Mathematical Modeling and Analysis of Flutter in Bending-Torsion Coupled Beams, Rotating Blades, and Hard Disk Drives

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Abstract: In this study I present a review of several research directions in the area of mathematical analysis of flutter phenomenon. Flutter is known as a structural dynamical instability that occurs in a solid elastic structure interacting with a flow of gas or fluid, and consists of violent vibrations of the structure with rapidly increasing amplitudes. The focus of this review is a collection of models of fluid-structure interaction, for which precise mathematical formulations are available. The main objects of interest are analytical results on such models, which can be used for flutter explanation, its qualitative and even quantitative treatments. This paper does not pretend to be a comprehensive review of the enormous amount of engineering literature on analytical, computational, and experimental aspects of the flutter problem. I present a brief exposition of the results obtained in several selected papers or groups of papers on the following topics: (1) bending-torsion vibrations of coupled beams; (2) flutter in transmission lines; (3) flutter in rotating blades; (4) flutter in hard disk drives; (5) flutter in suspension bridges; and (6) flutter of blood vessel walls. Finally, I concentrate on the most well-known case of flutter, i.e., flutter in aeroelasticity. The last two sections of this review are devoted to the precise analytical results obtained in my several recent papers on a specific aircraft wing model in a subsonic, inviscid, incompressible airflow.

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Introduction

The present work is the first part of a three-part survey devoted to the flutter phenomenon in engineering and its mathematical modeling and analysis. The object of this study is to present a review of several selected research directions in the area of flutter analysis. The current paper, Part I, contains, in addition to the Introduction, the following four sections: Bending-torsion coupled beams under deterministic and random loads; Flutter of transmission lines; Flutter of helicopter, propeller, and turbine blades; and Flutter in hard disk drives. The second paper (in this issue), Part II, contains two sections: Flutter of long-span suspension bridges and Flutter of tubes conveying fluids—biomedical setting. The last paper of the series (to be published at a later date), Part III, contains three remaining sections. They are "Flutter Problem in Aeroelasticity," "Mathematical Analysis of Coupled Euler-Bernoulli/Timoshenko Beam Model," and "Mathematical Analysis of Aircraft Wing Model in Subsonic Air Flow."

To create a complete picture, I briefly outline the content of every section. Each section has two parts (the only exception is the section concerning flutter in transmission lines, which contains only the descriptional part). The first part of each section contains a more or less detailed description of the model associated to unstable vibrations. My choice of model is based on two facts: First, in the chosen paper, there is a mathematical formulation and at least partial treatment of the problem; secondly, in the aforementioned paper, I present either a new model or a new analytical/computational approach to the solution of the known problem. In the second part of each section, I present a brief description of a number of different papers containing some advances in the area. Only in the section related to flutter of long-span suspension bridges, this order has been reversed.

Recalling that flutter is a physical phenomenon, one that occurs in a solid elastic structure interacting with a flow of gas or fluid. Flutter is a structural dynamical instability, which consists of violent vibrations of the solid structure with rapidly increasing amplitude. It usually results either in serious damage to the structure or in its complete destruction. Flutter occurs when the parameters characterizing fluid-structure interaction reach certain critical values. The physical reason for this phenomenon is that, under special conditions, the energy of the flow is rapidly absorbed by the structure and transformed into the energy of mechanical vibrations.

In engineering practice, flutter must be avoided either by design of the structure or by introducing control mechanisms capable of suppressing harmful vibrations. Flutter is known as an inherent feature of fluid-structure interaction, and thus it cannot be eliminated completely. However, the critical conditions for the flutter onset can be shifted to the safe range of the operating parameters. This is an ultimate goal for designing flutter control mechanisms.

The most well-known cases of flutter are related to the flutter in aircraft wings, tails, and control surfaces. Flutter is an in-flight event, which happens beyond some speed-altitude combinations. However, flutter vibrations occur in a variety of engineering and

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J. Aerosp. Eng. 2004.17:56-69.

even biomedical situations. A representative listing of such situations could be made from the section headings of the entire survey. Namely, in aeronautic engineering, the flutter of helicopter, propeller, and turbine blades is a serious problem. It also affects electric transmission lines, high-speed hard disk drives, and long-spanned suspension bridges. Flutter of cardiac tissue and blood vessel walls is of special concern to medical practitioners.

Flutter is an extremely complex physical phenomenon whose complete theoretical explanation is openly a problem. Its control and suppression is an obvious engineering problem as well. An enormous amount of engineering literature—theoretical, computational, and experimental—is devoted to this issue. I would like to emphasize at this point that it is not an objective of this paper to provide an exhaustive review of all literature on the subject. I have selected for this review several groups of papers related to theoretical analysis of different settings of flutter. So my choice is subjective and does not pretend to be complete. The criteria for selecting material for this review were the following. This review does not focus on computational methods used in flutter analysis. This review does not deal with experimental techniques either. It is not devoted to engineering designs of flutter control mechanisms. The focus of this review is a collection of models of fluid-structure interaction, for which precise mathematical formulations are available. My main objects of interest are analytical results on such models, which can be used for flutter explanation. I believe that analytical treatment of flutter problems is an important component of this area of research. Such treatment can provide insights not available from purely computational or experimental results. It is certainly important for designing flutter control mechanisms.

In ideal, a complete picture of fluid-structure interaction should be described by a system of partial differential equations, a system that contains both the equations governing the vibrations of an elastic structure and the hydrodynamic equations governing the motion of gas or fluid flow. The system of equations of motion should be supplied with appropriate boundary and initial conditions. The structural and hydrodynamic parts of the system must be coupled in the following sense. The hydrodynamic equations define a pressure distribution on the elastic structure. This pressure distribution in turn defines the so-called aerodynamic loads (lift and moment in aeroelastic setting), which appear as forcing terms in the structural equations. On the other hand, the parameters of the elastic structure enter the boundary conditions for the hydrodynamic equations.

The above picture is mathematically very complicated, and to make a particular problem tractable, it is necessary to introduce simplifying assumptions. In my selection of papers for this review, I have chosen the ones containing precisely formulated mathematical models of structures in gas/fluid flows and partial analytical results on solutions of these models. Even though most of the results are not completely rigorous mathematically, they contain important qualitative information. In the aforementioned models (with the exception of biomedical applications), the hydrodynamical parts of equations are missing. Instead, the aerodynamic loads are just introduced as forcing terms in the structural equations and reasonable assumptions about these terms are made.

The last two sections of the entire three-part survey (Mathematical analysis of coupled Euler-Bernoulli/Timoshenko beam model, and Mathematical analysis of aircraft wing model in subsonic air flow, in Part III) contain a series of recent analytical results on two aircraft wing models, which have been obtained by me. The first model describes ground vibrations of a long slender wing, which is modeled as a coupled Euler-Bernoulli/Timoshenko beam with boundary conditions representing control action of self-straining actuators. My results are rigorous and include the following information. The system is treated as a single linear evolution equation in a Hilbert state space of the system. The dynamics generator of the system is an *unbounded nonself-adjoint operator* in the state space. I give explicit asymptotic formulas for the complex eigenvalues of this operator, which represent the natural frequencies and the rates of energy dissipation for the ground vibrations of the wing. In particular, it follows from my asymptotic formulas that the spectrum consists of two branches, the bending and torsion eigenvalues. Asymptotic formulas for the mode shapes of the ground vibrations are also obtained in my papers. I have also proven that the entire set of the mode shapes (the eigenfunctions of the dynamics generator) and the associate mode shapes (the associate functions) forms a natural *unconditional basis* (the so-called Riesz basis). Using the Riesz basis property, I present the solution of the original initial boundary-value problem in the form of expansion with respect to the Riesz basis of the mode shapes, which may be very efficient in computational analysis.

The second model describes a wing with high aspect ratio in a subsonic, inviscid, incompressible air flow. In this model, the hydrodynamic equations have been solved explicitly and aerodynamic loads are represented as forcing terms in the structural equations as time convolution–type integrals with very complicated kernels. Thus, the model is described by a system of integro-differential equations. The integral convolution parts of these equations vanish if a speed of an air stream is equal to zero, and I obtain the equations of motion for the previous ground vibration model. For the second model, I give explicit asymptotic formulas for the bending and torsion branches of the aeroelastic modes and mode shapes. (To the best of my knowledge, these are the first such formulas in the literature on aeroelasticity.) I have also proven that the set of all mode shapes forms a nonorthogonal basis (Riesz basis) of the state space of the system, and present the solution of the original initial boundary-value problem in the form of expansion with respect to the mode shapes.

Remark. I suggest possible explanation of the flutter phenomenon in the framework of a specific solid structure–gas/fluid interaction model. Flutter should be viewed as a sharp increase of the amplitude of the solution of the corresponding initial boundary-value problem. The first and most obvious reason for such an increase consists of the following: In a specific model, the aeroelastic modes (or their analogs in a nonaeroelastic setting) are the eigenvalues of a certain operator (the dynamics generator). These modes are the functions of such parameters of the model as the air speed, the control gains parameters in the boundary conditions, etc. If all aeroelastic modes are the complex points located in the left half-plane, then the solution of the problem is stable. However, for some critical combination of the parameters (e.g., for sufficiently high speed of the air stream), one or several modes may cross the imaginary axis and move into the right half-plane. This shift causes a flutter instability of the solution of the corresponding initial boundary-value problem. The value of the air speed, at which at least one of the aeroelastic modes crosses the imaginary axis, is defined as the *flutter speed*.

The aforementioned reason for the flutter instability is not the only one. It is accepted in extensive engineering literature that flutter in a wing (or any beam-like structure) may occur when two eigenmodes corresponding to bending and torsional vibrations become coalescent. More precisely, classic coupled bending/torsion flutter in wings occurs when with increase of an air speed, the

J. Aerosp. Eng. 2004.17:56-69.

bending frequency increases and the torsional frequency falls until both frequencies become coincident. Such a flutter is well documented based on experimentation. The natural question is, *Why does the coincidence cause instability*? The coalescence of two eigenmodes can lead to an appearance of the eigenmode, which has a nontrivial Jordan block in the spectral decomposition of the dynamics generator. In other words, the corresponding eigenvector has a chain of associate vectors. In that case, equations of motion have a solution whose amplitude as a function of time can be represented in the form $f(t)=Ct^n\exp(-\alpha t)$, where $(n+1)$ is the size of the Jordan block (n is the number of the associate vectors) and $(-\alpha)<0$ is the real part of the corresponding eigenmode. Clearly, the function $f(t)$ has a peak of the amplitude $A=C(n/\alpha e)^n$ before it goes to zero as $t\to\infty$. This peak may be large enough to cause flutter-type instability. (The amplitude A may be large even if $n=1$ but $\alpha>0$ is small, i.e., the aeroelastic mode is stable but is close to the imaginary axis.) Also, the dynamics of the elastic structure are subject to nonlinear effects, which are neglected for small amplitude vibrations. When the amplitude suddenly increases, the vibrations could interact with nonlinearities and destroy the stability of the system.

Now I will describe the organization of the three-part survey.

1. In the section ‘‘Bending-Torsion Coupled Beams under Deterministic and Random Loads,’’ I discuss an approach of Eslimy-Isfahany et al. (1996) to the well-known problem on analysis of the so-called bending-torsion coupled vibrations. In flutter analysis, the most important is fluid/gas-structure interaction. However, to understand behavior of a flexible structure in a stream of gas or fluid, one has to know characteristics of the natural vibrations, i.e., frequencies and mode shapes when the structure is not in fluid/gas stream. This is exactly the topic of the first part of this section. The problem, which is known as bending/torsion vibration analysis, is a difficult mathematical problem. In ‘‘Mathematical Analysis of Euler-Bernoulli/Timoshenko Beam Model,’’ I present rigorous mathematical results on the aforementioned bending torsion vibrations problem, and in the section ‘‘Mathematical Analysis of an Aircraft Wing Model in Subsonic Air Flow,’’ I apply those results to the analysis of a flexible wing model in an airflow. The novelty of the paper by Eslimy-Isfahany et al. (1996) is the fact that the writers have introduced structural damping in the model and have analyzed damped bending/torsion vibrations with different types of loads (deterministic and random). Analysis in Eslimy-Isfahany et al. (1996) has been done at the ‘‘engineering’’ level of accuracy, which means that mathematically rigorous justification is an open (and quite challenging) problem. The second paper (Adam 1999), discussed with some details in the first part of the present section, is related to undamped bending/torsion vibrations. A new approach suggested by the writer looks promising.
2. In the section ‘‘Flutter of Transmission Lines,’’ an interesting question concerning flutter of transmission lines is discussed. The modeling is extremely difficult in this case because the shapes of ice-accreted conductor lines depend strongly on weather conditions.
3. The section ‘‘Flutter of Helicopter, Propeller, and Turbine Blades’’ is devoted to the fluttering vibrations of rotating helicopter and turbine blades. The dominant effect in this case is the load due to rotation. In the model discussed by Dzygadlo and Sobieraj (1977), the aforementioned load is modeled by a complimentary axial force. The writers suggest a mathematical model in the form of a system of two coupled hyperbolic partial differential equations. To solve their system, the writers develop a new numerical method, which they call the dynamic finite-element method. Thus, analytical study of the system is an interesting open problem.
4. The section ‘‘Flutter in Hard Disk Drives’’ is devoted to the analysis of such flutter. The problem of harmful vibrations for the case of magnetic disks is extremely important since minor transversal vibrations can distort information significantly. Hosaka and Crandall (1992) have derived an operator equation for small amplitude transversal oscillations of a flat spinning disk. Using acoustic excitation, the writers arrived at a very interesting analytical result. Namely, they found out that there exist two different types of waves propagating in a disk, which they called backward-traveling waves (BTW) and forward-traveling waves (FTW). The major finding in the paper is that the aerodynamic pressure is the cause for FTW to be more highly damped than BTW. The difference becomes visible at very high speeds (at subcritical speeds). At a certain rotation speed, the damping of BTW can vanish, which is exactly the onset of flutter. The mathematical model studied in Hosaka and Crandall (1992) is very complicated, and rigorous mathematical analysis would be very desirable. In the second part of this section, a number of papers on controlling vibrations of rotating magnetic disks is discussed.
5. ‘‘Flutter of Long-Span Suspension Bridges’’ is devoted to a very important question concerning divergent vibrations of suspension bridges, vibrations that may lead to total destruction. I present analysis of a bridge deck motion from Wilde et al. (2001), Part II of this three-part survey (Shubov 2004b), where the writers analyze the so-called *active flutter control* that use additional control surfaces (flaps) attached beneath both edges of the deck through aerodynamically shaped pylons. The rotational displacement of the control surface is actively adjusted by feedback control such that the generated aerodynamic forces provide a stabilizing action on the deck. In this section, I also present with some detail an analysis suggested in Kwon et al. (2000). The writers of this paper present and discuss a mathematical model for a new *passive aerodynamical control* method by utilizing a *tuned mass damper mechanism* to activate the control plates, which can modify airflow around the deck to stabilize bridge motion. They also present the description of experimental setting to verify analytical results.
6. ‘‘Flutter of Tubes Conveying Fluids—Biomedical Setting’’ (in Part II; Shubov 2004b) is devoted to the problem of divergent oscillations that occur in blood vessel walls under specific conditions. Huang (1998), discussed in this section, deals exactly with this type of oscillation. The writer of the paper derives and then investigates a boundary-value problem for a pressure perturbation induced in shear flows through elastic channels, whose walls undergo symmetric wave motions of phase speed less than maximum flow speed. The main result of this study is that the fluid pressure always has a component in phase with the wave slope, causing wave drag and energy transfer from the flow to the perturbation waves. This is a mechanism for traveling wave flutter, which occurs when the rate of this energy transfer exceeds the rate of energy dissipation in the walls. Approximations for the critical flow velocity and the wave speed for the flutter onset are also suggested in the paper.
7. The third part of this survey, to be published at a later late, will include the section on ‘‘Flutter Problem in Aeroelasticity’’ (in Part III; Shubov 2004a) is devoted to the flutter

J. Aerosp. Eng. 2004.17:56-69.

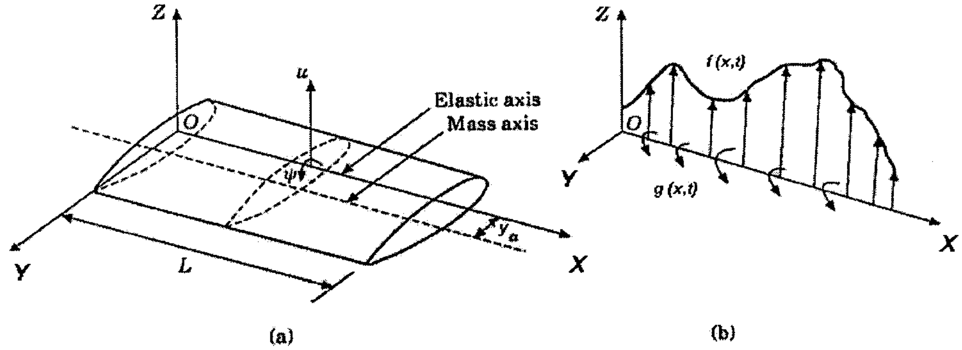

Fig. 1. (a) Coordinate system and notation for coupled bending-torsional vibrations of a beam (——=elastic axis; – – – –=mass axis); (b) distribution of bending and torsional loads

problem in aeronautics, where the flutter phenomenon is one of the most important criteria for designers of new aircrafts (Shubov 2004a). I outline relevant information from two well-known survey-type papers (Friedmann 1999; Garrick and Reed III 1981). The writer of the first paper considers flutter analysis an essential part of the broader scientific field of *aeroelasticity*, which deals with the behavior of an elastic vehicle in an airstream with significant feedback, i.e., significant reciprocal interaction between structural deformation and flow. The second paper (Garrick and Reed III 1981) presents aircraft flutter occurrences in historical retrospect and addresses such practical question as historical developments and changes in flutter-testing equipment, from the wind-tunnel model, through the replica model concept, and passing to the design and fabrication of contemporary aeroelastically scaled flutter models. In addition, I present a brief review of different papers dealing with such topics as (1) the *panel flutter*, which occurs at supersonic speeds; (2) the stability of an aircraft carrying external stores; (3) the effect of the trailing-edge flap on flutter control; and (4) the power and efficiency of smart structures for prevention of undesirable aeroelastic effects.

8. Finally, also in Shubov (2004a), in the sections "Mathematical Analysis of Euler-Bernoulli/Timoshenko Beam Model" and "Mathematical Analysis of an Aircraft Wing Model in Subsonic Airflow," I present my results on the rigorous solution of an initial boundary-value problem describing a specific aircraft wing model in a subsonic, incompressible, inviscid air flow. In particular in that section, I discuss the problem of natural frequencies and mode shapes of the wing when an aircraft is not in flight—i.e., I study the so-called *ground vibrations*. I allow smart-material inclusions into the wing structure, which provide a stabilizing effect. The problem considered in this section is a modification of the bending-torsion vibration model studied in papers discussed in the section "Bending-Torsion Coupled Beams under Deterministic and Random Loads." The difference in the statements of the problems in prior sections is that I consider the case when the damping coefficients c_1 and c_2 are zeros, while in "Bending-Torsion Coupled Beams under Deterministic and Random Loads" those coefficients are positive constants; and secondly, the boundary conditions in that section are conservative, while in "Mathematical Analysis of Euler-Bernoulli/Timoshenko Beam Model," the boundary conditions take into account the action of self-straining actuators; i.e., energy exchange with the environment is allowed. To obtain my results, I use the methods and ideas of rigorous mathematical analysis. In the section "Mathematical Analysis of an Aircraft Wing Model in Subsonic Airflow," I apply the results of the previous section to analyze the response of an aircraft wing to a turbulent air flow, and clarify the notion of a flutter frequency.

Bending-Torsion Coupled Beam under Deterministic and Random Loads

As has already been mentioned, in this section, I present some analytical but mostly engineering results concerning the coupled beam vibration model. It turns out that such results provide the necessary important step for flutter analysis.

I start with the paper by Eslimy-Isfahany et al. (1996), which addresses the question of force vibrations of a bending-torsion coupled beam. To obtain the response of a uniform coupled beam, which is subjected to either deterministic or random load, the writers use the normal mode method. The deterministic load is assumed to be harmonically varying, whereas the random load is assumed to be Gaussian, having both stationary and ergodic properties. The theory developed in Eslimy-Isfahany et al. (1996) is applied to a cantilever aircraft wing, for which there is a substantial coupling between the bending and torsional modes of deformation. The deterministic load is a harmonically varying concentrated force at the tip of the wing, and the random load is atmospheric turbulence represented by the Von Karman spectrum (Houbolt et al. 1964; Perry et al. 1990), which is uniformly distributed over the length of the wing (see also Banerjee and Price 1997; Dokumaci 1977).

Eslimy-Isfahany et al. start with establishing the free natural vibration characteristics of a bending-torsion coupled beam by analyzing the basic governing differential equations of motion. The effects of shear deformation, rotary inertia, and warping stiffness are small enough to be neglected. The main assumption made in this study is that the linear small deflection theory is valid. At this moment, I mention that the writers study the most complete version of the bending-torsion model, i.e., they incorporate into their system of equations the effect of viscous damping in the structure, which makes analysis much more complicated.

Governing Differential Equations of Motion

The uniform straight beam of the length L, and of the airfoil section shown on Fig. 1(a), has its X-axis coincident with the elastic axis. The allowable displacements consist of a flexural translation $u(x,t)$ in the Z-direction and a torsional rotation $\psi(x,t)$ about the X-axis, where x and t denote the distance from

J. Aerosp. Eng. 2004.17:56-69.

the origin and time, respectively. The cross-sectional loads are represented by a force per unit length $f(x,t)$ acting parallel to OZ, and applied through the shear center, together with a torque per unit length $g(x,t)$ about OX [Fig. 1(b)]. The mass and the elastic axis (i.e., the loci of the mass center and the shear center of the cross section) of the wing are separated by a distance y_α.

In what follows, I will use the notation "prime" to denote the derivative with respect to the spatial variable, and the notation "overdot" to denote the derivative with respect to time. Then the governing differential equations of motion are taken in the following forms (Dokumaci 1977, Hallauer and Liu 1982, Friberg 1983; Bishop et al. 1989; Banerjee and Price 1997):

$$EI\,u'''' + m\ddot{u} + c_1(\dot{u} - y_\alpha\dot{\psi}) - my_\alpha\ddot{\psi} = f(x,t) \tag{1}$$

$$GJ\,\psi'' + my_\alpha\ddot{u} + I_\alpha\ddot{\psi} - c_2\dot{\psi} + c_1 y_\alpha\dot{u} = g(x,t) \tag{2}$$

Here EI and GJ=bending and torsion rigidities; m=mass per unit length; and I_α=mass moment of inertia per unit length about the X-axis (i.e., the axis through the shear center of the beam). The coefficients c_1 and c_2 in Eqs. (1) and (2) are viscous damping terms per unit length in flexure and torsion, respectively. [Each point of the cross section moves with a different local velocity so that in Eq. (1), the local damping force sums over the section to the given expression containing the c_1 term. Similarly, in Eq. (2), the expression containing the c_2 term is a torque about the elastic axis due to the elemental damping forces. No other sources of damping are taken into account.]

This system [(1) and (2)] is equipped with conservative boundary conditions. Eslimy-Isfahany et al. (1996) look for the solutions of different initial boundary-value problems in terms of expansions with respect to the normal modes. The normal modes of the beam in free undamped vibrations are found by setting the damping parameters c_1 and c_2 and the external force and torque $f(x,t)$ and $g(x,t)$ in Eqs. (1) and (2) to zero. The equations for undamped vibrations in conjunction with the conservative boundary conditions yield the eigenvalues (natural frequencies) and eigenfunctions (mode shapes) of the bending-torsion coupled beam. The solutions of the system (with $c_1=c_2=f=g=0$) are sought in the forms (see also Hallauer and Liu 1982)

$$u_n(x,t) = U_n(x)e^{i\omega_n t}; \quad \psi_n(x,t) = \Psi_n(x)e^{i\omega_n t}; \quad n=1,2,3,... \tag{3}$$

After straightforward calculations, the following expressions are found for the amplitudes

$$\begin{pmatrix} U_n(\xi) \\ \Psi_n(\xi) \end{pmatrix} = \begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix} \begin{pmatrix} \cosh\alpha_n\xi \\ \sinh\alpha_n\xi \end{pmatrix} + \begin{bmatrix} A_3 & A_4 \\ B_3 & B_4 \end{bmatrix} \begin{pmatrix} \cos\beta_n\xi \\ \sin\beta_n\xi \end{pmatrix} + \begin{bmatrix} A_5 & A_6 \\ B_5 & B_6 \end{bmatrix} \begin{pmatrix} \cos\gamma_n\xi \\ \sin\gamma_n\xi \end{pmatrix} \tag{4}$$

where $\xi = x/L$, the two sets of constants $\{A_i\}_{i=1}^{6}$ and $\{B_i\}_{i=1}^{6}$ are not all independent; the constants α_n, β_n, and γ_n are given by some complicated formal expressions. The following orthogonality condition of the principal modes of free vibrations of the beam can be derived as (Banerjee and Fisher 1992):

$$\int_0^1 \{(mU_mU_n + I_\alpha\Psi_m\Psi_n) - my_\alpha(U_m\Psi_n + U_n\Psi_m)\}d\xi = \mu_n\delta_{mn} \tag{5}$$

where μ_n=generalized mass in the nth mode; and δ_{mn}=Kronecker symbol.

With the free vibration modes, natural frequencies, and orthogonality conditions, Eq. (5), it is now possible to find the general solution of the forced vibration problem.

Response to Deterministic Loads

The writers seek solutions for the general forced vibration problem in the forms

$$u(x,t) = u(\xi L,t) = \sum_{n=1}^{\infty} q_n(t)U_n(\xi),$$

$$\psi(x,t) = \psi(\xi L,t) = \sum_{n=1}^{\infty} q_n(t)\Psi_n(\xi) \tag{6}$$

where $q_n(t)$=time-dependent generalized coordinate for the nth mode. Substituting representations (6) into Eqs. (1) and (2), one obtains

$$\sum_{n=1}^{\infty} [m\omega_n^2(U_n - y_\alpha\Psi_n)q_n + mU_n\ddot{q}_n + c_1U_n\dot{q}_n - c_1\Psi_n\dot{q}_n - mx_\alpha\Psi_n\ddot{q}_n] = f(\xi,t) \tag{7}$$

$$\sum_{n=1}^{\infty} [\omega_n^2(my_\alpha U_n - I_\alpha\Psi_n)q_n + my_\alpha U_n\ddot{q}_n - c_2\Psi_n\dot{q}_n + c_1U_ny_\alpha\dot{q}_n - I_\alpha\Psi_n\ddot{q}_n] = g(\xi,t) \tag{8}$$

where an overdot denotes differentiation with respect to time.

Multiplying Eq. (7) by U_m and Eq. (8) by $(-\Psi_m)$, and summing up the resulting equations, integrating both sides from 0 to L, and making use of the orthogonality conditions Eq. (5), one obtains the differential equation for q_n:

$$\ddot{q}_n(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2q_n(t) = [F_n(t) + G_n(t)] \tag{9}$$

where

$$F_n(t) = \frac{1}{\mu_n}\int_0^L U_n(x)f(x,t)dx = \frac{1}{\mu_n}P_n(t) \tag{10}$$

$$G_n(t) = -\frac{1}{\mu_n}\int_0^L \Psi_n(x)g(x,t)dx = -\frac{1}{\mu_n}Q_n(t) \tag{11}$$

$$\zeta_n = c_1/(2m\omega_n) = c_2/(2mr^2\omega_n) \tag{12}$$

$P_n(t)$ and $Q_n(t)$ from Eqs. (10) and (11) are the nth generalized forces in bending and torsional motions, respectively; and ζ_n in Eq. (9) is the nondimensional damping coefficient in the nth mode with r as the radius of gyration defined by $\sqrt{I_\alpha/m}$. Note, Eslimy-Isfahany et al. (1996) have chosen $c_2 = c_1r^2$ and thus Eq. (9) can be derived from Eqs. (1) and (2). The validity of this assumption is currently under the writers' investigation.

The solution of Eq. (9) is obtained by using Duhamel's integral in the form

$$q_n(t) = e^{-\zeta_n\omega_n t}\{A_n\cos(\omega_{nd}t) + B_n\sin(\omega_{nd}t)\} + \frac{1}{\omega_{nd}}\int_0^t \{F_n(\tau) + G_n(\tau)\}e^{-\zeta_n\omega_n(t-\tau)}\sin\{\omega_{nd}(t-\tau)\}d\tau \tag{13}$$

where $\omega_{nd} = \omega_n(1-\zeta_n^2)^{1/2}$, and A_n and B_n are coefficients related to the initial conditions.

Using Eq. (13) for Eq. (6) yields the general solution for bending and torsional displacements [due to arbitrary flexural and torsional loading $f(y,t)$ and $g(y,t)$]:

J. Aerosp. Eng. 2004.17:56-69.

$$u(\xi,t)=\sum_{n=1}^{\infty} U_n(\xi)\left[e^{-\zeta_n\omega_n t}\{A_n\cos(\omega_{nd}t)+B_n\sin(\omega_{nd}t)\}\right.$$
$$+\frac{1}{\omega_{nd}}\int_0^t\{F_n(\tau)+G_n(\tau)\}e^{-\zeta_n\omega_n(t-\tau)}$$
$$\left.\times\sin\{\omega_{nd}(t-\tau)\}d\tau\right] \tag{14}$$

$$\psi(\xi,t)=\sum_{n=1}^{\infty} \Psi_n(\xi)\left[e^{-\zeta_n\omega_n t}\{A_n\cos(\omega_{nd}t)+B_n\sin(\omega_{nd}t)\}\right.$$
$$+\frac{1}{\omega_{nd}}\int_0^t\{F_n(\tau)+G_n(\tau)\}e^{-\zeta_n\omega_n(t-\tau)}$$
$$\left.\times\sin\{\omega_{nd}(t-\tau)\}d\tau\right] \tag{15}$$

As an example, the writers take the externally applied bending and torsional loads in the forms $f(x,t)=\delta(x-a_i)F_i\sin\Omega_i t$ and $g(x,t)=\delta(x-b_i)G_i\sin\Omega_i t$, which represent a system of harmonic concentrated forces and torques with circular frequencies Ω_i applied at the points a_i and b_i, $i=1,2,3,...N$, and $\delta(x)$ is the Dirac delta function. Then the dynamic response for bending and torsional displacements according to Eqs. (14) and (15) is

$$u(\xi,t)=\sum_{n=1}^{\infty} U_n(\xi)e^{-\zeta_n\omega_n t}\{A_n\cos(\omega_{nd}t)+B_n\sin(\omega_{nd}t)\}$$
$$+\sum_{n=1}^{\infty} U_n(\xi)\sum_{i=1}^{N}\frac{\{U_n(a_i)F_i+\Psi(b_i)G_i\}}{\mu_n\{(\omega_n^2-\Omega_i^2)^2+(2\zeta_n\omega_n\Omega_i)^2\}}$$
$$\times\{(\omega_n^2-\Omega_i^2)\sin\Omega_i t-2\zeta_n\omega_n\Omega_i\cos\Omega_i t\}, \tag{16}$$

$$\psi(\xi,t)=\sum_{n=1}^{\infty} \Psi_n(\xi)e^{-\zeta_n\omega_n t}\{A_n\cos(\omega_{nd}t)+B_n\sin(\omega_{nd}t)\}$$
$$+\sum_{n=1}^{\infty} \Psi_n(\xi)\sum_{i=1}^{N}\frac{\{U_n(a_i)F_i+\Psi(b_i)G_i\}}{\mu_n\{(\omega_n^2-\Omega_i^2)^2+(2\zeta_n\omega_n\Omega_i)^2\}}$$
$$\times\{(\omega_n^2-\Omega_i^2)\sin\Omega_i t-2\zeta_n\omega_n\Omega_i\cos\Omega_i t\} \tag{17}$$

If there is only one external force and only one external torque acting at $x=a$ and $x=b$, respectively, then $f(x,t)=\delta(x-a)F\sin\Omega t$ and $g(x,t)=\delta(x-b)G\sin\Omega t$, and the steady-state bending and torsional responses, for given initial conditions, can be obtained from Eqs. (16) and (17) as

$$u(\xi,t)=\sum_{n=1}^{\infty} U_n(\xi)\{U_n(a)F+\Psi_n(b)G\}\{A_n/(\mu_n\omega_n^2)\}$$
$$\times\sin(\Omega t-\phi) \tag{18}$$

$$\psi(\xi,t)=\sum_{n=1}^{\infty} \Psi_n(\xi)\{U_n(a)F+\Psi_n(b)G\}\{A_n/(\mu_n\omega_n^2)\}$$
$$\times\sin(\Omega t-\phi) \tag{19}$$

where

$$\tan\phi=2\zeta_n(\Omega/\omega_n)/\{1-(\Omega/\omega_n)^2\};$$
$$A_n=[\{1-(\Omega/\omega_n)^2\}^2+\{2\zeta_n(\Omega/\omega_n)\}^2]^{-1/2} \tag{20}$$

Response to Random Loads

As the next result, response of the bending-torsion coupled beam to stationary, ergodic random excitation with zero initial conditions has been considered. The receptances (complex frequency response functions) $H_u(\xi,\xi_1,\Omega)$ and $H_\psi(\xi,\xi_1,\Omega)$ for bending displacement u and torsional rotation ψ are defined by their amplitudes at the point ξ when a harmonically varying force and/or torque of circular frequency Ω and amplitude 1 is applied at the point ξ_1. Thus, for the purposes of computing receptances, the externally applied loadings $f(\xi,t)$ and $g(\xi,t)$ have been represented in the forms

$$f(\xi,t)=\delta(\xi-\xi_1)e^{i\Omega t};\quad g(\xi,t)=\delta(\xi-\xi_1)e^{i\Omega t} \tag{21}$$

Substituting representations (21) into Eqs. (10) and (11) yields

$$F_n(t)=(1/\mu_n)U_n(\xi_1)e^{i\Omega t};\quad G_n(t)=-(1/\mu_n)\Psi_n(\xi_1)e^{i\Omega t} \tag{22}$$

If the solution of Eq. (9) for the above loading is taken in the form

$$q_n(t)=\hat{q}_n e^{i\Omega t} \tag{23}$$

then by substituting Eq. (21) into Eq. (9) and using Eqs. (22), one finds

$$\hat{q}_n(\xi_1,\Omega)=V_n(\xi_1)/\{\mu_n(\omega_n^2-\Omega^2+2i\zeta_n\Omega\omega_n)\} \tag{24}$$

in which $V_n(\xi_1)=a_F U_n(\xi_1)-a_G\Psi_n(\xi_1)$. The values a_F and a_G can be either 1 or 0 depending upon whether or not bending and/or torsional loads are present.

The receptances for u and ψ can now be obtained from Eqs. (16), (17), and (24) as

$$H_u(\xi,\xi_1,\Omega)=\sum_{m=1}^{\infty}\hat{q}_n(\xi_1,\Omega)U_n(\xi);$$
$$H_\psi(\xi,\xi_1,\Omega)=\sum_{m=1}^{\infty}\hat{q}_n(\xi_1,\Omega)\Psi_n(\xi) \tag{25}$$

Once the receptances are known, the response to stationary, ergodic random loads can be found by superposition with the use of normal mode method (Grandall and Mark 1963; Newland 1984).

The objective of the next paper (Adam 1999) is to propose a new numerically more efficient method, developed by the writer, for analysis of coupled bending-torsional vibrations of distributed parameter beams. The governing partial differential equations [which are similar to Eqs. (1) and (2) when $c_1=c_2=0$] are solved by splitting the dynamic responses in two parts, i.e., in a quasistatic and a complimentary dynamic responses. The quasistatic part may contain discontinuities because of the sudden load changes, and this part is determined in a closed form. The remaining complimentary dynamic part is nonsingular and can be approximated by a truncated modal series of fast-accelerated convergence. The solution of the resulting generalized decoupled single-degree-of-freedom oscillators is given by the Duhamel's convolution integral. The procedure proposed by the author is illustrated for a dynamically loaded simply supported beam with a thin-walled monosymmetric open cross section. The problem studied in Eslimy-Isfahany et al. (1996) is more general than the one from Adam (1999) and Adam et al. (1997), since the latter problem does not contain the internal damping. Nevertheless, the approach described in Adam (1999) is totally different. Namely, the approach of Eslimy-Isfahany et al. leads to a solution represented as a series, which might be very slowly convergent. In Adam, the modal expansion is performed only for the dynamic part of the solution. The quasistatic part is determined separately

J. Aerosp. Eng. 2004.17:56-69.

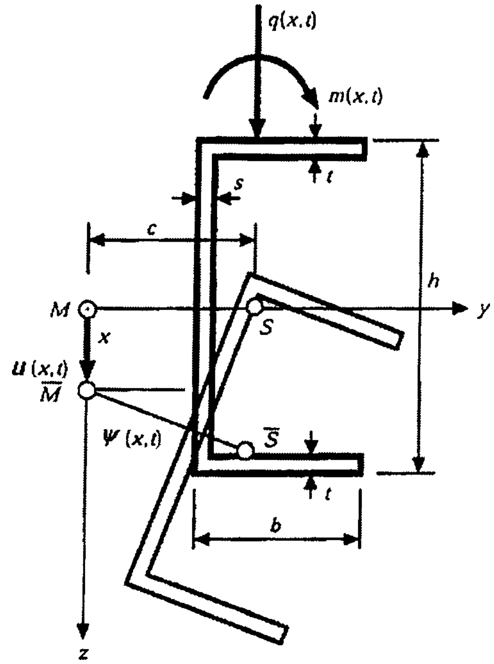

Fig. 2. Channel cross section

and in a closed form by means of weighted integration of the corresponding influence function. Such a splitting is numerically efficient and also more accurate since the quasistatic part may contain singularities or discontinuities that are properly accounted for and would be poorly modeled by a truncated modal series solution. The remaining complementary dynamic response is nonsingular and can be approximated by a finite modal series of fast-accelerated convergence. The procedure suggested in the paper is illustrated for a simply supported beam with channel cross section, and improvement in comparison to the classical modal analysis is shown.

Governing Equations

Coupled bending and torsional vibrations of a thin-walled monosymmetric beam with open cross section are considered. The beam consists of a linear elastic material with mass density ρ. The centroid and the shear-center, denoted by S and M respectively, are separated by the distance c (Fig. 2). The beam is referenced to a Cartesian system of coordinates x, y, z, where the x-axis coincides with the shear center and the y-axis coincides with the symmetry axis of the cross section. It is assumed that the beam is loaded by a given transverse force per unit length $q(x,t)$, distributed along the centroidal axis, and an external torque of intensity $m(x,t)$, (Fig. 2). The deformation is determined by the lateral deflection $u(x,t)$ of the shear center axis and by the angle of twist $\psi(x,t)$ of the cross section. I note here that in the original paper (Adam 1999), the writer uses the notation w for the lateral displacement and the notation θ for the angle of twist. We have decided to use the uniform notation throughout the paper [as well as in Parts II and III (Shubov 2004a,b)] in order to enable the reader to make connections between different model equations and see the underlying resemblance between the various applications where flutter appears. As was shown in Weaver et al. (1990), the response is governed by the following set of coupled differential equations:

$$EIu'''' + \rho A(\ddot{u} + c\ddot{\psi}) = q \tag{26}$$

$$(EA_{\varphi\varphi})\psi'''' - GJ\psi'' + \rho(I_0 + c^2 A)\ddot{\psi} + \rho A C\ddot{u} = m + cq \tag{27}$$

where EI=bending rigidity of the beam with respect to the y-axis; $EA_{\varphi\varphi}$=warping rigidity; GJ=torsional rigidity for uniform torsion; I_0=centroidal polar moment of inertia of the cross section; and A=cross-sectional area. Overdot stands for the time derivatives. It is assumed that the beam is of length l so the boundary conditions are imposed at the sections $x_b = 0$ and $x_b = l$. The following three classical boundary conditions are summarized (Bishop et al. 1989):

<u>Simply supported end</u>

$$u(x_b,t) = 0, \quad u''(x_b,t) = 0, \quad \psi(x_b,t) = 0, \quad \psi''(x_b,t) = 0 \tag{28}$$

<u>Clamped end</u>

$$u(x_b,t) = 0, \quad u'(x_b,t) = 0, \quad \psi(x_b,t) = 0, \quad \psi'(x_b,t) = 0 \tag{29}$$

<u>Free end</u>

$$u''(x_b,t) = 0, \quad u'''(x_b,t) = 0, \quad \psi''(x_b,t) = 0, \quad -(EA_{\varphi\varphi})\psi''' + GJ\psi' = 0 \tag{30}$$

The free-vibration analysis of Eqs. (26) and (27) can be found, e.g., in Bishop et al. (1989).

Dynamic Response Analysis

The solution of the system [(26) and (27)] is represented in Adam (1999) as a combination of quasistatic and dynamic parts, i.e.

$$u(x,t) = u^S(x,t) + u^D(x,t); \quad \psi(x,t) = \psi^S(x,t) + \psi^D(x,t) \tag{31}$$

The quasistatic part may contain singularities or discontinuities, while the dynamic part is nonsingular. The dynamic part, being smooth, can be described by means of a relatively small number of mode shapes. Using Eq. (31), one can modify Eqs. (26) and (27) as

$$EI(u^S)'''' + EI(u^D)'''' + \rho A\ddot{u}^S + \rho A\ddot{u}^D + \rho Ac\ddot{\psi}^S + + \rho Ac\ddot{\psi}^D = q \tag{32}$$

$$(EA_{\varphi\varphi})(\psi^S)'''' + (EA_{\varphi\varphi})(\psi^D)'''' - GJ(\psi^S)'' - GJ(\psi^D)'' + \rho(I_0 + c^2 A)\ddot{\psi}^S + \rho(I_0 + c^2 A)\ddot{\psi}^D + \rho Ac\ddot{u}^S + \rho Ac\ddot{u}^D = m + cq \tag{33}$$

Assuming that the differential equations of the quasistatic response have the forms

$$EI(u^S)'''' = q, \quad (EA_{\varphi\varphi})(\psi^S)'''' - GJ(\psi^S)'' = m + cq \tag{34}$$

the equations of the dynamic response can be separated from Eqs. (32) and (33):

$$EI(u^D)'''' + \rho A(\ddot{u}^D + c\ddot{\psi}^D) = -\rho A(\ddot{u}^S + c\ddot{\psi}^S) \tag{35}$$

$$(EA_{\varphi\varphi})(\psi^D)'''' - GJ(\psi^D)'' + \rho A\left[\left(\frac{I_0}{A} + c^2\right)\ddot{\psi}^D + c\ddot{u}^D\right] = -\rho A\left[\left(\frac{I_0}{A} + c^2\right)\ddot{\psi}^S + c\ddot{u}^S\right] \tag{36}$$

The solutions of Eqs. (32) and (33) are found by modal analysis. Thereby, the quasistatic parts u^S, ψ^S and the complementary dynamic portions u^D, ψ^D are transformed to a set of modal amplitudes. These transformations are expressed as follows:

J. Aerosp. Eng. 2004.17:56-69.

$$u^S(x,t) = \sum_{n=1}^{\infty} Y_n^S(t) U_n(x); \quad \psi^S(x,t) = \sum_{n=1}^{\infty} Y_n^S(t) \Psi_n(x) \tag{37}$$

$$u^D(x,t) = \sum_{n=1}^{\infty} Y_n^D(t) U_n(x); \quad \psi^D(x,t) = \sum_{n=1}^{\infty} Y_n^D(t) \Psi_n(x) \tag{38}$$

The modal series (37) and (38) are inserted into Eqs. (35) and (36), multiplied by U_m and Ψ_m, respectively, and added. Integration over the beam length l together with the orthonormality relations (D'Angelo and Mote 1993a)—

$$\int_0^1 \left[U_n U_m + c(\Psi_n U_m + U_n \Psi_m) + \left(\frac{I_0}{A} + c^2 \right) \Psi_n \Psi_m \right] dx = \delta_{nm} \tag{39}$$

—leads to a decoupled system of single-degree-of-freedom oscillator equations for the complementary dynamic variables $Y_n^D(t)$:

$$\ddot{Y}_n^D + \omega_n^2 Y_n^D = -\ddot{Y}_n^S \tag{40}$$

where ω_n has the same meaning as in Eq. (3). In the next step, the equations in (37) are inserted into Eq. (34), and the quasistatic modal amplitudes become

$$Y_n^S(t) = \frac{1}{\rho A \omega_n^2} P_n(t) \tag{41}$$

with

$$P_n(t) = \int_0^1 [q(x,t) U_n(x) + m(x,t) \Psi_n(x) + c q(x,t) \Psi_n(x)] dx \tag{42}$$

being the generalized loading associated with the mode shapes U_n and Ψ_n. Finally, the solution of Eq. (40) is given by the Duhamel's convolution integral

$$Y_n^D(t) = Y_n^D(0) \cos \omega_n t + \frac{\dot{Y}_n^D(0)}{\omega_n} \sin \omega_n t - \frac{1}{\rho A \omega_n^3} \int_0^t \ddot{P}_n(\tau) \sin[\omega_n (t-\tau)] d\tau \tag{43}$$

where $Y_n^D(0)$, $\dot{Y}_n^D(0)$ represent the initial conditions (at $t=0$):

$$Y_n^D(0) = Y_n(0) - \frac{1}{\rho A \omega_n^2} P_n(0); \quad \dot{Y}_n^D(0) = \dot{Y}_n(0) - \frac{1}{\rho A \omega_n^2} \dot{P}_n(0) \tag{44}$$

In Eqs. (43) and (44), $Y_n(0)$, $\dot{Y}_n(0)$ are derived from the initial conditions at $t=0$ by modal decomposition of $u(x,0)$, $\psi(x,0)$, $\dot{u}(x,0)$ and $\dot{\psi}(x,0)$.

In the conclusion of his paper, Adam (1999) considers an example of a simply supported beam under harmonic excitation and shows a numerical improvement in comparison with the standard modal analysis approach.

The remaining part of the current section is related to the optimization problem for different beam structures. Civil, mechanical, and aeronautics engineers often need to size structural elements such as beams so that a certain range of natural frequencies is avoided, while a minimum mass is achieved. Such design is relatively straightforward when the cross section of the beam is doubly symmetric, in which case the bending and torsional modes of vibration are uncoupled and may be treated separately. However, for a great deal of practical beam problems the shear center and mass center do not coincide, and the bending and torsional modes are inherently coupled. The optimum design of a nonuniform beam with bending-torsion coupling in one plane has been considered elsewhere (Hanagud et al. 1987; Grandhi and Moradmand 1989). Most frequently, the Rayleigh-Ritz method has been used in optimization problems. In this method, a beam is modeled as a series of finite elements connected at the nodes. Hanagud et al. (1987) have developed an optimality method, which can either minimize mass subject to a single frequency constraint, or maximize frequency subject to a mass constraint. Grandhi and Moradmand (1989) have used the mathematical programming techniques and considered both multiple frequency constraints and the effects of nonstructural mass.

In Butler and Banerjee (1996), the optimum design of a nonuniform beam with couplings in one plane is considered using a different approach. Analysis is based on exact dynamic stiffness theory developed in Banerjee (1989), while for optimization the standard mathematical programming techniques have been used. The effects of multiple frequency constraint and nonstructural mass have been taken into account and presented results illustrate optimum design subject to either a constraint on the fundamental frequency or a constraint on the separation of the first bending and the first torsional frequencies. The latter are compared with designs produced when optimizing the same problem subject to a minimum flutter-speed constraint using the aeroelastic optimization programs developed and presented in Lilico et al. (1994). One has to take into account the flutter speed because classical wing flutter involves the combined bending and torsional oscillations of a wing when it is subject to the unsteady aerodynamic loading produced by an airflow. (The flutter speed is defined as the critical value of airspeed, above which the effect becomes unstable and the oscillations diverge.) Therefore, an optimum design approach, which seeks to separate the bending and torsional natural frequencies of a wing while reducing a wing mass, should also produce a favorable flutter design. Essentially, the dynamic stiffness matrix method used in Lilico et al. (1994) involves the assembly of a single frequency-dependent stiffness matrix, the elements of which are transcendental functions of both mass and stiffness terms. This is different from the traditional finite-element approaches, in which mass and stiffness matrices are obtained separately. The method of Butler and Banerjee (1996) is exact in the sense that the dynamic stiffness matrix is obtained from the exact solution of the governing differential equations of motion of the structure. Butler and Banerjee (1996) have imposed the assumptions that the rotary inertia, shear deformation, and warping stiffness are small enough to be neglected. Under these assumptions, the main system of coupled partial differential equations coincides with system of Eqs. (1) and (2) if one assumes that $f=g=0$ and $c_1=c_2=0$. The aforementioned assumptions are quite reasonable for, e.g., metallic beams having either solid cross sections or hollow closed cross sections.

Concluding Remark. As has already been mentioned in the "Introduction," the main idea for my choice of research papers for this review is to present analytical results in order to find an explanation of the extremely complicated flutter phenomenon. The crucial part of the flutter analysis consists of investigation of unstable vibrations of a flexible structure in the presence of a coupling between different degrees of freedom, the vibrations which are known as bending-torsion vibrations. In all of the aforementioned papers, the modeling and derivation of the corresponding initial boundary-value problems have been done in an interesting and professional manner. However, mathematical

J. Aerosp. Eng. 2004.17:56-69.

analysis has been performed at "the engineering level" of accuracy, which means that some statements are likely to be valid but have not been proven rigorously. Moreover, in many papers the main tool of research is numerical analysis. Different writers suggest new or improved known numerical schemes. Even though numerical approach can provide valuable information, it is still desirable to carry out rigorous analysis by using methods and ideas of applied mathematics and physics, since analytical results can be a unique source of information not obtainable from numerical simulations.

Flutter of Transmission Lines

An interesting and practically important flutter phenomenon is related to high-amplitude galloping of conductors of the overhead transmission lines. It is generally agreed that conductor galloping is a dynamic aeroelastic instability (Nakamura 1980, 1996), often called flutter, which can occur for conductors experiencing a severe icing storm. In recent years, it has been recognized that overhead conductor galloping is not, in general, a purely vertical motion and that the dynamic torsion of the conductor or conductor bundle usually occurs and is of fundamental importance. The aforementioned flutter of bending-torsion systems with a frequency ratio close to one, which are inertially and elastically balanced about the center, is investigated analytically and experimentally in Nakamura (1996). The governing system of coupled partial differential equations has the structural part, which is not as complicated as Eqs. (1) and (2), but the right-hand sides of the equations represent the generalized forces and moments due to a strong airflow. Special attention is paid to the galloping of bundled transmission conductors. It is shown that flutter of different systems is typically characterized by resonance between bending and torsion, where extraordinary instability can be brought about when resonance wind speed is passed. The growth rate of oscillations in instability critically depends on the product of the in-phase components of the unsteady lift and moment coefficients. Based on the results of Nakamura (1996), it can be seen that any efficient means of control or prevention of conductor galloping should rest on increasing reduced mass and mass moment of inertia and/or controlling the detuning. It may be worth it to mention that it is enormously difficult to control aerodynamic forces and moments efficiently because the shapes of ice-accreted conductors vary widely according to weather conditions.

Flutter of Helicopter, Propeller, and Turbine Blades

Helicopter, propeller, and turbine blades of high aspect-ratio all qualify (at least for their low-energy vibration modes) as axially loaded beams, which usually have noncoincident elastic and inertial axes. Moreover, some complete plane and space frames can be represented with reasonable accuracy as assemblages of aerially loaded beams connected together. The effect of the important parameter, namely the axial force (which is usually negligible or nonexistent for some structures like as aircraft wings but not so for helicopters, turbine, or propeller blades), have to be appropriately taken into account. Applications of such coupled beams include also the aeroelastic calculations, for which coupled bending-torsional frequencies and modes are very essential requirements (Banerjee 1984, 1988; Friedmann and Straub 1980). Dzygadlo and Sobieraj (1977) suggest a modification of the finite-element method which they call the *dynamic finite element* approach in order to find accurate numerical solutions for coupled bending-torsional vibration of axially loaded beams. Using coupled Bernoulli–St-Venant bending-torsion beam theory (i.e., neglecting shear deformation, rotary inertia, and warping of the cross section), the governing partial differential equations of motion of the beam can be given as (Banerjee and Fisher 1992; Friberg 1983, 1985)

$$EIu'''' - Pu'' + m\ddot{u} + (Py_\alpha\psi'' - my_\alpha\ddot{\psi}) = 0 \tag{45}$$

$$(GJ + P(I_\alpha/m))\psi'' - I_\alpha\ddot{\psi} - (Py_\alpha u'' - my_\alpha\ddot{u}) = 0 \tag{46}$$

In Eqs. (45) and (46), all notation is the same as in Eqs. (1) and (2) with the additional terms containing parameter P, which is an axial force (constant per unit element). Note, if it is assumed that $c_1 = c_2 = 0$ in Eqs. (1) and (2), and $P = 0$ in Eqs. (45) and (46), then both homogeneous systems $(f = g = 0)$ will be identical. In Dzygadlo and Sobieraj (1977), analytical expressions of uncoupled bending and torsional dynamic shape function of an axially loaded uniform beam (derived in the exact sense) have been used to obtain the frequency-dependent stiffness matrix corresponding to the coupled vibrations of this type of a beam. It leads to an expression for the clamped-clamped (the boundary conditions) natural frequencies, which has been also investigated. The influence of the axial force on the coupled bending-torsional frequencies of a cantilever beam of a thin-walled section has been demonstrated by numerical results. Application of the suggested theory includes, e.g., frequency and mode calculations of helicopter, turbine, and propeller blades, plane and space frames consisting of axially loaded beams with noncoincident mass and shear centers. Numerical results on natural frequencies showed good agreement with published results and confirmed the correctness of the theory. The advantage of this approach in comparison with exact dynamic stiffness matrix method is that it can be extended to more complex cases as the coupled bending-torsional vibration of nonuniform axially loaded or centrifugally stiffened beams.

Aeroelastic Flutter in Hard Disk Drives

The new generation of hard disk drives is expected to pack high track densities and rotate at very high speeds [more than 20,000 rotations per minute (rpm)]. At rotation speed near and beyond 20,000 rpm, the aeroelastic coupling between the disk vibration and the air around the disk is significant. This coupling leads to disk flutter, which can contribute noticeably to the track misregistration and disk-drive failure. Thus, the prediction of the aeroelastic flutter speed is crucial for the design of the new generation of disk drives. The stability of the equilibrium configuration of floppy disks coupled through thin gas films to a rigid enclosure has been studied by a number of researchers (Chonan et al. 1992; Hosaka and Crandall 1992; Huang and Mote 1995; Adam et al. 1997). The onset of aeroelastic flutter in disks spinning without enclosure (unbounded domains) has also been studied both analytically and experimentally (D'Angelo and Mote 1993b; Renshaw et al. 1994; Yasuda et al. 1992).

The aeroelastic coupling problems in a hard disk are distinct from those of a disk in an unbounded enclosure or in a floppy disk, for the following reasons:

- The flow boundary conditions in the disk drives are bounded radially and axially. The acoustic wavelengths are larger than the characteristic gap widths and this can cause a significant difference between flutter speed of the enclosed disk and the open disk.

J. Aerosp. Eng. 2004.17:56-69.

- The rotation speeds in hard disk drives are greater than in floppy disks, and the gap widths are greater, which leads to a Reynolds number of substantially greater flow. Therefore, the use of hydrodynamic lubrication theory can result in an inaccurate model of the aerodynamic pressure. The Reynolds number for a commercially available 3.5 in. hard disk rotating at 10,000 RPM is about 600, which is substantially greater than 1. Note that the Reynolds number of the flow between two corotating disks in a disk stack will be even greater than this value because the inter-disk spacing is usually greater than the spacing between disk and the wall.
- The hard disks have high bending stiffness and for the satisfactory operations, the disk must maintain a near flat equilibrium.
- The aluminum substrate in the disk has significant material damping, which must be modeled.
- The multiple disk stacks are normal and the outer disks in a stack are subject to a dissimilar flow condition on their faces. A shear flow between the disk and rigid enclosure exists on one surface, and flow in a corotating enclosure is present on the other side, which raises the question of *whether the outer disks are more susceptible to flutter than the inner ones*.

Kim et al. (2000) present an experimental estimation technique for the flutter speed of a hard disk by utilizing the measurements taken at a subcritical speed. The technique is based on a simple fluid pressure model represented by a distributed, viscous pressure that rotates with respect to the disk (Hansen et al. 1999). This model approaches the special case of the pressure generated in a narrow gap at low Reynolds numbers. A very interesting observation has been made by Hansen et al. (1999). Namely, it is shown analytically that this specific type of aerodynamic loading differentially damps the forward and backward traveling waves, i.e., it damps the forward traveling waves much stronger than the backward ones. This differential damping exists primarily due to airflow effects because in vacuum the damping of the forward and backward traveling waves is identical (Hansen et al. 1999). At higher speeds, damping of the backward-traveling waves can vanish entirely leading to a *traveling wave flutter instability*. This result is exploited to extract model parameters from the frequency response function (FRF) of the acoustically excited disk at subcritical speeds. The method, developed in Hansen et al. (1999), is used to predict the supercritical speed, at which the damping of a backward-traveling wave vanishes and aeroelastic flutter occurs. In addition, the results of the paper show that the flutter speed and mode are strongly dependent on the enclosure gap width. The flutter speed can be as low as 35,000 rpm. A strong dependence of the flutter speed on the gap width confirms that the onset of flutter can be affected by means of airflow control through the enclosure design. The results also indicate that the onset of the aeroelastic flutter should not be a concern for the impending generation of 20,000 rpm drives, but possibly for the next generation of 30,000+ rpm drives. Lastly, the technique of Kim et al. (2000) and Hansen et al. (1999) can be applied to predict the flutter speed of optical disk systems used in the CD-ROM and DVD drives. Since the bending stiffness and natural frequencies are much smaller in these polycarbonate substrate disks, their flutter speed will also be lower.

Hansen et al. (1999) consider theoretical analysis related to the model they suggested. This analysis is briefly outlined below. Rotation of a single annular disk of the thickness h, the clamping radius a, and the outer radius b, is considered. Each disk in a stack can be modeled as isolated if the effects of spindle flexibility and bearing clearance are neglected. The disk substrate is assumed to be isotropic, with the Young's modulus E, the Poisson's ratio ν, and the density ρ. The disk spins at the constant speed Ω^*. A ground-fixed cylindrical coordinate frame (r^*,φ,z^*) is introduced (Fig. 3).

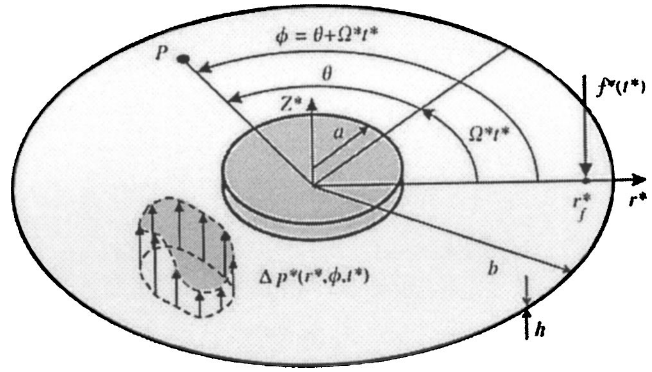

Fig. 3. Schematic of spinning disk system and definition of coordinate systems

The acoustic waves from a speaker, over a small circular area centered about the point (r_f^*,φ_f), are used to excite the disk transversely. The aerodynamic pressure difference across the two faces of the disk is Δp^*. Due to the large bending stiffness, the disk remains flat at equilibrium in the presence of aerodynamic pressure gradients. The governing equations for small amplitude transverse oscillations $u(r^*,\varphi,t^*)$ of a flat, spinning, linearly elastic disk has been presented by several authors (Hosaka and Crandall 1992; Renshaw 1998; Hansen et al. 1999). With the inclusion of the aerodynamic loading and acoustic excitation and the introduction of the dimensionless quantities—

$$u=\frac{u^*}{h},\quad r=\frac{r^*}{b},\quad \kappa=\frac{a}{b},\quad t=\frac{t^*}{t_0},$$

$$\Omega=\Omega^* t_0,\quad \Delta p^*=\frac{\Delta p^* t_0^2}{\rho h^2} \tag{47}$$

—where t^* represents time, and $t_0=(12(1-\nu)\rho b^4/(Eh^2))^{1/2}$ is the characteristic time constant, the field equations become

$$\ddot{u}+2\Omega\dot{u}_\varphi+\Omega^2 u_{\varphi\varphi}+D[\dot{u}]+K[u]=\Delta p+\frac{1}{r}f(t)|_A \tag{48}$$

where $f(t)$=acoustic excitation and A=surface of excitation. Further,

$$K[u]=\nabla^4 u-\frac{1}{r}(rN_{rr}u_r)_r-\frac{1}{r^2}(N_{\varphi\varphi}u_{\varphi\varphi}) \tag{49}$$

is the self-adjoint stiffness operator modeling the bending stiffness and stiffness caused by the membrane stresses $(N_{rr},N_{\varphi\varphi})$ of rotation. Substrate material viscoelastic effects are introduced through the self-adjoint, positive definite operator $D[\dot{u}]=\nabla^4\dot{u}$, where the material damping is assumed to be proportional to the rate of bending strain (Hosaka and Crandall 1992; Hansen et al. 1999). Finally, the boundary and the periodicity conditions for the plate deflection and the aerodynamic pressure are given in Hansen et al. (1999) by

$$u|_{r=k}=0,\quad [u_{rr}+\nu(u_{\varphi\varphi}+u_r)]|_{r=1}=0,$$

$$u_r|_{r=k}=0,\quad [(\nabla^2 u)_r+(1-\nu)(u_r-u)_{\varphi\varphi}]|_{r=1}=0,$$

$$u(r,\varphi,t)=u(r,\varphi+2\pi,t),\quad \Delta p(\kappa,\varphi,t)=\Delta p(1,\varphi,t)=0 \tag{50}$$

J. Aerosp. Eng. 2004.17:56-69.

Aeroelastic Coupling

The main feature of the lubrication model used by Hansen et al. (1999) is that Δp can be described in terms of a distributed viscous damping that rotates at half the rotation speed of the disk (Hosaka and Crandall 1992). It is a custom to assume that the rotating damping speed, being half of the disk rotation speed, arises because at very low Reynolds numbers, the mean flow speed in the gap is approximately half of the disk speed. The writers suggest retaining the rotating damping model because it describes qualitatively many experimental observations. However, in this study, contrary to the usual assumption that the viscous damping rotates at half the rotation speed of the disk, no prescription of the speed of the rotating damping is provided. Instead, the speed will be reduced experimentally. Further, in the lubrication model, the speed of rotating damping is independent of the mode of vibration of the disk. The writers now allow the rotating damping speed to depend on the number of modal diameters of the excited mode, which leads to the following generalization of the aerodynamic loading (in a ground-fixed frame):

$$\nabla^2(\Delta p) = -\alpha[\dot{u} + (\Omega - \Omega_{dmn})u_\varphi] \tag{51}$$

where ∇^2=Laplacian operator; α=positive parameter dependent on the viscosity of the fluid, the rotation speed Ω, and the gap width. Further, (m,n) are the nodal circle and nodal diameter number of the particular mode of vibration, respectively. Therefore, this generalization admits the rotating damping speed to depend on the excited mode, allowing a nonlinear variation of the rotating damping speed with disk speed as well. Choosing the speed of rotating damping $\Omega_{dmn}=\Omega/2$ yields the lubrication theory model for pressure loading. The effects of radial flow (Huang and Mote 1996) are also neglected in this analysis because the radial flow effect in this shrouded disk is shown experimentally not to be significant. Lastly, Hansen et al. (1999) address another question: *whether outer disks in a disk stack are more susceptible to flutter*. The speed of rotating damping with respect to the disk is expected to be greater in the gap between the disk and rigid enclosure than between the two corotating disks since the mean flow in the disk-rigid enclosure gap is smaller due to the high transverse shear. For this reason, the highest probability for flutter occurs in a single disk with a base plate and a rigid cover on each side. Based on this reasoning, the writers focus on a single disk in a stack.

Coupled Eigenvalue Problem

Taking into account that the Laplacian together with the boundary conditions from Eq. (50) defines a self-adjoint operator, one can find Δp from Eq. (51) by using the Green's function and have $\Delta p = -C[u_t + (\Omega - \Omega_{dmn})u_\varphi]$, with C being a self-adjoint operator. A solution taken in a separable form is

$$u(r,\varphi,t) = R_{mn}(r)e^{in\varphi + \lambda t} \tag{52}$$

where $R_{mn}(r)$=normalized complex valued functions, and (m,n)=number of nodal circles and diameters of the mode. Substituting Eq. (52) into Eq. (48), in the absence of acoustic excitation, yields the coupled aeroelastic eigenvalue problem

$$(\lambda^2 + i2\lambda n\Omega - n^2\lambda^2)R_{mn} + K_n^r[R_{mn}] + \lambda D_n^r[R_{mn}] + [\lambda + in(\Omega - \Omega_{dmn})]C_n^r[R_{mn}] = 0 \tag{53}$$

where (K_n^r, D_n^r, C_n^r)=one-dimensional differential operators obtained by the substitution of the assumed mode shape into the spatial operators (K,D,C). Aeroelastic coupling between the modes possessing different number of nodal circles is neglected (Hansen et al. 1999). Taking the inner products of Eq. (53) with each normalized eigenfunction gives

$$(\lambda_{mn} + in\Omega)^2 + \omega_{mn}^2 + c_{mn}[\lambda_{mn} + in(\Omega - \Omega'_{dmn})] = 0 \tag{54}$$

where $\omega_{mn}^2 = \langle R_{mn}, K_r^n[R_{mn}]\rangle$=uncoupled natural frequencies of the modes (corotating frame), and are approximated in (D'Angelo and Mote 1993b) as

$$\omega_{mn}^2 \approx (\omega_{mn}^{st})^2 + s_{mn}\Omega^2 \tag{55}$$

where ω_{mn}^{st}=natural frequency of the same mode in a stationary disk, and s_{mn} are corrective positive constants. Further

$$c_{mn} = \langle R_{mn}, D_r^n[R_m]\rangle + \langle R_{mn}, C_r^n[R_m]\rangle \tag{56}$$

represents the combined effects of structural and corotating fluid dampings, and

$$\Omega'_{dmn} = \Omega_{dmn}\frac{\langle R_m, C_r^n[R_m]\rangle}{c_{mn}} \tag{57}$$

where Ω'_{dmn}=effective rotating damping speed modified by the ratio of aerodynamic corotating damping to the total corotating damping of the mode. Thus, the greater the structural dissipation of the substrate material, the lower the effective rotation damping speed. Solution for the eigenvalues under assumption of weak damping yields

$$\lambda_{mn}^F = -\frac{c_{mn}}{2}\left(1 + \frac{n\Omega'_{dmn}}{\omega_{mn}}\right) - i\{\omega_{mn} + n\Omega\},$$

$$\lambda_{mn}^B = -\frac{c_{mn}}{2}\left(1 - \frac{n\Omega'_{dmn}}{\omega_{mn}}\right) + i(\omega_{mn} - n\Omega) \tag{58}$$

where the superscripts F and B refer to forward or backward-traveling waves that propagate in the direction of disk rotation and against it. These waves are abbreviated FTW and BTW. Thus, the immediate effect of the circulatory term in the aerodynamic pressure is to cause FTW to be more highly damped than BTW. This phenomenon has been observed in a hard disk drive at subcritical speeds. As can be seen from Eq. (58), the damping of a BTW can vanish at the onset of aeroelastic flutter. The condition for the onset of aeroelastic traveling wave flutter is

$$\frac{\omega_{mn}}{n} = \Omega'_{dmn} \tag{59}$$

or when the effective rotating damping speed equals an uncoupled wave speed of the disk. Similarly, from Eq. (58), one can see that the frequencies of the backward and forward-traveling waves split and at a critical speed, $\Omega_c = \omega_{mn}/n$, the frequency of a BTW equals zero. Lastly, the frequency response function of the disk, acoustically excited at $(r_f^*, 0)$ and measured at (r_x^*, φ_x), corresponding to the eigenvalue problem Eq. (53), can be approximated by (Lee and Kim 1995)

$$H(\omega) \approx R_{mn}(r_f^*)R_{mn}(r_x^*)(H_{mn}e^{in\varphi_x} + \overline{H}_{mn}(-\omega e^{in\varphi_x}) \tag{60}$$

where

$$H_{mn}(\omega) = 1/[\omega_{mn}^2 - (\omega + n\Omega)^2 + ic_{mn}(\omega + n\Omega - n\Omega'_{dmn})] \tag{61}$$

So, as follows from the calculations and has been verified experimentally, the damping of forward-traveling waves increases with the rotation speed, while the damping of backward-traveling waves decreases. An experimental technique for estimation of the flutter speed of a hard disk drive is also presented in the paper. Modal parameters are extracted from the frequency response function of an acoustically excited disk spinning at a subcritical speed ranging from 6,000 to 19,800 rpm. The flutter speed was estimated through the extrapolation of the effective rotating damping speed and the wave speed. The results indicate that the aeroelastic flutter speed can be as low as 35,000 rpm and depend significantly on the air gap between the disk and the cover.

Investigation initiated in Kim et al. (2000) has been continued in Hansen et al. (2001), where an experimental method for predicting flutter of a spinning disk has been developed and illustrated by an experiment with a steel disk spinning in air. In the experiment, it has been observed that the damping of forward-traveling waves increases with the rotation speed, while the damping of backward-traveling waves decreases, which is in good agreement with theoretical predictions of Kim et al. (2000). A reference experiment, performed on the disk in near vacuum, showed no systematic difference in the damping between these traveling wave pairs. From experiments conducted in air, the speeds and magnitudes of the damping forces for the four lowest modes have been estimated at increasing rotation speeds. It is shown that the damping speeds increase nonlinearly with the rotation speed. By assuming that the trends of the damping speeds continue until flutter occurs, it was possible to predict the flutter speed and mode of the disk.

In Guo and Chen (2001), a very important question concerning *data storage capacity* of magnetic hard disks was raised. In recent years, this storage capacity of hard disk drives has grown by over 100% every year, which in turn requires track density to grow very rapidly. While current drives typically have track densities around 35,000 tracks per inch, there is a strong upward push, which will require significant improvement of actuator track following control to reduce track misregistration (TMR). At the same time, as the spin speed increases, disk flutter becomes an increasingly dominant contributor to the TMR. In the past, some investigations have been carried out aiming at characterizing disk flutter and its impact on head off track. Bouchard and Talke (1986) have investigated disk flutter magnitude with respect to radius of the disk and found that the flutter amplitude increases almost linearly with the disk radius. They have also concluded that the flutter amplitude density distribution is very close to Gaussian while flutter peak value has a Rayleigh distribution. More recently, these writers proposed a method to reduce disk flutter by altering the air flow path inside a drive.

In Heo et al. (2001), the writers address the same question of controlling severe turbulent excitations from the surrounding air in high-speed driving. Past and present research on disk flutter includes flow visualization (Abrahamson et al. 1991; Lennemann 1974) and disk flutter reduction (Imai et al. 1999). Results of flow visualization show the existence of large-scale turbulent eddies between outer parts of the disks, in addition to three-dimensional toroidal vortices, which appear in the turbulent boundary layer between the disk outer rim and the shroud (Abrahamson et al. 1991). It has been shown that reducing the disk-to-shroud spacing and the shroud opening-angle can effectively reduce disk flutter (Heo et al. 2001). The purpose of Heo et al. (2001) is to demonstrate additional novel aerodynamic designs to reduce flutter.

Concluding Remark. The model studied by Hansen et al. (1999) is reasonably complicated, and rigorous mathematical analysis would be a challenging objective for future research. However, using their model, the writers were able to find satisfactory approximations for such model parameters as $\tilde{\omega}_{0n}$, c_{0n}, and $\Omega'_{d_{on}}$ as functions of a rotational speed, which is necessary for the prediction of the onset of flutter of a spinning disk. Furthermore, the writers suggested and conducted experiments for obtaining the frequency response functions. From the experimental results, the model parameters have been estimated as functions of the rotational speed at preflutter speeds and then those parameters have been used for prediction of the flutter speed, which is an ultimate goal for any flutter analysis. Evaluating their model analytically, numerically, and verifying it experimentally, the writers have indeed proven that when a disk is spinning in vacuum, no systematic difference in the damping characteristics for FTW and BTW exists. However, the damping factors of the wave pairs (obtained numerically and experimentally for a disk spinning in the air) clearly indicate that FTWs are more highly damped than the RTWs due to influence of the air flow. The difference between the damping factors increases with the rotational speed. The aforementioned observation can be explained by representing the aerodynamic pressure as a distributed rotating damping force.

Conclusions

In the present paper, modeling and partial analytical and numerical studies of flutter phenomena in different engineering areas have been discussed. Even though the mathematical investigation is not rigorous enough, qualitative (and sometimes quantitative) results are interesting and important. In each area, the appropriate theory has been applied to clarify some practical questions. For example, the theory of the section ‘‘Bending-Torsion Coupled Beams under Deterministic and Random Loads’’ (see Eslimy-Isfahany et al. 1996) has been developed to carry out the response analysis of a flexible structure when it is subjected to deterministic and/or random loads. Applications include aircraft wings, for which bending and torsional motions are coupled due to noncoincident mass and shear centers. The writers have presented numerical simulations for a cantilever aircraft wing under both deterministic and random loads, for which simple Euler-Bernoulli theory cannot be applied. Adam (1999) has also studied the initial boundary-value problem of coupled bending-torsion vibrations of elastic monosymmetric beams. The new numerical approach developed by the author is extremely efficient for numerical simulations. An illustrating example is given for a simply supported composite beam under harmonic excitation. The obtained results have shown high improvement when compared to the standard modal analysis approach.

The problem discussed in ‘‘Flutter of Transmission Lines,’’ regarding flutter of coupled bending-torsion vibrations when a frequency ration is close to one (Nakamura 1996), has been given analytically in the form of a system of two coupled equations. However, the writer makes no attempt to solve this system. He just simplifies the model using certain physical and engineering information in order to find acceptable formulas for vibration frequencies assuming that coupling between different types of motions is very small. The writer has discovered (and then supported the discovery experimentally) that the stability of the system is vitally related to the resonance between the two degrees of freedom. On the basis of their results, the writer has suggested that although the growth rate of vibration amplitude tends to infinity at resonance due to the crude approximations in the work, the formulas could still be useful for qualitative discussion. He has shown that there exists a transition in instability mode from one branch to another when increasing the wind speed through resonance. For example, if the bending branch is strongly unstable before resonance, the torsion branch is strongly stable. The situation is reversed when the resonance speed is exceeded, i.e., the torsion branch becomes strongly stable while the bending branch becomes strongly unstable. In my opinion, it would be extremely important to obtain the aforementioned results by carrying out analytical investigation of the corresponding initial boundary-value problem. Moreover, analytical study can provide some ideas of how to implement efficient control and prevention of conductor galloping.

The problem considered in ''Flutter of Helicopter, Propeller, and Turbine Blades'' is known as an extremely important one in the area of rotoctrafts and turbomachinary. To analyze the mathematical setting of the problem, the writers have used the Galerkin's approximations method and then proposed a new dynamic finite-element numerical method to calculate the natural frequencies and mode shapes of coupled bending-torsion vibrations of a uniform axially loaded beam. Numerical results showed very good agreement with the available literature results, which confirms the correctness of the theory. The advantage of the new method is that it can be extended to more complex cases such as coupled bending-torsion vibrations of nonuniform axially loaded or centrifugally damped beams.

A model of a high-speed rotating disk presented in ''Flutter in Hard Disk Drives'' is very difficult from an engineering point of view and really sophisticated from mathematical point of view. Hansen et al. (1999) have systematically studied their model both analytically and experimentally, and have obtained deep results. Namely, they were able to predict and confirm that the reason for different damping of FTWs and BTWs is exactly because of the aerodynamic loading. The writers also gave their version on which parameters of the model should be taken into account in order to predict the flutter speed and mode.

Summarizing all of the above, I would like to emphasize that even though many researchers working in the field of flutter analysis have made attempts to carry out mathematical analysis of existing and newly derived models, the accuracy and rigorousness were often insufficient. However, the importance of mathematical and physical analysis of the problem should not be underestimated, since theoretical results can generate insights not available from purely numerical simulations or experimental verification of the faithfulness of the models.

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