(*): Assigned to weekly problem set.

2D LoTUS and Covariance/Correlation

1. A random point is chosen uniformly in the unit disk $\{(x, y) : x^2 + y^2 \leq 1\}$. Let R be its distance from the origin.
   (a) Find $\mathbb{E}[R]$ using 2D LOTUS. Polar coordinates may be helpful here...
   (b) Find the CDF of R^2 and R without integrating, using the fact that the probability that the randomly chosen point is in a particular region is proportional to the area of the region.
   (c) Then obtain the PDFs of R and R^2 by differentiating, and then use the results to calculate $E[R]$ in two more ways: by using the definition of expectation on R, and by using 1D LOTUS and thinking of R as a function of R^2.
2. (*) Let X and Y be iid Unif$(0, 1)$.
   (a) Compute the covariance of $(X + Y)$ and $(X - Y)$.
   (b) Are $X + Y$ and $X - Y$ independent? Explain (you do not have to do this mathematically, but rather explain in words or via counter-example).
3. (*) (Great practice for midterm 2 in terms of setting up the problem; covariance won't be on midterm of course). Each of $n \geq 2$ people puts their name on a slip of paper (no two have the same name). The slips of paper are shuffled in a hat, and then each person draws one (uniformly at random at each stage, without replacement).
   (a) Find the expected value of the number of people who draw their own names.
   (b) Find the standard deviation of the number of people who draw their own names.