Naive Definition of Probability + Counting

1. Consider the same assumptions as the original birthday problem: no leap years, no twins/triplets, and each day of the year is equally likely to be a birthday. Now suppose there is a room full of $n+1$ people: you and n others. What is the probability that at least one other person shares *your* birthday?

   (a) Discuss: what is the the event of interest A? *A is the event hat at least one person shares* your *birthday.*

   (b) Determine the sample space and its size.*Sample space is the birthdays of the remaining n people. Sampling with replacement the 365 days of year, so* $|\mathcal{S}| = 365^n$.

   (c) Discuss: Is it easy to find $P(A)$? If not, what other tools do you have? *It's hard to find* $|A|$ *because so many outcomes contribute to it. Instead, easier to find* $P(A^c)$*: the probability that no one shares your birthday!*

   (d) Now actually find the probability of interest $P(A)$!$|A^c| = 364^n$ *since each other person has free choice of birthdays except yours. So* $P(A) = 1 - P(A^c) = 1 - \frac{364^n}{365^n}$.

2. A committee has professors representing each of the divisions at Middlebury: Natural Sciences has 4, Social Sciences has 3, Humanities has 2, and Languages has 2. The committee members are all asked to sit in a row of chairs. How many unique ways can the professors sit if we ask that professors within a division sit together (e.g. all Natural Sciences professors, followed by Social Sciences, Humanities, and Languages)?

   We can first choose the ordering of the divisions. This is $4!$. *Once we have chosen the ordering, we simply need to count the ways the professors within a division can sit. This is simply permuting the professors within division. So we have* $4! \cdot (4!3!2!2!) = 13824$.

3. The United States has 100 senators: 2 senators from each of the 50 states. Suppose a senate committee is comprised of 8 members of the Senate. What is the probability that at least one of the two senators from Vermont is part of the committee?

   Let A be the event of interest. It's easier to think of the complement A^c *which is that neither of the Vermont senators are chosen. This occurs if we take the 8 committee members from the remaining 98 senators. So* $P(A) = 1 - \binom{98}{8}/\binom{100}{8}$.

4. To fulfill the requirements for a certain degree, a student can choose to take any 7 out of a list of 20 courses, with the constraint that at least 1 of the 7 courses must be a statistics course. Suppose that 5 of the 20 courses are statistics courses.

   (a) I want to know how many choices there are for which 7 courses to take. Explain intuitively why the answer is not $\binom{5}{1} \cdot \binom{19}{6}$.

   The (incorrect) argument for $\binom{5}{1}\binom{19}{6}$ *is to say that the student first chooses one statistics course, then choose six courses among the remaining 19. However, this is incorrect because it doesn't appropriately account for the schedules where a student takes more than one statistics course. For example, suppose a student takes STAT*

1 and STAT 2, with five other non-statistics course. Then this pairing would be counted twice in the $\binom{5}{1}\binom{19}{6}$*: once when STAT 1 is the statistics course and STAT 2 is in the remainder, and vice versa.*

(b) How many choices are there for which 7 courses to take?

There are $\binom{20}{7}$ *ways to choose 7 classes out of the 20, but* $\binom{15}{7}$ *are schedules without a single statistics course. So the choices are* $\binom{20}{7} - \binom{15}{7} = 71085$.