(**) Tentatively

This problem set covers material from Week 11, dates 11/28- 12/01. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding chapter. "AP" stands for additional problems not found in the book.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Tuesday 11/28

- **AP 1**: Suppose emails arrive in your inbox according to a Poisson process with rate $\lambda = 1$. What is the probability that you have to wait at least 2 hours until you have received two emails?

  You can either use calculus, or check Section 8.8 for some possibly helpful code...

- **AP 2**: Let $X \sim \text{Gamma}(a, \lambda)$ and $Y \sim \text{Gamma}(b, \lambda)$ be independent. Show that $X + Y \sim \text{Gamma}(a + b, \lambda)$ in three ways:
  a) Using MGFs.
  b) Using a story about a Poisson process. (For this part only, assume a and b are positive integers).
  c) By going through the following process:
     i) Define $Z = X + Y$. Find the conditional CDF of Z given $X = x$. Then, differentiate this CDF to obtain the conditional PDF of Z given $X = x$, Don't forget to specify the support of the PDF and define the CDF everywhere.
     ii) Use continuous LoTP (Theorem 7.1.18) to express the marginal PDF of Z as an integral in terms of the conditional PDF of Z given $X = x$ and the marginal PDF of X.
     iii) Evaluate the integral from above by recognizing it as a Beta integral (you may need to make an appropriate u-substitution first).
     iv) Verify that the marginal PDF of Z is indeed the PDF for $\text{Gamma}(a + b, \lambda)$.

- **AP 3**: Suppose $U \sim \text{Unif}(0, 1)$. Define $Y = U^{1/a}$ for some $a > 0$. Find the PDF of Y (along with its support). What named distribution does Y follow?

Thursday 11/30

- TBD

Friday 12/01

- TBD

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.