

(\*): Assigned to weekly problem set.

## Indicators and Fundamental Bridge

1. A group of  $n \geq 4$  people are comparing their birthdays (assume the usual set-up of the birthday problem). Let  $\mathbf{1}_{ij}$  be the indicator random variable that persons  $i$  and  $j$  have the same birthday ( $i < j$ ). Is  $\mathbf{1}_{12}$  independent of  $\mathbf{1}_{34}$ ? Is  $\mathbf{1}_{12}$  independent of  $\mathbf{1}_{13}$ ? Are all the  $\mathbf{1}_{ij}$  independent of each other?
2. (\*) Two researchers independently select simple random samples from a population of size  $N$ , with sample sizes  $m$  and  $n$  (for each researcher, the sampling is done without replacement, with all samples of the prescribed size equally likely). For clarity: once the first researcher samples without replacement their  $m$  elements, the researcher puts them all back into the pool and then the second researcher chooses their  $n$  elements without replacement. Find the expected size of the overlap of the two samples by:
  - (a) Using the fundamental bridge.
  - (b) Using a distribution we have learned in this class.
3. Suppose you are watching cows walk around Addison County. Every minute, a cow walks by, which is equally likely to be any one of the  $n$  cows in the county. What is the expected number of distinct cows you have seen after  $t$  minutes?