(*): Assigned to weekly problem set.

Exponential Distribution

1. (*) Let T be the time until a radioactive particle decays, and suppose (as is often done in physics and chemistry) that $T \sim \text{Expo}(\lambda)$.

   (a) The half-life of the particle is the time at which there is a 50% chance that the particle has decayed (in statistical terminology, this is the median of the distribution of T). Find the half-life of the particle.

   (b) Show that for ϵ a small, positive constant, the probability that the particle decays in the time interval $[t, t+\epsilon]$, given that it has survived until time t, does not depend on t and is approximately proportional to ϵ. *Hint:* $e^x \approx 1 + x$ if $x \approx 0$.

   (c) Now consider n radioactive particles, with i.i.d. times until decay $T_1, \ldots, T_n \sim \text{Expo}(\lambda)$. Let L be the first time at which one of the particles decays. Find the CDF of L. Also, find $\mathbb{E}[L]$ and $\text{Var}(L)$.

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2. (*) The textbook states the expected value and variance of an Exponential(1) random variable can be found using integration by parts, then uses the scaling property of exponential RVs to show that if $Y \sim \text{Exponential}(\lambda)$, then $\mathbb{E}[Y] = \frac{1}{\lambda}$ and $\text{Var}(Y) = \frac{1}{\lambda^2}$. Here, you will rigorously find the expectation!

   Let $Y \sim \text{Exponential}(\lambda)$. Show that $\mathbb{E}[Y] = \frac{1}{\lambda}$. *Integration by parts will be helpful:* $\int_a^b u\,dv = uv|_a^b - \int_a^b v\,du$