

# **Master of Aerospace Engineering Research Project**

## **Manufacturing Constraints in Topology Optimization Framework**

### **S2 Project report**

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## **ABSTRACT**

This project deals with the understanding of the methods that are available for the Topology Optimization and its usage in the Additive Manufacturing(AM). The methods like Moving Node Approach(MNA), Geometric Projection(GP) and Moving Morphable Components(MMC) are widely used based on the requirement. These approaches have the same output but methods employed are different. In order to simplify the Topology Optimization, a new approach is introduced called Generalized Geometric Projection(GGP). Reviewing the work done on GGP and understanding its implementation and implementing the proposed method in to the Additive Manufacturing.

**KEYWORDS:** Additive Manufacturing, Generalized Geometry Projection, Topology Optimization

## 1 INTRODUCTION

Structural Topology optimization is a particular method that aims at optimizing the distribution of material in any specific design domain for given set of loads, constraints such that certain properties like compliance, stress bearing ability are optimized. Topology optimization is widely used in industries and in any real world problems where the structure has to be optimized. There are several methods that are implemented in the whole world of Topology optimization. These methods have received lot of focus since the work done by Bendose and Kikuchi [1] who are considered to be the pioneers for topology optimization. Apart from the structural optimization, the methods are spread onto different fields such as acoustics, electromagnetics and others [2]. Most of these existing methods either use pixel or node point-based framework. Considering the pixel based framework, even though lot of developments have been made in this, further work has to be done to solve some challenging issues like the fact that pixel based approaches are not consistent with Computer-aided design(CAD) modeling systems [2]. Where as in node point based approach, level-set method is used [2,8]. The most famous approach is the Solid Isotropic Material with Penalization (SIMP) [1,7]. This method uses a penalty factor which makes the densities upon which it is used, to be closer to 0/1. Another approach is the Geometry Projection [9] was initially introduced for shape optimization. In this approach, a sampling window is considered which computes the volume fraction enclosed by this sampling window. Another latest method is the Moving Node Approach (MNA) [5] in which the components are considered as nodes and the move around to find an optimum solution. Moving Morphable Components (MMC) is an optimization technique [2] which uses movable components that can be deformed as needed in a design space. This is used to define the density of the structure, as a result of which the computation time has been optimized.

These methods are known as explicit topology optimization, where in the components of the structure can change different parameters to give an optimized solution. The above methods give a similar solution but the method used to implement the solution are different. In order to make it easier, a new method is proposed called Generalized Geometry Projection (GGP) [10]. This uses all the 3 methods and presents it as one single formulation.

Additive Manufacturing (AM) is a fast developing technology as it carries high value in terms of weight and time for production. Also most of the AM technologies are able to produce highly complex geometries with high accuracy and efficiency. The process of AM is to add materials layer by layer to obtain a final 3D model. This technique is free-form as compared to other conventional manufacturing techniques.

## 2 SEMESTER SECTION

### 3.1 Context and Key issues

During the initial study of the project framework, it was understood that the main study has to be done on the Generalized Geometry Projection (GGP) and Additive Manufacturing (AM)

- Understanding of the exact constraints that are associated with the manufacturing.
- Understanding on the ways to use the different Topology Optimization methods into this particular project.
- Using of the MATLAB code and optimizer code effectively for this specific problem.
- Understanding the process of 3 main topology optimization framework.
- Studying the methods associated with Additive manufacturing technologies and the previous work done on it.

### 3.2 Main bibliography and State of the Art

Topology Optimization can be understood by realizing the scheme of the problem. The main idea for this is declare the domain in which the work has to be done (in  $\mathbb{R}^2$  or  $\mathbb{R}^3$ ). In this domain, the optimal material distribution has to be figured. This can be done by defining the objective function and constraint definition. The mathematical expression for any load bearing structure is given by, [7]

$$\min_{u \in U, \theta} \int_{\Omega} \rho u \cdot d\Omega + \int_{\Gamma_T} t u \cdot ds$$

Subject to,  $\int_{\Omega} C_{ijkl}(x) \varepsilon_{ij}(u) \varepsilon_{kl}(v) \cdot d\Omega = \int_{\Omega} \rho v \cdot d\Omega + \int_{\Gamma_T} t v \cdot ds$  for all  $v \in U$

$$C_{ijkl}(x) = \Theta(x) C_{ijkl}^0$$

$$\Theta(x) = \begin{cases} 1 & \text{if } x \in \Omega^m, \\ 0 & \text{if } x \in \Omega \setminus \Omega^m, \end{cases}$$

$$Vol(\Omega^m) = \int_{\Omega} \Theta(x) \cdot d\Omega \leq V$$

$$Geo(\Omega^m) \leq K$$

$U$  = Kinematically admissible displacement fields

$u$  = The equilibrium displacement

$\rho$  = body forces

$t$  = boundary forces

$\varepsilon(u)$  = linearized strains

$Geo(\Omega)$  = constraint function to limit geometric complexity of the domain

$\Theta(x)$  = Pointwise volume fraction of the material

$C_{ijkl}^0$  = stiffness tensor of a given elastic material

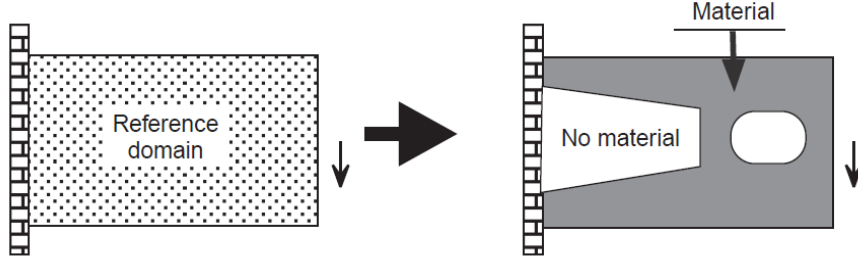


Figure 1 The generalized shape design problem of finding the optimal material distribution

Solutions obtained from the above relation given the density of each cell as between 0 or 1 i.e., having full density for 1 and empty for 0. For this estimation, there are many methods that can be used to understanding and optimizing this solution.

### 3.2.1 The Solid Isotropic Material with Penalization(SIMP) model

This method implements a correction factor called penalty, which makes sure that the material density is restricted to either 0 or 1. The reason for doing this is to make sure the boundary can be clearly justified. So the value of this penalty  $p$  is generally taken as 3 or more and can be changed accordingly. The value of 3 is used to make sure that the problem of grey area can be resolved. Since a relative high value of  $p$  is used, optimizer is forced to find the best result. A square finite elements are generated in the design domain and density based method are used for optimization. The element  $e$  is assigned a density  $x_e$  and gives Young's modulus  $E_e$ ,

$$E_e(x_e) = E_{min} + x_e^p(E_0 - E_{min}), x_e \in [0,1]$$

Where,

$E_0$  = stiffness of the material

$E_{min}$  = Small stiffness to avoid singular matrices

$p$  = Penalisation factor

And therefore, the optimisation model then is formulated as:

$$\begin{aligned} \min c(x) &= U^T K U = \sum_{e=1}^N E_e(x_e) u_e^T k_0 u_e \\ \text{Subject to: } &\frac{V(x)}{V_0} = f \\ &K U = F \\ &0 \leq x \leq 1 \end{aligned}$$

$c$  = compliance

$U, F$  = Global displacement and force vectors

Even though SIMP is reliably used for optimization process, it is not able to implement in Computer aided design system.

### 3.2.2 Moving Morphable Components(MMC)

The primary aim of this method is to establish a direct connection between the structural topology optimization and Computer aided design modelling systems. It helps in conduction the optimization

in a more explicit and a flexible way. As result Moving Morphable components is proposed. It can engulf more geometry and mechanical information into topology optimization directly. Also it has a great potential to reduce the computational burden associated with the topology optimization.

In this method, morphable components are used as primary building blocks. It inherits from level set method, where the value is positive inside the domain, zero on boundary and negative outside. One continuous path is taken as a component and are allowed to overlap.

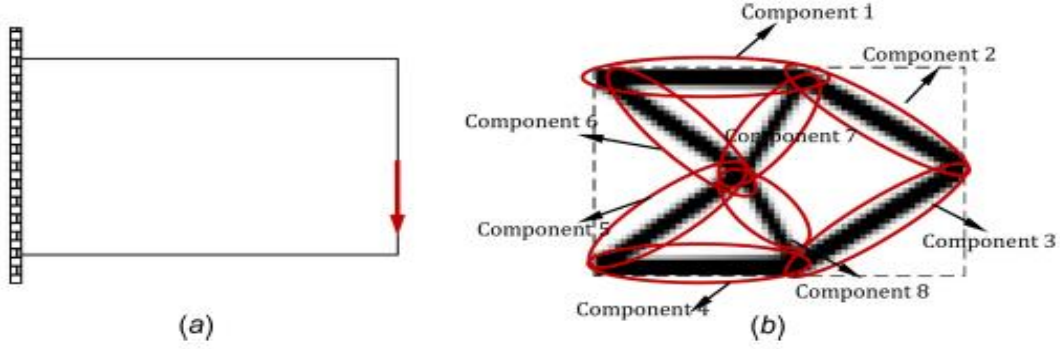


Figure 2 Structural topology represented by the layout of structural components

Considering the above beam which is designed to transfer vertical load to the clamped support with the least structural compliance i.e., maximum stiffness. The optimal manufacturable solution is of the form show in the above fig(b). 8 structural components are used to make optimal structure. This component is an object that contains the solid material occupying a specific volume in the domain of design.

### 3.2.3 Moving Node Approach(MNA)

In this method the structures are optimized by varying different parameters such as the length, thickness, orientation, location and connectivity between components. The main thought process for this method is to reduce the number of input variables and also to minimize the degrees of freedom. The components for which various parameters are defined, tends to rearrange themselves. In this the density is determined by the position of the mass containing nodes. These data of the density field that has been generated is fed into the FEM solver, which in turn gives the optimized compliance through iterative process over a fixed boundary domain.

The density function is given by,

$$\rho(x) = \sum_{l=1}^n m^l W(x, \mu^l)$$

$n$  = Total number of mass nodes

$m^l$  = Mass associated to each mass node

$\mu^l = [x^l, y^l, \theta^l, L_x^l, L_y^l]^T$  = Material variables vector containing the x co-ordinate, y co-ordinate, angle of component, length and breadth of each component.

$W(x, \mu^l) = w(|\xi(x, \mu^l)| \left(\frac{L_x^l}{2}\right)) * w(|\eta(x, \mu^l)| \left(\frac{L_y^l}{2}\right))$  = The density function



While the objective function remains the same, the constraints for this is modeled as

$$(m_{Max} - \sum_{l=1}^N \beta L_x^l L_y^l) \geq 0$$

$m_{Max}$  = Maximum allowed structural mass

### 3.2.4 Method of Moving Asymptotes(MMA)

This method for structural optimization is based on the special type of convex approximation [4]. It was introduced in 1987 to optimize non-linear programming in general and structural optimization in particular. This is done by iteration where a sub-problem is generated and continued until the convergence criteria or if a satisfactory solution is met. Controlling the generation of these sub-problems are done by ‘Moving Asymptotes’. A good feature of this asymptote is that it speeds up and stabilizes the convergence. This method can hold both the shape variables and material orientation as design variable apart from element size.

$$\begin{aligned} \min f_o(x) \quad (x \in R^n) \\ \text{s.t. } f_i(x) \leq \hat{f}_i \quad \text{for } i = 1, \dots, m \\ \underline{x}_j \leq x_j \leq \bar{x}_j \quad \text{for } j = 1, \dots, n \end{aligned}$$

$x = (x_1, \dots, x_n)^T$  = Vector of design variable

$f_o(x)$  = Objective function

$f_i(x) \leq \hat{f}_i$  = Behaviour Constraints limiting stresses and displacements

$\underline{x}_j$  and  $\bar{x}_j$  = Gives upper and lower bounds on design variable

### 3.2.5 Generalized Geometry Projection (GGP)

There are several methodologies that are present in structural optimization [10]. The main focus is on the geometric feature based approach which is also known as explicit topology optimization. In this, the components can change parameters like size, position and its orientation. The review for this methodologies is done based on Geometric Projection (GP), Moving Node Approach (MNA) and Moving Morphable Components (MMC) frameworks. Considering these 3 frameworks, Generalized Geometric Projection is formulated.

The geometric components that are considered are the round ended bars which is defined by 5 parameters. These are position of the centre of the component  $\{X, Y\}$ , dimensions of the components  $\{L, h\}$  and finally the orientation of the component  $\{\theta\}$ . In the final configuration, the topology will be the superposition of multiple such basic components.

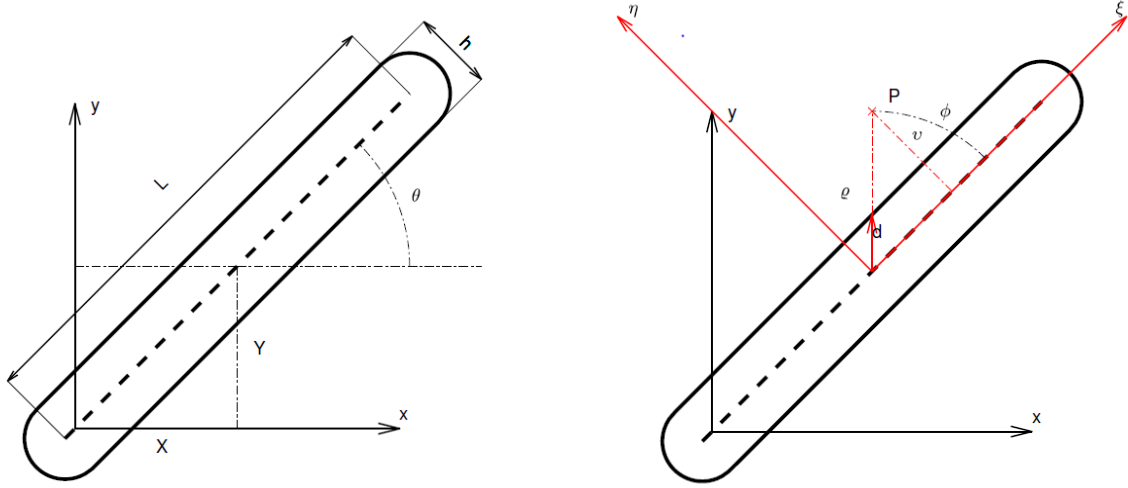


Figure 3 (a) Geometric parameters of the component (b) Plot in Local coordinate system with the distance from component boundary and distance from components middle axis.

The classical equation which minimized the compliance of the structure is considered which are subjected to different constraints of volume fraction and design space.

$$\min_{\{x\}} C = \{U\}^T \{F\}$$

$$\text{S.t. } V = \frac{\sum_{el=1}^N \rho^{el}}{N} \leq V_0$$

$$\{l_b\} \leq \{x\} \leq \{u_b\}$$

In the above equation,  $\{l_b\}$  and  $\{u_b\}$  are the lower and upper bound vector of design variable and  $V_0$  is the maximum volume fraction.

In Moving Morphable Components (MMC) [2], a Topology Distribution Function (TDF) is introduced which is positive inside the component, equal to zero on the boundary and negative outside. The definition of TDF is

$$X_i = 1 - \left( \frac{4v_i^2}{h_i^2} \right)^\alpha \text{ with } \alpha \geq 1$$

A heaviside function  $H(x)$  is applied to TDF which determines the presence or absence of material inside the design domain. A regularized version  $H_\epsilon(x)$  which replaces  $H(x)$  is used.

$$H_\epsilon(x) = \begin{cases} 1, & \text{if } x > \epsilon, \\ \frac{3(1-\beta)}{4} \left( \frac{x}{\epsilon} - \frac{x^3}{3\epsilon^3} \right) + \frac{1+\beta}{2}, & \text{if } -\epsilon \leq x \leq \epsilon, \\ \beta, & \text{otherwise} \end{cases}$$

The smooth variation of 1 inside to  $\beta$  outside is denoted by a transition zone  $D_g^{MMC}$  given by,

$$D_g^{MMC} = \left\{ \{X_g\} \mid \frac{h}{2}(1-\epsilon)^{\frac{1}{2\alpha}} \leq v \leq \frac{h}{2}(1+\epsilon)^{\frac{1}{2\alpha}} \right\}$$

The width of this transition zone is denoted by  $w_g$ . In this, the width is directly proportional to the thickness  $h$ , as a result, a smaller components varies faster from full material to void.

$$w_g^{MMC} = \frac{h}{2} \left[ (1-\epsilon)^{\frac{1}{2\alpha}} - (1+\epsilon)^{\frac{1}{2\alpha}} \right]$$

In Geometric Projection (GP) [9], it computes the signed distance between each element central point and the surface of the component. Here a sampling window is considered in order to compute the volume fraction and subsequently, density and Young's modulus is computed based on the volume fraction. The sampling window considered can be of any shape but here a circle  $B_p^r$  with radius  $r$ . The local volume fraction is the ratio of the volume in the sampling window to the total area.

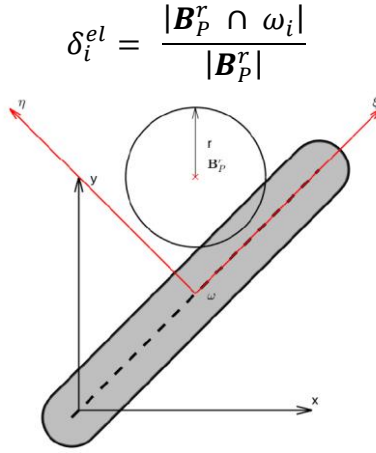


Figure 4 Component with sampling window

The transition zone associated with GP is in relation w.r.t the thickness of the component and the radius of the sampling window. The width of this transition zone is  $2r$  and to achieve regular distribution of density, the thickness should be greater than or at least equal to  $2r$ .

$$D_g^{GP} = \left\{ \{X_g\} \mid \frac{h}{2} - r \leq v \leq \frac{h}{2} + r \right\}$$

$$w_g^{GP} = 2r$$

In Moving Node Approach (MNA) [5], the building blocks are mass nodes. The center of each mass node is computed again w.r.t the local coordinate system. The local density contribution is estimated with the application of the weighting function. To make sure the total density does not exceed 1, asymptotic density was proposed by Overvelde [5] in his work. The geometry of each component drives the finite element model to update.

$$w(v, h, \varepsilon) = \begin{cases} 1, & \text{if } v \leq l, \\ a_3 v^3 + a_2 v^2 + a_1 v + a_0, & \text{if } l < v < u, \\ 0, & \text{otherwise} \end{cases}$$

$$l = \frac{h}{2} - \frac{\varepsilon}{2}; u = \frac{h}{2} + \frac{\varepsilon}{2}; a_3 = \frac{2}{\varepsilon^3}; a_2 = -\frac{3h}{\varepsilon^3}; a_1 = 3 \left( \frac{h^2 - \varepsilon^2}{\varepsilon^3} \right); a_0 = -\frac{(h + \varepsilon)^2 (h - 2\varepsilon)}{4\varepsilon^3}$$

The transition zone and the thickness of this transition zone is defined as given below,

$$D_g^{MNA} = \left\{ \{X_g\} \mid \frac{h - \varepsilon}{2} \leq v \leq \frac{h + \varepsilon}{2} \right\}$$

$$w_g^{GP} = \varepsilon$$

In Generalized Geometry Projection (GGP), the three methods are considered and are taken into account but it is a generalization of the geometry projection. The geometric component has to be choose and in this case it is the round ended bar. A characteristic function  $\gamma$  which is defined for set of points inside the geometric primitive.

$$\gamma(\{X_g\}, \omega_i) = \begin{cases} 1 & \text{if } \{X_g\} \in \omega_i. \\ 0, & \text{otherwise} \end{cases}$$

In GGP, the finite element model can be updated depending on the various component configuration. The results obtained in [10], we understand that, depending on the characteristic function and the Gauss points used, several mesh induced phenomena impacts the model responses. Few strategies that reduces such phenomena are to increase the component Transition region width, refining the mesh, using a multi resolution approach, using GGP etc.

### 3.5.6 Additive Manufacturing (AM)

Additive manufacturing or Additive Layer Manufacturing is method of layer by layer manufacturing where the materials are added one layer after another. Unlike the conventional manufacturing methods, where the material is removed, this process is more efficient, light and is able to generate complex structural components with more efficiency and accuracy. Since the freedom for the design in AM is much higher that of conventional manufacturing, the topology of the component is re imagined, as a result Topology Optimization is the great tool for AM [11].

In this, main problem is the overhang constraint in AM, for which several support structures are also printed. Generating structures with the support results in many post processes and might create rough surface. So one would like to eliminate this support material by limiting the overhang to certain value of this angle. It is studied that the maximum achievable overhang of printed parts is around  $45^\circ$  [11]. So for this formulation, 2D components used in MNA is considered. The primary idea for this is to represent the solution using printed pathways and for this 1D components are considered. This simplification might not be used for complex structures but it can be used for basic structures as it is practical and straightforward.

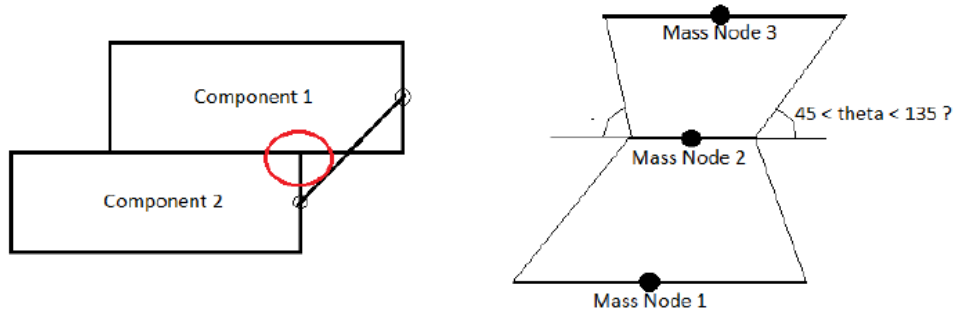


Figure 5 1D and 2D components comparisons

For structure to support itself, the overhang angle must be  $45^\circ$  to  $135^\circ$  and the above picture represents that.

Using 1D components, the variables are position  $(x, y)$ , length  $(l)$ , and component angle  $(\theta)$ . The density function is,

$$\rho(x) = \sum_{l=1}^n m^l W(x, \mu^l)$$

Where  $n$  = Total components

$m^l$  = Each component mass

$u^l = [x^l, L_x^l]^T$  = Component variables vector

$$W(\xi) = \begin{cases} 1 - 6\xi^2 + 6\xi^3 & \text{if } \xi \leq 0.5 \\ 2 - 6\xi + 6\xi^2 - 2\xi^3 & \text{if } \xi > 0.5 \end{cases} = \text{The shape functions}$$

$$\xi(x, x_\alpha, L_\alpha) = \frac{2 \cdot \text{abs}(x - x_\alpha)}{L_\alpha} = \text{The normalising function}$$

$$x_\alpha = (1 - \alpha)x_i + \alpha x_{i+1} = \text{Position of equivalent node}$$

$$L_\alpha = (1 - \alpha)L_i + \alpha L_{i+1} = \text{Length of equivalent node}$$

$$\alpha = \text{Normalised vertical distance.}$$

Two components are chosen one above the other and distance between them is calculated after its linear normalization. The Pseudo centres ( $x_\alpha$ ) and pseudo length ( $L_\alpha$ ) are calculated based on the distance. Using this, normalized distance of each FEM element centre at every layer up to the 2<sup>nd</sup> component. The shape function takes in the distance and gives the density of material.

This constraint of overhang angle needs to be stipulated and applied to solve it. The main idea is to have 2 constraints for each overhang angle limit. One for  $\theta \geq 45^\circ$  and another for  $\theta \leq 135^\circ$ .

$$C_1 : x_1 - \frac{L_1}{2} - x_2 + \frac{L_2}{2} \leq \tan(135)$$

$$\rightarrow x_1 - \frac{L_1}{2} - x_2 + \frac{L_2}{2} - N_{layers} \leq 0$$

$$C_2 : x_2 + \frac{L_2}{2} - x_1 - \frac{L_1}{2} \leq \tan(45)$$

$$\rightarrow x_2 + \frac{L_2}{2} - x_1 - \frac{L_1}{2} - N_{layers} \leq 0$$

The lateral position ( $x_1$ ) and length ( $L_1$ ) are of a given component and the corresponding ( $x_2$ ) and ( $L_2$ ) are that of the next level.

### 3 RESULTS AND ANALYSIS

The main aim of the work was to fully understand the working of the new method and to understand the previous work done on the additive manufacturing field. In the previous work we can see that the usage of 1D components make it much easier and faster to arrive at the solution. The framework of Generalized Geometry Projection will help in making the implementation of the constraints of AM easier. For this particular reason a thorough study was done.

The main code of GGP is run for different conditions of method employed, number of gauss points used, size of the sampling window used and based on these variables the output changes



Moving Morphable Bars(MBB) topology optimization using GGP for  $N_{GP} = 2$  and  $R = 0.5$  showing component plot, Density plot and Convergence plot



MBB topology optimization using GGP for  $N_{GP} = 1$  and  $R = 0.5$  showing component plot, Density plot and Convergence plot



MBB topology optimization using GGP for  $N_{GP} = 16$  and  $R = 0.5$  showing component plot, Density plot and Convergence plot.

## 4 CONCLUSION

Initially during the start of this project the objective was to implement the manufacturing constraints on to the GGP code of the Additive Manufacturing. The first set of objective is to understand the methodology that are available in the topology optimization domain and the work done on those fields. Then we studied the new method that was proposed Generalized Geometry Project on its implementation. This combines the already existing methods into one. Then further study was made on Additive Layer Manufacturing. The problem of Additive Manufacturing which being the Overhang angle was considered and the previous work that was done on the solution for this is studied.

The future work to be done is to implement the constraints of the Additive Manufacturing into the code and developing the knowledge, understanding and implementing the constraints for different manufacturing methods.

## 5 FUTURE WORK

The constraints used are based on the ability of structure to be manufactured or machined. For the future work many cases of constraint implementation will be studied and implemented. One such case is introducing manufacturing and machining constraints into the topology optimization [3]. The objective and constraint are having non-convex behaviour, so the design space is also non convex. A 2D design with and without manufacturing and machining constraints is shown below. Design space is approximated using the convex approximation.

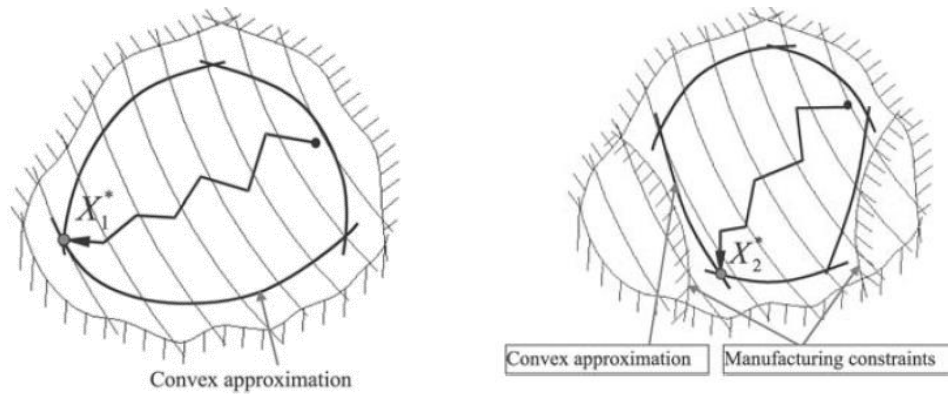


Figure 6 2D design space without and with manufacturing constraints

For instance, considering a finite element model of 3D cantilever beam with certain given boundary conditions, minimize the compliance with minimum machining hole size constraint. This hole might not have any use and during topology optimization it will be generated considering this hole, which might be difficult to machine in the later stages. As a result, this constraint does not consider the hole and manufactures the part without it. Such constraints can be worked on for future work.

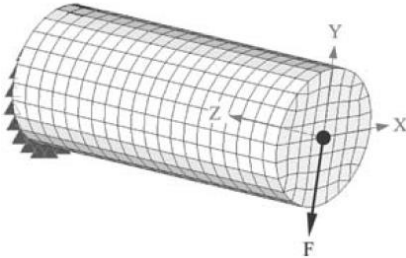


Figure 9 FEM model



Figure 8 Result without min hole constraint



Figure 7 Result with Min hole constraint

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