(*): Assigned to weekly problem set.

Order statistics and Conditional Expectation

1. (*) Let $X_1, X_2, \ldots$ be iid random variables with CDF F_X. For any positive integer n, denote $M_n = \max\{X_1, \ldots, X_n\}$. Find the joint distribution of M_n and M_{n+1} for each integer $n \geq 1$. *Note: both the joint CDF and the joint PDF specify a "joint distribution", so you can find either one for this problem*

2. A fair six-sided die is rolled once. Find the expected number of additional rolls needed to obtain a value at least as large as that of the first roll.

3. A researcher studying crime is interested in how often people have gotten arrested. Let $X \sim \text{Poisson}(\lambda)$ be the number of times that a random person got arrested in the last 10 years. However, data from police records are being used for the researcher's study, and people who were never arrested in the last 10 years do not appear in the records. In other words, the police records have a *selection bias*: they only contain information on people who *have* been arrested in the last 10 years.

   So averaging the numbers of arrests for people in the police records does not directly estimate $\mathbb{E}[X]$; it makes more sense to think of the police records as giving us information about the conditional distribution of how many times a person was arrest, given that the person was arrested at least once in the last 10 years. The conditional distribution of X, given that $X \geq 1$, is called a *truncated Poisson distribution.*

   Find $\mathbb{E}[X|X \geq 1]$.