Indicators and Fundamental Bridge

1. A group of $n \geq 4$ people are comparing their birthdays (assume the usual set-up of the birthday problem). Let $\mathbf{1}_{ij}$ be the indicator random variable that persons i and j have the same birthday ($i < j$). Is $\mathbf{1}_{12}$ independent of $\mathbf{1}_{34}$? Is $\mathbf{1}_{12}$ independent of $\mathbf{1}_{13}$? Are all the $\mathbf{1}_{ij}$ independent of each other?

2. Suppose you are watching cows walk around Addison County. Every minute, a cow walks by, which is equally likely to be any one of the n cows in the county. What is the expected number of distinct cows you have seen after t minutes?