

This problem set covers material from Week 4, dates 3/03- 3/07. Textbook problems (if assigned) can be found at the end of the corresponding chapter.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Monday 3/03

1. Recall de Montmort's matching problem from Chapter 1: in a deck of n cards labeled 1 through n , a match occurs when the number on the card matches the card's position in the deck. Let X denote the number of matching cards. In this problem, assume $n \geq 2$.
 - (a) What is the support of X ?
 - (b) Is X Bernoulli? Binomial? Hypergeometric? Discrete Uniform? None of these? Explain your reasoning for each one of these distributions/cases, including "none of these".
2. 3.4
3. 3.9

Wednesday 3/05

4. 3.33
5. 3.39
6. 3.44 (a and b only)

Friday 3/07

7. Let X be a discrete random variable.
 - (a) First suppose $S_X = \{1, 2, 3\}$. Show that $\mathbb{E}[X] = \sum_{k=0}^2 P(X > k)$.
 - (b) Now let $S_X = \mathbb{Z}^* = \{0, 1, 2, \dots\}$. Show $\mathbb{E}[X] = \sum_{k=0}^{\infty} P(X > k)$. *Generalize what you did in (a) by re-arranging a sum in a "smart" way.*
8. Suppose we are flipping two fair coins. Let X be the number of Heads in the two independent flips of the coins. Find $\mathbb{E}[X^3]$ in two ways:
 - a) Finding the PMF of $Y = X^3$, then obtaining $\mathbb{E}[Y]$.
 - b) Using LoTUS.

General rubric

Points	Criteria
5	The solution is correct <i>and</i> well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.