(*): Assigned to weekly problem set.

Variance

1. Bobo the amoeba currently lives alone in a pond. After one minute, Bobo will either die, remain the same, or split into two amoebas, with equal probability. Find the variance for the number of amoebas in the pond after one minute.

2. Suppose X is a random variable with support $S_X = \{1, 2, \ldots\}$ and PMF $P(X = x) = \frac{1}{x(x+1)}$. What is $\text{Var}(X)$?

3. (*) Show that $\text{Var}(X + c) = \text{Var}(X)$, where $c \in \mathbb{R}$ is a constant.

4. (*) Show that $\text{Var}(cX) = c^2 \text{Var}(X)$, where $c \in \mathbb{R}$ is a constant.