(*): Assigned to weekly problem set.

MGFs continued

1. (*) Use MGFs to determine whether X + 2Y is Poisson if X and Y are i.i.d. Poisson(λ). *Hint: if $X+2Y$ was indeed Poisson, which specific Poisson distribution would it have?*

2. (*) Suppose $X_1, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ random variables. Define $Y = \frac{1}{n}\sum_{i=1}^{n} X_i$.

   (a) Find the MGF of Y.

   (b) Based on your answer in (a), state the exact distribution of Y.

3. Use the MGF of the Geometric(p) distribution to give another proof that the mean of this distribution is $\frac{1-p}{p}$ and the variance is $\frac{1-p}{p^2}$.