This problem set covers material from Week 8, dates 4/07- 4/11. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding chapter.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Monday 4/07

1. 6.10 *Note, you don't have to re-prove things that we've already shown! If we've shown something before, you can simply reference the result here.*

2. We're going to find the skewness of the Exp(λ) distribution. Rather than go via LoTUS immediately (which would lead to us integrating by parts several times), we will break down the problem into a few smaller steps:

   (a) Let $X \sim \text{Exp}(\lambda)$. Define Y as the standardized version of X, so $Y = \frac{X-\mu}{\sigma}$ where $\mu = \mathbb{E}[X]$ and $\sigma = \text{SD}(X)$ (use the appropriate mean and SD for this distribution). Find the PDF of Y (don't forget the support).

   (b) Use Y to find Skew(X).

Wednesday 4/09

3. The continuous random variable L has the *Laplace distribution* if its PDF is

$$f_L(l) = \frac{1}{2}e^{-|l|}, \qquad l \in \mathbb{R}$$

   (a) Find the MGF of the Laplace distribution. Don't forget to define where it is finite.

   (b) Let $X, Y \overset{\text{iid}}{\sim} \text{Exp}(1)$. Define $L = X - Y$. Use MGFs to show that the distribution of L is Laplace.

4. Suppose $X_i \overset{\text{indep}}{\sim} N(\mu_i, \sigma_i^2)$ for $i = 1, \ldots, n$.

   (a) Define the r.v. $Y = \sum_{i=1}^n X_i$. Use MGFs to find the exact distribution of Y.

   (b) Now suppose that the $\mu_i = \mu$ and $\sigma_i^2 = \sigma^2$ for all i. Define the random variable $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$. Find the exact distribution of $\bar{X}$.

   (c) In 1-2 sentences, clearly state what (a) and (b) tell us.

5. We will see that sometimes, we can get the moments of a distribution directly from the MGF without needing to take derivatives.

   For this problem, let $X \sim \text{Exp}(\lambda)$.

(a) We found that $M_X(t) = \frac{\lambda}{\lambda - t}$ for $t < \lambda$. Working backwards, find a Taylor series representation of this $M_X(t)$. That is, find the form of the coefficients a_n such that $M_X(t) = \sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$.

(b) What are these a_n equivalent to?!

(c) Now, let's re-find the skew of an Exp(λ) distribution. Starting with the definition of skew, use your work from (a) and (b) to re-find the skew. (You should get the same value as in 2b!)

Friday 4/11

None!

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.