(*): Assigned to weekly problem set.

Topic

1. There has been a malware attack on university data management system, which stores student CRFs. To lose one semester's CRFs, it is enough for two computer worms to modify/infect two important lines of code, or for one virus to overwrite the host program. Let p_1 be the probability of a worm infecting the first important line of code, p_2 the probability of a worm infecting the second important line of code, and p_3 the probability of a virus overwriting the host program. It is assumed that all three places are targeted independently of each other. What is the probability that the semester's CRFs are lost?

2. (*-ish) Middlebury College currently does not require college applicants to send standardized test scores. So let us consider a hypothetical school called Medianbury College which does require SAT scores. Medianbury will accept a student if and only if they score well on the SAT, have big dreams, or both.

   (a) *Among the entire US population*, do you think doing well on the SAT and having big dreams are independent or dependent? If dependent, do you think there is a negative association (i.e. conditioning on a good SAT score decreases the 'size' of your dream)?

   (b) *Among students at Medianbury College*, do you think doing well on the SAT and having big dreams are independent or dependent? If dependent, do you think there is a negative association (i.e. conditioning on a good SAT score decreases the 'size' of your dream)? Provide a brief explanation.

   (c) Suppose we have two independent events A and B. Define $C = A \cup B$. Assuming $P(A \cup B) < 1$ and $P(A \cap B) > 0$, demonstrate that A and B are conditionally *dependent* given C, and $P(A|B, C) < P(A|C)$.

   This phenomenon is known as *Berkson's paradox* or *collider bias*.

   This problem is adapted from Blitzstein and Hwang 2.36.

3. A device has four parts, each of which operates independently. It was discovered that two of the parts failed. What is the probability that it was parts 1 and 2 that failed if the probability of part i failing is $\frac{i}{10}$, for $i = 1, 2, 3, 4$?