Naive Definition of Probability + Counting

1. Consider the same assumptions as the original birthday problem: no leap years, no twins/triplets, and each day of the year is equally likely to be a birthday. Now suppose there is a room full of $n+1$ people: you and n others. What is the probability that at least one other person shares *your* birthday?

   (a) Discuss: what is the the event of interest A?

   (b) Determine the sample space and its size.

   (c) Discuss: Is it easy to find $P(A)$? If not, what other tools do you have?

   (d) Now actually find the probability of interest $P(A)$!

2. A committee has professors representing each of the divisions at Middlebury: Natural Sciences has 4, Social Sciences has 3, Humanities has 2, and Languages has 2. The committee members are all asked to sit in a row of chairs. How many unique ways can the professors sit if we ask that professors within a division sit together (e.g. all Natural Sciences professors, followed by Social Sciences, Humanities, and Languages)?

3. The United States has 100 senators: 2 senators from each of the 50 states. Suppose a senate committee is comprised of 8 members of the Senate. What is the probability that at least one of the two senators from Vermont is part of the committee?

4. To fulfill the requirements for a certain degree, a student can choose to take any 7 out of a list of 20 courses, with the constraint that at least 1 of the 7 courses must be a statistics course. Suppose that 5 of the 20 courses are statistics courses.

   (a) I want to know how many choices there are for which 7 courses to take. Explain intuitively why the answer to (a) is not $\binom{5}{1} \cdot \binom{19}{6}$.

   (b) How many choices are there for which 7 courses to take?