Introduction to Hypothesis Testing

10/14/24

Testing

We are now entering into second branch of inference-related tasks: testing.

- We have some “claim”/question about the target population, and we use sampled data to provide evidence for or against the claim/question.
- Especially important in experiments where we want to learn the effect of some new drug
- We will use the *hypothesis testing framework* to formalize the process of making decisions about research claims.
  - Because claim is about target population, we will almost always formulate claims in terms of population parameters
  - Then we use sample statistics to provide the evidence for/against

Step 1: Define hypotheses

A **hypothesis test** is a statistical technique used to evaluate competing claims using data

- We define hypotheses to translate our research question/claim into statistical notation
- We always define two hypotheses *in context*: a null hypothesis and an alternative hypothesis
- **Null hypothesis** H_0: the hypothesis that represents “business as usual”/status quo/nothing unusual or noteworthy
- **Alternative hypothesis** H_A: claim the researchers want to demonstrate

It will not always be obvious what H_0 should be, but you will develop intuition for this over time!

Defining hypotheses in context

Research question: do the majority of STAT 201A/STAT 201B students get more than 7 hours of sleep?

- Define p as the true proportion of STAT 201A/STAT 201B who get at least 7 hours of sleep on average

- $H_0 : p \geq 0.5$
- $H_A : p < 0.5$

Step 2: Collect and summarize data

Suppose I collect a sample of $n = 10$ students from each class:

In STAT 201A sample: 6 students received at least 7 hours of sleep, and 4 received less than 7 hours

- Sample statistic: $\hat{p}$: 0.6

In STAT 201B sample: 7 students received at least 7 hours of sleep, and 3 received less than 7 hours

- Sample statistic: $\hat{p}$: 0.7

- Are we prepared to answer our research question based on this evidence?
- Due to variability in data and $\hat{p}$ we should ask: do the data provide ***convincing evidence*** that the majority of students get at least 7 hours of sleep?

Step 3: Determine if we have "convincing evidence"

"Convincing evidence" for us means that it would be highly unlikely to observe the data we did (or data even more extreme) ***if*** H_0 were true!

- We will calculate a **p-value**: the probability of observing data as or more extreme than we did *assuming* H_0 true
  - Note: p is not the same as true proportion p!
- Highly unlikely is vague and needs to defined by the researcher, ideally before seeing data.
  - If we want to answer the research question with a binary yes/no, we need some threshold to compare the p-value to. This is called a **significance level** α
  - Common choices are $\alpha = 0.05, \alpha = 0.01$ (more on this later)!
- For our example, we will choose $\alpha = 0.05$

How to obtain p-value?

- How to obtain this probability? It depends!
  - Option 1: if we have assumptions about how our data behave, we can obtain this probability using theory/math (next week)
  - Option 2: if we don't want to make assumptions, why not apply the bootstrap technique and simulate?
    - We will call this option "simulating under H_0"
- This is the step that requires the most "work", and what exactly you do will depend on the the type of data and the research question/claim you have
- Remark: hypothesis tests, like confidence intervals, are not unique!

Simulating under H_0 (step 3 cont.)

- We have to simulate our data under the assumption that H_0 is true (recall H_0: $p \leq 0.5$)
- Imagine a big bag with pink and purple slips of paper
  - Pink = people who got at least 7 hours of sleep
  - Purple = people who got less than 7 hours
- What proportion of the slips in the bowl should be pink vs purple?
  - **To simulate under** H_0, no more than 50% of the slip should be pink
    - We want convincing evidence even in the most "borderline" case, so we will choose 50% of the slips to be pink.

Simulating under H_0 (step 3 cont.)

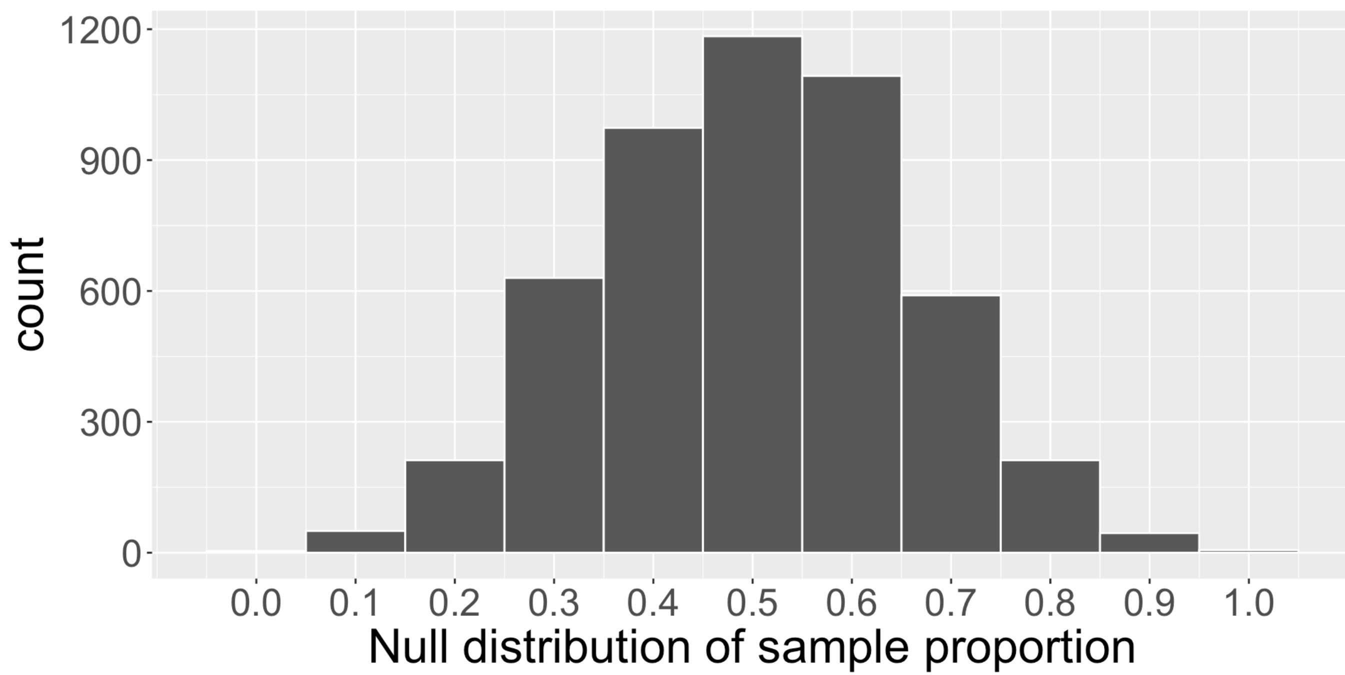
- Activity: we now replicate our original sample, this time sampling from this bag of paper slips
  - We repeatedly take samples from the null distribution, using original sample size $n = 10$
  - For each sample, calculate the simulated proportion of pink slips, $\hat{p}_{sim}$
- Live code?

```
set.seed(2) # reproducibility
B <- 5000 # number of simulations to do to gather enough evidence
n <- 10 # size of our original sample
p_null_vec <- rep(NA, B) # vector to store the simulated proportions
for(b in 1:B){
  # sample() takes a random sample
  null_samp <- sample(x = c("pink", "purple"), # pink and purple slips
                      size = n, # sample of size n
                      replace = T, # tell R that my bowl has infinitely many marbles
                      prob = c(0.5, 0.5)) # 50% of slips are pink and 50% are purple

  # calculate and store the proportion of pink slips in this simulation
  p_null_vec[b] <- sum(null_samp == "pink")/n
}
```

Null distribution of statistic

We can visualize the distribution of $\hat{p}_{sim}$ assuming H_0 true:



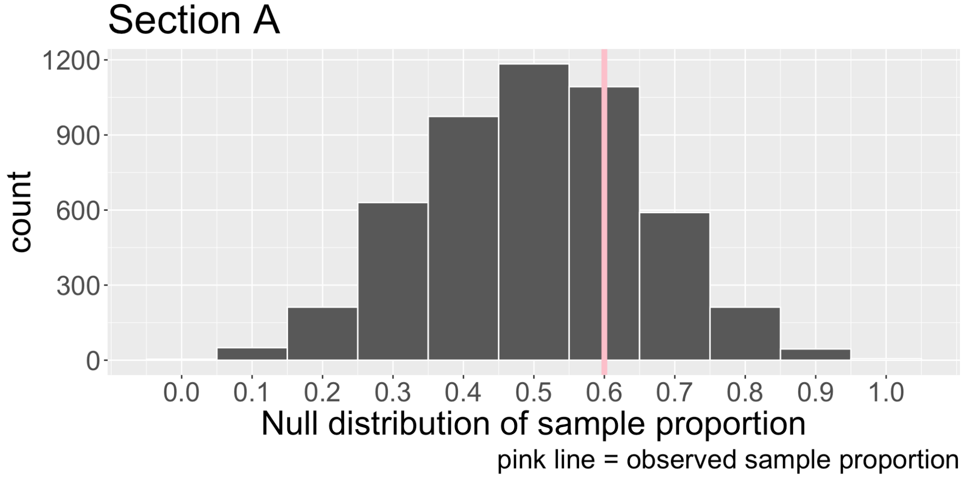
- This is called the **null distribution** of the sample statistic, which is the distribution of the statistic assuming H_0 is true
- Where is the null distribution of $\hat{p}$ centered? Why does that "make sense"?

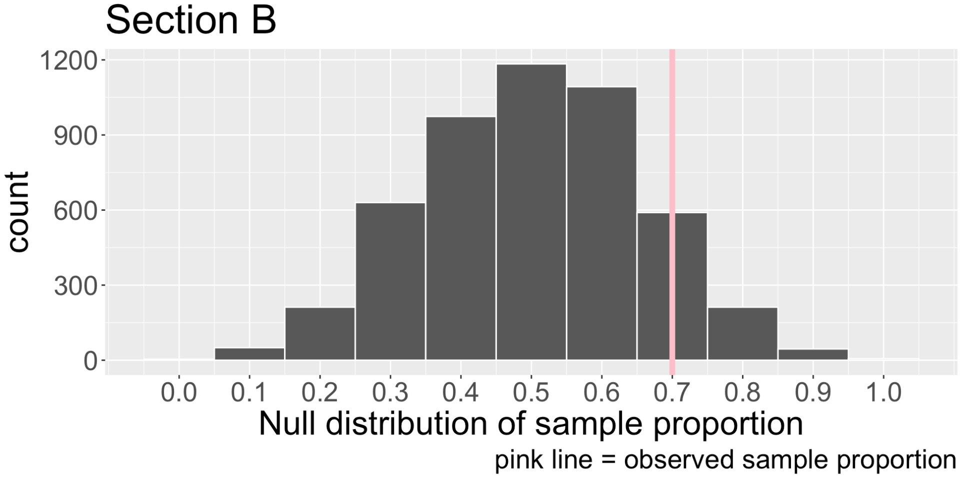
Comparing null to observed

Let's return to our original goal of Step 3! We need to find the **p-value**: the probability of observing data as or more extreme as ours, assuming H_0 were true.

- Our observed data were $\hat{p} = 0.6$ (STAT 201A) or $\hat{p} = 0.7$ (STAT 201B)
- $H_0 : p \leq 0.5$ and $H_A : p > 0.5$
- What does "as or more extreme" mean in this context?

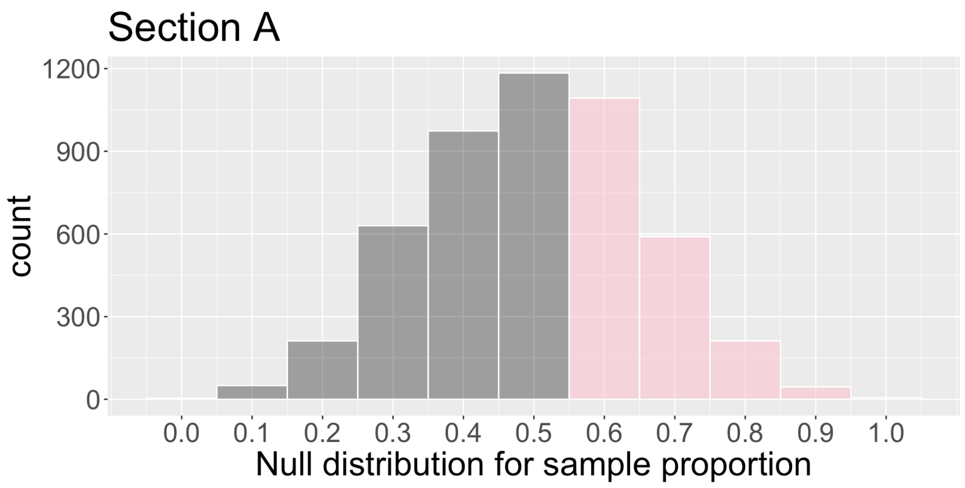
  How can we use the null distribution to obtain this probability?

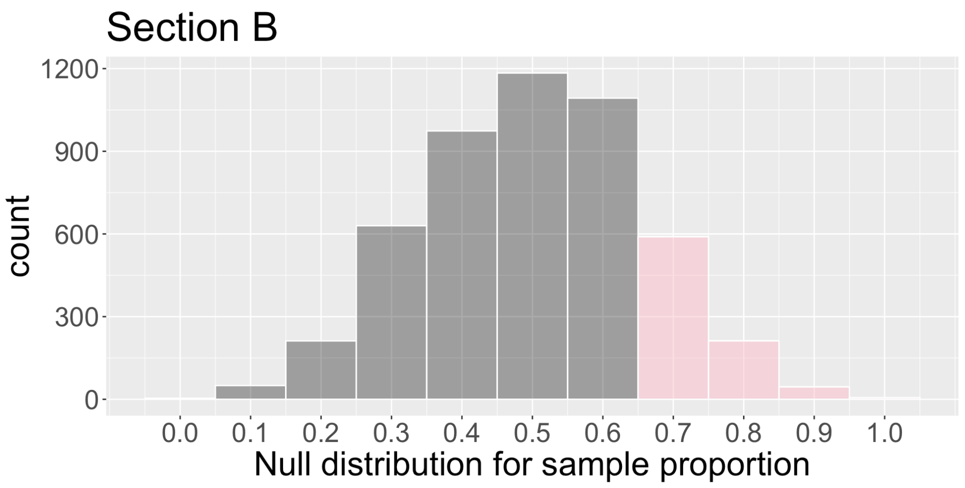




Obtain p-value (step 3 cont.)

We can directly obtain (technically estimate) the p-value using our null distribution and our observed $\hat{p}$!





- Out of 5000 replications, we saw 1946 instances of $\hat{p}_{sim} \geq \hat{p}$
- p-value is $\frac{1946}{5000} \approx 0.39$

- Out of 5000 replications, we saw 853 instances of $\hat{p}_{sim} \geq \hat{p}$
- p-value is $\frac{853}{5000} \approx 0.17$

Step 4: Interpret p-value and make decision

1. Interpret the p-value in context

   - Assuming H_0 true, the probability of observing a sample proportion as or more extreme as ours (0.6 or 0.7) is 0.39 or 0.17

2. Make a decision about research claim/question by comparing p-value to significance level α

   - If p-value $< \alpha$, we *reject* H_0 (it was highly unlikely to observe our data given our selected threshold)
   - If p-value $\geq \alpha$, we *fail to reject* H_0 (we did not have enough evidence against the null)
     - Note: we never "accept H_A"!
   - Since our p value is less than $\alpha = 0.05$, we fail to reject H_0. The data do not provide sufficient evidence to suggest that the majority of STAT 201A/STAT 201B students get less than 7 hours of sleep.

Summary of testing framework

Four steps for hypothesis test:

1. Define null and alternative hypotheses H_0 and H_A in context
2. Collect data and set significance level α
3. Obtain/estimate p-value by modeling randomness that would occur if the H_0 were true
   - We did this using by simulating under the null distribution
4. Interpret p-value and make a decision in context

Errors in decision

- In Step 4, we make a decision but it could be wrong! (Unfortunately, we will never know)
- We always fall into one of the following four scenarios:

		State of world	
		H_0 true	H_0 false
Decision	Fail to reject H_0		
	Reject H_0		

- Identify which cells are good scenarios, and which are bad

Errors in decision

		State of world	
		H_0 true	H_0 false
Decision	Fail to reject H_0	Correct	Type II error
	Reject H_0	Type I error	Correct

- What kind of error could we have made in our example?
- It is important to weight the consequences of making each type of error!
  - We have some control in this. How?

Comprehension questions

- What are the similarities/differences between the bootstrap distribution of a sample statistic and the simulated null distribution?
- Do you understand what a p-value represents, and how we obtain it from the null distribution?
- What role does α play? Why is it important to set α early on?