(*): Assigned to weekly problem set.

Functions and Independence of Random Variables

1. Let $X \sim$ Discrete Unif$(1, 9)$, and suppose we are interested in how far X is from the middle of the distribution, namely, 5. Define $Y = |X - 5|$. What is the PMF of Y, $p_Y(y)$?

2. (*) For x and y binary digits (0 or 1), let $x \oplus y$ be 0 if $x = y$ and 1 if $x \neq y$ (this operation is called *exclusive or*, often abbreviated *XOR*).

   (a) Let $X \sim$ Bern(p) and $Y \sim$ Bern$(1/2)$, where $X \perp\!\!\!\perp Y$ (the $\perp\!\!\!\perp$ stands for "independent"). What is the distribution of $Z = X \oplus Y$?

   (b) With notation as in (a), is Z independent of X? Is Z independent of Y? Be sure to consider both the case $p = 1/2$ and the case $p \neq 1/2$.

3. Use the `plot` function in R (or `ggplot` if you're familiar with it) to plot the CDF of a Binomial$(8, 0.72)$ random variable. Explain how to use the values displayed in the CDF plot to recover the PMF of this random variable. Based on the plot of the CDF, which value of the random variable has the highest probability of occurring? Why? Verify your method works by plotting the PMF of this same random variable.

4. (*) A book has n typos. Two proofreaders, Barbie and Ken, independently read the book. Barbie catches each typo independently with probability p_1. Likewise for Ken, who has probability p_2 of catching each typo, independently. Let X be the number of typos caught by Barbie, Y the number caught by Ken, and Z the number caught by at least one of the two proofreaders.

   (a) Find the distribution of Z.

   (b) For this part only, assume that $p = p_1 = p_2$. Find the conditional distribution of X given $X + Y = t$.