This problem set covers material from Week 8, dates 10/31- 11/03. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding chapter. "AP" stands for additional problems not found in the book.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Tuesday 10/31

- **Chapter 6**: 4, 10. *While not necessary, calculations for number 4 might be sped up using `R`! It's also a good refresher!*
- **AP 1**: Let $U \sim \text{Unif}(a, b)$.
  a) Find the kurtosis of U. Simplify as much as possible. (Don't be scared! The Uniform distribution is very kind to us!).
  b) Recall that the -3 in the formula for kurtosis is added to make a Normal random variable have a kurtosis of 0. Knowing this and the interpretation of kurtosis, explain why the kurtosis you found in (a) makes sense.

Thursday 11/02

- **AP 2**: Suppose X is a discrete random variable with PMF

$$f_X(k) = P(X = k) = c\frac{p^k}{k}, \qquad k = 1, 2, 3, \ldots$$

  where $p \in (0, 1)$ and c is some constant that does not depend on k.
  a) Use the Taylor series for $\log(1 - x)$ to find the value of c.
  b) Compute the mean of X.
- **AP 3**: The *Laplace distribution* has PDF

$$f_X(x) = \frac{1}{2}e^{-|x|}, \qquad -\infty < x < \infty$$

  Find the MGF of the Laplace distribution. Don't forget to define where it exists.
- **AP 4**: We have previously seen that the location-scale transformation applies for the Uniform. However, let us pretend we don't know that! Let $U \sim \text{Unif}(a, b)$, and define $Y = cU + d$ for some constants $c, d \in \mathbb{R}$. Using MGFs, find the distribution of Y.

Friday 11/03

- **Chapter 6**: 16, 19, 24 (note AP 3)
- **AP 5**: Suppose $X_1, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ random variables. Define $Y = \frac{1}{n}\sum_{i=1}^{n} X_i$.
  a) Find the MGF of Y.
  b) Based on your answer in (a), state the exact distribution of Y.

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.