Pre-class preparation

Please read the following textbook sections from Blitzstein and Hwang's *Introduction to Probability* (second edition) OR watched the indicated video from Blitzstein's Math 110 YouTube channel:

- Textbook: Sections 8.3 (everything before Story 8.3.3), 8.4 (everything before Story 8.4.5), 8.5
- Video:
  - Lecture 23: Beta Distribution (beginning to 8:00, then 17:45 to 25:30)
  - Lecture 24: Gamma Distribution and Poisson Process
  - Lecture 25: Order Statistics and Conditional Expectation (beginning to 24:00)

Objectives

By the end of the day's class, students should be able to do the following:

- State the PDF for the beta distribution with parameters a and b, and describe the shape of the distribution for various values of these parameters.
- Calculate the normalizing constant $\beta(a, b)$ in the beta distribution **without using calculus** via the 'billiard ball' story.
- State the integral definition of the gamma function.
- State the PDF for the Gamma distribution with parameters a and λ, and describe the shape of the distribution for various values of these parameters.
- Compute the mean, variance, and other moments of the Gamma distribution by pattern recognition.
- Describe the relation between the Beta and Gamma distributions.
- Compute the mean of a Beta distributed random variable.

Reflection Questions

Please submit your answers to the following questions to the corresponding Canvas assignment by 7:45AM:

1. For what values of a, b will the $\text{Beta}(a, b)$ distribution be symmetric around $x = 0.5$?
2. In what ways is the Beta distribution a generalization of the uniform distribution on $(0, 1)$

3. Suppose a and b are positive real numbers. Use properties of the Gamma function to show the following (without doing ANY integrals):

$$\frac{\Gamma(a+b+2)}{\Gamma(a+1)\Gamma(b+1)} \cdot \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a+b+1)(a+b)}{ab}$$

   Do not assume that a and b are integers.

4. Suppose the number of hours between the arrival of consecutive emails in my inbox is Expo(3), independent of other waiting times. What is the distribution for the amount of time I need to wait until I have 3 emails in my inbox? What is the expected wait time?

5. True or false: Suppose that $X_i \sim \text{Exp}(\lambda_i)$ for $i = 1, \ldots, n$, and each X_i is independent of the others. Then $X_1 + \cdots + X_n \sim \text{Gamma}(n, \sum_{i=1}^{n} \lambda_i)$.

6. (Optional) Is there anything from the pre-class preparation that you have questions about? What topics would you like would you like some more clarification on?