# Conditional independence and Independence

1. There has been a malware attack on university data management system, which stores student CRFs. To lose one semester's CRFs, it is enough for two computer worms to modify/infect two important lines of code, or for one virus to overwrite the host program. Let $p_1$ be the probability of a worm infecting the first important line of code, $p_2$ the probability of a worm infecting the second important line of code, and $p_3$ the probability of a virus overwriting the host program. It is assumed that all three places are targeted independently of each other. What is the probability that the semester's CRFs are lost?

2. Middlebury College currently does not require college applicants to send standardized test scores. So let us consider a hypothetical school called Medianbury College which does require SAT scores. Medianbury will accept a student if and only if they score well on the SAT, have big dreams, or both.

   (a) *Among the entire US population*, do you think doing well on the SAT and having big dreams are independent or dependent? If dependent, do you think there is a negative association (i.e. conditioning on a good SAT score decreases the 'size' of your dream)?

   (b) *Among students at Medianbury College*, do you think doing well on the SAT and having big dreams are independent or dependent? If dependent, do you think there is a negative association (i.e. conditioning on a good SAT score decreases the 'size' of your dream)? Provide a brief explanation.

   (c) Suppose we have two independent events $A$ and $B$. Define $C = A \cup B$. Assuming $P(A \cup B) < 1$ and $P(A \cap B) > 0$, demonstrate that $A$ and $B$ are conditionally *dependent* given $C$, and $P(A|B, C) < P(A|C)$.

   This phenomenon is known as *Berkson's paradox* or *collider bias.*

   *This problem is adapted from Blitzstein and Hwang 2.36.*

3. A device has four parts, each of which operates independently. It was discovered that two of the parts failed. What is the probability that it was parts 1 and 2 that failed if the probability of part $i$ failing is $\frac{i}{10}$, for $i = 1, 2, 3, 4$?