Discrete Uniform and CDFs

1. Once upon a time, a famous statistician offered tea to a lady. The lady claimed that she could tell whether milk had been added to the cup before or after the tea. The statistician decided to run some experiments to test her claim.

   The lady is given 6 cups of tea, where it is known in advance that 3 will be milk-first and 3 will be tea-first, in a completely random order. The lady gets to taste each and then guess which 3 were milk-first. Assume for this part that she has no ability whatsoever to distinguish milk-first from tea-first cups of tea. Find the probability that at least 2 of her 3 guesses are correct.

2. For each of the following, determine if the function $F(x)$ is a valid CDF of some random variable X. If not, explain why not. If so, provide the corresponding PMF.

   (a)
   $$F(x) = \begin{cases} 0 & x < 0 \\ 0.5 & 0 \leq x < 1 \\ 0.25 & 1 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

   (b)
   $$F(x) = \begin{cases} 0 & x < 0 \\ 0.5 & 0 \leq x < 1 \\ 0.65 & 1 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

3. Recall de Montmort's matching problem from Chapter 1: in a deck of n cards labeled 1 through n, a match occurs when the number on the card matches the card's position in the deck. Let X denote the number of matching cards. In this problem, assume $n \geq 2$.

   (a) What is the support of X?

   (b) Is X Bernoulli? Binomial? Hypergeometric? Discrete Uniform? None of these? Explain your reasoning for each one of these distributions/cases, including "none of these".