This problem set covers material from Week 1, dates 2/09- 2/13. Textbook problems (if assigned) can be found at the end of the corresponding chapter.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Monday 2/09

None!

Wednesday 2/11

1. We roll three six-sided dice. The sides of each die are numbered 1-6. Let A be the event that the first die shows an even number, B the event that second die shows an even number, and C the event that the third die shows an even number. Also define event A_i as the event that the first die rolls the number i, for $i = 1, \ldots, 6$. Define B_i and C_i similarly for the second and third dice, respectively.
   Express each of the following events in terms of the events described above:
   (a) All three dice show even numbers
   (b) No die shows an even number
   (c) At least one die shows an odd number
   (d) At most two dice show odd numbers
   (e) The sum of the three die is no greater than 4
2. Prove DeMorgan's Law Part 2: For every two sets A and B in sample space $\mathcal{S}$,
   $$(A \cap B)^c = A^c \cup B^c$$
   There are multiple ways to prove this, but 'proof by picture' doesn't count (though it's good for building intuition)!
3. Prove the following theorem (both parts): For every two sets A and B in some sample space $\mathcal{S}$:
   i) $A \cap B$ and $A \cap B^c$ are disjoint, and
   ii) $A = (A \cap B) \cup (A \cap B^c)$

   Then, in a single sentence/phrase, describe what this theorem says about $(A \cap B)$ and $(A \cap B^c)$.

Friday 2/13

4. Suppose we have an experiment with sample space $\mathcal{S} = \{a, b, c, d, e, f\}$.

   (a) How many ways are there to form two groups of three elements, where order doesn't matter? That is, how many ways are there to form two subsets of three outcomes each? Briefly justify your solution. *Please use counting methods, not brute force!*

   (b) How many ways are there to form three groups of two elements each, where order doesn't matter? Briefly justify your solution. *Please use counting methods, not brute force!*

5. **Chapter 1**: 35 *Briefly justify/explain your solution.*

6. In a standard deck of US playing cards, there are cards values Ace, 2, $\ldots$, 10, Jack, Queen, King. Each value shows up once for each of 4 suits: Hearts, Diamonds, Clubs, and Spades. This yields a deck of 52 cards. Let's consider a subset of this deck of cards: just the Jacks, Queens, Kings, and Aces.

   (a) We will draw cards one by one from this smaller deck, until the first time an Ace appears. What's the probability that no Kings, Queens, or Jacks appear before the first Ace?

   (b) What is the probability that exactly two Queens appear before the first Ace? *Briefly justify/explain your solution.*

   (c) What is the probability that exactly one King, exactly one Queen, and exactly one Jack appear (in any order) before the first Ace? *Briefly justify/explain your solution.*

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.