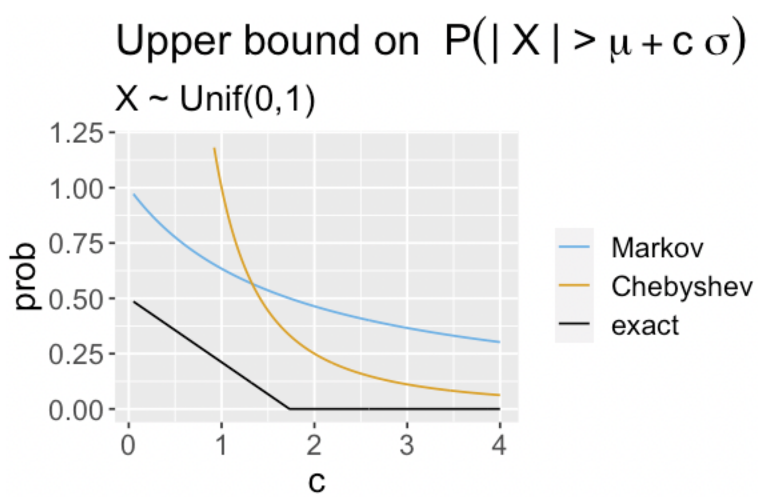
Suppose $X \sim \text{Unif}(0,1)$. We are interested in the probability that X is more than $c > 0$ standard deviations above its mean. Useful information:

$$\mathbb{E}[X] = \frac{1}{2} \qquad \text{Var}(X) = \frac{1}{12} \qquad F_X(x) = x, \text{ for } x \in (0,1)$$

1. Write the probability of interest as a probability statement in the context of this problem. Then use the CDF to find a formula for this probability.

2. Using Markov, what is an upper bound on the probability of interest?

3. Using Chebyshev, what is an upper bound on the probability of interest?



Inequalities and LLN

1. The **frequentist** definition of the probability of an event A occurring is that it is the long-run proportion of times the event occurs in a sequence of independent trials. In other words,

$$P_{\text{freq}}(A) = \lim_{n\to\infty} \frac{I_1 + \cdots + I_n}{n}$$

where I_k is the indicator that event A occurred in the kth trial. Use the Law of Large Numbers (and the Fundamental Bridge!) to prove that the frequentist definition of probability is equivalent to the axiomatic definition we've been using throughout the term.

2. Suppose $X \sim \text{Exp}(1)$. Use each of the Markov, Chebyshev, and Chernoff inequalities to give upper bounds on the probability that X is more than 3 standard deviations above its mean. Then compute the actual probability of this event. How do each of your bounds compare to the true value?