(*): Assigned to weekly problem set.

Normal Distribution

1. (*) Let $Y = |X|$, with $X \sim N(\mu, \sigma^2)$. This is a well-defined continuous RV, even though the absolute value function is not differentiable at 0.

   (a) Find the CDF of Y in terms of Φ. Be sure to specify the CDF everywhere.

   (b) Find the PDF of Y.

   (c) Is the PDF of Y continuous at 0? If not, is this a problem as far as using the PDF to find probabilities?

2. (*) The distance between two points need to be measured, in meters. The true distance between the points is 10 meters, but due to measurement error we can't measure the distance exactly. Instead, we will observe a value of $10 + \epsilon$, where the error ϵ is distributed $N(0, 0.04)$. Find the probability that the observed distance is within 0.4 meters of the true distance (10 meters). Give both an exact answer in terms of Φ and an approximate numerical answer (not using R).

3. Assume the following result is true (we will show later why): if X_1 and X_2 are independent with $X_i \sim N(\mu_i, \sigma_i^2)$ for $i \in \{1, 2\}$, then $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. Using this result, find $P(X < Y)$ for $X \sim N(a, b)$ independent of $Y \sim N(c, d)$. Does your answer make sense in the special case when X and Y are i.i.d?