(*): Assigned to weekly problem set.

Monty Hall

1. Consider the following 7-door version of the Monty Hall problem. There are 7 doors, behind one of which there is a car (which you want), and behind the rest of which there are goats. Initially, all possibilities are equally likely for where the car is. Without loss of generality, suppose you choose Door 1. Monty then opens three goat doors and offers you the option of switching to any of the remaining three doors.

   Assume that Monty knows which door has the car, will always open three goat doors and offer the option of switching, and that Monty chooses with equal probabilities from all his choices of which goat doors to open. Should you switch? What is your probability of success if you switch to one of the remaining three doors?

2. (*) Consider the following variation of the Monty Hall problem: Before each show, Monty secretly flips a coin with probability p of Heads. If the coin lands Heads, Monty resolves to open a door with a goat (with equal probability if there is a choice). Otherwise, Monty resolves to open a random door with equal probabilities. Of course, Monty will not open the door that the contestant initially chooses. The contestant knows p, but does not know the outcome of the coin flip.

   When the show starts, the contestant chooses a door. Monty (who knows where the car is) then opens a door. If the car is revealed, the game is over! If a goat is revealed, the contestant is offered the option of switching.

   Now suppose the contestant chooses Door 1 and then Monty opens Door 2, revealing a goat. What is the contestant's probability of success if they switch to Door 3?

First-Step Analysis and Gambler's Ruin

1. There are 100 equally-spaced points around a circle. At 99 of the points, there are sheep, and at 1 point there is a wolf. At each time step, the wolf randomly moves either clockwise or counterclockwise by 1 point. If there is a sheep at that point, he eats it. The sheep don't move (so silly!). What is the probability that the sheep who is initially opposite the wolf is the last one remaining?