to shared design variables.

MDOIS: Applicable to MDO problems with no shared objectives, constraints, or design variables. Discipline subproblems are solved independently assuming fixed coupling variables, and then a multidisciplinary analysis is performed to update the coupling.

ASO: System subproblem is like that of MDF, but some disciplines solve a discipline optimization subproblem within the multidisciplinary analysis with respect to local variables subject to local constraints. Coupled post-optimality derivatives from the discipline subproblems are computed to guide the system subproblem.