Pre-class preparation

Please read the following textbook sections from Blitzstein and Hwang's *Introduction to Probability* (second edition) OR watched the indicated video from Blitzstein's Math 110 YouTube channel:

- Textbook: Sections 8.6 and 9.1
- Video:
  - Lecture 25: Order Statistics and Conditional Expectation (23:30 through end)
  - Lecture 26: Conditional Expectation (beginning through 29:00)

Objectives

By the end of the day's class, students should be able to do the following:

- State the definition of order statistics.
- Derive the CDF and PDF of an order statistic generally, and in the special case of Uniform random variables.
- State the definition of conditional expectation given an event.
- Apply the Law of Total Expectation to compute expected values.
- Resolve the 2 envelope paradox.

Reflection Questions

Please submit your answers to the following questions to the corresponding Canvas assignment by 7:45AM:

1. Let $X_1, \ldots, X_n$ be iid continuous RVs with PDF $f(x)$ and CDF $F(x)$. Explain in your own words the rational/justification of each component of/term in the PDF of an order statistic $X_{(j)}$:
$$f_{X_{(j)}}(x) = n\binom{n-1}{j-1} f(x) F(x)^{j-1} (1 - F(x))^{n-j}$$
2. Suppose X is a random variable and let $Y = X^3$. Evaluate $\mathbb{E}[Y|X = 2]$. Does your answer agree with your intuition?
3. In your own words, explain what the Law of Total Expectation means. Limit your answer to 2 - 3 sentences.
4. (Optional) Is there anything from the pre-class preparation that you have questions about? What topics would you like would you like some more clarification on?