

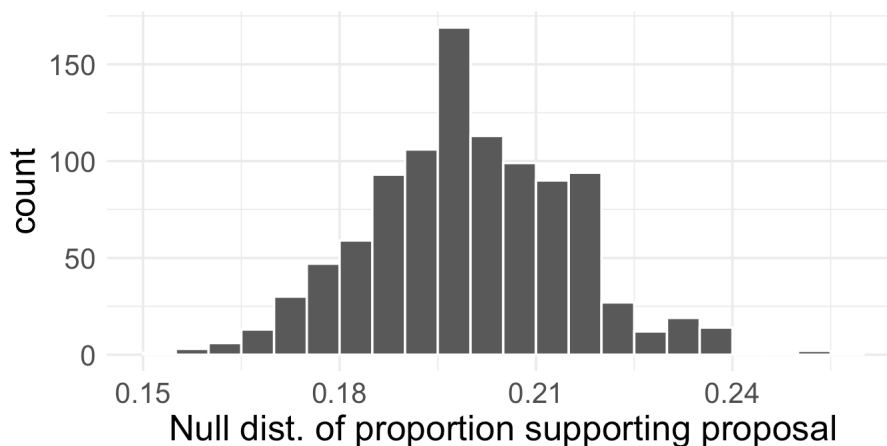
## Intro Hypothesis Testing

1. For each of the research statements below, determine whether it represents a null hypothesis claim or an alternative hypothesis claim.
  - (a) The number of hours that grade-school children spend doing homework predicts their future success on standardized tests. *Alternative*
  - (b) King cheetahs on average run the same speed as standard spotted cheetahs. *Null*
  - (c) For a particular student, the probability of correctly answer a 5-option multiple choice test is larger than 0.2 (i.e. better than guessing) *Alternative*
  - (d) The probability of getting in a car accident is the same if using a cell phone then if not using a cell phone. *Null*
2. Write out the null and alternative hypotheses in words and also in statistical notation for each of the following situations. When writing in statistical notation, be sure to define quantities in context.
  - (a) New York is known as “the city that never sleeps”. A random sample of 25 New Yorkers were asked how much they sleep they get per night. Does these data providing convincing evidence that New Yorkers on average sleep less than 8 hours per night?  
 *$H_0 : \mu = 8$  (On average New Yorkers sleep 8 hours a night) versus  $H_A : \mu < 8$  (On average New Yorkers sleep less than 8 hours a night), where  $\mu$  is the mean hours of sleep New Yorkers receive.*
  - (b) A study suggests that 25% of 25 year-olds have gotten married. You believe that this is incorrect and decide to collect your own data to conduct a hypothesis test.  
 *$H_0 : p = 0.25$  (True proportion of 25 year-olds who have gotten married is 25%) versus  $H_A : p \neq 0.25$  (True proportion of 25 year-olds who have gotten married is not 25%)*
3. A Survey USA poll conducted in Seattle, WA in May 2021 reports that of the 650 respondents (adults living in this area), 159 support proposals to defund police departments.
  - (a) A journals writing a news story on the poll results wants to use the headline: “More than 1 in 5 adults living in Seattle support proposals to defund police departments”. You caution the journalist that they should first conduct a hypothesis test to see if the poll data provide convincing evidence for this claim. Write the hypotheses for this test using proper notation, defining any necessary quantities.  
 *$H_0 : p = 0.20$  versus  $H_A : p > 0.20$  where  $p$  is the true proportion of Seattle adults who support proposals to defund.*

- (b) Describe in words a simulation scheme that would be appropriate for this situation. Also describe how the p-value can be calculated using the simulation results.

*Example solution:* . Take 100 cards, 20 black cards representing those who support proposals to defund police departments and 80 red cards representing those who do not. Shuffle the cards and draw with replacement (shuffling each time in between draws to get our “infinite population”) 650 cards representing the 650 respondents to the poll. After each iteration, calculate  $\hat{p}_{sim}$ , the proportion of black cards which represents the simulated proportion of adults in favor. The p-value will be the proportion of simulations where  $\hat{p}_{sim} \geq 0.245$ .

- (c) The histogram below shows the distribution of 1000 simulated proportions under  $H_0$ . Estimate the p-value using the plot and use it to evaluate your hypotheses (i.e. make a conclusion). Assume a significance level of 0.05.



*There is only one simulated proportion that is at least 0.245, therefore the approximate p-value is 0.001. Since  $0.001 < 0.05$ , reject  $H_0$ . The data provide convincing evidence that the proportion of Seattle adults who support proposals to defund police departments is greater than 0.20.*

4. A study conducted in 2020 found that the U.S. adjusted divorce rate was 14 per 1000 married women. Joe is suspicious and disagrees with the stated divorce rate. Joe somehow collected data from 323 married or previously-married women, and asked them if they had a divorce in 2020. 55 of the women responded that they indeed had a divorce in 2020.

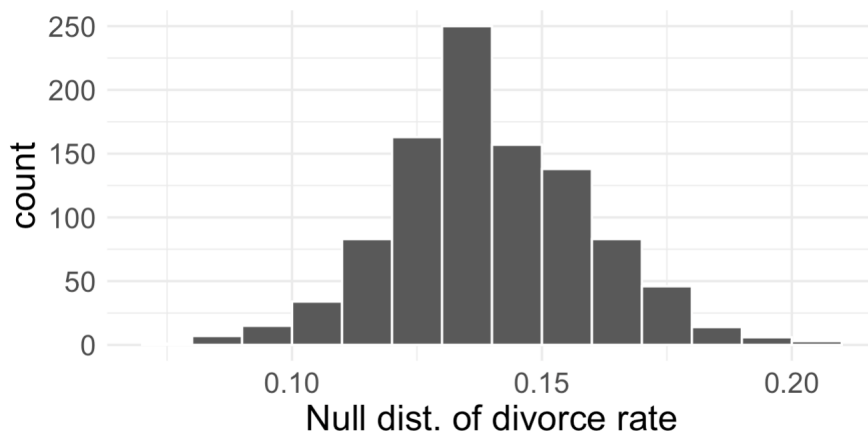
- (a) Write out the hypotheses corresponding to this scenario.

*$H_0 : p = 0.14$  versus  $H_A : p \neq 0.14$  where  $p$  is the true divorce rate among married women.*

- (b) Describe in words a simulation scheme that would be appropriate for this situation. Also describe how the p-value can be calculated using the simulation results.

*Similar to previous problem.*

- (c) The histogram below shows the distribution of 100 simulated proportions under  $H_0$ . Estimate the p-value using the plot and use it to evaluate Joe's hypotheses (i.e. make a conclusion). Assume a significance level of 0.05.



*The observed proportion was  $\hat{p}_{obs} = 55/323 = 0.17$ . Since the alternative is two-sided, the p-value is approximately 0.13. Since this is larger than 0.05, fail to reject. The data do not provide convincing evidence that the divorce rate among married women is different from 0.14.*

- (d) Joe has some free time and also created a 90% bootstrap confidence interval for the divorce rate.

He obtained the following interval: (0.136, 0.207). Interpret this interval in context.

*Joe is 90% confidence that the true divorce rate among married women is between 0.136 and 0.207.*

- (e) Based on this interval, would it be appropriate for Joe to conclude that the study's reported rate was wrong? Explain your reasoning.

*No! 0.14 is included in the interval, so it is a plausible value.*

- (f) How do your conclusions from (c) and (e) compare?

*They agree!*