Ecodesign and 3D topology optimization

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Summary

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Context

- Algorithm to simultaneously optimize topology and fiber orientation of 3D printed structures
- Variables of interest: compliance, CO_2 footprint, mass
- Main code written in Python, using Ansys finite element solver
- Utilization of 3D elements

Problem formulation

- **First approach to 3D optimization:**
  - Choose printing direction, which defines printing layers
  - Fiber orientations defined as a rotation inside the printing plane
  - Design variables (for each element): density ρ and orientation θ

- Optimization for compliance minimization, based on SIMP method:

$$\min_{\boldsymbol{\rho},\boldsymbol{\theta}} c(\boldsymbol{\rho}, \boldsymbol{\theta}) = \sum_{e} \rho_e^p \, \boldsymbol{u_e^T} \, \boldsymbol{k_0}(\theta_e) \, \boldsymbol{u_e}$$

$$\text{s.t.} \begin{cases} \frac{V(\boldsymbol{\rho})}{V_0} \leq f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \leq \boldsymbol{\rho} \leq 1 \\ -\frac{\pi}{2} \leq \boldsymbol{\theta} \leq \frac{\pi}{2} \end{cases}$$

- **Second approach to 3D optimization:**
  - Orientations freely oriented in space, hard to achieve with current manufacturing methods
  - Fiber orientations defined by spherical angles
  - Design variables (for each element): density ρ, in-plane orientation θ and out-of-plane orientation α
- Optimization for compliance minimization, based on SIMP method:

$$\min_{\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}} c(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}) = \sum_e \rho_e^p \, \boldsymbol{u_e^T} \, \boldsymbol{k_0}(\theta_e,\alpha_e) \, \boldsymbol{u_e}$$

$$\text{s.t.} \begin{cases} \frac{V(\boldsymbol{\rho})}{V_0} \leq f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \leq \boldsymbol{\rho} \leq 1 \\ -\frac{\pi}{2} \leq \boldsymbol{\theta} \leq \frac{\pi}{2} \\ -\frac{\pi}{2} \leq \boldsymbol{\alpha} \leq \frac{\pi}{2} \end{cases}$$

Variable updating

- Method of Moving Asymptotes - MMA
- Sensitivity with respect to the density:

$$\frac{\partial c}{\partial \rho_e} = -p\,\rho_e^{p-1}\,\boldsymbol{u_e^T}\,\boldsymbol{k_0}\,\boldsymbol{u_e}$$

- Sensitivity with respect to the orientations:

$$\frac{\partial c}{\partial \theta_e} = -\rho_e^p\,\boldsymbol{u_e^T}\,\frac{\partial \boldsymbol{k_0}}{\partial \theta_e}\,\boldsymbol{u_e^T}, \quad \frac{\partial c}{\partial \alpha_e} = -\rho_e^p\,\boldsymbol{u_e^T}\,\frac{\partial \boldsymbol{k_0}}{\partial \alpha_e}\,\boldsymbol{u_e^T}$$

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- Sensitivity with respect to the density:

$$\frac{\partial c}{\partial \rho_e} = -p\, \rho_e^{p-1}\, \boldsymbol{u_e^T}\, \boldsymbol{k_0}\, \boldsymbol{u_e} = -\frac{p}{\rho_e}\, \underbrace{\rho_e^p \boldsymbol{u_e^T}\, \boldsymbol{k_0}\, \boldsymbol{u_e}}_{\substack{2\,\times\,\text{element}\\ \text{strain energy}}}$$

- Sensitivity with respect to the orientations:

$$\frac{\partial c}{\partial \theta_e} = -\rho_e^p\, \boldsymbol{u_e^T}\, \frac{\partial \boldsymbol{k_0}}{\partial \theta_e}\, \boldsymbol{u_e^T}, \quad \frac{\partial c}{\partial \alpha_e} = -\rho_e^p\, \boldsymbol{u_e^T}\, \frac{\partial \boldsymbol{k_0}}{\partial \alpha_e}\, \boldsymbol{u_e^T}$$

- Method of Moving Asymptotes - MMA
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- Sensitivity with respect to the orientations:

$$\frac{\partial c}{\partial \theta_e} = -\rho_e^p\,\boldsymbol{u_e^T}\,\frac{\partial \boldsymbol{k_0}}{\partial \theta_e}\,\boldsymbol{u_e^T}, \qquad \frac{\partial c}{\partial \alpha_e} = -\rho_e^p\,\boldsymbol{u_e^T}\,\frac{\partial \boldsymbol{k_0}}{\partial \alpha_e}\,\boldsymbol{u_e^T}$$

$$\frac{\partial \boldsymbol{k_0}}{\partial \theta_e}(\theta_e, \alpha_e) = \iiint \boldsymbol{B_e^T}\,\boldsymbol{T_\alpha}\left(\frac{\partial \boldsymbol{T_\theta}}{\partial \theta_e}\,\boldsymbol{C}\,\boldsymbol{T_\theta^T} + \boldsymbol{T_\theta}\,\boldsymbol{C}\,\frac{\partial \boldsymbol{T_\theta^T}}{\partial \theta_e}\right)\boldsymbol{T_\alpha^T}\,\boldsymbol{B_e}\,d\Omega$$

$$\frac{\partial \boldsymbol{k_0}}{\partial \alpha_e}(\theta_e, \alpha_e) = \iiint \boldsymbol{B_e^T}\left(\frac{\partial \boldsymbol{T_\alpha}}{\partial \alpha_e}\boldsymbol{T_\theta}\boldsymbol{C}\boldsymbol{T_\theta^T}\boldsymbol{T_\alpha^T} + \boldsymbol{T_\alpha}\boldsymbol{T_\theta}\boldsymbol{C}\boldsymbol{T_\theta^T}\frac{\partial \boldsymbol{T_\alpha^T}}{\partial \alpha_e}\right)\boldsymbol{B_e}\,d\Omega$$

- Two independent convolution filters with weights linear on the distance between element centers $\Delta(e, i)$, applied in the end of each iteration

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$$\rho_e \widetilde{\frac{\partial c}{\partial \rho_e}} = \frac{1}{\sum_i H^{\rho}_{ei}} \sum_i H^{\rho}_{ei}\, \rho_i \frac{\partial c}{\partial \rho_i}$$

$$H^{\rho}_{ei} = \max(0, r_\rho - \Delta(e, i))$$

Filtering

- Two independent convolution filters with weights linear on the distance between element centers $\Delta(e, i)$, applied in the end of each iteration
- **Density filter:** avoid checkerboard pattern by setting a minimum feature size, applied on the sensitivities:

$$\rho_e \widetilde{\frac{\partial c}{\partial \rho_e}} = \frac{1}{\sum_i H^{\rho}_{ei}} \sum_i H^{\rho}_{ei}\, \rho_i \, \frac{\partial c}{\partial \rho_i}$$

$$H^{\rho}_{ei} = \max(0, r_{\rho} - \Delta(e, i))$$

- **Orientation filter:** ensure fiber continuity by setting a minimum curvature radius

$$\begin{pmatrix} \tilde{\theta}_e \\ \tilde{\alpha}_e \end{pmatrix} = \frac{1}{\sum_i H^{\theta}_{ei}\, \rho_i} \sum_i H^{\theta}_{ei}\, \rho_i \begin{pmatrix} \theta_i \\ \alpha_i \end{pmatrix}$$

$$H^{\theta}_{ei} = \max(0, r_{\theta} - \Delta(e, i))$$

- Rule of mixtures - transversely isotropic material

$$E_x = E_f V_f + E_m(1 - V_f)$$

$$E_y = \frac{E_f E_m}{E_f(1 - V_f) + E_m V_f}$$

$$\nu_{xy} = \nu_f V_f + \nu_m(1 - V_f)$$

$$\nu_{yz} = \nu_{xy} \frac{1 - \nu_{xy}\frac{E_y}{E_x}}{1 - \nu_{xy}}$$

$$G_{xy} = \frac{G_f G_m}{G_f(1 - V_f) + G_m V_f}$$

$$\rho = \rho_f V_f + \rho_m(1 - V_f)$$

Material properties

- Material CO_2 intensity

$$CO^{i}_{2,mat} = \frac{\rho_f \, V_f \, CO^{i}_{2,f} + \rho_m \, (1 - V_f) \, CO^{i}_{2,m}}{\rho}$$

- Total CO_2 footprint: material production + use phase

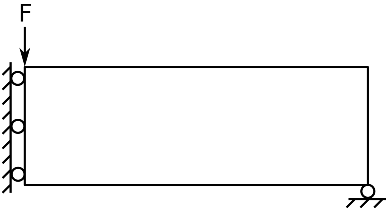
$$CO_{2,mat} = M \cdot CO^{i}_{2,mat}$$

$$CO_{2,use} = M \cdot 98.8 \ \mathrm{tCO_2/kg}$$

$$CO_{2,tot} = CO_{2,mat} + CO_{2,use}$$

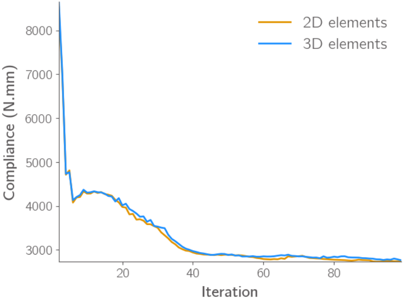
Summary

- **Reference problem:** 168 mm $\times$ 80 mm $\times$ 8 mm half MBB beam, divided in elements with side length 4 mm and subject to a vertical force $F = 1$ kN

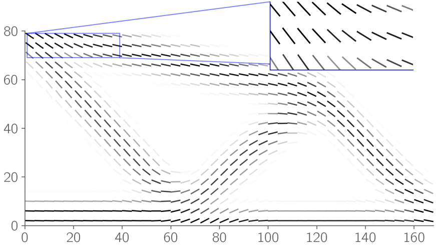


- **Parameters:**
  - $r_\rho = 8$ mm, $r_\theta = 20$ mm
  - $f = 0.3$
  - Initial orientation: -30°
  - 100 iterations
  - Bamboo + cellulose, fiber volume fraction 0.5

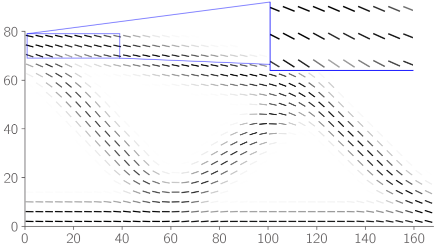
2D-3D equivalence for thin structures



- 2D:



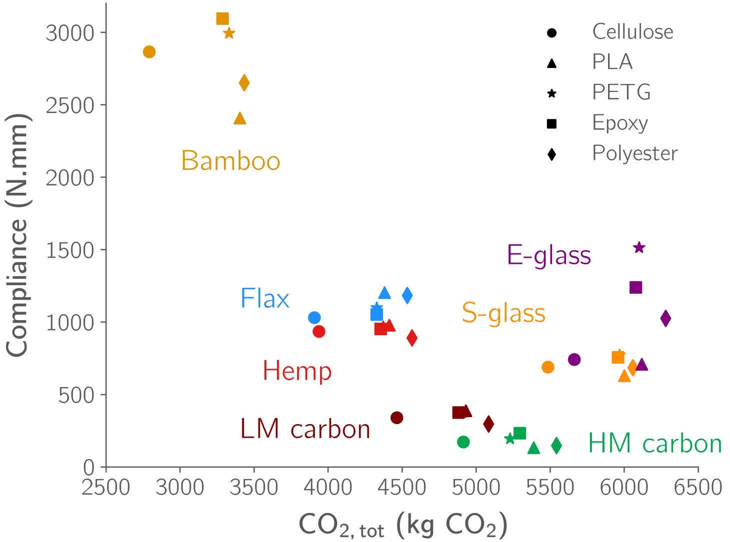
- 3D:



- Same MBB configuration, multiple fiber/resin couples

		ρ (kg/m^3)	E (GPa)	ν	CO_2^i (kg CO_2/kg)
Fibers	Bamboo	700	17.5	0.04	1.0565
	Flax	1470	53.5	0.355	0.44
	Hemp	1490	62.5	0.275	1.6
	HM Carbon	2105	760	0.105	68.1
	LM Carbon	1820	242.5	0.105	20.3
	S-Glass	2495	89.5	0.22	2.905
	E-Glass	2575	78.5	0.22	2.45
Resins	Cellulose	990	3.25	0.355	3.8
	PLA	1290	5.19	0.39	2.115
	PETG (abs)	1270	2.06	0.403	4.375
	Epoxy	1255	2.41	0.399	5.94
	Polyester	1385	4.55	0.35	4.5

- $f = 0.3$, random initial orientations

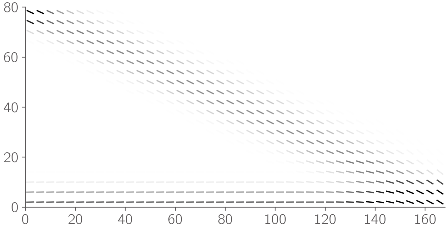


3D with material optimization

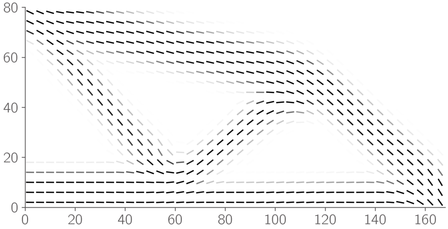
- Replace **carbon/epoxy** by **hemp/cellulose** in structural components
- Varying f to achieve a compliance of 500 N.mm

	f	Compliance (N.mm)	Mass (g)	$CO_{2,tot}$ (kg CO_2)	$CO_{2,mat}$ (kg CO_2)
Carbon	0.116	503	20.9	2060	2.25
Hemp	0.425	503	56.6	5581	0.14

- Carbon (16.1x more $CO_{2,mat}$)



- Hemp (2.7x more $CO_{2,tot}$)

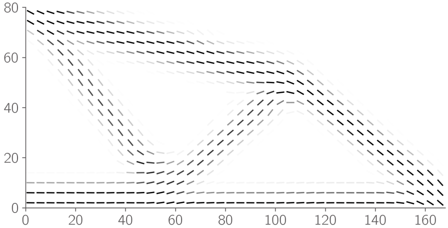


3D with material optimization

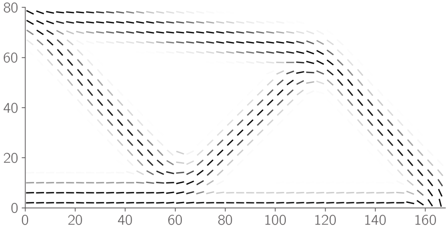
- Replace **glass/polyester** by **bamboo/cellulose** in non structural components
- $f = 0.3$

	Compliance (N.mm)	Mass (g)	$CO_{2,tot}$ (kg CO_2)
Glass	574	65.2	6160
Bamboo	2136	27.2	2682

- Glass (2.4x more CO_2)



- Bamboo (3.7x more compliance)



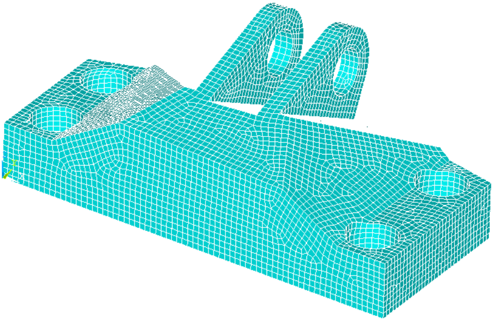
- Formulation for multiple load cases

$$\min_{\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}} C(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha}) = \left(\sum_{i\in LC} c_i(\boldsymbol{\rho},\boldsymbol{\theta},\boldsymbol{\alpha})^n\right)^{\frac{1}{n}}$$

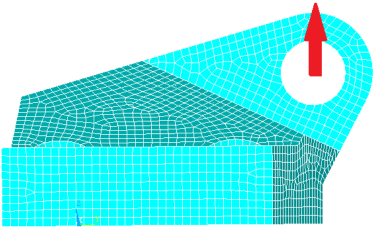
$$= \left(\sum_{i\in LC}\left(\sum_e \rho_e^p\, \boldsymbol{u}_{e,i}^T\, \boldsymbol{k_0}(\theta_e,\alpha_e)\, \boldsymbol{u}_{e,i}\right)^n\right)^{\frac{1}{n}}$$

$$\text{s.t.} \begin{cases} \frac{V(\rho)}{V_0} \leq f \\ \boldsymbol{KU} = \boldsymbol{F} \\ 0 < \rho_{min} \leq \boldsymbol{\rho} \leq 1 \\ -\frac{\pi}{2} \leq \boldsymbol{\theta} \leq \frac{\pi}{2} \\ -\frac{\pi}{2} \leq \boldsymbol{\alpha} \leq \frac{\pi}{2} \end{cases}$$

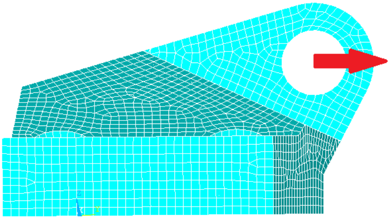
Bracket



- Load case 1: 35.6 kN

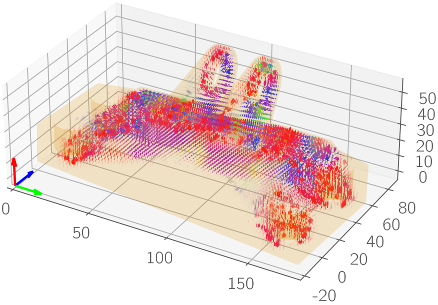


- Load case 2: 37.8 kN

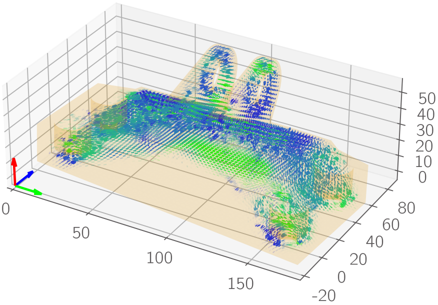


Bracket

- ρ, θ, α



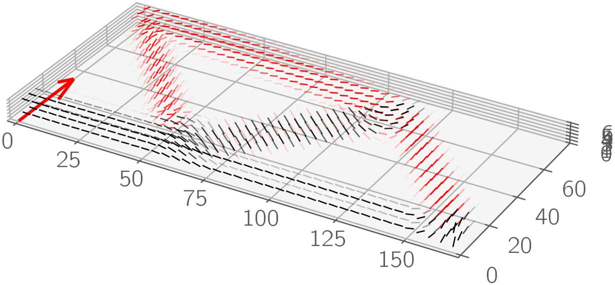
- ρ, θ



Design variables	c_1 (10^4 N.mm)	c_2 (10^4 N.mm)
ρ, θ, α	14.544	10.393
ρ, θ	12.282	8.687

Next steps

- Printability
  - Define if element is supported by elements below



  - Add printability constraint to optimization
  - Compare printable and non printable compliance
- Failure
  - Composite failure (Hashin's criteria)
  - Fiber buckling