This problem set covers material from Weeks 5 and 6, dates 3/10- 3/26. Unless otherwise noted, all problems are taken from the textbook.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Monday 3/10

1. Let $X \sim \text{NegBinom}(r, p)$ and $t \in \mathbb{R}$ a constant. Find $\mathbb{E}[e^{tX}]$ and specify the values of t for which this expectation is finite. (This may feel silly now, but we'll use it later in the semester!)

   You will probably need the following theorem: Let $n \in \mathbb{Z}^+$. Then

   $$\sum_{x=0}^{\infty} \binom{x+n-1}{x} (-1)^x v^x = (1+v)^n \qquad \text{for } |v| < 1$$

2. 4.26

Wednesday 3/12

3. 4.43

4. Suppose that we have a series of n independent coin flips with probability p of Heads.

   (a) What is the expected number of occurrences of the pattern HTH (consecutively)? Note that overlap is allowed, so for example, $HTHTH$ has two occurrences.

   (b) The sequence of outcomes can be divided into "runs", which are blocks of consecutive H's or T's. For example, the sequence $HHHTTHTT$ becomes $HHH|TT|H|TT$ which is 4 runs. Find the expected number of runs in n coin flips.
   Hint: think about what it means to start a new run.

Monday 3/24

5. 4.53

6. 4.86

Wednesday 3/26

7. Five million people enter a certain lottery. For each person, the chance of winning is one in five million, independent of each other. It is possible for multiple people to win the lottery.

   (a) Find a simple, good approximation for the PMF of the number of people who win the lottery.

   (b) Congrats! You won the lottery. Assume now that the number of winners other than you is $W \sim \text{Poisson}(1)$, and that if there is more than one winner, then the prize is awarded to one randomly chosen winner. Given this information, find the probability that you win the prize. Simplify as much as possible.

8. In a group of 73 people, find a simple, good approximation for the probability that there is at least one pair of people such that they share a birthday *and* their biological mothers share a birthday. (Note, the mothers can have a different birthday from the children.) Assume our usual birthday assumptions. Further assume that no one among the 73 people in the room is the biological mother of another one of the people, nor do any two people have the same biological mother.

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.