

(*): Assigned to weekly problem set.

Change of Variable

1. (*) Find the PDF of $Y = e^{-X}$ for $X \sim \text{Exp}(1)$.
2. Find the PDF of $|Z|$ where $Z \sim N(0, 1)$.
3. (*) Suppose U and $V \stackrel{\text{iid}}{\sim} \text{Exp}(\lambda)$. We will find the joint distribution of $X = U + V$ and $Y = \frac{U}{U+V}$, as well as the marginal distribution of Y alone.
 - (a) Define the function $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $(x, y) = g(u, v) = (u + v, \frac{u}{u+v})$. Find a formula for the inverse transformation $(u, v) = g^{-1}(x, y)$.
 - (b) Calculate the Jacobian of the transformation and its determinant.
 - (c) Use the change-of-variables formula to express the joint PDF $f_{X,Y}(x, y)$ in terms of the joint PDF $f_{U,V}(u, v)$. Don't forget the support!
 - (d) Based on your previous answer, are X and Y independent?
 - (e) Find a formula for the marginal PDF of Y . What named distribution is this?