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INTEGRATED STRUCTURE-CONTROL OPTIMIZATION OF SPACE STRUCTURES

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Abstract

In the past few years there has been substantial interest in the integrated design of large space structures and their vibration control systems. This paper overviews two aspects of integrated control-structure design. The first is the calculation of derivatives of control system stability margins with respect to structural parameters, and the effect of modal truncation on the accuracy of these derivatives. The second aspect is the various formulations used to define an integrated design optimization problem.

Introduction

Traditionally, the process of designing a structure and a control system for damping vibration in the structure has been sequential. First the structure is optimized to minimize an objective such as cost or weight subject to constraints on response quantities such as displacements, stresses, or vibration frequencies. Then the control system is optimized to minimize some measure of control performance such as control effort, subject to constraints on other performance measures such as stability margins or robustness.

This process of sequential design of the structure and its control system can result in sub-optimal performance. For large space structures the weight of the system consisting of the structure, the control system and the power source for the controls is extremely critical because of the high cost of transferring mass to orbit. Therefore, there is merit in considering the simultaneous or integrated design of the structure and control system.

One of the key ingredients in the integrated design procedure is cross sensitivity information. This includes the sensitivity of structural performance and response to changes in control system parameters and the sensitivity of control system performance to changes in structural parameters. This sensitivity information can be used to assess the potential gain, or synergistic effect, associated with the integrated design. For example, Haftka et al.[1] showed that for a laboratory structure, designed to simulate large space structures, the performance of the control system could be significantly enhanced by minor modifications to the structure. This enhancement of the control system performance was predicted well by cross sensitivity derivatives. Sensitivity derivatives are also, of course, the life blood of most optimization procedures. Therefore the calculation of sensitivity derivatives has received much attention in papers dealing with integrated structure-control design, and this is also the first topic discussed in this paper. One major issue associated with derivative calculation is the accuracy of these derivatives when they are calculated from a reduced structural model.

A second major issue associated with integrated design is the choice of objective function. Here the problem is how to reconcile the different objectives used in the design of the structure and control system. Several approaches to this problem will be discussed in the second part of this paper.

Most of the work on integrated structure-control system design was performed for linear-quadratic Gaussian (LQG) control laws. While other control laws may be equally appropriate, this historical precedent also guides this paper, with the discussion focused on LQG control.

Analysis and Sensitivity

The discretized equations of motion for a flexible structure are usually written as

$$M\ddot{q} + D\dot{q} + KQ = L \qquad [1]$$

where M, D and K are the mass, damping and stiffness matrices, respectively, L is the load vector, q is the structural response vector and a dot denotes differentiation with respect to time. The structural response q can be written in physical coordinates if Eq. (1) is directly obtained from a finite-element model. However, in most applications the order of the finite-element model of the structure is too high for control system design, and so it is reduced by employing some type of modal reduction. When the natural vibration modes of the structure are employed as reduced-basis vectors M and K become diagonal matrices. Here, however, we will assume that they can be fully populated.

Equation (1) can be reduced to a more general first-order form

$$\dot{x} = Ax + f \tag{2}$$

One way to accomplish this reduction to a first-order system is to define $x^T = [\dot{q}, q]$ and then

$$A = \begin{bmatrix} -M^{-1}D & -M^{-1}K \\ I & 0 \end{bmatrix} \quad f = \begin{Bmatrix} M^{-1}L \\ 0 \end{Bmatrix} \tag{3}$$

where I denotes the unit matrix. In the consideration of the control problem we limit ourselves to linear control so that f is given as

$$f = Bu + w \tag{4}$$

where u is the control vector, and w is a vector of white noise in the control commands assumed to be zero-mean, Gaussian stationary process. The control law is assumed to be linear and time-invariant, that is

$$u = -Gx \tag{5}$$

where G is a matrix of constant gains which is found by solving an optimal control problem. Substituting Eqs. (4) and (5) into Eq. (2) we get

$$\dot{x} = (A - BG)x + w \tag{6}$$

so that the stability of the closed-loop system is determined by the eigenvalues λ_i of $A - BG$.

The derivatives of these eigenvalues with respect to structural or control parameters is given as

$$\lambda_i' = x_{Li}^T(A' - B'G - BG')x_{Ri} \tag{7}$$

where a prime denotes a derivative with respect to the parameter, and where x_{Li} and x_{Ri} are the left and right eigenvectors of the matrix $A - BG$, respectively, corresponding to λ_i and normalized so that

$$x_{Li}^T x_{Ri} = 1 \tag{8}$$

In calculating A' we need to calculate derivatives of K, D and M. When q respresents a vector of modal amplitudes these matrices are obtained from equations of the form

$$M = \Phi^T \bar{M} \Phi \tag{9}$$

where $\bar{M}$ is a mass matrix obtained from a finite-element analysis and Φ is a matrix of modes. Differentiating Eq. (9) we get

$$M' = \Phi'^T \bar{M} \Phi + \Phi^T \bar{M}' \Phi + \Phi^T \bar{M} \Phi' \tag{10}$$

So that even though we do not need derivatives of the eigenvectors x_{Li} and x_{Ri} for calculating λ_i' we may need derivatives of the vibration modes. Calculating derivatives of vibration modes is expensive. Fortunately, however, Φ' may often be neglected (see, for example, Sandridge and Haftka[2]).

Control Design Formulations

With linear-quadratic regulators the gain matrix G in Eq. (5) is found by minimizing a quadratic performance index J

$$J = E(x^T Q x + x^T R x) \tag{11}$$

where E is the expected-value operator and Q and R are positive definite matrices.

The gain matrix which minimizes the quadratic performance index is given as

$$G = R^{-1} B^T S \tag{12}$$

where S is the solution of a matrix Riccati equation

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \tag{13}$$

The sensitivity of the gain matrix can be calculated by differentiating Eq. (12) and using the relation

$$(R^{-1})' = -R^{-1} R' R^{-1} \tag{14}$$

to obtain

$$\begin{aligned} G' = &-R^{-1}R'R^{-1}B^T S + R^{-1}B'^T S \\ &+ R^{-1}B^T S' \end{aligned} \tag{15}$$

where S' is obtained by differentiating Eq. (13) and using Eq. (12) and the symmetry of S

$$\begin{aligned} S'(A - BG) + (A - BG)^T S' + SA' + A'^T S + Q' \\ - S(BR^{-1}B^T)' S = 0 \end{aligned} \tag{16}$$

Equation (16) is a Lyapunov equation that can be solved for S' and then G' is calculated from Eq. (15) and the eigenvalue derivatives from Eq. (7). The optimum value of J is given as

$$J^* = tr\{SW\} \tag{17}$$

where W is the noise covariance matrix, that is

$$E[w(t)w^T(\tau)] = W\delta(t-\tau) \tag{18}$$

An interesting property of J^* (see Gilbert[3]) is that its derivatives with respect to changes in the gains are zero.

It is usually impossible to measure all the elements of x. Instead we have sensors that observe a vector z

$$z = Cx + v \tag{19}$$

where v is observation noise with covariance matrix V. We assume that the full vector x is estimated as $\hat{x}$ from z by a Kalman filter that integrates

$$\dot{\hat{x}} = A\hat{x} + Bu - F(z - Cx) \tag{20}$$

where the filter matrix F is found as the solution to another Riccati matrix equation. For expressions of the sensitivity of the observation solution, see Gilbert.[3] The combination of linear quadratic regulator with a Kalman filter is called linear-quadratic Gaussian (LQG) control.

LQG control minimizes a quadratic performance index J (Eq. (11)) which is a compromise between vibration levels and control effort. The choice of the weighting matrices Q and R, however, is somewhat arbitrary and these are typically adjusted by the designer to achieve satisfactory response characteristics.

The primary measures of the response of the controlled structure are the closed-loop eigenvalues or the damping ratios associated with them. The Q and R matrices are often adjusted to satisfy limits (referred to as stability margins) on the real part of the eigenvalues or their damping ratios. Alternatively the gain matrix G can be calculated to obtain desired eigenvalues in a process called pole placement (the closed-loop eigenvalues are poles of the system transfer matrix).

When the control system is designed subject to stability margin limits, it may be reasonable to minimize only a measure of control effort because the response is limited by the stability margins. One measure of control effort used (e.g. Knot et al.[4]) is the Frobenius norm of the control gains $\|G\|_f$.

$$\|G\|_f = \text{trace}\{G^T RG\} \tag{21}$$

The structural model is often a poor representation of the structure because of either modelling errors or deliberate model truncation. Therefore it is important to design a robust control system, that is one which is not sensitive to small variations in the properties of the structure. Control systems are sometimes designed to maximize robustness. One measure of robustness (e.g. Junkins and Rew[5]) is the condition number of the closed-loop modal matrix. Another method used to maximize robustness is to minimize a norm of the sensitivity of the closed-loop eigenvalues to a set of specified parameters (e.g. Lim and Junkins[6]).

Truncation Error in Reduced Models

As noted earlier the size of the structural model is reduced so as to make the design of the control system tractable. This is typically done by using a subset of the vibration modes of the structure as reduced-basis vectors. Modal cost techniques (e.g. Skelton et al.[7]) are often used to select the vibration modes to be included in the reduced model. However, the vibration modes of the structure are smooth functions and so they cannot represent well the response to point actuators which introduce spatial discontinuities. In the presence of such point actuators a reduced model based on vibration modes can be virtually useless.

The difficulty is demonstrated by a five-span beam example[2] shown in Figure 1. For this simple example direct rate feedback controllers are used which can be modeled as dashpots. One controller is a displacement rate controller on the second span, and the second is a rotation-rate controller on the middle span. The controllers are designed to produce damping ratios of one to ten percent in the first six modes. Derivatives of the damping ratios are calculated with respect to an added mass at point A on the beam.

Figure 2 shows the error in the fourth damping ratio and its derivative with respect to the mass for a finite-element model of 3 beam finite elements per span. The model has 26 unrestrained degrees of freedom and it is seen that even for 24 out of 26 modes the error in the derivative is very high.

To check the convergence beyond 24 modes, more refined models with 6, 9 and 12 elements per span were also analyzed. Additionally, analytical modes were calculated in closed form from a continuum model of the beam, and these exact modes were also used to reduce the model. We now have three types of models. Two use a reduced model based on vibration modes (one with finite-element modes and one with continuum modes).

The third is the full finite-element model. The order of the model for the first two types is equal to the number of modes, while for the third it is equal to the number of finite-element degrees of freedom.

The errors in the derivative of the fourth damping ratio for all three types of model are given in Figure 3 as a function of the order of the model. The top solid line is the error in the derivative calculated using the continuum modes. The dashed lines are the errors in the reduced-model derivatives for modes obtained from the four finite-element models, and the bottom solid line is the error in the finite-element derivative.

From Figure 3 it is clear that the convergence of the continuum, reduced-model derivative is much slower than the finite-element derivative. For the same order, the finite-element derivative is two to four orders of magnitude more accurate. It is obvious that for this example, the approach of using vibration modes to obtain a reduced model is counter productive in that it requires a higher order model for the same accuracy.

As expected, finite-element, reduced-order results are close to the continuum results when the number of modes is small compared to the number of finite-element degrees of freedom. As the number of modes approaches the number of finite-element degrees of freedom, the reduced-order results approach the full-finite-element results. This produces the paradoxical result that for a fixed number of modes, the accuracy of the derivative deteriorates as the finite-element model is refined. For example, with 48 modes, the 30-element (56 dof) has an error of 60%, the 45-element model (86 dof) has an error of 158%, and the 60-element model (116 dof) has an error of 177%.

The solution to this convergence problem is to include in the reduced basis modes that can model the discontinuity associated with the actuator action. One possibility is to include the static response vectors due to a unit force by each actuator. These vectors are referred to as Ritz vectors.

Two Ritz vectors corresponding to the static response to a unit load at the translation-rate actuator and a unit torque at the rotation-rate actuator were added to the exact eigenvectors to form the reduced model. Thus, for a six-mode model, the two Ritz vectors and the first four vibration modes were used to form the reduced model. The calculations for the reduced-model derivatives were the same as before except that the reduced mass and stiffness matrices were no longer diagonal.

Figure 4 shows the effect of the additional Ritz vectors on the convergence of the fourth damping ratio and its derivative. It can be seen that the convergence is greatly improved with the Ritz vectors for both the damping ratio and derivative. Without the Ritz vectors, there is a 34% error in the derivative with 24 out of 26 modes used. However, with the Ritz vectors included, the derivative converges to within 10% error with only 8 modes.

These results were obtained with a very simple control system. However, the same problems of modal truncation for point actuators were demonstrated for LQG control (Sandridge and Haftka[8]). Also the use of Ritz vector was shown to alleviate such problems for LQG control.

Integrated Design Formulations

The optimization of structural parameters for a space structure can be formulated as

$$\begin{aligned} &\text{minimize} \quad m(v) \\ &\text{such that} \quad g_j(v) \geq 0 \quad j{=}1,\dots,n_g \end{aligned} \qquad [22]$$

where v is a vector of structural sizes and m is the structural objective function—typically the mass. The constraint function $g_j(v)$ represents limits on stresses, displacements, vibration frequencies, buckling loads, etc.

When we consider the integrated design of the structure and control system many formulations are possible. It is possible to design first the structure, and then permit small changes in structural parameter so as to optimize the control objective function (e.g. Khot et al.,[9,10] Venkayya and Tischler[11]).

Similarly, it is possible (e.g. Hanks and Skelton[12]) to use the design of the control system to infer about required structural changes, because some parts of the gain matrix are equivalent to modifications in the structural stiffness matrix.

It is also possible to treat the structural variables as primary, and assume that for a given structure we use an optimal LQR or LQG control system. Khot and Grandhi[13–16] used this approach with the weight or Frobenius norm of the gains as objective functions with limits on closed-loop eigenvalues.

More truly integrated structure-control design procedures operate simultaneously on a set of structural and control variables to optimize a combined structure-control problem. The mass of the structure can be combined with the quadratic performance index J (Eq. (11)) as

$$m^* = \theta m + (1-\theta)J \qquad 0 \leq \theta \leq 1 \qquad [23]$$

to obtain a composite objective m^*. The weighting parameter θ may be chosen so as to emphasize the importance of m or J. This type of an approach was employed by Salama et al.[17,18] Hale et al.[19] and Miller and Shim.[20] Alternatively the quadratic performance index can be optimized for a given structural mass (Messac et al.[21]), or the structural objective minimized for a given J (Dracopoulos and Öz[22]). Similar formulations are possible with control design procedures which do not work with the quadratic performance measure but with other performance measures such as stability margins (Bodden and Junkins,[23] Haftka et al.[1]) robustness measures (Bendsoe et al.,[24] Junkins and Rew,[5] Lim and Junkins[6]), or actuator force magnitudes (Thomas and Schmit[25]).

Instead of combining the structure and control objectives as in Eq. (23), or relegate one of them to a constraint it is possible to approach the problem in the context of multi-objective optimization (e.g. Rao[26]) or game theory (Rao et al.[27]). When multi-objective optimization is used we try to optimize all objectives. For example, if our objectives are the structural mass and quadratic performance index the problem may be formulated as

$$\begin{aligned} &\text{minimize} && m(v),\ J(G,v) && \\ &\text{such that} && g_j(v) \geq 0 && j=1,...,n_g \\ & && h_i(v,G) \geq 0 && i=1,...,n_h \end{aligned} \qquad [24]$$

where $h_i(v,G)$ represent a set of constraints on the closed-loop system such as stability margins and robustness requirements. Multi-objective optimization procedures seek, so called, efficient designs. These are designs where one objective cannot be improved without a loss in the other objective. Under some mild conditions all the efficient designs can be obtained by minimizing m^* of Eq. (23) subject to the constraints of Eq. (24) for all values of θ. Homotopy continuation methods can be used to generate efficiently the optima for the entire range of θ (e.g. Junkins and Rew,[5] Milman et al.[28]). This weighting approach can be extended to more than two objectives, but generating all the efficient solutions becomes much more expensive.

The multi-objective formulation, Eq. (24), has two shortcomings. First, the effect of the control system on the mass and structural constraints is neglected. This effect can be significant when the magnitude of the control effort determines the mass of the control system and its power supply. Second, the Q and R matrices used in the quadratic performance index are somewhat arbitrary and are typically tuned by the control designer. It is therefore unreasonable to select them a priori and leave them unchanged in the design procedure.

A formulation that attempted to address these two concerns was suggested by Onoda and Haftka.[29] This formulation is summarized in the next section.

Onoda's Formulation

We assume that the mass of the control system m_c is related to the control effort as

$$m_c = \alpha C_E^{\beta} \qquad [25]$$

where α and β are constants and C_E is the quadratic measure of the control effort

$$C_E = E(u^T R u) \qquad [26]$$

The optimization problem is to minimize the total mass of the system consisting of the structural mass m_s and control mass m_c subject to a constraint on the magnitude of the response, as measured by the quadratic measure

$$r = E(x^T Q x) \qquad [27]$$

The optimization problem is then formulated as

$$\begin{aligned} &\text{find } v \text{ and } G \text{ to minimize} && m = m_s(v) + m_c(v,G) \\ &\text{such that} && r(v,G) \leq \sigma_1 \end{aligned} \qquad [28]$$

where σ_1 is a response magnitude allowable. In this formulation it may seem that the matrices Q and R are specified, but an alternate formulation of the same problem reveals that the control system is designed with some flexibility of Q and R. The alternative formulation is

$$\begin{aligned} &\text{find } v \text{ and } k \text{ to minimize} && m = m_s(v) + m_c(v,G) \\ &\text{such that} && r(v,G) \leq \sigma_1 \end{aligned} \qquad [29]$$

where G is found by minimizing

$$J^* = E[x^T Q x + u^T (kR) u] = r + kC_E \qquad [30]$$

It can be shown that the two formulations are equivalent. The variable k plays the role of a scaling factor that permits just enough control effort to meet the response limit. This formulation also has the advantage that the gains G can be obtained by a standard Riccati equation solver that minimizes J^*. This is a two-level optimization solution where the control system is reoptimized for each structural design.

It is possible to add to the problem additional structural constraints which do not depend on G. However, the addition of structural constraints which depend on G (e.g. via the mass of the control system) or of stability margins would preclude the use of the Riccati equation to obtain the gains G.

Reference 29 presented a free-free beam (see Fig. 5) example under Gaussian white-noise disturbance. The beam supports a uniform payload and is controlled by two altitude control units with torque actuators symmetrically installed on the structure. The beam was modeled by finite elements and its cross-sectional thickness as well as the actuator location used as structural design variables. Five elements were used to model one half of the beam. In the results presented here it is assumed that the control mass m_c is proportional to the control effort C_E (i.e., $\beta = 1$), that the beam thickness is proportional to the thickness, and that the payload is equal to the structural mass. It is assumed that the entire vector x is sensed so that there is no need for a Kalman filter.

Table 1 compares four cases with varying response requirements (σ_1) and control cost (α). As can be seen from the table, the average thickness (proportional to the structual mass m_s) is very sensitive to the mass cost of the control. When that cost is reduced by a factor of 10 the structural mass is reduced by a factor of 2 to 3. Also, both control effort and mass are sensitive to the required response limit σ_1. It thus appears to be quite important to do the combined optimization. Another demonstration of this importance is shown in Figure 5 which shows the total mass as a function of the structural mass for a uniform beam. The figure indicates that even with an optimal control system the control effort is sensitive to the structural stiffness. If fact if the structure is made too flexible the total mass becomes very high.

Onoda and Watanabe[30] extended this approach to the case where the entire state vector x is not sensed, so that there is a need for an observer. Padula et al.[31] extended Onoda's formulation in that the control mass was calculated based on the maximum required power rather than the somewhat artificial control effort of Eq. (26). Also, instead of using Eqs. (29) and (30) to coordinate the structural and control optimization, these were coordinated by a two-level optimization process.

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References

1. Haftka, R. T., Martinovic, Z. N., and Hallauer, W. L., "Enhanced Vibration Controllability by Minor Structural Modification," AIAA Journal, Vol. 23, No. 8, 1985, pp. 1260–1266.

2. Sandridge, C. A. and Haftka, R. T., "Accuracy of Derivatives of Control Performance Using a Reduced Structural Model," Proceedings AIAA/ASME/ASCE/AHS 28th Structures, Structural Dynamics and Materials Conference and AIAA Dynamics Specialists Meeting, Monterey, CA, April 1987, Part 2B, pp. 622–628.

3. Gilbert, M. G., "Results of an Integrated Structure/ Control Law Design Sensitivity Analysis," in Recent Advances in Multidisciplinary Analysis and Optimization Proceedings of a Symposium held in Hampton, Virginia, September 28–30, 1988 (J-F.M. Barthelemy, editor), NASA CP-3031, Part 2, pp. 727–746, 1988.

4. Khot, N. S., Öz, H., Grandhi, R. V., and Venkayya, V. B., "Optimal Structural Design with Control Gain Norm Constraint," AIAA Journal, Vol. 26, No. 5, 1988, pp. 604–611.

5. Junkins, J. L. and Rew, D. W., "Unified Optimization of Structures and Controllers," in Large Space Structures: Dynamics and Control (S. N. Atluri and A. K. Amos, editors), Springer-Verlag, 1987.

6. Lim, K. B. and Junkins, J. L., "Robust Optimization of Structural and Controller Parameters," J. Guidance, Vol. 12, January 1989, pp. 89–96.

7. Skelton, R. E., Hughes, P. C., and Hablani, H. B., "Order Reduction for Models of Space Structures Wings Modal Cost Analysis," J. Guidance, 5, pp. 351–357, 1982.

8. Sandridge, C. A. and Haftka, R. T., "Effect of Modal Truncation on Derivatives of Closed-Loop Damping Ratios in Optimal Structural Control," in Computer Aided Optimum Design of Structures: Applications (C. A. Brebbia and S. Hernandez, editors), Computational Mechanics Publications, Springer-Verlag, Berlin, Heidelberg, pp. 159–168, 1989.

9. Khot, N. S., Venkayya, V. B., and Eastep, F. E., "Structural Modifications to Reduce the LOS-Error in Large Space Structures," AIAA Paper 84-0997-CP, proceedings of the AIAA/ASME/ASCE/AHS 25th, Structures, Structural Dynamics and Material Conference, Palm Springs, CA, May 1984, Part 2, pp. 296–305.

10. Khot, N. S., Venkayya, V. B., and Eastep, F. E., "Optimal Structural Modifications to Enhance the Active Vibration Control of Flexible Structures," AIAA Journal, Vol. 24, No. 8, 1986, pp. 1368–1374.

11. Venkayya, V. B. and Tischler, V. A., "Frequency Control and Its Effects on the Dynamic Response of Flexible Structures," AIAA Journal, Vol. 22, No. 9, pp. 1293–1298, 1984.

12. Hanks, B. R. and Skelton, R. E., "Designing Structures for Reduced Response by Modern Control Theory," Paper presented at the AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Material Conference, Lake Tahoe, NV, May 1983.

13. Khot, N. S., "Minimum Weight and Optimal Control Design of Space Structures," in Computer Aided Optimal Design: Structural and Mechanical Systems (C. A. Mota Soares, editor), Springer-Verlag, 1987, pp. 389–403.

14. Khot, N. S., "Structures/Control Optimization to Improve the Dynamic Response of Space Structures," Computational Mechanics, 3, pp. 179–186, 1988.

15. Grandhi, R. V., "Structure and Control Optimization of Space Structures," Computers and Structures, 31, 2, pp. 139–150, 1989.

16. Khot, N. S., and Grandhi, R. V., "Structural and Control Optimization with Weight and Frobenius Norm as Performance Functions," in Structural Optimization (G. I. N. Rozvany and B. L. Karihaloo, eds.), Kluwer Academic Publishers, pp. 151–158, 1988.

17. Salama, M., Hamidi, M., and Demestz, L., "Optimization of Controlled Structures," Proceedings of the JPL Workshop on Identification and Control of Flexible Space Structures, JPL Publication 85-29, Vol. II, April 1985, pp. 311-327.

18. Salama, M., Garba, J., and Demesetz, L., "Simultaneous Optimization of Controlled Structures," Computational Mechanics, 3, pp. 275–282, 1988.

19. Hale, A. L., Lisowski, R. J., and Dahl, W. G., "Optimum Simultaneous Structural and Control Design of Maneuvering Flexible Spacecraft," Journal of Guidance Dynamics and Control, Vol. 8, No. 1, 1985, pp. 86–93.

20. Miller, D. F. and Shim, J., "Gradient-Based Combined Structural and Control Optimization," J. Guidance, Vol. 10, No. 3, 1987, pp. 291–298.

21. Messac, A., Turner, J., and Soosaar, K., "An Integrated Control and Minimum Mass Structural Optimization Algorithm for Large Space Structures," Proceedings of the JPL Workshop on Identification and Control of Flexible Space Structures, JPL Publication 85-29, Vol. II, April 1985, pp. 231–236.

22. Dracopoulos, T. N. and Öz, H., "Integrated Aeroelastic Control Optimization," paper presented at 7th VPI&SU/AIAA Symposium on Dynamics and Control of Large Structures, Blacksburg, Virginia, May 1989.

23. Bodden, D. S. and Junkins, J. L., "Eigenvalue Optimization Algorithms for Structure/Controller Design Interactions," Journal of Guidance Dynamics and Control, Vol. 8, No. 6, 1985, pp. 697–706.

24. Bendse, M. P., Olhoff, N., and Taylor, J. E., "On the Design of Structure and Control for Optimal Performance of Actively Controlled Flexible Structures," Mech. Struct. & Mach. 15 (3), pp. 265–295, 1987.

25. Thomas, H. L. and Schmit, L. A., Jr., "Control Augmented Structural Synthesis with Dynamic Stability Constraints," AIAA Paper 89-1216-CP, Proceedings AIAA/ASME/ASCE/AHS/ASC 30th Structures, Structural Dynamics and Materials Conference, Mobile, AL, April 3-5, 1989, Part I, pp. 521–531.

26. Rao, S. S., "Combined Structural and Control Optimization of Flexible Structures," Engineering Optimization, Vol. 13, 1988, pp. 1–16.

27. Rao, S. S., Venkayya, V. B., and Khot, N. S., "Game Theory Approach for the Integrated Design of Structures and Controls," AIAA Journal, Vol. 26, No. 4, 1988, pp. 463–469.

28. Milman, M., Scheid, R. E., and Salama, M., "Methods for Combined Control-Structure Optimization," Paper presented at the VPI&SU 7th Symposium on Dynamics and Control of Large Structures," Blacksburg, VA, May 8–10, 1989.

29. Onoda, J. and Haftka, R. T., "An Approach to Simultaneous Structure-Control Optimization of Large Flexible Spacecraft," AIAA Journal, Vol. 25, No. 8, 1987, pp. 1133–1138.

30. Onoda, J. and Watanabe, N., "Integrated Direct Optimization of Structure/Regulator/Observer for Large Flexible Spacecraft," AIAA Paper 89-1313CP, Proceedings AIAA/ASME/ASCE/AHS/ASC 30th Structures, Structural Dynamics and Materials Conference, Mobile, AL, April 3–5, 1989, Part 3, pp. 1336–1344.

31. Padula, S. L., Sandridge, C. A., Walsh, J. L., and Haftka, R. T., "Integrated Controls-Structures Optimization of a Large Space Structure," AIAA Paper 90-1058, Presented at the AIAA/ASME/ASCE/AHS/ASC 31st Structures, Structural Dynamics and Materials Conference, Long Beach, CA, April 2–4, 1990.

Table 1

Effect of control cost and response limit on combined optimum design for controlled beam

response limit	σ_1	0.01	0.001	0.01	0.001
control cost	α	1.0	1.0	0.1	0.1
normalized	$\xi 1$	0.262	0.822	0.120	0.342
beam	$\xi 2$	0.620	1.785	0.184	0.506
thickness	$\xi 3$	0.811	2.23	0.156	0.408
	$\xi 4$	0.639	1.14	0.159	0.430
	$\xi 5$	0.114	0.713	0.242	1.27
actuator location	$\bar{x}_c$	0.786	0.622	0.386	0.391
average thickness	$\bar{\xi}$	0.489	1.338	0.172	0.591
total normalized mass	m_T	1.063	2.179	0.268	0.603

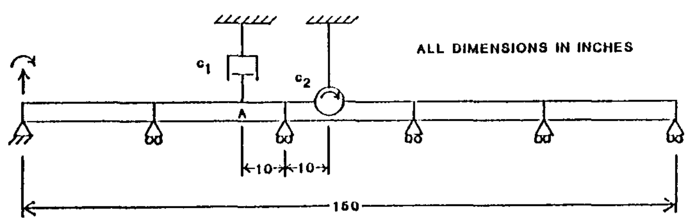

Figure 1: Multispan beam geometry.

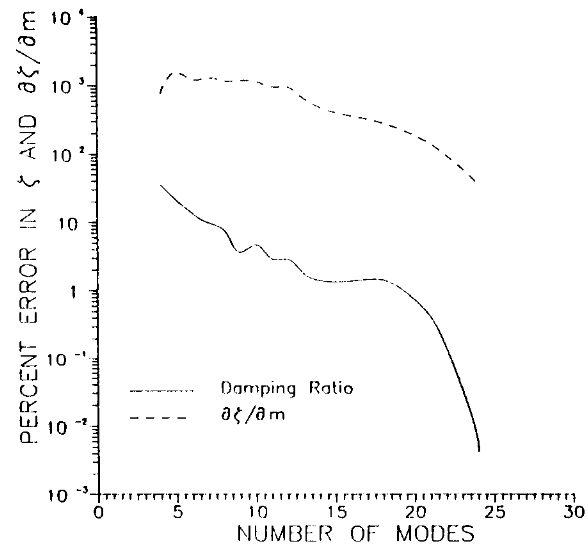

Figure 2: Error in fourth damping ratio and its derivative versus order of reduced model.

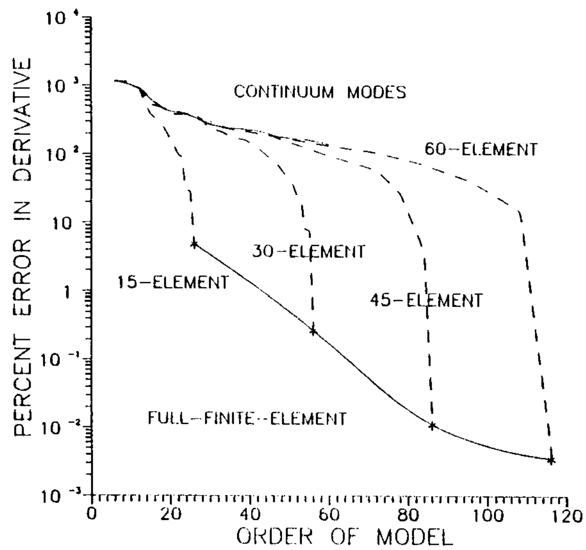

Figure 3: Errors in fourth-mode derivatives, reduced models, finite-element models and continuum model.

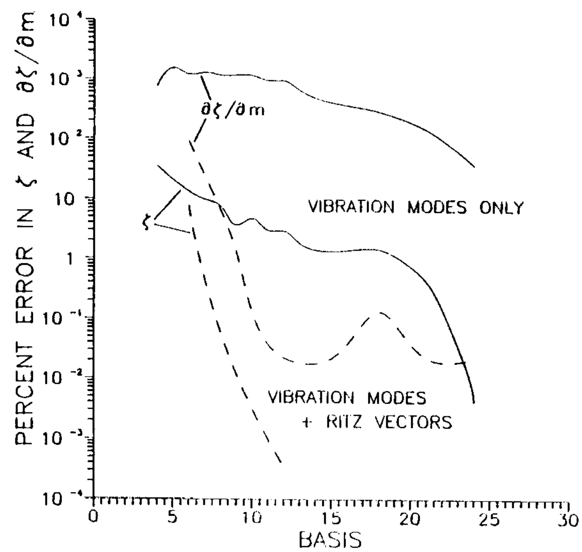

Figure 4: Effect of ritz vectors on convergence of fourth damping ratio and its derivative.

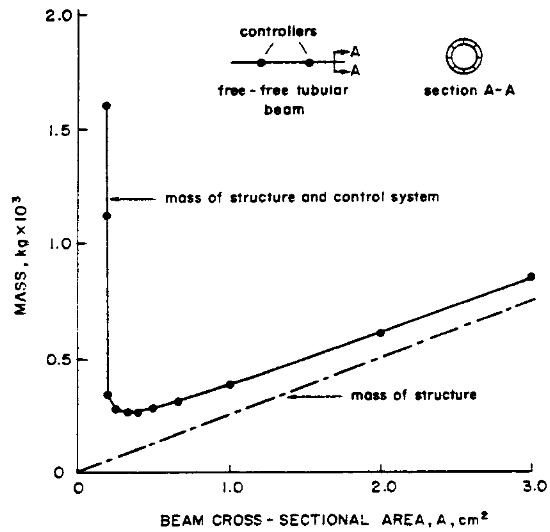

Figure 5: Effect of structural mass on total mass for uniform thickness beam.