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The design of actively controlled structures is considered with constraints on the damping parameters of the closed loop system. The cross sectional areas of the members are treated as design variables. The purpose of control is to effectively suppress structural vibrations due to initial excitations. Starting with a baseline structural design, the objective of the integrated structural/control optimization is to produce a light weight structure with reduced vibrational and control energy content (when excited). The structural weight and the controlled system energy are considered as objective functions for minimization. The effect of using different control weighting matrices, constraints, and objective functions in optimization is illustrated through numerical simulations using two truss structures.

KEYWORDS: Structures, design, control, dynamics, active damping.

INTRODUCTION

The design of efficient structural systems is of fundamental interest to both structural and control engineers. Systematic methodologies for both structural and active control system synthesis are receiving increased application. However, these design techniques, for the most part, have been applied independently within the overall design process. Usually, the structure is designed subject to prescribed strength and stiffness requirements with the active control system being subsequently designed.

The design challenge presented by large flexible space structures has generated interest in interdisciplinary approaches to the design problem. Because of the requirement of low weight, such structures will lack the stiffness and damping necessary for the passive control of vibrations. Hence, current research is directed toward the design of active vibration control systems for such structures. The objective of vibration control is to optimally design the structure and its controls either to eliminate vibration completely or to reduce the mean square response of the system to a desired level within a reasonable span of time.

A great deal of research is currently in progress on designing active vibration control systems for large flexible space structures[1–3]. In Refs [4-6], an optimal structure is initially designed to satisfy constraints on weight, strength, displacements and frequency distribution, and then an optimum control system is designed to improve the dynamic response of the structure. In Refs [7-9], a simultaneous integrated design of the structure and vibration control system is achieved by improving the configuration as well as the control system. A unified approach to achieve satisfaction of eigenspace constraints is presented in Ref. [10]. The modeling, dynamics and control issues are discussed in Refs [11-13]. The weight of the structure is minimized with constraints on the distribution of the eigenvalues and/or damping parameters of the closed-loop system by Khot, Eastep and Venkayya[14]. Salama[8] and Miller and Shim[15] considered the simultaneous minimization, in structural and control variables, of the sum of structual weight and the infinite horizon linear regulator quadratic control cost. The frequency control, the effect on the dynamic response of flexible structures and the associated computational issues were discussed by Venkayya and his associates[5,16]. The structure/control system optimization problem was formulated by Khot *et al.*[17] with constraints on the closed-loop eigenvalue distribution and the minimum Frobenius norm of the control gains. The effect of minor structural modifications in enhancing the vibration controllability of flexible structures was discussed in Ref. [18].

In this work the problem of integrated structural/control design is formulated as an optimization problem and the following studies are conducted:

1) Effect of using different objective functions for minimization.

2) Effect of constraining the smallest damping ratio of the system.

3) Effect of minimizing the structural mass with upper bounds on the other objective functions.

4) Effect of using different control weighting matrices.

The designs of two actively controlled truss structures are considered for the numerical study.

EQUATIONS OF MOTION

The equations of motion of a large space structure with active controls under external forces are given by

$$[M]\ddot{\mathbf{U}} + [C]\dot{\mathbf{U}} + [K]\mathbf{U} = [D]\mathbf{F} \tag{1}$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix and $[K]$ is the stiffness matrix. These matrices are of order r where r denotes the number of degrees of freedom of the structure. $\mathbf{U}$ represents the vector of displacements and a dot over a symbol denotes differentiation with respect to time. $[D]$ is the $r \times p$ matrix denoting the applied load distribution that relates the control input vector $\mathbf{F}$ to the coordinate system. The number of components in $\mathbf{F}$ is assumed to be p. By introducing the coordinate transformation

$$\mathbf{U} = [\phi]\boldsymbol{\eta} \tag{2}$$

where $[\phi]$ is the $r \times r$ modal matrix whose columns are the eigenvectors and $\boldsymbol{\eta}$ is the vector of modal coordinates, Eq. (1) can be transformed into a system of uncoupled differential equation as

$$[\bar{M}]\ddot{\boldsymbol{\eta}} + [\bar{C}]\dot{\boldsymbol{\eta}} + [\bar{K}]\boldsymbol{\eta} = [\boldsymbol{\phi}]^T[D]\mathbf{F} \tag{3}$$

where

$$[\bar{M}] = [I] \tag{4}$$

$$[\bar{C}] = [2\zeta\omega] \tag{5}$$

$$[\bar{K}] = [\omega^2] \tag{6}$$

$\boldsymbol{\zeta}$ is the vector of modal damping factors and $\boldsymbol{\omega}$ is the vector of natural frequencies of the structure. Equations (3) can be converted into a state space representation by using the transformation

$$\mathbf{x} = \begin{Bmatrix} \boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}} \end{Bmatrix} \tag{7}$$

where $\mathbf{x}$ is the $n \times 1$ state variable vector with $n = 2r$. This gives the state input equation

$$\dot{\mathbf{x}} = [A]\mathbf{x} + [B]\mathbf{F} \tag{8}$$

where $[A]$ is the $n \times n$ plant matrix and $[B]$ is the $n \times p$ input matrix given by

$$[A] = \begin{bmatrix} [O] & [I] \\ [-\omega^2] & [-2\zeta\omega] \end{bmatrix} \tag{9}$$

and

$$[B] = \begin{bmatrix} [O] \\ [\phi]^T[D] \end{bmatrix} \tag{10}$$

The state output equation is given by

$$\mathbf{y} = [\underset{\sim}{C}]\mathbf{x} \tag{11}$$

where y is $q \times 1$ output vector and $[\underset{\sim}{C}]$ is the $q \times n$ output matrix. If the sensors and actuators are co-located, then $q = p$ and

$$[\underset{\sim}{C}] = [B]^T \tag{12}$$

In order to design a controller using a linear quadratic regulator, a performance index J is defined as

$$J = \int_0^t (\mathbf{x}^T[Q]\mathbf{x} + \mathbf{F}^T[R]\mathbf{F})\,\mathrm{d}t \tag{13}$$

where $[Q]$ is the state weighting matrix which has to be positive semidefinite and $[R]$ is the control weighting matrix which has to be positive definite. It is possible to control the damping response time, amplitudes of vibration, etc., of the system by proper selection of the elements of the matrices $[Q]$ and $[R]$. If $[Q]$ and $[R]$ are chosen as

$$[Q] = \begin{bmatrix} [K] & [O] \\ [O] & [M] \end{bmatrix} \tag{14}$$

and

$$[R] = [D]^T[K]^{-1}[D] \tag{15}$$

then Eq. (13) provides a measure of total system strain, kinetic and potential energies.

The result of minimizing the quadratic performance index and satisfying the state equation gives the state feedback control law[19]

$$\mathbf{F} = -[G]\mathbf{x} \tag{16}$$

where $[G]$ is the optimum gain matrix given by

$$[G] = [R]^{-1}[B]^T[P] \tag{17}$$

with $[P]$ representing a symmetric positive definite matrix called the Riccati matrix and is found by solving the following algebraic Riccati equation

$$[A]^T[P] + [P][A] + [Q] - [P][B][R]^{-1}[B]^T[P] = [O] \tag{18}$$

Substituting Eq. (16) into Eq. (8) gives the governing equations for the optimal closed-loop system in the form

$$\dot{\mathbf{x}} = [A_{cl}]\mathbf{x} \tag{19}$$

where

$$[A_{cl}] = [A] - [B][G] \tag{20}$$

The eigenvalues of the closed-loop matrix, $[A_{cl}]$, are a set of complex conjugate pairs written as

$$\lambda_i = \bar{\sigma}_i \pm j\bar{\omega}_i; \qquad i = 1, 2, \ldots, n \tag{21}$$

where $j = \sqrt{-1}$. The damping factors ξ_i and the damped frequencies $\bar{\omega}_i$ are related to the complex eigenvalues through

$$\xi_i = -\frac{\bar{\sigma}_i}{\sqrt{\bar{\sigma}_i^2 + \bar{\omega}_i^2}} \tag{22}$$

OPTIMIZATION PROBLEM

The cross-sectional areas of the members of the structure are chosen as the design variables. The weight of the structure, f_1, is given by

$$f_1(\mathbf{z}) = \sum \rho_i A_i l_i \tag{23}$$

where ρ_i is the weight density, A_i is the cross sectional area, l_i is the length of element i, and $\mathbf{z}$ is the design vector. For any specified initial condition $\mathbf{x}_0$, it is well known that

$$\min_{\mathbf{F}} \frac{1}{2}\int_0^\infty (\mathbf{x}^T[Q]\mathbf{x} + \mathbf{F}^T[R]\mathbf{F})\,dt = \frac{1}{2}\mathbf{x}_0^T[P]\mathbf{x}_0 \tag{24}$$

and the corresponding optimal control is given by Eq. (16). The minimum value of the quadratic performance index is taken as the second objective function, $f_2(\mathbf{z})$, as

$$f_2(\mathbf{z}) = \mathbf{x}_0^T[P]\mathbf{x}_0 \tag{25}$$

Other possible objective functions are the Frobenius norm and the effective damping response time. The Frobenius norm of the control gains, $f_3(\mathbf{z})$, is given by

$$f_3(\mathbf{z}) = \mathrm{trace}\{[G]^T[R][G]\} \tag{26}$$

Essentially, the Frobenius norm of the control gains represents the expected value of the integrand of the quadratic control effort. By minimizing the expected value of the integrand, one can hope to minimize the required control effort. The effective damping response time, $f_4(\mathbf{z})$, under the action of an initial disturbance $\mathbf{x}_0 = \mathbf{x}(t = 0)$ can be expressed as

$$f_4(\mathbf{x}) = \frac{\mathbf{x}_0^T[P]\mathbf{x}_0}{\mathbf{x}_0^T[Q]\mathbf{x}_0} \tag{27}$$

The magnitude of f_4 indicates the effect of the control system in reducing vibrations.

Constraints are placed on the closed loop damping ratios as

$$\xi_i - \xi_i^{(0)} = 0 \tag{28}$$

or as

$$\xi_i - \xi_i^{(0)} \geq 0 \tag{29}$$

where $\xi_i^{(0)}$ is a specified value. Bounds are placed on the cross-sectional areas of members as

$$A_i^{(l)} \leq A_i \leq A_i^{(u)} \tag{30}$$

where the superscripts l and u indicate the lower and upper bound values.

The design problem was formulated as a standard multiobjective optimization problem. The goal programming and game theory techniques were used for solving the problem. These techniques transform the problem into an equivalent single objective optimization problem. The subroutine VMCON, which is based on Powell's algorithm for nonlinear constraints that uses Lagrangean functions[20], has been used for solving the single objective optimization problem.

ILLUSTRATIVE EXAMPLES

The multiobjective optimization techniques outlined above were applied to two truss structures. The dimensions of the structure were defined in unspecified consistent units. The elastic modulus of the members was assumed to be 1.0 and the density of the structural material was assumed to be 0.001.

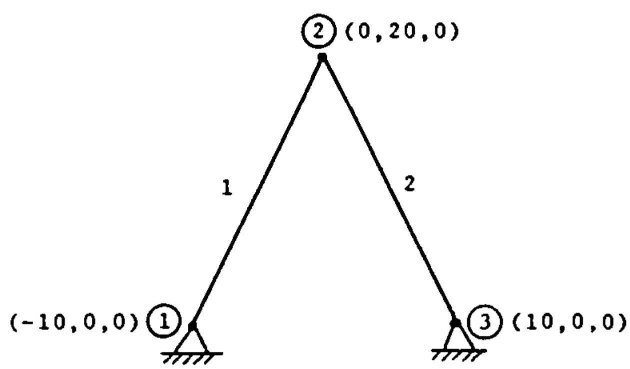

Figure 1 Two bar truss.

Example 1 2-bar Truss

The design of the two bar truss shown in Figure 1 is considered with a nonstructural mass of 2 units attached at node 2. The sensor and the actuator are located in element 1 connecting nodes 1 and 2. The lower and upper bounds on the design variables (cross sectional areas of the members) are assumed to be $z_i^{(l)} = 10$ and $z_i^{(u)} = 2000$. The objective functions are redefined as F_i such that $F_i = c_i f_i = 100$ at $\mathbf{z}_0$, $i = 1, 2, 3, 4$. The lowest damping ratio of the closed loop system is constrained either as $\zeta_1 = \zeta_1^{(0)}$ or as $\zeta_1 \geq \zeta_1^{(0)}$ where the value of $\zeta_1^{(0)}$ is chosen as the value of ζ_1 at the starting design vector, $\mathbf{z}_0$.

When the matrices $[Q]$ and $[R]$ were selected as per Eqs (14) and (15), i.e. when the objective function F_2 is made to represent the energy, the minimizations of F_1 and F_3 yielded the same results. Similarly the minimizations of F_2 and F_4 led to the same optimum point as indicated in Table 1. The results obtained by using the inequality constraint $\zeta_1 \geq \zeta_1^{(0)}$ instead of $\zeta_1 = \zeta_1^{(0)}$ are also given in Table 1. It can be seen that when the constraint $\zeta_1 = \zeta_1^{(0)}$ is used, z_2 attained its lower bound value when F_1 and F_3 are minimized while z_1 reached its maximum possible value when F_2 and F_4 are minimized. When the inequality constraint $\zeta_1 \geq \zeta_1^{(0)}$ is used, both the design variables attained their lower (upper) bound values when F_1 and F_3 (F_2 and F_4) are optimized.

When the matrices $[Q]$ and $[R]$ were chosen as identity matrices, the minimization of different F_i resulted in different solutions when the constraint $\zeta_1 > \zeta_1^{(0)}$ is imposed (see Table 2). It can be seen that the minimum possible values of F_1, F_2, F_3 and F_4 are,

Table 1 Results of 2-bar truss optimization, $[Q]$ and $[R]$ given by Eqs (14) and (15).

Quantity	Starting point	Optimum point			
		With $\zeta_1 = \zeta_1^{(0)}$		With $\zeta_1 > \zeta_1^{(0)}$	
		Minimization of F_1 or F_3	Minimization of F_2 or F_4	Minimization of F_1 or F_3	Minimization of F_2 or F_4
Design variables					
z_1	1000	100	2000	10	2000
z_2	100	10	200	10	2000
Eigenvalues (open loop)					
ω_1^2	1.17	0.371	1.66	0.299	4.23
ω_2^2	4.82	1.52	6.81	0.598	8.46
Eigenvalues (closed loop)					
λ_1	$-0.112 \pm 1.18j$	$-0.0355 \pm 0.372j$	$-0.159 \pm 1.66j$	$-0.105 \pm 0.315j$	$-0.148 \pm 4.46j$
λ_2	$-3.20 \pm 4.72j$	$-1.01 \pm 1.49j$	$-4.52 \pm 6.67j$	$-0.286 \pm 0.573j$	$-4.04 \pm 8.11j$
Damping ratios					
ζ_1	0.09499	0.09499	0.09499	0.3153	0.3153
ζ_2	0.5608	0.5608	0.5608	0.4462	0.4462
Objective functions					
F_1	100	10.000	200.000	1.818	363.640
F_2	100	3162.300	35.355	92015.000	32.532
F_3	100	47.070	158.810	38.987	585.790
F_4	100	316.230	70.711	920.150	65.065

Table 2 Results of 2-bar truss optimization, $[Q]$ and $[R]$ taken as identity matrices.

Quantity	Starting point	Optimum point with $\xi_1 > \xi_1^{(0)}$			
		Minimization of F_1	Minimization of F_2	Minimization of F_3	Minimization of F_4
Design variables					
z_1	1000	10	2000.000	1023.579	10.000
z_2	100	10	1945.660	525.981	42.497
Eigenvalues (open loop)					
ω_1^2	1.17	0.299	4.20	2.45	0.361
ω_2^2	4.82	0.598	8.41	5.35	1.06
Eigenvalues (closed loop)					
λ_1	$-0.0228 \pm 1.17j$	$-0.139 \pm 0.394j$	$-0.159 \pm 4.20j$	$-0.098 \pm 2.45j$	$-0.492 \pm 0.550j$
λ_2	$-0.361 \pm 4.81j$	$-0.586 \pm 0.635j$	$-0.320 \pm 8.40j$	$-0.347 \pm 5.34j$	$-0.278 \pm 0.970j$
Damping ratios					
ξ_1	0.01944	0.3313	0.03793	0.04008	0.6665
ξ_2	0.07483	0.6777	0.03803	0.06486	0.2753
Objective functions					
F_1	100	1.8182	358.700	140.870	4.772
F_2	100	81595.000	42.211	88.703	60843.000
F_3	100	459.970	82.799	83.747	288.680
F_4	100	8.159	168.840	92.936	6.084

Table 3 Results of 2-bar truss optimization, $[Q]$ and $[R]$ chosen as identity matrices.

Quantity	Starting point	Optimum point with $\xi_1 = \xi_1^{(0)}$			
		Minimization of F_1	Minimization of F_2	Minimization of F_3	Minimization of F_4
Design variables					
z_1	1000	657.233	2000.000	2000.000	758.835
z_2	100	10.000	533.881	533.881	33.493
Eigenvalues (open loop)					
ω_1^2	1.17	0.377	2.62	2.62	0.687
ω_2^2	4.82	3.84	7.06	7.06	4.15
Eigenvalues (closed loop)					
λ_1	$-0.0228 \pm 1.17j$	$-0.00734 \pm 0.377j$	$-0.0509 \pm 2.62j$	$-0.0509 \pm 2.62j$	$-0.0134 \pm 0.687j$
λ_2	$-0.361 \pm 4.81j$	$-0.365 \pm 3.83j$	$-0.354 \pm 7.05j$	$-0.354 \pm 7.05j$	$-0.364 \pm 4.14j$
Damping ratios					
ξ_1	0.01944	0.01944	0.01944	0.01944	0.01946
ξ_2	0.07483	0.09497	0.05013	0.05013	0.08751
Objective functions					
F_1	100	60.658	230.350	230.350	72.030
F_2	100	238.020	45.222	45.222	154.160
F_3	100	327.970	79.058	79.058	150.730
F_4	100	102.810	180.890	180.890	88.772

Table 4 Results of 2-bar truss optimization, with bounds on F_2, F_3, and F_4.

Quantity	Starting point		Optimum point			
			With $\xi_1 = \xi_1^{(0)}$		With $\xi_1 < \xi_1^{(0)}$	
	[Q] and [R] given by Eqs (14) and (15)	[Q] = [R] = [I]	[Q] and [R] given by Eqs (14) and (15)	[Q] = [R] = [I]	[Q] and [R] given by Eqs (14) and (15)	[Q] = [R] = [I]
Design variables						
z_1	1000	1000	250.0	703.757	250.0	143.325
z_2	100	100	25.0	20.279	25.0	84.181
Eigenvalues (open loop)						
ω_1^2	1.17	1.17	0.578	0.536	0.587	0.964
ω_2^2	4.82	4.82	2.41	3.99	2.41	2.04
Eigenvalues (closed loop)						
λ_1	$-0.112 \pm 1.18j$	$-0.02284 \pm 1.17j$	$-0.0561 \pm 0.588j$	$-0.0104 \pm 0.536j$	$-0.0561 \pm 0.588j$	$-0.143 \pm 0.966j$
λ_2	$-3.20 \pm 4.72j$	$-0.361 \pm 4.81j$	$-1.60 \pm 2.36j$	$-0.364 \pm 3.97j$	$-1.60 \pm 2.36j$	$-0.373 \pm 2.02j$
Damping ratios						
ξ_1	0.09499	0.01944	0.09499	0.01944	0.09499	0.1465
ξ_2	0.05608	0.07483	0.5608	0.09135	0.5608	0.1820
Objective functions						
F_1	100	100	25.000	65.821	25.000	20.682
F_2	100	100	800.000	184.700	800.000	800.000
F_3	100	100	55.891	200.000	55.891	125.650
F_4	100	100	200.000	91.476	200.000	16.434

respectively, 1.8182, 42.211, 82.747 and 6.084. The worst (or maximum) values of the objective functions are 358.7, 81595.0, 459.97 and 168.84. The penalties associated with other objective functions while minimizing a particular objective function can be seen from the results given in Table 2. The results obtained with the minimization of different objective functions F_i, when the constraint $\zeta_1 = \zeta_1^{(0)}$ is used in the problem formulation, are shown in Table 3. In this case, the minimum values achieved for the four objective functions are 60.658, 45.222, 79.058 and 88.772. The maximum values of F_1, F_2, F_3, and F_4 are 230.35, 238.02 327.97 and 180.89, respectively.

In the final set of problems, the structural mass is minimized by placing upper bounds on the other three objective functions. The problem is stated as follows:

$$\begin{aligned}
&\text{Minimize} \quad F_1(\mathbf{z})\\
&\text{subject to}\\
&F_i(\mathbf{z}) \leq F_i^{(u)}; \quad i = 2, 3, 4\\
&\zeta_1(\mathbf{z}) = \zeta_1^{(0)} \quad \text{or} \quad \zeta_1(\mathbf{z}) \geq \zeta_1^{(0)}\\
&z_j^{(l)} \leq z_j \leq z_j^{(u)}; \quad j = 1, 2
\end{aligned}$$

The upper bounds on the objective functions are taken as $F_2^{(u)} = 800$, $F_3^{(u)} = F_4^{(u)} = 200$. The solution of the problem obtained with different choices of the constraint on ζ_1 and with different definitions of the matrices $[Q]$ and $[R]$ are given in Table 4. It can be seen that the constraints $\zeta_1 = \zeta_1^{(0)}$ and $\zeta_1 \geq \zeta_1^{(0)}$ yielded the same results when $[Q]$ and $[R]$ are selected according to Eqs (14) and (15). The constraints on F_2 and F_4 have been found to be active at the optimum solution. When the matrices $[Q]$ and $[R]$ are taken as identity matrices, the optimum values of F_1 and F_3 were found to be larger and of F_2 and F_4 to be smaller compared to those obtained when $[Q]$ and $[R]$ were given by Eqs (14) and (15).

Example 2 12-bar Truss

The design of the 12-bar truss shown in Figure 2 is considered. The edges of this tetrahedral truss are 10 units long. The structure has 12 degrees of freedom and 4 masses of 2 units each, which are attached at nodes 1 through 4. The actuators and the sensors are located in six bipods and are assumed to coincide with each other. Thus, the matrix $[D]$ in Eq. (1) would consist of the direction cosines relating the forces in the six bipods with their components in the coordinate directions. Since the sensors and the actuators are co-located, matrices $[B]$ and $[C]$ in Eqs (8) and (11) would satisfy Eq. (12). The weighting matrix $[R]$ for this case would be of order 6×6. The passive damping parameters ζ in Eq. (9) were assumed to be zero.

The optimization problem, in the case of minimization of weight, can be stated as:

$$\begin{aligned}
&\text{Minimize}\, f_1(\mathbf{z}) \text{ subject to}\\
&\zeta_1(\mathbf{z}) \geq \zeta_1^{(0)}\\
&z_i^{(l)} \leq z_i \leq z_i^{(u)}
\end{aligned}$$

The lower bound on the closed loop damping ratio coresponding to the lowest frequency, $\zeta_1^{(0)}$, is taken as 0.25. The values of $z_i^{(l)}$ and $z_i^{(u)}$ are taken as 10 and 2000, respectively, for $i = 1, 2, \ldots, 12$.

At the starting design, the cross-section areas of the members were taken to be the same as those assigned by the Charles Stark Draper Laboratory (ACOSS FOUR

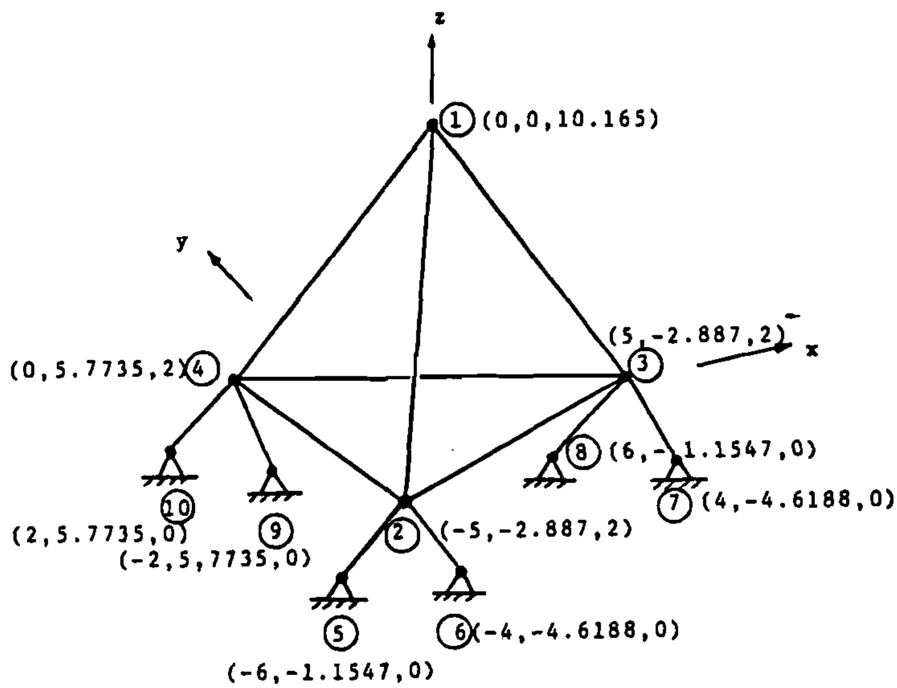

Figure 2 Twelve bar truss.

model) for their investigation[4]. The characteristics of this design, along with those of the designs found in this investigation are shown in Tables 5 and 6. It can be seen that the minimum values of f_1 and f_2 are 34.4735 and 59.3872 respectively. When f_1 is minimized, the values of the other objective functions were $f_2 = 308.2618$, $f_3 = 169.5061$ and $f_4 = 0.4940$. When f_2 is minimized, the values of f_1, f_3 and f_4 were found to be 45.9741, 465.1829 and 0.3463 respectively. These results indicate the penalty associated with the other objective function(s) while minimizing a particular objective function. All the open loop eigenvalues attained lower values when f_1 is minimized compared to those obtained with the minimization of f_2.

The sensitivities of a typical objective function and a typical response quantity with variations in design variables at the optimum solution of f_2 are shown in Figures 3

Table 5 Values of objective functions of 12-bar truss

Quantity	Initial design	Minimization of F_1	Minimization of F_2
f_1	43.6976	34.4735	45.9741
f_2	232.5418	308.2618	59.3872
f_3	346.5160	169.5061	465.1829
f_4	1.4640	0.4940	0.3463

Table 6 Eigenvalues and damping ratios of 12-bar truss

ω_i^2 (open loop), $i = 1, 2, \ldots, 12$			λ_i (closed loop), $i = 1, 2, \ldots, 12$			ξ_i, $i = 1, 2, \ldots, 12$		
Initial design	Minimization of F_1	Minimization of F_2	Initial design	Minimization of F_1	Minimization of F_2	Initial design	Minimization of F_1	Minimization of F_2
1.34	0.494	1.41	$-0.351 \pm 1.36j$	$-0.129 \pm 0.501j$	$-0.376 \pm 1.46j$	0.2503	0.2503	0.2496
1.66	0.751	1.92	$-0.572 \pm 1.71j$	$-0.132 \pm 0.755j$	$-0.800 \pm 1.99j$	0.3146	0.1718	0.3728
2.89	1.01	2.70	$-1.44 \pm 2.93j$	$-0.672 \pm 0.995j$	$-1.12 \pm 2.71j$	0.4412	0.5599	0.3817
2.96	1.37	3.32	$-1.70 \pm 2.96j$	$-0.791 \pm 1.34j$	$-1.12 \pm 3.38j$	0.4967	0.5071	0.3136
3.40	1.97	4.89	$-2.14 \pm 3.37j$	$-1.29 \pm 1.94j$	$-2.98 \pm 4.97j$	0.5359	0.5530	0.5151
4.20	2.01	5.73	$-2.84 \pm 4.11j$	$-1.27 \pm 1.98j$	$-2.34 \pm 5.77j$	0.5685	0.5408	0.3759
4.66	2.65	6.79	$-2.80 \pm 4.56j$	$-1.67 \pm 2.61j$	$-4.03 \pm 6.80j$	0.5233	0.5379	0.5100
4.76	2.92	7.18	$-2.73 \pm 4.61j$	$-1.95 \pm 2.86j$	$-2.02 \pm 6.86j$	0.5093	0.5627	0.2826
8.54	7.41	9.09	$-2.33 \pm 8.48j$	$-1.30 \pm 7.38j$	$-2.02 \pm 9.33j$	0.2650	0.1734	0.2112
9.25	8.15	10.3	$-2.20 \pm 9.19j$	$-1.26 \pm 8.14j$	$-2.19 \pm 10.4j$	0.2329	0.1527	0.2051
10.3	9.27	14.7	$-1.67 \pm 10.3j$	$-0.941 \pm 9.27j$	$-9.33 \pm 14.4j$	0.1602	0.1010	0.5446
12.9	11.8	17.1	-0.581 ± 12.9	$-0.354 \pm 11.8j$	$-8.86 \pm 16.2j$	0.0451	0.0301	0.4801

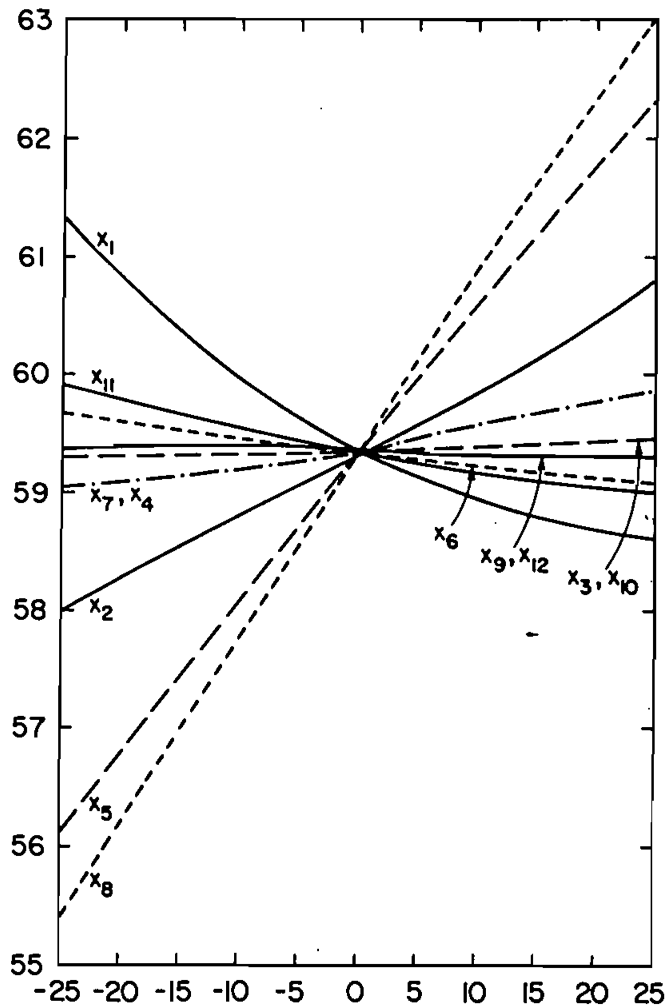

Figure 3 Sensitivity of f_2.

and 4. The sensitivity analysis results can be used in (i) eliminating the less sensitive design variables from the design vector in the design of similar structures, (ii) rounding the theoretical optimum solution to practically feasible values, and (iii) studying the effect of changing the bounds on the response quantities. Figure 3 indicates that the objective function f_2 is most influenced by variations in x_8 and least by changes in x_9 and x_{12}. It is interesting to note that the variable x_1, which has significant influence on the objective function f_2 (and also on f_1, f_3 and f_4), does not have any effect on the response quantity ξ_1. The variations of the objective functions with the progress of optimization (when f_2 is minimized) is shown in Figure 5.

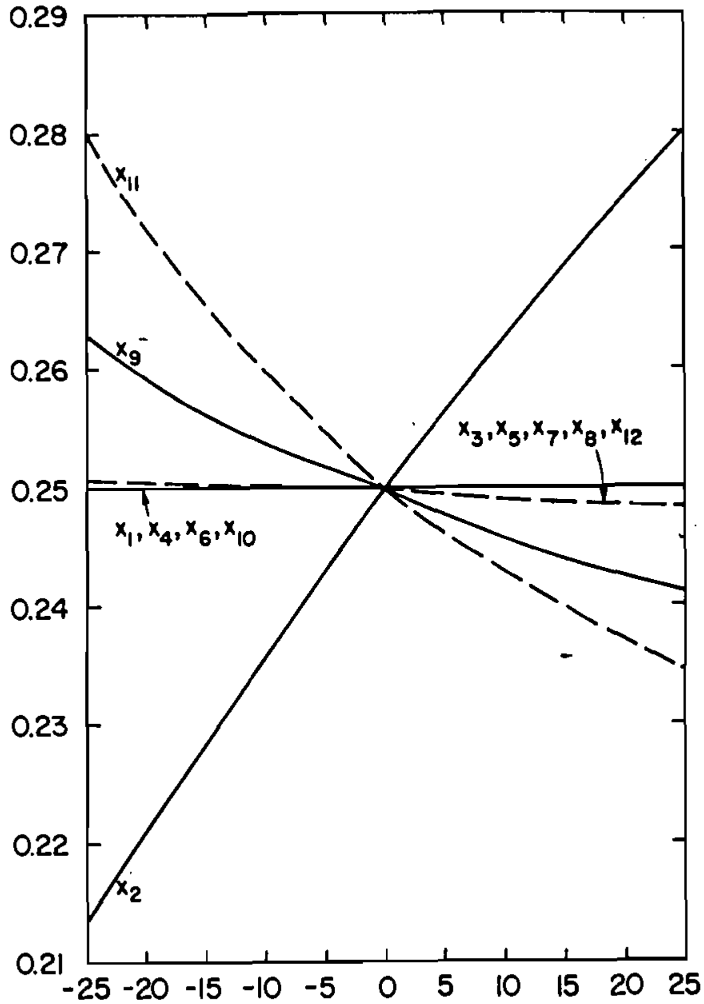

Figure 4 Sensitivity of ξ_1.

CONCLUSION

It was observed that the minimization of structural weight (control energy) yielded the same results as the minimization of the Frobenius norm of the control gains (effective damping response time) when the weighting matrices [Q] and [R] were selected so as to make f_2 represent the control energy. The optimization results obtained with the minimization of different objective functions were found to be different when [Q] and [R] were chosen to be identity matrices. The sensitivity analysis results given in this work are expected to be useful in (a) eliminating the least sensitive design variables in the optimization of similar structures, (b) rounding the theoretically obtained

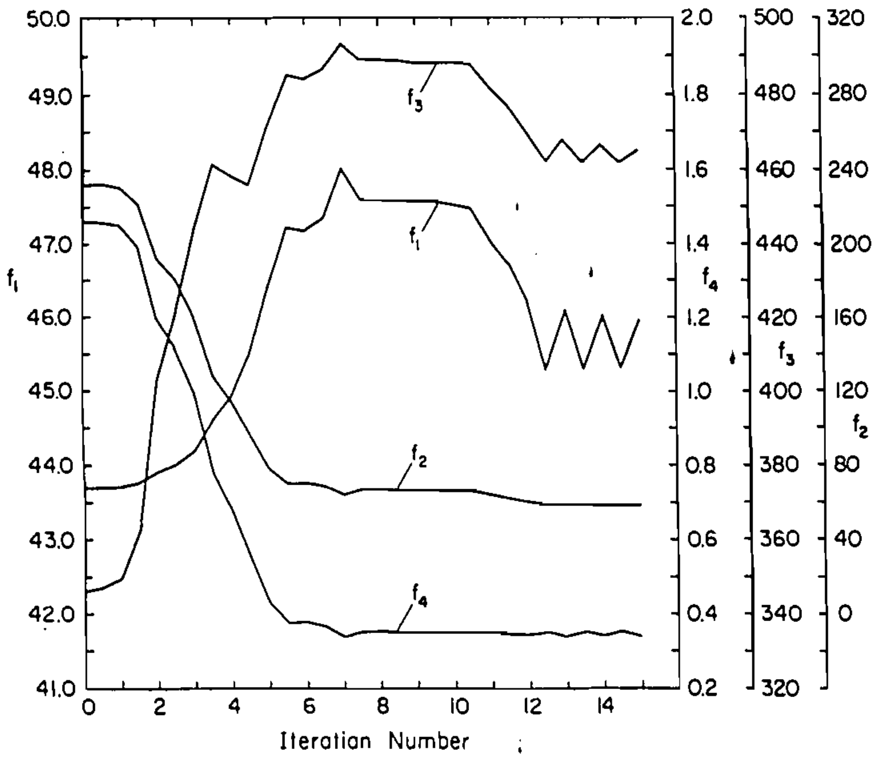

Figure 5 Progress of optimization.

optimum values of the design variables to the nearest practically feasible values, and (c) studying the effect of changing the bounds on the response quantities. In the case of 12-bar truss, x_{12} was found to be the least sensitive design variable.

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