Ecodesign and 3D topology optimization

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Summary

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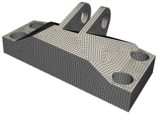
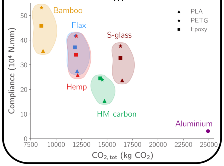
Proposed workflow



STEP 1
MATERIAL SELECTION

Variables
- Fiber
- Resin

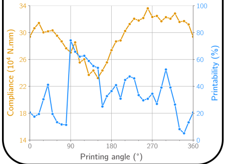
Objectives
- Min compliance
- Min CO_2 footprint
- Min cost
- …



STEP 2
PRINTING DIRECTION

Variables
- Printing direction

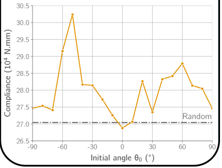
Objectives
- Min compliance
- Max printability w/o supports

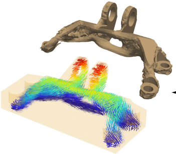


STEP 3
RESULTS REFINEMENT

Variables
- Initial conditions for design variables

Objectives
- Min compliance





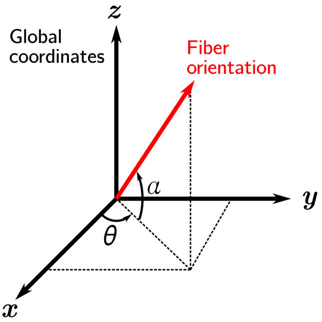
$$\min_{\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}} C(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}) = \left(\sum_{i\in LC} c_i(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha})^n\right)^{\frac{1}{n}}$$

$$= \left(\sum_{i\in LC}\left(\sum_{e} \rho_e^p \boldsymbol{u}_{\boldsymbol{e,i}}^{\boldsymbol{T}} \boldsymbol{k_0}(\theta_e,\alpha_e)\boldsymbol{u}_{\boldsymbol{e,i}}\right)^n\right)^{\frac{1}{n}}$$

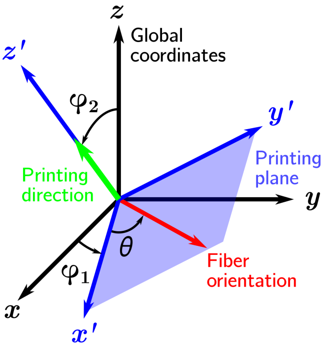
$$\text{s.t.}\quad \begin{cases} \frac{V(\rho)}{V_0} \le f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \le \boldsymbol{\rho} \le 1 \\ -\frac{\pi}{2} \le \boldsymbol{\theta} \le \frac{\pi}{2} \\ -\frac{\pi}{2} \le \boldsymbol{\alpha} \le \frac{\pi}{2} \end{cases}$$

Problem formulation

Free



In layers



$$\text{Printing direction} = [n_x, n_y, n_z]^T$$

$$\varphi_1 = -\arctan \frac{n_x}{n_y}$$

$$\varphi_2 = -\arctan \frac{\sqrt{n_x^2 + n_y^2}}{n_z}$$

Filtering

- Two independent convolution filters with weights linear on the distance between element centers $\Delta(e, i)$, applied in the end of each iteration

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- **Density filter:** avoid checkerboard pattern by setting a minimum feature size, applied on the sensitivities:

$$\rho_e \widetilde{\frac{\partial c}{\partial \rho_e}} = \frac{1}{\sum_i H^\rho_{ei}} \sum_i H^\rho_{ei} \, \rho_i \frac{\partial c}{\partial \rho_i}$$

$$H^\rho_{ei} = \max(0, r_\rho - \Delta(e, i))$$

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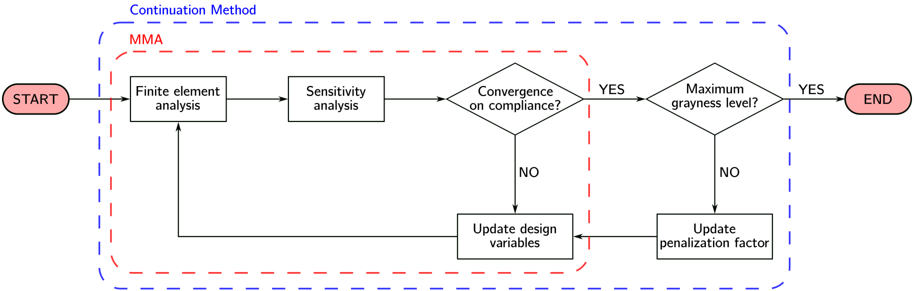
$$H_{ei}^{\rho} = \max(0, r_{\rho} - \Delta(e,i))$$

- **Orientation filter:** ensure fiber continuity by setting a minimum curvature radius, applied on the angles

$$\begin{pmatrix} \tilde{\theta}_e \\ \tilde{\alpha}_e \end{pmatrix} = \frac{1}{\sum_i H_{ei}^{\theta} \rho_i} \sum_i H_{ei}^{\theta} \rho_i \begin{pmatrix} \theta_i \\ \alpha_i \end{pmatrix}$$

$$H_{ei}^{\theta} = \max(0, r_{\theta} - \Delta(e,i))$$

- Variable updating: MMA - Python
- Finite element analysis: Ansys - PyMAPDL
- Continuation method on the penalization factor p



- Rule of mixtures – 2-phase micromechanical model

$$E_x = E_f V_f + E_m(1 - V_f)$$

$$E_y = \frac{E_f E_m}{E_f(1 - V_f) + E_m V_f}$$

$$\nu_{xy} = \nu_f V_f + \nu_m(1 - V_f)$$

$$\nu_{yz} = \nu_{xy} \frac{1 - \nu_{xy}\frac{E_y}{E_x}}{1 - \nu_{xy}}$$

$$G_{xy} = \frac{G_f G_m}{G_f(1 - V_f) + G_m V_f}$$

$$\rho = \rho_f V_f + \rho_m(1 - V_f)$$

- Constitutive matrix - transversely isotropic material

$$
\boldsymbol{C} = \begin{bmatrix}
\frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 & 0 & 0 \\
-\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{yz}}{E_y} & 0 & 0 & 0 \\
-\frac{\nu_{xy}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_y} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2(1+\nu_{yz})}{E_y} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}}
\end{bmatrix}^{-1}
$$

- Constitutive matrix - transversely isotropic material

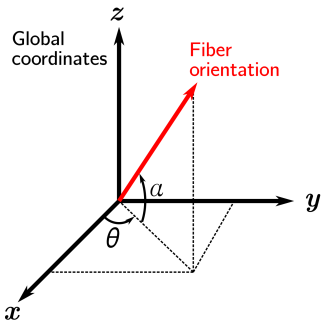
$$\boldsymbol{C} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{yz}}{E_y} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_y} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{yz})}{E_y} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix}^{-1}$$

- Rotation matrices around x and z

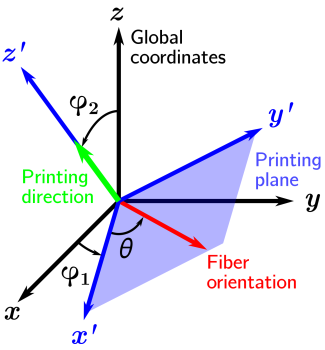
$$\boldsymbol{T_x}(\phi) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos^2\phi & \sin^2\phi & -2\sin\phi\cos\phi & 0 & 0 \\ 0 & \sin^2\phi & \cos^2\phi & 2\sin\phi\cos\phi & 0 & 0 \\ 0 & \sin\phi\cos\phi & -\sin\phi\cos\phi & \cos^2\phi - \sin^2\phi & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\phi & \sin\phi \\ 0 & 0 & 0 & 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$\boldsymbol{T_z}(\phi) = \begin{bmatrix} \cos^2\phi & \sin^2\phi & 0 & 0 & 0 & -2\sin\phi\cos\phi \\ \sin^2\phi & \cos^2\phi & 0 & 0 & 0 & 2\sin\phi\cos\phi \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\phi & \sin\phi & 0 \\ 0 & 0 & 0 & -\sin\phi & \cos\phi & 0 \\ \sin\phi\cos\phi & -\sin\phi\cos\phi & 0 & 0 & 0 & \cos^2\phi - \sin^2\phi \end{bmatrix}$$

Free



In layers



$$\boldsymbol{T} = \boldsymbol{T_x}(\alpha_e)\,\boldsymbol{T_z}(\theta_e)\,\boldsymbol{T_x}(\varphi_2)\,\boldsymbol{T_z}(\varphi_1)$$

$$\boldsymbol{C}_{xyz} = \boldsymbol{T}\,\boldsymbol{C}\,\boldsymbol{T}^{\boldsymbol{T}}$$

- Sensitivity with respect to the density:

$$\frac{\partial c_i}{\partial \rho_e} = -p\,\rho_e^{p-1}\,\boldsymbol{u_{e,i}^T}\,\boldsymbol{k_0}\,\boldsymbol{u_{e,i}}$$

Sensitivity analysis

- Sensitivity with respect to the density:

$$\frac{\partial c_i}{\partial \rho_e} = -p\,\rho_e^{p-1}\,\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}}^{\boldsymbol{T}}\,\boldsymbol{k}_0\,\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}} = -\frac{p}{\rho_e}\,\underbrace{\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}}^{\boldsymbol{T}}\,(\rho_e^p\,\boldsymbol{k}_0)\,\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}}}_{\substack{2\,\times\,\text{elemental}\\ \text{strain energy}}}$$

- Sensitivity with respect to the density:

$$\frac{\partial c_i}{\partial \rho_e} = -p\,\rho_e^{p-1}\,\boldsymbol{u_{e,i}^T}\,\boldsymbol{k_0}\,\boldsymbol{u_{e,i}} = -\frac{p}{\rho_e}\,\underbrace{\boldsymbol{u_{e,i}^T}\,(\rho_e^p\,\boldsymbol{k_0})\,\boldsymbol{u_{e,i}}}_{\substack{2\,\times\,\text{elemental}\\ \text{strain energy}}}$$

- Sensitivity with respect to the orientations:

$$\frac{\partial c_i}{\partial \theta_e} = -\rho_e^p\,\boldsymbol{u_{e,i}^T}\,\frac{\partial \boldsymbol{k_0}}{\partial \theta_e}\,\boldsymbol{u_{e,i}}, \qquad \frac{\partial c_i}{\partial \alpha_e} = -\rho_e^p\,\boldsymbol{u_{e,i}^T}\,\frac{\partial \boldsymbol{k_0}}{\partial \alpha_e}\,\boldsymbol{u_{e,i}}$$

$$\frac{\partial \boldsymbol{k_0}}{\partial \theta_e} = \iiint \boldsymbol{B_e^T}\,\frac{\partial \boldsymbol{C}_{xyz}}{\partial \theta_e}\,\boldsymbol{B_e}\,d\Omega, \qquad \frac{\partial \boldsymbol{k_0}}{\partial \alpha_e} = \iiint \boldsymbol{B_e^T}\,\frac{\partial \boldsymbol{C}_{xyz}}{\partial \alpha_e}\,\boldsymbol{B_e}\,d\Omega$$

- Sensitivity with respect to the density:

$$\frac{\partial c_i}{\partial \rho_e} = -p\,\rho_e^{p-1}\,\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}}^{\boldsymbol{T}}\,\boldsymbol{k}_0\,\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}} = -\frac{p}{\rho_e}\,\underbrace{\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}}^{\boldsymbol{T}}\,(\rho_e^p\,\boldsymbol{k}_0)\,\boldsymbol{u}_{\boldsymbol{e},\boldsymbol{i}}}_{\substack{2\,\times\,\text{elemental}\\ \text{strain energy}}}$$

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$$\frac{\partial \boldsymbol{k}_0}{\partial \theta_e} = \iiint \boldsymbol{B}_{\boldsymbol{e}}^{\boldsymbol{T}}\,\frac{\partial \boldsymbol{C}_{xyz}}{\partial \theta_e}\,\boldsymbol{B}_{\boldsymbol{e}}\,d\Omega, \qquad \frac{\partial \boldsymbol{k}_0}{\partial \alpha_e} = \iiint \boldsymbol{B}_{\boldsymbol{e}}^{\boldsymbol{T}}\,\frac{\partial \boldsymbol{C}_{xyz}}{\partial \alpha_e}\,\boldsymbol{B}_{\boldsymbol{e}}\,d\Omega$$

- Final sensitivities

$$\frac{\partial C}{\partial\,\cdot} = \sum_{i\in LC} c_i^{n-1}\,C^{1-n}\,\frac{\partial c_i}{\partial\,\cdot}$$

CO_2 footprint assessment

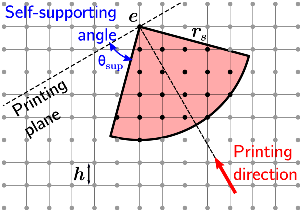
- Material CO_2 intensity

$$CO^i_{2,mat} = \frac{\rho_f\, V_f\, CO^i_{2,f} + \rho_m\,(1 - V_f)\, CO^i_{2,m}}{\rho}$$

- Total CO_2 footprint: material production + use phase

$$CO_{2,mat} = M \cdot CO^i_{2,mat}$$
$$CO_{2,use} = M \cdot 98.8\ \mathrm{tCO_2/kg}$$
$$CO_{2,tot} = CO_{2,mat} + CO_{2,use}$$

- Overhang support neighborhood N_e^S



- Support variable ρ_e^S

$$\rho_e^S = \frac{\tanh(\beta_T T) + \tanh(\beta_T(\mu_e^S - T))}{\tanh(\beta_T T) + \tanh(\beta_T(1 - T))}$$

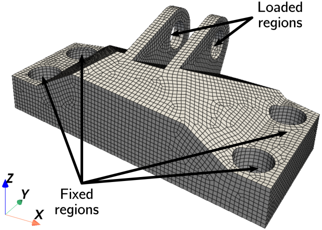
$$T = \frac{3}{2\pi(1 - \sin\theta_{sup})} \frac{h^2}{r_S^2}$$

$$\mu_e^S = \frac{1}{\#N_e^S} \sum_{i \in N_e^S} \rho_i \cdot \rho_i^S$$

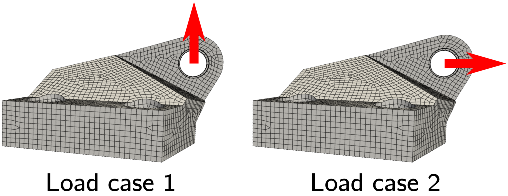
- **Printability score:** average of ρ_e^S of all elements, weighted by ρ_e

Studied problem

- Domain:



- Load cases: 1) 35586 N vertical force, 2) 37810 N horizontal force



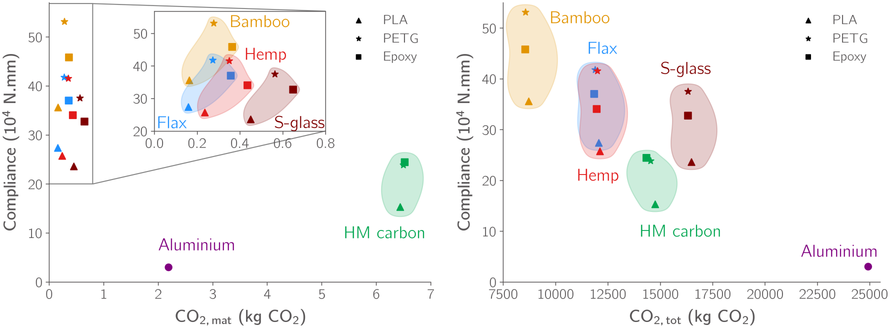
- Optimization parameters

Parameter	Value
Prescribed volume fraction (f)	0.2
Density filter radius (r_ρ)	5 mm
Orientation filter radius (r_θ)	7 mm
Support set radius (r_S)	$1.5r_\rho = 7.5$ mm
Minimum self-supporting angle (θ_{sup})	45°
Heaviside parameter - support evaluation (β_T)	25
Density threshold - void element	0.1
Density threshold - filled element	0.9
Maximum greyness	0.3
Initial penalization factor	1.0
Penalization factor step - continuation method	0.5
Norm for compliance aggregation (n)	8

- Multiple fiber/resin couples, fiber volume fraction $V_f = 0.5$

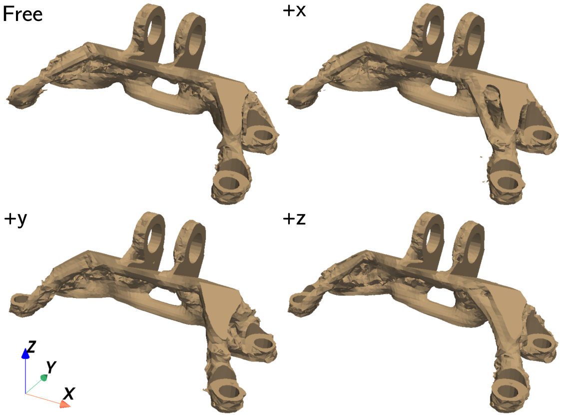
	Material	ρ (kg/m^3)	E (GPa)	ν	CO_2^i (kg CO_2/kg)
Fibers	Bamboo	700	17.5	0.39	1.0565
	Flax	1470	53.5	0.355	0.44
	Hemp	1490	62.5	0.275	1.6
	HM Carbon	2105	760	0.105	68.1
	S-Glass	2495	89.5	0.22	2.905
Resins	PLA	1255	3.45	0.39	2.28
	PETG	1270	2.06	0.403	4.375
	Epoxy	1255	2.41	0.399	5.94

- Selected material: flax/PLA

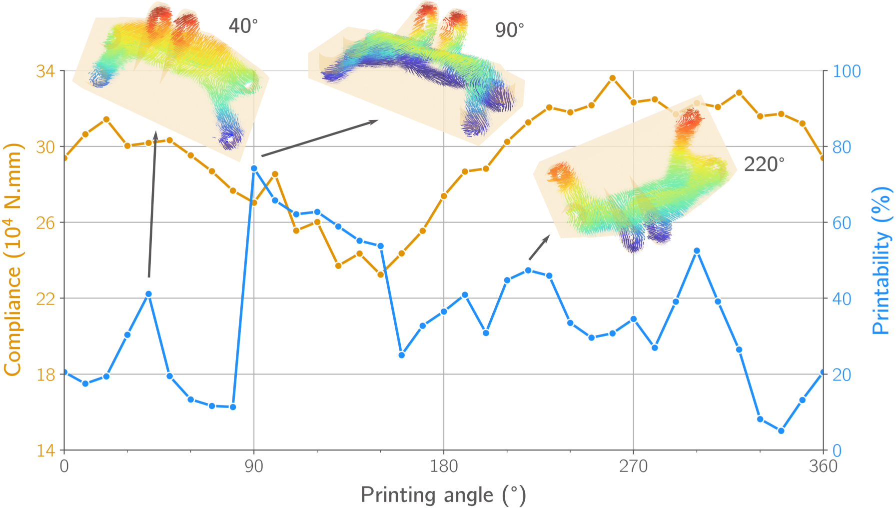


Printing direction

Printing direction	C (10^4 N.mm)	c_1 (10^4 N.mm)	c_2 (10^4 N.mm)	Printability (%)
Free	33.86	32.99	27.48	–
$+x$	32.53	32.13	24.20	70.3
$+y$	26.38	25.71	21.38	18.8
$+z$	27.34	26.80	21.54	73.7



- Printing direction inside *yz* plane: $+y = 0°$ and $+z = 90°$
- Chosen printing direction: 90°, i.e. along $+z$



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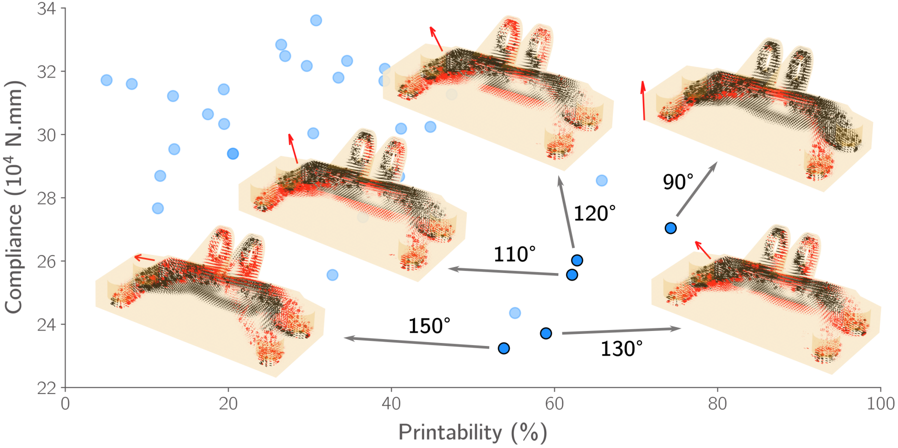
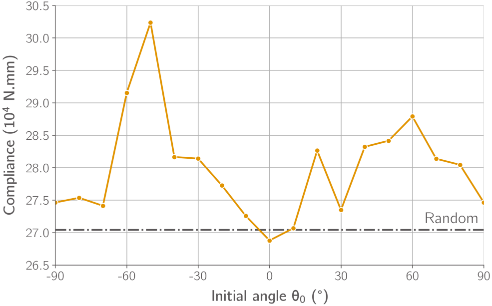
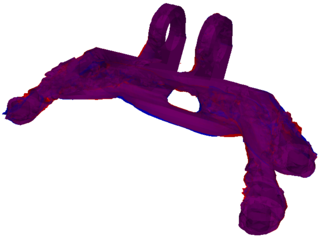


- Influence of the initial orientations



Random initial orientations
Compliance: 27.04×10^4 N.mm
Printability: 74.2%

Initial orientation: 0°
Compliance: 26.88×10^4 N.mm
Printability: 73.1%



Summary

Conclusions

- There is a clear separation of the effects of each parameter on the objectives: steps present design choices that are almost uncoupled
  - Material selection: compliance - CO_2 footprint compromise
  - Printing direction selection: compliance - printability compromise
- Using random initial orientations is a good strategy: refinement gives compliance gains in the order of 1%