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## Continuous Random Variables and Uniform

1. Determine whether the following statements are true or false. Briefly explain.
  - (a) If  $X$  is a continuous random variable and  $a \in \mathbb{R}$ , then  $P(X \leq a) = P(X < a)$ .
  - (b) The density function  $f$  for any continuous random variable must satisfy  $0 \leq f(x) \leq 1$  for all  $x \in \mathbb{R}$ .
  - (c) The support of a continuous random variable  $X$  is the set of all real numbers  $x$  so that  $P(X = x) > 0$ .
  - (d) The CDF  $F$  for a continuous random variable is an antiderivative for its density function.
  - (e) If  $X$  is a continuous random variable with density  $f$  and  $\epsilon > 0$  is small, then  $f(x) \cdot \epsilon \approx P(x - \epsilon/2 < X < x + \epsilon/2)$ .
2. Let  $X$  be a continuous random variable with PDF  $f_X(x) = 4xe^{-2x^2}$  for  $x > 0$ . Find the CDF of  $X$  and use it to obtain  $P(X > 1)$ .
3. Let  $U \sim \text{Uniform}(-1, 1)$ . Find the CDF and PDF of  $U^2$ . Is the distribution of  $U^2$   $\text{Uniform}(0, 1)$ ? Why or why not?