Assignment 5

1. Probability mass function :

total $= 10!$ $\Rightarrow$ $P(x=1) = \frac{5\times 9!}{10!} = \frac{5}{10}$ $P(x=2) = \frac{5\times 5\times 8!}{10!} = \frac{25}{90}$ $P(x=3) = \frac{\binom{5}{2}\cdot 7!}{10!} = \frac{50}{720}$

$P(x = 1 \text{ to } 6)$ $P(x=4) = \frac{\binom{5}{3}\cdot 6!}{10!} = \frac{50}{5040}$ $P(x=5) = \frac{\binom{5}{4}\cdot 5!}{10!} = \frac{25}{30240}$ $P(x=6) = \frac{\binom{5}{5}\cdot 4!}{10!} = \frac{5}{60480}$

$E(x) = \sum_{k=1}^{6} k\cdot P(x=k)$

$= 1\cdot 0.5 + 2\cdot\frac{25}{90} + 3\cdot\frac{50}{720} + 4\cdot\frac{50}{5040} + 5\cdot\frac{25}{30240} + 6\cdot\frac{5}{60480} = 1.8208$

2. X_i could be $\begin{cases} 1 & \text{success} \\ 0 & \text{fail} \end{cases}$ $\quad P(X_i=1) = p$, $P(X_i=0) = 1-p$ $\quad E[x] = 1\cdot p + 0\cdot(1-p) = p$ $\quad E(x) = E(x_1) + E(x_2) + \dots + E(x_n) = np$

every x has same probability p

$Var(x) = Var(X_1 + X_2 + \dots + X_n)$

$\because$ each X_i is independant

$Var(x) = \sum_{i=1}^{n} Var(X_i) = n\cdot Var(X_1)$ and $Var(X_i) = E(x^2) - (E(x))^2 = p - p^2 = p(1-p)$

so $Var(x) = np(1-p)$

3. Since it has dd, dr, rd, rr , getting dominant outward appearance is $\frac{3}{4}$

$P(x=3) = \binom{4}{3}(\frac{3}{4})^3\cdot(\frac{1}{4})^1 = \frac{27}{64}$

4. $p = \frac{1}{365}\times\frac{1}{365} = \frac{1}{365^2}$

since n large p small

a. $p = \frac{1}{365}\times\frac{1}{365}\cdot\frac{1}{365^2}$ $np = 8000\times 7.5\times 10^{-6} \approx 0.6 = \lambda$

$P(x\geq 1) = 1 - P(x=0) = 1 - e^{-\lambda} = 1 - 0.5488 = 0.4512$

b. $p = \frac{1}{365}$ $\lambda = \frac{8000}{365} \approx -219.178$

$P(x\geq 1) = 1 - P(x=0) = 1 - e^{-219.178} \approx 1 - 0 = 1$

almost 100%

5. $\mu = 10$, $\sigma^2 = 36\ (\sigma = 6)$ $Z = \frac{X-\mu}{\sigma}$

(1) $P(x>5)$ $Z = \frac{5-10}{6} = -\frac{5}{6}$ $P(Z > -0.833) = 1 - P(Z \leq -0.833) = 1 - \Phi(-0.83)$

$\Phi(-0.83) \approx 1 - 0.7967$ $\therefore$ $P(x>5) = 1 - (1 - 0.7967) = 0.7967$

(2) $P(4<x<16)$ left: $\frac{4-10}{6} = -1$ right: $\frac{16-10}{6} = 1$ So $P(-1<Z<1)$

equal to $\underbrace{P(Z<1)}_{\Phi(1)} - \underbrace{P(Z<-1)}_{1-\Phi(1)} = 2\Phi(1) - 1 = 2\times 0.8413 - 1 = 0.6826$

(3) $P(x<8)$ $Z = \frac{8-10}{6} = -0.333$ $P(Z<-0.333) = 1 - \Phi(0.333)$

$\approx 1 - 0.6293$

$= 0.3707$

(d) $P(X<20)$ $z=\frac{20-10}{6}=\frac{5}{3}$ $\therefore$ ~~$P(Z<1.667)$~~ $P(Z<1.67)=\Phi(1.67)=0.9525$

(e) $P(X>16)$ $z=1$ $P(Z>1)=1-P(Z\le 1)=1-\Phi(1)$

$1-0.8413=0.159$

6. $\mu=1000$, $\sigma=200$

(a). $P(X<1100)$ is $Z=\frac{1100-1000}{200}=0.5$ $P(Z<0.5)=\Phi(0.5)=0.6915$

so two weeks is $(0.6915)^2=0.4782$

(b). $P(X_1+X_2>2200)$ $\underbrace{X_1+X_2}_{\text{independent normal}}$ has mean 2000 and σ^2 $40000+40000=80000$

so σ is $200\sqrt{2}$

$Z=\frac{2000-2200}{200\sqrt{2}}=-\frac{1}{\sqrt{2}}$

$P(Z>-\frac{1}{\sqrt{2}})=1-\Phi(0.7071)=1-0.7611=0.2389$

7. econ: $\mu=60$, $\sigma=20$ stat: $\mu=55$, $\sigma=10$

(a) econ: $Z_e=\frac{70-60}{20}=\frac{1}{2}=0.5$ stat: $Z_s=\frac{62-55}{10}=0.7$

$P(Z_e<0.5)=\Phi(0.5)$ $P(Z_s<0.7)=\Phi(0.7)$

$=0.6915$ $=0.7580$

$0.6915<0.7580$, so greater percentile in statistics

(b) it's 69.15%

(c) it's 75.80%