1. Determine whether the following statement is always true or sometimes false (If true, explain why. If false, give a counter-example):

   If A, B, C are events so that A and B are conditionally independent given C, then A and B are independent.

2. A box has three coins in it. One has heads on both sides, one has tails on both sides, and one is a fair coin. A coin is selected at random and flipped twice.

   (a) If the result of the first flip is heads, what is the probability that the coin is two-headed?

   (b) If the result of the first flip is heads, what is the probability that the second flip is also heads?

   (c) If the result of both flips are heads, what is the probability that the coin is two-headed?

3. Suppose $X \sim \text{Binomial}(8, 0.25)$ and let $Y = 2X + 1$. Find and simplify a formula for the PMF of Y. Then compute the expected value of Y.

4. Suppose A and B are events. Use axioms of probability to show that if $A \subseteq B$, then $P(A) \leq P(B)$.

5. Suppose A and B are events with $0 < P(B) < 1$. Show that if $P(A|B) > P(A|B^c)$, then

$$P(A|B) > P(A) > P(A|B^c).$$

6. On a certain statistics test (not necessarily this one!), 10 out of 100 key terms will be randomly selected to appear on an exam. A student then must choose 7 of these 10 to define. Since the student knows the format of the exam in advance, the student is trying to decide how many of these terms to memorize.

   (a) Suppose the student studies s key terms, where s is an integer between 0 and 100. What is the distribution of X? Give the name and parameters of the distribution, in terms of s.

   (b) Write an expression involving the explicit formula for the PMF or CDF of the named distribution for the probability that the student knows at least 7 of the 10 key terms, assuming the student studies $s = 75$ key terms.