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Master of Aerospace Engineering
Research Project

Manufacturing Constraints in Topology Optimization Framework

S2 Progress report

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1 Goal of the project

Structural Topology optimization is a particular method that aims at optimizing the distribution of material in any specific design domain for given set of loads, constraints such that certain properties like compliance, stress bearing ability are optimized. Topology optimization is widely used in industries and in any real world problems where the structure has to be optimized. The several methods are implemented in the whole world of Topology optimization.

The main goal of this particular project is to initially find the constraints in the manufacturing process and finding a way to eliminate these using the Topology Optimization technique. Then, implementing the different issues into the constraints of the Optimization. Also, simulating various boundary conditions by further studying the literatures which has been done on this optimization and improving the usage of the Optimizer.

2 Project issues

During the initial study of the project on the finding the constraints of manufacturing few issues are found,

- Understanding of the exact constraints that are associated with the manufacturing.
- Understanding on the ways to use the different Topology Optimization methods into this particular project.
- Using of the MATLAB code and optimizer code effectively for this specific problem.
- Studying the constraints and finding out if there are any particular issues with that constraint.
- Finding the best method that is suitable for the implementation of the manufacturing constraints.

3 Main bibliography and State of the Art

Topology Optimization can be understood by realizing the scheme of the problem. The main idea for this is declare the domain in which the work has to be done (in R^2 or R^3). In this domain, the optimal material distribution has to be figured. This can be done by defining the objective function and constraint definition. The mathematical expression for any load bearing structure is given by,

$$\min_{u \in U, \Theta} \int_{\Omega} \rho u . d\Omega + \int_{\Gamma_T} tu . ds$$

Subject to, $\int_{\Omega} C_{ijkl}(x)\varepsilon_{ij}(u)\varepsilon_{kl}(v) \cdot d\Omega = \int_{\Omega} \rho v . d\Omega + \int_{\Gamma_T} tv . ds \; for \; all \; v \in \mathrm{U}$

$$C_{ijkl}(x) = \Theta(x) C^0_{ijkl}$$

$$\Theta(x) = \begin{cases} 1 \; if \; x \in \Omega^m, \\ 0 \; if \; x \in \Omega \backslash \Omega^m, \end{cases}$$

$$Vol(\Omega^m) = \int_{\Omega} \Theta(x) . d\Omega \leq V$$

$$Geo(\Omega^m) \leq K$$

U = Kinematically admissible displacement fields

u = The equilibrium displacement

ρ = body forces

t = boundary forces

$\varepsilon(u)$ = linearized strains

$Geo(\Omega)$ = constraint function to limit geometric complexity of the domain

$\Theta(x)$ = Pointwise volume fraction of the material

C^0_{ijkl} = stiffness tensor of a given elastic material

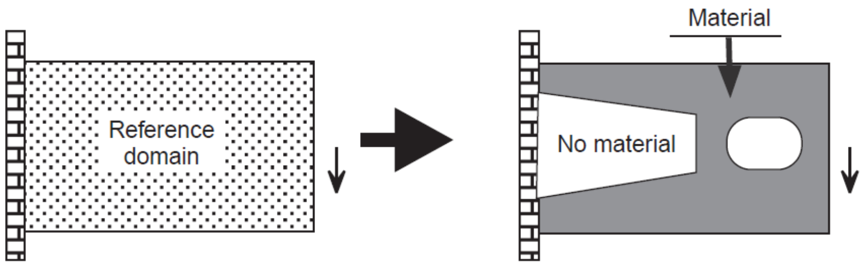

Figure 1 The generalized shape design problem of finding the optimal material distribution

Solutions obtained from the above relation given the density of each cell as between 0 or 1 i.e., having full density for 1 and empty for 0. For this estimation, there are many methods that can be used to understanding and optimizing this solution.

3.1. The Solid Isotropic Material with Penalization(SIMP) model

This method implements a correction factor called penalty, which makes sure that the material density is restricted to either 0 or 1. The reason for doing this is to make sure the boundary can be clearly justified. So the value of this penalty p is generally taken as 3 or more and can be changed accordingly. The value of 3 is used to make sure that the problem of grey area can be resolved. Since a relative high value of p is used, optimizer is forced to find the best result. A square finite elements are generated in the design domain and density based method are used for optimization. The element e is assigned a density x_e and gives Young's modulus $E_{e,}$

$$E_e(x_e) = E_{min} + x_e^p(E_0 - E_{min}),\ x_e \in [0,1]$$

Where,

E_0 = stiffness of the material

E_{min} = Small stiffness to avoid singular matrices

p = Penalisation factor

And therefore, the optimisation model then is formulated as:

$$\min c(x) = U^T K U = \sum_{e=1}^{N} E_e(x_e) u_e^T k_0 u_e$$

$$\text{Subject to:} \quad \frac{V(x)}{V_0} = f$$

$$KU = F$$

$$0 \le x \le 1$$

c = compliance

U, F = Global displacement and force vectors

Even though SIMP is reliably used for optimization process, it is not able to implement in Computer aided design system.

3.2. Moving Morphable Components(MMC)

The primary aim of this method is to establish a direct connection between the structural topology optimization and Computer aided design modelling systems. It helps in conduction the optimization in a more explicit and a flexible way. As result Moving Morphable components is proposed. It can engulf more geometry and mechanical information into topology optimization directly. Also it has a great potential to reduce the computational burden associated with the topology optimization.

In this method, morphable components are used as primary building blocks. It inherits from level set method, where the value is positive inside the domain, zero on boundary and negative outside. One continuous path is taken as a component and are allowed to overlap.

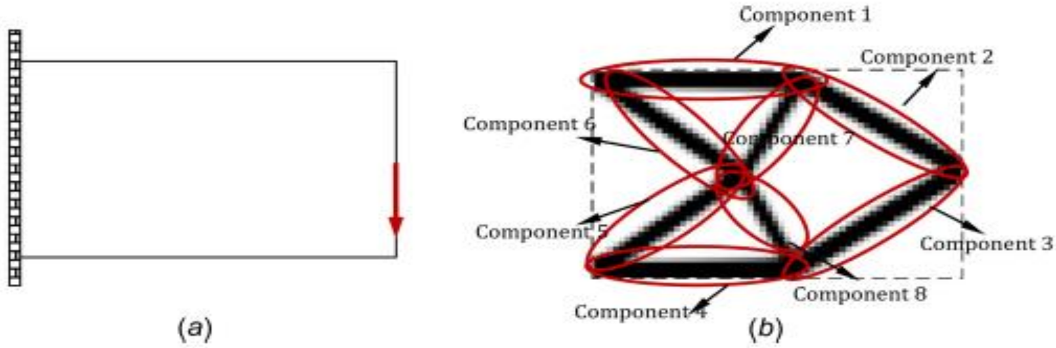

Figure 2 Structural topology represented by the layout of structural components

Considering the above beam which is designed to transfer vertical load to the clamped support with the least structural compliance i.e., maximum stiffness. The optimal manufacturable solution is of the form show in the above fig(b). 8 structural components are used to make optimal structure. This component is an object that contains the solid material occupying a specific volume in the domain of design.

3.3. Moving Node Approach(MNA)

In this method the structures are optimized by varying different parameters such as the length, thickness, orientation, location and connectivity between components. The main thought process for this method is to reduce the number of input variables and also to minimize the degrees of freedom. The components for which various parameters are defined, tends to rearrange themselves. In this the density is determined by the position of the mass containing nodes. These data of the density field that has been generated is fed into the FEM solver, which in turn gives the optimized compliance through iterative process over a fixed boundary domain.

The density function is given by,

$$\rho(x) = \sum_{I=1}^{n} m^I W(x, \mu^I)$$

n = Total number of mass nodes

m^I = Mass associated to each mass node

$u^I = [x^I, y^I, \theta^I, L_x^I, L_y^I]^T$ = Material variables vector containing the x co-ordinate, y co-ordinate, angle of component, length and breadth of each component.

$W(x, \mu^I) = w(|\xi(x, \mu^I)| \left(\frac{L_x^I}{2}\right) * w(|\eta(x, \mu^I)| \left(\frac{L_y^I}{2}\right)$ = The density function

While the objective function remains the same, the constraints for this is modeled as

$$(m_{Max} - \sum_{I=1}^{N} \beta L_x^I L_y^I) \geq 0$$

m_{Max} = Maximum allowed structural mass

3.4. Method of Moving Asymptotes(MMA)

This method for structural optimization is based on the special type of convex approximation. It was introduced in 1987 to optimize non-linear programming in general and structural optimization in particular. This is done by iteration where a sub-problem is generated and continued until the convergence criteria or if a satisfactory solution is met. Controlling the generation of these sub-problems are done by 'Moving Asymptotes'. A good feature of this asymptote is that it speeds up and stabilizes the convergence. This method can hold both the shape variables and material orientation as design variable apart from element size.

$$\min f_o(x) \quad (x \in R^n)$$
$$\text{s.t. } f_i(x) \leq \widehat{f_i} \quad for\ i = 1, \dots\dots, m$$
$$\underline{x}_j \leq x_j \leq \overline{x}_j \quad for\ j = 1, \dots\dots, n$$

$x = (x_1, \dots., x_n)^T$ = Vector of design variable

$f_o(x)$ = Objective function

$f_i(x) \leq \widehat{f_i}$ = Behaviour Constraints limiting stresses and displacements

$\underline{x}_j\ and\ \overline{x}_j$ = Gives upper and lower bounds on design variable

4 Milestones of the project

During the entire course of this project, gradual progress is to be made in regards with method that has be implemented. Also a further thorough bibliographic work has to be made in order to better understand the methods that can be implemented for this particular project. The below mentioned are the projected milestones,

- The Bibliography of the project has been initiated (Mar-Apr 19)
- Realizing the constraints for different manufacturing process. (Apr 19)
- Further study has to be made on the particular problem of the constraints. (Apr-May 19)
- Better understanding of the MATLAB code and also its implementation. (Apr-May 19)
- Finalizing on the method which will be best suited based on constraints. (May 19)
- Translating the constraints of this problem on to the optimizer. (June 19)

5 Bibliographic Study

5.1. Studying the various work that has been previously done

Initially to understand the project, there is a need to understand the concept behind it. There are many works that interests this project. So an extensive study has to be made for the understanding the work. After this initial study, there are lot of methods that can is used for different types of problems. So a further in depth study is to be made to understand every method.

The methods include SIMP, where a penalization technique is used to make sure that density will be closer to 0/1 by penalizing the intermediate densities. This is widely used and a very effective method. Another method is Moving Morphable Components(MMC) where morphable components are used as preliminary building blocks. It is very helpful when using with CAD system.

Apart from these, there are several methods which has its advantages, all of which are explained above. The initial study has been made, where the understanding of the project is underway and also the detailed study is being made.

5.2. Plan versus achievements

The plan of this task is to start the study of the concepts which is already done and is on time for this bibliographic study. Also further in-depth study has been initiated.

5.3. Planned work for the next 2 months

For the bibliographic study, the future plan is to continue with the detailed study of the different methods. Also in the coming months, the plan is to further the knowledge on the particular method so that, while implementing the constraints it can be clearly defined as to what method is the best and optimizes the defined problem.

6 Constraint definition and learning the MATLAB code

6.1. Defining the manufacturing constraints and understanding the code

As mentioned earlier, the important task is to identify the constraints that will be associated with the different manufacturing process like moulding, milling or casting. There is a need to understand the way these manufacturing process takes place and find the issue that can be optimized with the help of topology optimization. Then the constraint will be defined and implemented onto the code.

Another major task on a need to achieve basis is the understanding the project code. The main crux of the code is to be learnt and has to be modified wherever it is necessary for this work. Especially in the constraint definition, these variables are identified and made sure that the execution of this in the code is correct.

6.2. Plan versus achievements

The plan for the constraint definition is to understand the problem that it is facing during the standard manufacturing process. For this to happen, study is being made in the problem identification and application of this constraint.

6.3. Planned work for the next 2 months

As mentioned earlier, for the coming months, completion of most of the study needs to be done. Then constraint has to be identified for the problem and the code with which the optimizer runs will be learnt.

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