Pre-class preparation

Please read the following textbook sections from Blitzstein and Hwang's *Introduction to Probability* (second edition) OR watched the indicated video from Blitzstein's Math 110 YouTube channel:

- Textbook: Sections 9.5-9.6
- Video: Lecture 27: Conditional Expectation given a R.V (from 30:00 to end)

Objectives

By the end of the day's class, students should be able to do the following:

- State the definition of conditional variance.
- Use "Eve's Law" to compute conditional variance.

Reflection Questions

Please submit your answers to the following questions to the corresponding Canvas assignment by 7:45AM:

1. Suppose X and Y are random variables. What is one circumstance where X **is not** a constant random variable, but where $\text{Var}(X|Y) = 0$?
2. Let $N \sim \text{Binom}(n, p)$ and for $i = 1, \ldots, n$, let $X_i \sim \text{Bern}(1/2)$ independently of each other, as well as of N. Define a variable S by $S = X_1 + X_2 + \cdots + X_N$ (that is, S is a variable containing a random sum of some subset of the variables $X_1, \ldots, X_n$). Explain what is wrong with the following argument:

$$\mathbb{E}[S] = E\left[\sum_{k=1}^{N} X_i\right] = \sum_{k=1}^{N} \mathbb{E}[X_i] = \sum_{i=1}^{N} \frac{1}{2} = \frac{N}{2}.$$

3. (Optional) Is there anything from the pre-class preparation that you have questions about? What topics would you like would you like some more clarification on?