(*): Assigned to weekly problem set.

Conditional Probability, Bayes' rule and LoTP

1. In poker, you are dealt a hand of five cards from a standard deck of 52 cards. What is the probability of getting a poker hand where all the cards are seven or higher, given that at least one of the cards is higher than ten (i.e. Jack, Queen, King, or Ace)?

2. (*) In deterministic logic, the statement "A implies B" is equivalent to its contrapositive: "not B implies not A". In this problem, we will consider analogous statements in probability, the logic of uncertainty. Let A and B be events with probabilities not equal to 0 or 1. Show that if $P(B|A) = 1$, then $P(A^c|B^c) = 1$. *Hint: Apply Bayes' rule and LoTP.*

3. Let A and B be two events where $P(B) > 0$.

   (a) Show that if $P(A) = 1$, then $P(A|B) = 1$.

   (b) Interpret what (a) means intuitively.

4. (*) **Game of Craps**. Craps is a casino dice rolling game where two dice are rolled. There are many bets that can be made, but we will focus on betting on the "Pass" line. The rules are as follows:

   - Roll two dice and sum the two values.
   - If the outcome is 7 or 11: you win!
   - If the outcome is 2, 3, or 12: you lose!
   - If the outcome is any other number (e.g. 5), call this number "the point".
     - Roll the pair of dice repeatedly until you roll a 7 or the point.
     - If the outcome is 7: you lose!
     - If the outcome is the point: you win!

   What is the probability of winning the game (round to thousandths place)?