

This problem set covers material from Weeks 10, dates 11/14 and 11/16. **Note the atypical due date!** Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding chapter. “AP” stands for additional problems not found in the book.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Tuesday 11/14

- Chapter 8: 1, 13
- **AP 1:** Suppose U and $V \stackrel{\text{iid}}{\sim} \text{Exp}(\lambda)$. We will find the joint distribution of $X = U + V$ and $Y = \frac{U}{U+V}$, as well as the marginal distribution of Y alone.
 - a) Define the function $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $(x, y) = g(u, v) = (u + v, \frac{u}{u+v})$. Find a formula for the inverse transformation $(u, v) = g^{-1}(x, y)$.
 - b) Calculate the Jacobian of the transformation and its determinant.
 - c) Use the change-of-variables formula to express the joint PDF $f_{X,Y}(x, y)$ in terms of the joint PDF $f_{U,V}(u, v)$. Don't forget the support!
 - d) Based on your previous answer, are X and Y independent?
 - e) Find a formula for the marginal PDF of Y . What named distribution is this?

Thursday 11/16

- **AP 2:** Suppose we roll five fair six-sided dice, all of which are independent of each other.
 - a) What is the probability that the number 1 and the number 4 will appear the same number of times?
 - b) Suppose it is known that two of the dice rolls resulted in values that were neither 1 nor 4. Given this information, what is the probability distribution for the number of 1's and 4's rolled?

General rubric

Points	Criteria
5	The solution is correct <i>and</i> well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.