Functions and Independence of Random Variables

1. Let X be a discrete random variable with PMF
$$p_X(x) = \frac{1}{2^{x+1}}, \qquad x = 0, 1, 2, \ldots$$
Let $Y = 2^X$. Find a formula for the PMF of Y $p_Y(y)$ (don't forget the support), and compute $P(Y = 1)$.

2. For every $p > 1$, define
$$c(p) = \sum_{k=1}^{\infty} \frac{1}{k^p}.$$
Suppose that the discrete r.v. X has the following PMF:
$$f_X(x) = \frac{1}{c(p)x^p}, \quad \text{for } x = 1, 2, \ldots$$
and 0 otherwise.

   (a) For each $n \in \mathbb{Z}^+$, determine the probability that X will be divisible by n.

   (b) Determine the probability that X will be odd.

   (c) Now suppose X_1, X_2 are two iid random variables with the PMF $f_X(x)$ defined above. Letting $Y = X_1 + X_2$, determine the probability that Y will be even.