(*): Assigned to weekly problem set.

Joint distributions: Continuous and Hybrid

1. Let X and Y have a joint PDF

$$f_{X,Y}(x,y) = cxy \quad \text{for } 0 < x < y < 1.$$

   (a) Find the value of c that makes this a valid joint PDF.

   (b) Find the marginal PDFs of X and Y.

   (c) Explain why X and Y are **not** independent (even though superficially, their joint PDF might appear to factor). *Hint: carefully consider the domain of* $f_{X,Y}$

   (d) Find the conditional PDF of Y given $X = x$.

2. (*) Alice and Bob arrange to meet for lunch on a certain day at noon. However, neither is known for punctuality. They both arrive independently at uniformly distributed times between noon and 1pm on that day. Each is willing to wait up to 15 minutes for the other to show up. What is the probability they will meet for lunch that day?

   Hint: Construct a graph of the region in the xy-plane corresponding to the event that Alice and Bob meet for lunch.