Geometric and Negative Binomial Distributions

1. A fair coin is tossed repeatedly until it has landed Heads at least once and Tails at least once. Find the expected number of tosses.

2. Suppose that $X \sim \text{Binom}(n, p)$ and that $Y \sim \text{NegBinom}(r, p)$. Show that $P(X < r) = P(Y > n - r)$ using the story set-up of the two distributions.

3. For $X \sim \text{Geometric}(p)$, find $\mathbb{E}[2^X]$ (if it is finite) and $\mathbb{E}[2^{-X}]$ (if it is finite). For each, make sure to clearly state what the values of p are for which it is finite.