This problem set covers material from Week 4, dates 9/30 – 10/03.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Monday 9/30

1. The American Community Survey (ACS) is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 ACS estimated that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English (i.e. a foreign language) at home, and 4.2% fall into both categories.

   (a) Are living below the poverty line and speaking a foreign language at home disjoint?

   (b) Draw a Venn diagram summarizing the probabilities and their associated probabilities. Be sure to complete the diagram by including a "bounding box".

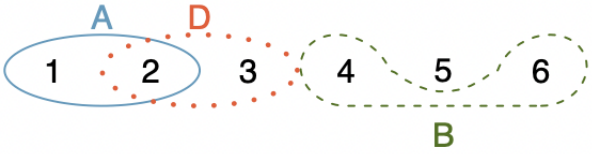
   (c) What percent of Americans live below the poverty line and only speak English at home?

   (d) What percent of Americans live below the poverty line or speak a foreign language at home?

   (e) What percent of Americans live above the poverty line and only speak English at home?

   (f) Is the event that someone lives below the poverty line independent of the event that the person speaks a foreign language at home?

2. In the following figure, the sample space is the set of outcomes $S = \{1, 2, 3, 4, 5, 6\}$. We define three different events A, B and D as denoted by the coloring and circling.



   (a) Which pairs of events are disjoint?

   (b) Suppose we have a random variable X whose sample space is S as defined above. Also suppose that $\mathrm{P}(B) = 0.2$, $\Pr(D^c) = 0.6$, and events A and D are independent. Based on this information, create a table that represents a valid probability distribution for X.

3. Every week you buy a ticket in a lottery that offers one chance in a million of winning. If you play for ten years, what is the probability that you never win? Be sure to state any assumptions you make.

Wednesday 10/02

4. Suppose a standard deck of 52 cards is shuffled well. You deal cards one a time until an Ace is dealt. What is the probability that more than 2 cards are dealt?

   Recall: a standard deck of cards has 4 suits, and each suit has 13 cards: Ace, 2, 3, . . ., 10, Jack, Queen, and King.

5. The global coronavirus pandemic illustrates the need for accurate testing of COVID-19, as its extreme infection rate poses a significant public health threat. Due to the time-sensitive nature of the situation, the FDA enacted emergency authorization of a number of serological tests for COVID-19. Full details of these tests may be found on its website.

   Let D be the event that a patient has COVID-19, and let T be the event that a test is positive for COVID-19. We will define some probabilities that are commonly used in these medical testing settings:

   - Prevalent: $\mathrm{P}(D)$
   - Sensitivity: $\mathrm{P}(T|D)$
   - Specificity: $\mathrm{P}(T^c|D^c)$
   - Positive predictive value: $\mathrm{P}(D|T)$
   - Negative predictive value: $\mathrm{P}(D^c|T^c)$

   According to the website, the Immunodiagnostic Systems COVID-19 test has a sensitivity of 97.6% and specificity of 99.6%. Suppose the prevalence of COVID-19 in the general US population 2%. What are the positive and negative predictive values of the Immunodiagnostic Systems test?

6. To get to Middlebury College, a professor uses their car 30% of the time, walks 20% of the time, and bikes 50% of the time. They are late 5% when walking, 10% of the time when driving (because this is Vermont and people stop for all pedestrians), and 2% of the time when biking.

   (a) What is the probability the professor drove to work if they were late?

   (b) What is the probability the professor walked to work if they were on time?

Thursday 10/03

None!

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.