
Functions and Independence of Random Variables

1. Let X be a discrete random variable with PMF

$$f_X(x) = \frac{1}{2^{x+1}}, \quad x = 0, 1, 2, \dots$$

Let $Y = 2^X$. Find a formula for the PMF of Y $f_Y(y)$ (don't forget the support), and compute $P(Y = 1)$.

2. For every $p > 1$, define

$$c(p) = \sum_{k=1}^{\infty} \frac{1}{k^p}.$$

Suppose that the discrete r.v. X has the following PMF:

$$f_X(x) = \frac{1}{c(p)x^p}, \quad \text{for } x = 1, 2, \dots$$

and 0 otherwise.

- (a) For each $n \in \mathbb{Z}^+$, determine the probability that X will be divisible by n .
- (b) Determine the probability that X will be odd.
- (c) Now suppose X_1, X_2 are two iid random variables with the PMF $f_X(x)$ defined above. Letting $Y = X_1 + X_2$, determine the probability that Y will be even.