Conditional Expectation given event

1. Suppose n people are bidding on a mystery prize that is up for auction. The bids are to be submitted in secret, and the individual who submits the highest bid wins the prize. The i-th bidder receives a random "signal" amount of money X_i, where the $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \text{Unif}(0, 1)$. The value of the prize, V, is defined to be the sum of the individual bidders' signals:
$$V = X_1 + \cdots + X_n$$
Each bidder i will learn their own value of X_i, but not the other $n - 1$ values.
   (a) Before receiving her signal, what is bidder 1's unconditional expectation for V?
   (b) Conditional on receiving the signal $X_1 = x_1$, what is bidder 1's expectation for V?
   (c) Suppose each bidder submits a bid equal to their conditional expectation for V (i.e., bidder i bids $\mathbb{E}[V|X_i = x_i]$). Conditional on receiving the signal $X_1 = x_1$ *and* winning the auction, what is bidder 1's expectation for $\mathbb{E}[V]$? Explain intuitively why this quantity is always less than the quantity calculated in (b). *Think about how to translate the information provided in words into statements involving random variables.*

2. A fair six-sided die is rolled once. Find the expected number of additional rolls needed to obtain a value at least as large as that of the first roll.