Simulation in R

One of these problems will be included on the problem set!

1. Let's re-visit de Montmort's matching problem: We have a well-shuffled deck of n cards, labeled 1 through n. You flip over the cards one by one, saying the numbers 1 through n as you go. You win the game if at some point, the number you say aloud is the same of the number on the card flipped over. For large n, we see that the probability is approximately $1 - e^{-1}$.

   (a) For $n = 5$, write `R` code that uses the `sample()` function to play one iteration of the game. Your code should output whether the game results in a win or loss. This can be done by reporting `TRUE` or 1 if you won and `FALSE` or 0 if you lost.

   (b) Optionally copy-and-pasting your code from (a), now use the `replicate()` function to simulate 10000 iterations of the game in order to approximate the probability of winning, still for $n = 5$.

   (c) Now generalize (b) by creating a function called `matching_prob` that returns the probability of winning. Your function should take in as input: the number of cards n and the number of iterations used to approximate the probability (call this input B).

   (d) Using a for loop and your new `matching_prob` function, simulate the probabilities of winning for $n = 2, \ldots, 25$ and $B = 10000$ iterations. Store these probabilities in a vector called `p`.

   (e) Visualize your results using the `plot()` function. Trying adding a horizontal line at $1 - e^{-1}$ using the `abline()` function and specifying the value for the horizontal line.

2. (a) What is the theoretical probability that in our classroom of 25 people (Prof. Tang included), at least one person has the same birthday *as you*? Obtain this in closed form (on paper) and then evaluate this probability in `R`.

   (b) Verify your answer in (a) using simulation in `R`.

   (c) How does the probability you found in this problem compare to the probability of at least one match in the usual birthday problem (with $k = 25$ people)? Explain intutively why this difference might be.