Pre-class preparation

Please read the following textbook sections from Blitzstein and Hwang's *Introduction to Probability* (second edition) OR watched the indicated video from Blitzstein's Math 110 YouTube channel:

- Textbook: Sections 10.1 (focus just on section 10.1.3 and onward), and 10.2
- Video:
  - Lecture 28: Inequalities (from 35:00 to end)
  - Lecture 29: Law of Large Numbers and Central Limit Theorem (from beginning to 15:00).

Objectives

By the end of the day's class, students should be able to do the following:

- Prove the Markov inequality, then derive the Chebyshev and Chernoff inequalities from Markov's inequality.
- Apply the Markov, Chebyshev, and Chernoff inequalities to find upper bounds on probabilities for random variables.
- State both the strong and the weak versions of the Law of Large Numbers.
- Prove the Weak Law of Large Numbers in the case when X has finite variance.

Reflection Questions

Please submit your answers to the following questions to the corresponding Canvas assignment by 7:45AM:

1. What is one circumstance where the Markov inequality can be used to estimate $P(|X| > a)$, but where the Chebyschev inequality cannot be used. *Hint: Think about what the assumptions made for the Chebyshev inequality, and compare to the assumptions made for the Markov inequality.*
2. Assume $X_1, X_2, \ldots$ are an iid sample of random variables with finite mean μ and variance σ^2. Using Chebyshev's inequality, what is an upper bound (as a function of n) of the probability that the sample mean $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ is more than 2 standard deviations away from the mean of the distribution μ? What is the interpretation of this probability as n gets larger?
3. In one or two sentences, summarize in your own words what the Law of Large Numbers means.
4. (Optional) Is there anything from the pre-class preparation that you have questions about? What topics would you like would you like some more clarification on?