This problem set covers material from Week 4, dates 10/03- 10/06. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding chapter. "AP" stands for additional problems not found in the book.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Tuesday 10/03

- **Chapter 3**: 9, 15
- **AP 1**: Recall de Montmort's matching problem from Chapter 1: in a deck of n cards labeled 1 through n, a match occurs when the number on the card matches the card's position in the deck. Let X denote the number of matching cards. In this problem, assume $n \geq 2$.

  a) What is the support of X?

  b) Is X Bernoulli? Binomial? Hypergeometric? Discrete Uniform? None of these? Explain your reasoning for each one of these distributions/cases, including "none of these".

- **AP 2**: Although the textbook says that the Hypergeometric distributions takes on integer values between 0 and n for some n, we should be a little more careful when describing the support. Suppose $X \sim$ Hyper Geometric(w, b, n). The true support of X is $S_X = \{\max(0, n-b), \ldots, \min(n, w)\}$, where the notation $\max(0, n-b)$ means that the larger of the two values 0 and $(n-b)$ is the first value in S_X.

  Clearly explain why the bounds of the support are $\max(0, n-b)$ and $\min(n, w)$, and not just 0 and n.

Thursday 10/05

- Chapter 3: 33, 39, 44 (parts a and b only)
- **AP 3**: Let X be a discrete random variable with PMF

$$p_X(x) = \frac{1}{2^{x+1}}, \qquad x = 0, 1, 2, \ldots$$

  Let $Y = 2^X$. Find a formula for the PMF of Y $p_Y(y)$ (don't forget the support), and compute $P(Y = 1)$.

Friday 10/06

- **AP 4**: Suppose we have two random variables X and Y. Let $V = \min(X, Y)$ and $W = \max(X, Y)$. So if we observe $X = x$ and $Y = y$, then V is observed as $\min(x, y)$ and similarly for W.

  a) Show the following (interesting but not necessarily useful) addition law for expectations:
  $$\mathbb{E}[W] = \mathbb{E}[X] + \mathbb{E}[Y] - \mathbb{E}[V]$$

  b) Let $X \sim \text{Binomial}(n, \frac{1}{2})$ and $Y \sim \text{Binomial}(n+1, \frac{1}{2})$. Find $\mathbb{E}[V] + \mathbb{E}[W]$.

- **AP 5**: Suppose we are flipping two fair coins. Let X be the number of Heads in the two independent flips of the coins. Find $\mathbb{E}[X^3]$ in two ways:

  a) Finding the PMF of $Y = X^3$, then obtaining $\mathbb{E}[Y]$.

  b) Using LoTUS.

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.