Variance

1. X is a random variable with PMF $f_X(x) = \frac{1}{x(x+1)}$ for $x \in \mathbb{Z}^+$. What is $\text{Var}(X)$?
2. An amoeba currently lives alone in a pond. After one minute, the amoeba will either die, remain the same, or split into two amoebas, with equal probability. Find the variance for the number of amoebas in the pond after one minute.
3. Show that $\text{Var}(X + c) = \text{Var}(X)$, where $c \in \mathbb{R}$ is a constant.
4. Show that $\text{Var}(cX) = c^2\text{Var}(X)$, where $c \in \mathbb{R}$ is a constant.