MGFs

1. Suppose that X is a random variable with MGF

$$M_X(t) = \frac{1}{4}\left(3e^t + e^{-t}\right), \qquad t \in \mathbb{R}$$

   Find the mean and variance of X.

2. Use the MGF of the Geometric(p) distribution to give another proof that the mean of this distribution is $\frac{1-p}{p}$ and the variance is $\frac{1-p}{p^2}$.

3. Let $X_1, \ldots, X_n \stackrel{\text{iid}}{\sim} \text{Exp}(\lambda)$. This means that each $X_i \sim \text{Exp}(\lambda)$ and they are independent of each other. Show that $Y = X_1 + \ldots + X_n$ is not an Exponential random variable.

4. Let X and Y be i.i.d. Poisson(λ) random variables. Use MGFs to determine whether X + 2Y is distributed Poisson or not. *Hint: if $X + 2Y$ was indeed Poisson, which specific Poisson distribution would it have?*