Mesh-less Methods for Structural Analysis

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I. INTRODUCTION

When the physical system is modeled in the computational space, it is required to solve the problem numerically that is defined in the form of partial differential equations(PDE). The Finite Element method is one of the first numerical methods being used successfully for solving the physical system by discretizing the computational space into uniform elements. But to define the uniform mesh or grid for complex structures or for a discontinuous structure or problem with moving boundaries or large deformations, FEM is not the best option available as it can result into very high errors requiring the re-meshing of the structure and impose the time burden on the solution.

That is when the mesh-less methods come into play as these methods ease the approximation of the problem by getting rid of its dependence on the mesh. Mesh-less Methods utilize the nodal mesh generation approach in which there is no need to define the elements linked by nodes to approximate the solution. The very first of the mesh-less method is Smooth Particle Hydrodynamics(SPH) that dates back to 1977 introduced by Lucy[2] and Gingold and Monaghan[3].

One of the major advantages[5] of the mesh-less methods over mesh-based methods is that the complex structure problems like moving discontinuities, large deformation etc can be solved easily with mesh-less methods but mesh-less methods are computationally costly and most of the mesh-less methods do not satisfy the Kronecker delta property, therefore the essential boundary conditions are needed to be imposed which is not as easy as in FEM.

The scope of the project is based on work done by Overvelde[1] in which a cantilever beam is subjected to parabolic traction forces at the free end. The previous work done on the problem with two of the mesh-less methods-EFG and MLPG using different approaches for the generation of nodes namely, uniformly distributed nodes, randomly distributed nodes, randomized nodes is reviewed and finally new techniques for nodal distribution: Latin Hypercube Sampling, Halton Sampling and Sobol Sampling are proposed.

II. STATE OF ART

The mesh-less methods came into existence in 1977 and from then there is huge evolution in their definition and applications as shown in Table 1[4]. The range of applications of the mesh-less methods is not limited to solid mechanics but is also extended to other engineering domains, for example, fracture mechanics, composite structures,etc. There are number of mesh-less methods proposed in literature as listed in Table 1 but the scope of this work is focused on two methods namely, Element Free Galerkin(EFG) method and Mesh-less Local Petrov Galerkin(MLPG) method.

Method	Year
Smooth Particle Hydrodynamics	1977
Diffuse Element Method	1992
Element free Galerkin Method	1994
Reproducing Kernel Particle Method	1995
Partition of Unity Method	1997
Meshless local Petrov Galerkin method	1998
Meshless Natural Element method	2004

TABLE I: Some Proposed Mesh-less Methods

EFG method was first introduced in 1994 by Belytschlo et al.[7] and is based on the global weak form where the test function is chosen similar to the displacement vector u and δu representing the infinitesimal variations of the displacement and the obtained function is integrated over the problem domain with the help of a background mesh which makes EFG not a complete mesh-less method. There are various methods to generate the background mesh[6] and most commonly used is the Gauss Quadrature. On the other hand, MLPG method was proposed by Atluri and Zhu[8] in 1998 is based on local weak form. The formulation of the MLPG method is similar to the EFG method except the test function is chosen in such a way that it is equal to the Dirac delta function and there is no requirement of background mesh for integration.

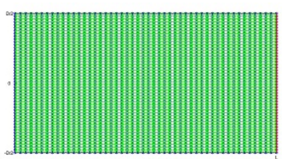
The Moving least squares(MLS) based on series representation has been used to approximate the solution in both the methods in which the weight functions can be varied from a simple exponential weight function to Gaussian Process kernel[6]. Overvelde[1] has used cubic spline weight function for the MLS approximation. As described earlier, the essential boundary conditions are imposed in MLS approximation and methods proposed in literature are penalty method, Lagrange's multiplier, etc.

III. PREVIOUS WORK DONE

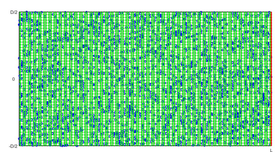
In this section, previous work done by Overvelde[1] on the techniques of nodal mesh generation are discussed namely, uniform, random and semi-random nodal distribution.

In uniform distribution, the nodes are defined uniformly all over the domain in both the axis as a rectangular grid; in random distribution, the problem domain is discretized by randomly distributed nodes; and lastly in semi-random distribution, the nodes are first distributed uniformly over the whole domain and then they are displaced from their position depending upon the perturbation parameter that has taken as 0.5 in the presented work as shown in Figure 1. There are an infinite number of orientations possible for random and semi-random distribution and therefore, 1000 simulations are done to obtain the mean result. The results are listed in Table 2.

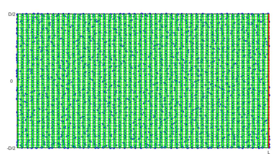
The solution is compared on the basis on the two scalar parameters: error norm and compliance. Error norm is calculated by determining the relative error between the elastic energy of the analytical solution and the approximated solution and the compliance is the inverse of the stiffness of the structure. The low values of both the parameters are expected for the best fit.



(a) Uniform



(b) Random



(c) Semi-Random

Fig. 1: Previous work done on Nodal Distributions

IV. Proposed Work and Results

In this section, new techniques of nodal mesh generation ae proposed namely, Latin Hyper-cube Sampling(LHS), Halton Sampling and Sobol Sampling.

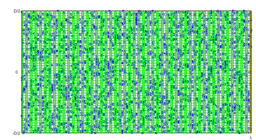
LHS is a sampling method based on stratified approach ensuring that all the fractions of the defined domain are sampled; The Halton sampling is a category of Quasi-Random Point sequence and provides the deterministic sampling by utilizing various prime bases to discretize the domain uniformly in each of the dimensions; The Sobol sampling is similar to Halton Sampling but there is only one major difference between the two that in Sobol sampling, the prime basis in every sequence is the same and is equal to 2. 1000 simulations are taken to calculate the mean value of error norm and compliance in LHS and Sobol sampling All these sampling are shown in Figure 2 and the results are listed are Table 2.

V. Conclusion

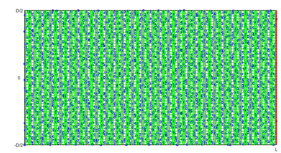
The displacement and stress analysis in the cantilever beam subjected to parabolic traction forces at the free end is performed with two mesh-less methods namely, EFG and MLPG with different nodal distribution techniques as listed in Table 2 and as shown in Figure 1 and Figure 2. The comparison among all the techniques is done on the basis of

Nodal Distribution	Error Norm	Compliance
Uniform	0.0015	2.6699
Random	0.4543	2.6702
Semi-Random	0.0402	2.6691
LHS	0.4592	2.67
Halton	0.3674	2.67
Sobol	0.382	2.67

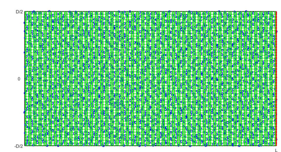
TABLE II: Error Norm and Compliance for different Nodal Distributions



(a) LHS



(b) Halton Sampling



(c) Sobol Sampling

Fig. 2: Proposed Nodal Distributions

two parameters i.e., error norm and compliance as defined in Section III.

From Table 2, it can be observed that with LHS and Random nodal distribution, the structure is very ill conditioned with very high values of error norm as 0.4592 and 0.4543 respectively. Similarly, Halton and Sobol sampling gives better results but yet the values of error norm are far deviated from that of the uniform distribution. The semi-random distribution has yielded acceptable results with error norm as 0.0402 and lastly, with value of error norm equal to 0.0015, the uniform nodal distribution represents the best fit to the analytical solution

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