(*): Assigned to weekly problem set.

Conditional expectation given a RV

1. (*) Let $X_1, X_2, \ldots$ be iid random variables with mean 0, and let $S_n = X_1 + X_2 + \ldots + X_n$ for all positive integers n.

   (a) For integer k such that $1 \leq k \leq n$, show that $\mathbb{E}[X_k|X_n] = \frac{S_n}{n}$. *Hint: consider* $\mathbb{E}[S_n|S_n]$ *and use linearity and symmetry.*

   (b) Use the preceding result to find $\mathbb{E}[S_k|S_n]$ for integer k such that $1 \leq k \leq n$.

2. Let $\mathbf{X} \sim \text{Multinom}_5(n, \mathbf{p})$. Recall this notation means that both $\mathbf{X}$ and $\mathbf{p}$ are vectors of length 5. Find $\mathbb{E}[X_1|X_2]$.

3. (*) Show that if $\mathbb{E}[Y|X] = c$ where c is a constant, then X and Y are uncorrelated. *Hint: use Adam's law to find* $\mathbb{E}[Y]$ *and* $\mathbb{E}[XY]$.

4. Show that for any random variables X and Y,

$$\mathbb{E}[Y|\mathbb{E}[Y|X]] = \mathbb{E}[Y|X]$$

   Hint: use Adam's law with extra conditioning.