(*): Assigned to weekly problem set.

Geometric and Negative Binomial Distributions

1. A fair coin is tossed repeatedly until it has landed Heads at least once and Tails at least once. Find the expected number of tosses.

2. (*) Bill and Steve are independently flipping coins (because they have nothing better to do with their time!). For concreteness, assume Bill flips a nickel that has a probability p_b of Heads, and Steve is flipping a penny with probability p_s of Heads. Let $X_1, X_2,, \dots$ be Bill's results and $Y_1, Y_2, \dots$ be Steve's results, with $X_i \sim \text{Bernoulli}(p_b)$ and $Y_j \sim \text{Bernoulli}(p_s)$, all independent.

   Find the distribution (i.e. PMF) of the first time at which both Bill and Steve are simultaneously successful (i.e., the smallest n such that $X_n = Y_n = 1$). Also find the expected value.

3. For $X \sim \text{Geometric}(p)$, find $\mathbb{E}[2^X]$ (if it is finite) and $\mathbb{E}[2^{-X}]$ (if it is finite). For each, make sure to clearly state what the values of p are for which it is finite.