Introduction to Hypothesis Testing

3/27/25

Testing

We are now entering into second branch of inference-related tasks: testing.

- We have some “claim”/question about the target population, and we use sampled data to provide evidence for or against the claim
    - Especially important in medicine
- We will use the *hypothesis testing framework* to formalize the process of making decisions about research claims.
    - Because the claim is about target population, we will almost always formulate claims in terms of population parameters
    - Then we use sampled data to provide the evidence for/against

Hypothesis testing framework

Four stages (we will step through each one):

1. Define your hypotheses
2. Collect data, set a significance level
3. Determine strength of evidence (null distribution, p-value)
4. Make decision and conclusion in context

Step 1: Define hypotheses

A **hypothesis test** is a statistical technique used to evaluate competing claims using data

- We define hypotheses to translate our research question/claim into statistical notation
- We always define two hypotheses *in context*: a null hypothesis and an alternative hypothesis
- **Null hypothesis** H_0: hypothesis that represents “business as usual”/status quo/nothing unusual or noteworthy
- **Alternative hypothesis** H_A: claim the researchers want to demonstrate

It will not always be obvious what the hypotheses should be, but you will develop intuition for this over time!

Practice in defining hypotheses

For each of the following, determine whether it represents a null hypothesis claim or an alternative hypothesis claim:

1. King cheetahs on average run the same speed as standard spotted cheetahs.
2. For a particular student, the probability of correctly answer a 5-option multiple choice test is larger than 0.2 (i.e. better than guessing)
3. The probability of getting in a car accident is the same if using a cell phone then if not using a cell phone.
4. The number of hours that grade-school children spend doing homework predicts their future success on standardized tests.

Practice in defining hypotheses

For each of the following, determine whether it represents a null hypothesis claim or an alternative hypothesis claim:

1. King cheetahs on average run the same speed as standard spotted cheetahs.
    - Null!
2. For a particular student, the probability of correctly answer a 5-option multiple choice test is larger than 0.2 (i.e. better than guessing)
    - Alternative!
3. The probability of getting in a car accident is the same if using a cell phone then if not using a cell phone.
    - Null!
4. The number of hours that grade-school children spend doing homework predicts their future success on standardized tests.
    - Alternative!

Practice in defining hypotheses

Write out the null and alternative hypotheses in words and also in statistical notation for the following situations:

1. New York is known as “the city that never sleeps”. A random sample of 25 New Yorkers were asked how much they sleep they get per night. Do these data providing convincing evidence that New Yorkers on average sleep less than 8 hours per night?
2. A study suggests that 25% of 25 year-olds have gotten married. You believe that this is incorrect and decide to conduct your own analysis.

Practice in defining hypotheses

New York is known as “the city that never sleeps”. A random sample of 25 New Yorkers were asked how much they sleep they get per night. Does these data providing convincing evidence that New Yorkers on average sleep less than 8 hours per night?

- Words
  - H_0: New Yorkers sleep an average of 8 hours per night
  - H_A : New Yorkers sleep an average of less than 8 hours per night
- Notation: let μ be the average hours of sleep of New Yorkers
  - $H_0 : \mu = 8$
  - $H_0 : \mu < 8$

Practice in defining hypotheses

A study suggests that 25% of 25 year-olds in the US have gotten married. You believe that this is incorrect and decide to conduct your own analysis.

- Words
  - H_0: the proportion of 25 year-olds in the US who are married is 0.25
  - H_A : the proportion of 25 year-olds in the US who are married is not 0.25
- Notation: let p be the proportion of 25 year-olds in the US who are married
  - $H_0 : p = 0.25$
  - $H_0 : p \neq 0.25$

Defining hypotheses in context

Research question: do the minority of Middlebury students drink coffee regularly?

- Try to write down our null and alternative hypotheses in statistical notation! This includes defining parameters!
  - Define p as the true proportion of Middlebury students who drink coffee regularly
  - $H_0 : \; p = 0.5$ versus $H_A : \; p < 0.5$

Step 2: Collect and summarize data

Our sample is the convenience sample I took of our class: 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, where 1 = "yes" and 0 = "no".

- Point estimate: $\hat{p}_{obs} = 0.35$
- Are we prepared to answer our research question based on this evidence?
- NO! Due to variability, we should ask: do the data provide ***convincing evidence*** that the minority of Middlebury students drink coffee regularly?

Step 3: Determine if we have "convincing evidence"

"Convincing evidence" for us means that it would be highly unlikely to observe the data we did (or data even more extreme) ***if*** H_0 were true!

- We will calculate a **p-value**: the probability of observing data as or more extreme than we did, *assuming* H_0 true
  - Note: p in "p-value" is not the same as parameter p!
  - This is a conditional probability: we condition on H_0 true
- Highly unlikely is vague and needs to defined by the researcher, ideally before seeing data.
  - If we want to provide a yes/no answer to the research question, we need some threshold to compare the p-value to. This is called a **significance level** α
  - Common choices are $\alpha = 0.05, \alpha = 0.01$ (more on this later)!
- For our example, we will choose $\alpha = 0.05$

How to obtain p-value?

- How to obtain this probability?
- Need access to a distribution that corresponds to a world where H_0 is true (i.e. the **null distribution**)
  - Option 1: if we have assumptions about how our data behave, we can obtain this distribution using theory/math (next week)
  - Option 2: if we don't want to make assumptions, why not simulate?
    - We will call this option "simulating under H_0"
- This is the step that requires the most "work", and what exactly you do will depend on the the type of data and the research question/claim you have

Simulating under H_0 (step 3 cont.)

- We have to simulate our data under the assumption that H_0 is true (recall H_0: $p = 0.5$)
- Imagine a big bag filled with many slips of pink and purple slips of paper
  - Pink = coffee-drinkers
  - Purple = non-coffee-drinkers

- **To simulate under H_0**, what proportion of the slips in the bag should be pink vs purple?

- To simulate under $H_0 : p = 0.50$, half of the slips should be pink!

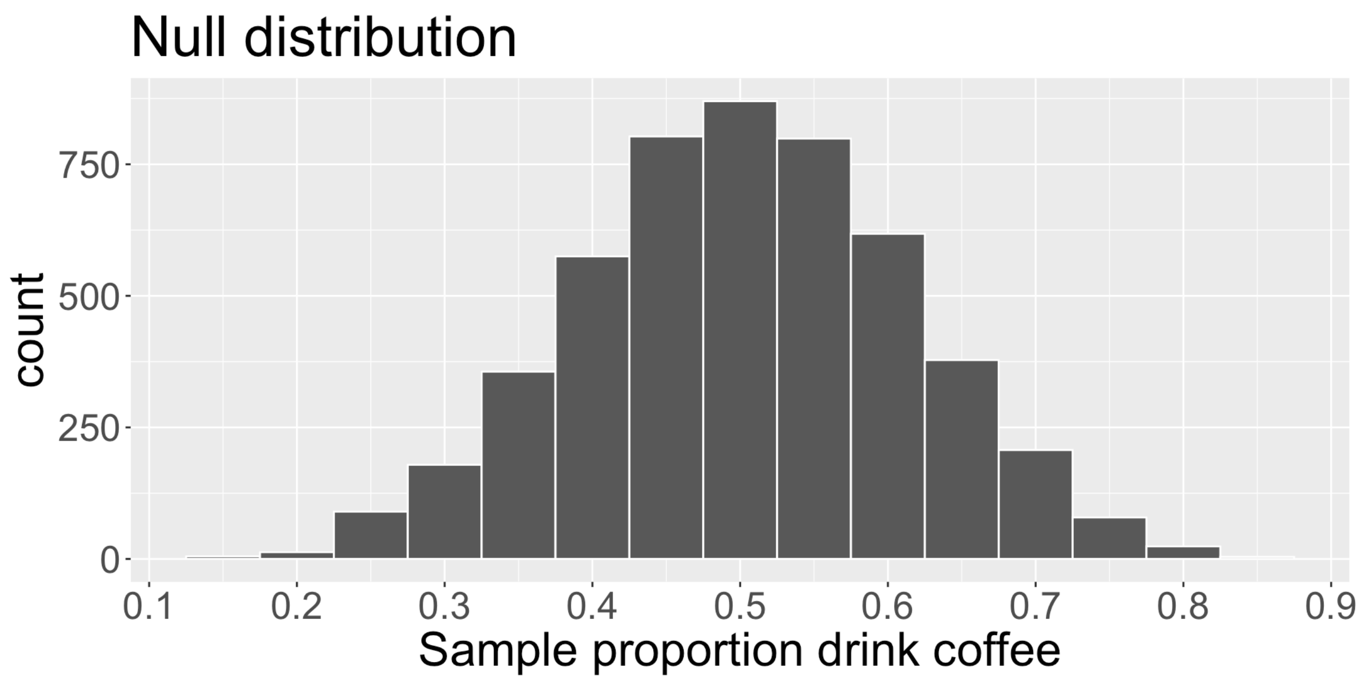
Simulating under H_0 (step 3 cont.)

- To simulate under H_0, we replicate our original sample, this time sampling from this "null world" bag of paper slips
  - Repeatedly take samples from this null distribution using original sample size $n = 20$
  - For each sample, calculate the simulated proportion of pink slips
- Live code?

```r
set.seed(2)
B <- 5000
n <- length(x)
p_null_vec <- rep(NA, B)
for(b in 1:B){
  null_samp <- sample(x = c("pink", "purple"),
                      size = n,
                      replace = T,
                      prob = c(0.5, 0.5))
  p_null_vec[b] <- sum(null_samp == "pink")/n
}
```

Null distribution of statistic

We can visualize the distribution of $\hat{p}$ assuming H_0 true:



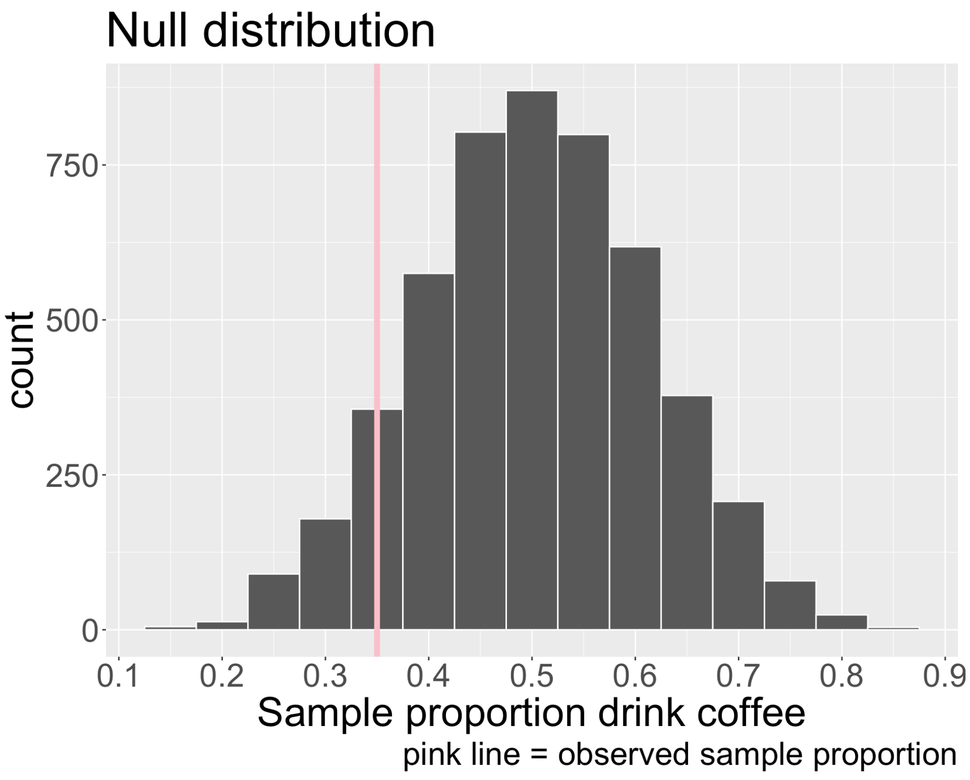
- This is called the **null distribution** of the sample statistic, which is the distribution of the statistic $\hat{p}$ assuming H_0 is true
- Where is this null distribution of $\hat{p}$ centered? Why does that "make sense"?

Comparing null to observed

Let's return to our original goal of Step 3! We need to find the **p-value**: the probability of observing data as or more extreme as ours, assuming H_0 were true.

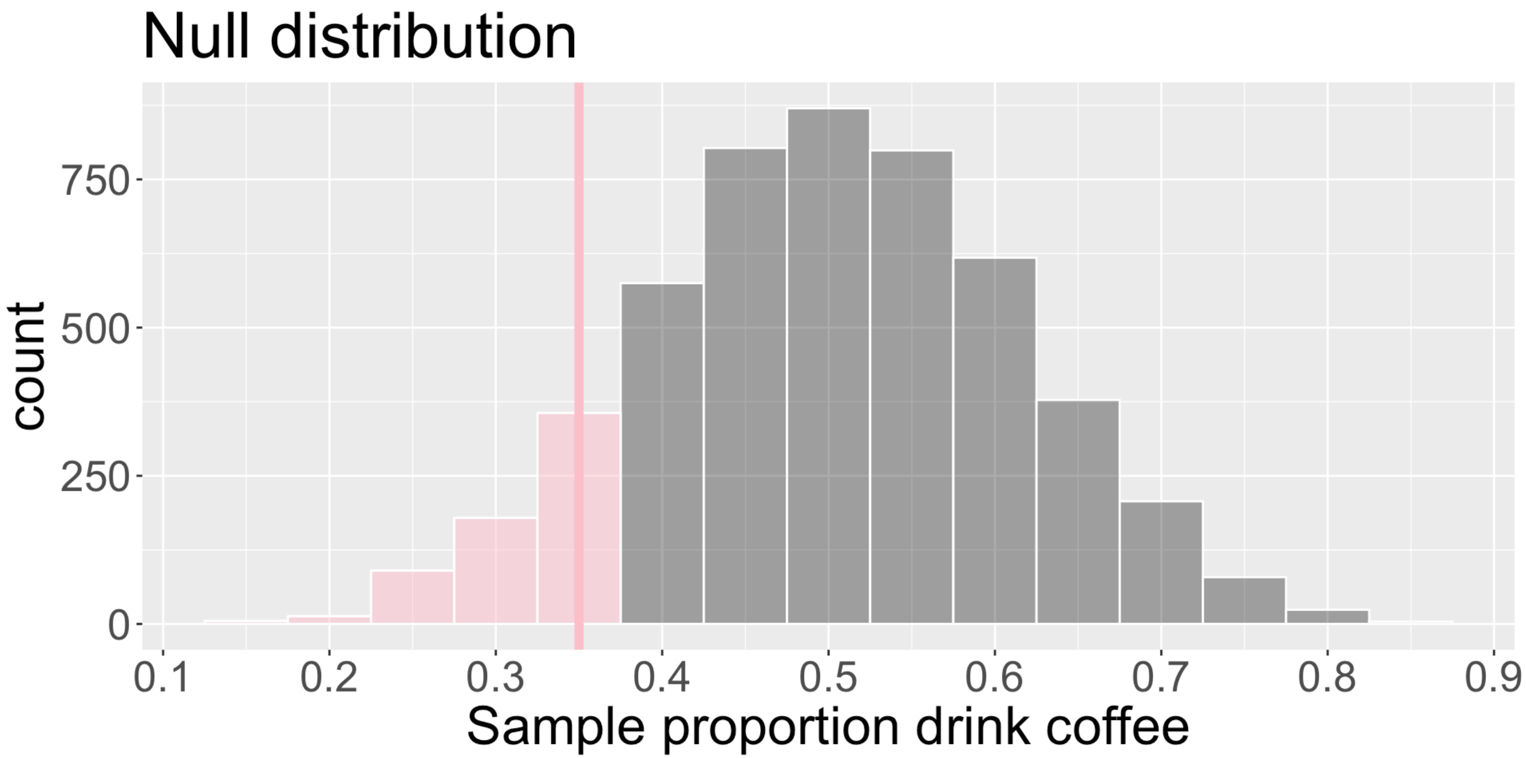
- Our observed point estimate was $\hat{p}_{obs} = 0.35$
- $H_0: p = 0.5$ and $H_A: p < 0.5$
- What does "as or more extreme" mean in this context?

  How can we use the null distribution to obtain this probability?



Obtain p-value (step 3 cont.)

We can directly estimate the p-value using our null distribution and our observed $\hat{p}$!



- Out of 5000 replications, we saw 643 instances of $\hat{p} \leq \hat{p}_{obs}$
- p-value is $\frac{643}{5000} \approx 0.13$

Step 4: Interpret p-value and make decision

1. Interpret the p-value 0.1286 in context

   - Assuming H_0 true, the probability of observing a sample proportion as or more extreme as our 0.35 is approximately 0.13

2. Make a decision about research claim/question by comparing p-value to significance level α

   - If p-value $< \alpha$, we *reject* H_0 (it was highly unlikely to observe our data given H_0 and our selected threshold)
   - If p-value $\geq \alpha$, we *fail to reject* H_0 (not have enough evidence against the null)
     - Note: we never "accept H_A"!
   - Since our p value is greater than $\alpha = 0.05$, we fail to reject H_0. The data do not provide sufficient evidence to suggest that the minority of Middlebury students drink coffee regularly.

Summary of testing framework

Four steps for hypothesis test:

1. Define null and alternative hypotheses H_0 and H_A in context
2. Collect data and set significance level α
3. Obtain the null distribution of the statistic and use it to obtain/estimate p-value
   - We did this using by simulating
4. Interpret p-value and make a decision in context

Errors in decision

- In Step 4, we make a decision but it could be wrong! (Unfortunately, we will never know)
- We always fall into one of the following four scenarios:

		State of world	
		H_0 true	H_0 false
Decision	Fail to reject H_0		
	Reject H_0		

- Identify which cells are good scenarios, and which are bad

Errors in decision

		State of world	
		H_0 true	H_0 false
Decision	Fail to reject H_0	Correct	Type II error
	Reject H_0	Type I error	Correct

- What kind of error could we have made in our example?
- It is important to weight the consequences of making each type of error!
  - We have some control in this - how? Through α!