1. A certain disease can pass from a mother to her children.

   - If the mother has the disease, her children will independently have the disease with probability $\frac{1}{2}$.
   - If the mother does not have the disease, the children will not have the disease either.

   Consider a mother who has the disease with probability $\frac{1}{3}$. She has two kids. Define the following events:

   - A = older child has the disease
   - B = younger child has the disease
   - M = mother has the disease

   (a) Are A and B (unconditionally) independent? Are A and B conditionally independent given M?

   (b) What is the probability that neither child has the disease?

2. **Monty Hall**. This is a game from "Let's Make a Deal". At the beginning of the game, there are 3 closed doors. Two doors each hide a goat, and the third hides a car. You want to win the car. The host, Monty, always knows where the car is. Here's how the game proceeds:

   - The player (you) pick an initial door at random. It remains closed.
   - Monty then opens one of the two non-car doors to reveal a goat. If both remaining doors have goats, he will pick one at random.
   - Monty offers you the choice: do you stay with your current door, or will you switch to the other closed door?

   What will you choose?

Let's number the doors 1, 2, and 3. Without loss of generality, assume that you chose Door 1 at the beginning of the game.

Unconditional probability of winning

Conditional probability of winning

3. **First-Step Analysis**.
   A single amoeba lives in a pond. Every minute, the amoeba will either die, split into two, or stay the same. All three options will occur with equal probability. All amoeba babies will behave the same, independent of other amoebas.

   What is the probability that all the amoebas die out?