Naive Definition of Probability + Counting

1. A committee has professors representing each of the major divisions at Middlebury: Natural Sciences has 4, Social Sciences has 3, Humanities has 2, and Languages has 2. The committee members are all asked to sit in a row of chairs. How many ways can the professors sit if we ask that professors within a division must sit together (e.g. all Natural Sciences professors, followed by Social Sciences, Humanities, and Languages)?

2. A box contains 24 light bulbs of which four are defective. Suppose that one person selects 10 bulbs from the box randomly without replacement, and then a second person takes the remaining 14 bulbs. How many ways are there for all four defective bulbs to be obtained by the same person?

3. The United States has 100 senators: 2 senators from each of the 50 states. Suppose a senate committee is comprised of 8 members of the Senate. What is the probability that at least of the two senators from Vermont is part of the committee?

4. There are thirty possible questions on an exam, where each question corresponds to one topic in the course. Students may choose between the following options of exams:

   i. The professor randomly selects 3 questions, and the student must answer 2 of them, or

   ii. The professor randomly selects 5 questions, and the student must answer 3 of them.

   Students pass the exam if they answer all of their selected questions correctly, and fail otherwise. Jack only knows 18 topics. Which option is better for Jack?