(*): Assigned to weekly problem set.

Moments

1. (*) Let $U \sim \text{Unif}(a, b)$.

    (a) Find the kurtosis of U. Simplify as much as possible. (Don't be scared! The Uniform distribution is very kind to us!).

    (b) Recall that the -3 in the formula for kurtosis is added to make a Normal random variable have a kurtosis of 0. Knowing this and the interpretation of kurtosis, explain why the kurtosis you found in (a) makes sense.

2. (*) A distribution is called *symmetric unimodal* if it is symmetric (about some point) and has a unique mode. For example, any Normal distribution is symmetric unimodal. Let X have a continuous symmetric unimodal distribution for which the mean exists. Show that the mean, median, and mode of X are all equal.