1. $\mu = 2.2mg$, $\sigma = 0.3mg$ $n = 100$

$P(\bar{x} \geq 3.1)$ CLT $\rightarrow \frac{0.3}{\sqrt{100}} = 0.03$ $E(\bar{x}) = 2.2mg$

$\bar{x} \sim N(2.2, 0.03)$ $Z = \frac{3.1-2.2}{0.03} = 30$

$P(Z \geq 30) = 1 - \Phi(30) \approx 0$, so it's nearly impossible

4. $n = 30$, $\bar{x} = \frac{346}{30} \approx 11.53$ $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \approx 3.833$

$\frac{s}{\sqrt{n}} = \frac{3.833}{\sqrt{30}} \approx 0.7$

$t_{0.025,29} = 2.045$ $2.045 \times 0.7 = 1.4315$

confidence interval is $(11.53 - 1.4315, 11.53 + 1.4315) = (10.0985, 12.9615)$

5. a. ① $n_1 = 36$, $\mu_1 = 120g$ $S_1^2 = 4$ ② $n_2 = 64$ $\mu_2 = 130g$ $S_2^2 = 5$

$\bar{x} - \bar{y} = 120g - 130g = -10g$, $t_{0.005,98} \rightarrow 36+64-2 = 98$ ≈ 2.626

$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{455}{98} \approx 4.643$

2.

a. use central limit theorem

It’s a kind of Bernoulli distribution for

Xi – 1. Rarely have breakfast, and 0. Otherwise

Xi ~ Bernoulli (P = 0.454)

X ~ N(E(x), var(x))

E(xi) = np = 300*0.454 = 136.2

Var(xi) = np(1-p) = 300*0.454*0.546 = 74.3652

Sd = sigma is approximately = 8.623

Z = (150 − 136.2) / 8.623 = 13.8 / 8.623 is approximately = 1.6

P(xi >= 150) = 1 – phi(1.6) = 1 – 0.9452 = 0.0548

b.

E(xi) = np = 300*0.284 = 85.2

Var(xi) = np(1-p) = 300*0.284*0.716 = 61.0128

Sd = sigma is approximately = 7.811

Z = (100 − 85.2) / 7.811 = 14.8 / 7.811 is approximately = 1.89

$P(xi < 100) = phi(1.895) = 0.9706$

3.
If Xi is given to be normally distributed, even if X bar is smaller than 30, we can also use central limit theorem.
(a). P(0.025 < x < 0.975)
According to the formula below

$$\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

X bar is 11.48 through calculation, sd is 0.08/ square root 10 is approximately = 0.0253
The interval is about [11.5304, 11.6296] ppm

(b) 95% lower bound
x bar + Z0.05 * sd/ square root n = x bar + 1.654 * 0.0253= 11.52
from negative infinity to 11.52 ppm

(c) 95% upper bound
X bar – Z0.05 * sd/ square root n = 11.44
From 11.44 to positive infinity ppm

5. a. ① $n_1 = 36$, $\mu_1 = 120g$ $S_1^2 = 4$ ② $n_2 = 64$ $\mu_2 = 130g$ $S_2^2 = 5$

$\bar{X} - \bar{Y} = 120g - 130g = -10g$, $t_{0.005, 98} \to 36+64-2 = 98$ ≈ 2.626

~~$\mu_1 - \mu_2 \in (-10 - 2.626$~~

$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{455}{98} \approx 4.643$

So $S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = \sqrt{4.643 \times (\frac{1}{36}+\frac{1}{64})} \approx \sqrt{0.20159} \approx 0.449$

so $\mu_1 - \mu_2 \in (-10 - 1.179, -10 + 1.179) = (-11.18, -8.82)\,g$

b. So it's $\sqrt{\frac{4}{36}+\frac{5}{64}} \approx 0.435$, $Z_{0.005} \approx 2.576$ by searching the table

CI is $-10 \pm 2.576 \times 0.435 \approx -10 \pm 1.121$

so CI $\in (-11.12, -8.88)\,g$

6. a. $p = \frac{204}{1200} = 0.17$ $n = 1200$ it's Bernoulli distribution

$\sigma = \sqrt{\frac{0.17 \times 0.83}{1200}} \approx 0.01084$ 90% CI is 1.645

$1.654 \times 0.01084 \approx 0.01784$ interval is $0.17 \pm 0.01784 = (0.152, 0.188)$

b. $\frac{48+80}{1200} \approx 0.1067$ same CI $\to 1.645$ $\sigma = \sqrt{\frac{0.1067 \times 0.8933}{1200}} \approx 0.0089$

$1.645 \times 0.0089 \approx 0.01466$

so interval is $0.1067 \pm 0.01466 = (0.0920, 0.1213)$

7. a. $\frac{67}{100} = 0.67$

b. $me = 0.02$ we know $me^2 = Z_{\alpha/2} \cdot \frac{P(1-P)}{n}$

So n is $\frac{P(1-P)}{me^2} \cdot Z_{\alpha/2} = \frac{1.96^2 \times 0.67 \times 0.33}{0.02^2}$

$n \approx 2123$ people so additional sample is $2123 - 100 = 2023$ people