

Pre-class preparation

If you'd like a refresher on change-of-variables from Calculus, please watch this video.

Please read the following textbook sections from Blitzstein and Hwang's *Introduction to Probability* (second edition) OR watched the indicated video from Blitzstein's Math 110 YouTube channel:

- Textbook: Section 8.1 (skim Example 8.1.10; it's good to see it, but you won't be assessed on it)
- Video: Lecture 22: Transformations and Convolutions (from 7:00 to 27:00). Also skim Section 8.1 in the text as there's a lot of information covered there that's not present in the video.

Objectives

By the end of the day's class, students should be able to do the following:

- State the change-of-variables formula in both one and several dimensions.
- Compute the PDF of the Log-Normal, the Chi-Square distribution and the Cauchy distribution using the single-variable change-of-variables formula.
- Use the multivariate change-of-variables formula to compute the joint PDF of transformations of 2 or 3 variables.

Reflection Questions

Please submit your answers to the following questions to the corresponding Canvas assignment by 7:45AM:

1. Suppose $y = x^3$. Compute $\frac{dx}{dy}$ as function of y in two ways (your final answers should be the same):
 - (a) By solving for x explicitly and differentiating.
 - (b) By calculating $\frac{dy}{dx}$, taking the reciprocal, and then substituting $x = \sqrt[3]{y}$ in the final formula.
2. Suppose U and T are two random variables with joint PDF $f_{U,T}$, and define $X = U + T$ and $Y = U - T$.
 - (a) Express U and T as explicit functions of X and Y .
 - (b) Calculate the Jacobian matrix $\frac{\partial(x,y)}{\partial(u,t)}$ and find the absolute value of its determinant.

- (c) Find a formula for the joint PDF $f_{X,Y}$ of (X, Y) in terms of $f_{U,T}$ and the variables x, y .
3. (Optional) Is there anything from the pre-class preparation that you have questions about? What topics would you like would you like some more clarification on?