(*): Assigned to weekly problem set.

Central Limit Theorem

1. Let $X \sim \text{Exp}(3)$, and define $Y = e^X$.

   (a) Find the mean and variance of Y.

   (b) For $Y_1, \ldots, Y_n$ iid with the same distribution of Y, what is the approximate distribution of the sample mean $\bar{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$ when n is large?

2. Suppose that a random sample of size $n = 12$ is taken from a $\text{Unif}(0, 1)$ distribution, i.e. $X_1, \ldots, X_{12} \overset{\text{iid}}{\sim} \text{Unif}(0, 1)$. Using the Central Limit Theorem (even though 12 is rather small), approximate the value of $P(|\bar{X}_n - \frac{1}{2}| \leq 0.1)$, where $\bar{X}_n$ is the sample mean of these 12 random variables. You may eventually find it useful to use $\Phi()$ notation!