1. Suppose A and B are disjoint events. Under what conditions are A^c and B^c disjoint?

2. Let $X \sim \text{Gamma}(4, 2)$. What bound does Chebyshev's inequality give for the probability $P(X \geq 4)$?

3. Suppose we have two random variables X and Y such that the conditional distribution of Y given X is $Y|X \sim N(X, X^2)$. (Put another way, when $X = x$ is observed/crystallizes, we have the conditional distribution $Y|X = x \sim N(x, x^2)$). Suppose the marginal distribution of X is $X \sim \text{Unif}(0, 1)$.

   (a) Find $\mathbb{E}[Y]$.

   (b) Find $\text{Var}(Y)$.

   (c) Find the covariance $\text{Cov}(X, Y)$.

(d) Define $U = \frac{Y}{X}$ and $V = X$. Using change of variables, find the joint PDF of U and V. Don't forget to specify support!

(e) Based on your previous answer, are U and V independent? Briefly explain why or why not.

(*) Extra credit: what is the marginal distribution of U?

4. Suppose A, B, C are iid Poisson(λ) and define $X = A + B$ and $Y = A + C$.

   (a) Compute $\text{Cov}(X, Y)$.

   (b) Are X and Y independent? Briefly explain. *Note: "Intuitive" explanation is not valid here!*

   (c) Find the a formula for the **conditional** joint distribution of X and Y, given $A = a$. That is, find a formula for $P(X = x, Y = y | A = a)$. *No need to simplify, and don't forget the support.*

(d) Find a formula for the **unconditional** joint distribution of X and Y. That is, find a formula for $P(X = x, Y = y)$. *No need to simplify, and don't forget the support.*

5. Suppose we are conducting an experiment where a fair coin is flipped repeatedly until a Heads is obtained for the first time, and we record the total number of flips required. Further suppose that we conduct this experiment three times, where each experiment is independent of the others. What is the probability that exactly the same number of tosses will be required for each of the three iterations? *Simplify as much as possible.*

6. Suppose you have a device that contains four batteries. The device will function if one battery dies, but the device will stop working if two or more batteries die. Suppose the lifetime of the four batteries are iid $\text{Exp}(\lambda)$ random variables. Find the PDF of the device's lifetime. *Simplify as much as possible.*