Covariance

1. Let X and Y be iid Unif$(0, 1)$.

   (a) Compute the covariance of $(X + Y)$ and $(X - Y)$.

   (b) Are $X + Y$ and $X - Y$ independent? Explain (you do not have to do this mathematically, but rather explain in words or via counter-example).

Multinomial Distribution

1. Regular M&M candies are produced by Mars, Inc. company. Each candy takes one of six color: red, orange, yellow, green, blue, or brown. (Fun fact: in 1995, the tan colored M&M was replaced by blue!) In 2008, Mars published that the color of the candies are produced in the following proportions:

Color	Red	Orange	Yellow	Green	Blue	Brown
Proportion	13%	20%	14%	16%	24%	13%

   It is assumed that each individual candy's color is determined by the above proportions, independent of the colors of other candy pieces.

   Suppose you have a bag of n candies. Let the vector (R, O, Y, G, B, Br) denote the number of candy pieces of the respective colors in your particular bag.

   (a) Explain why the vector (R, O, Y, G, B, Br) has a multinomial distribution, and explicitly write down the parameters of this distribution.

   (b) Suppose that I am red-green color blind, and so I treat the red and green candies as the same brown-ish color, which unfortunately also look like the brown M&Ms. Let S denote the number of red, green, and brown candies. What is the joint distribution of (B, O, Y, S) (note the ordering!)? What is the marginal distribution of S? Be sure to specify the parameters for each distribution.

2. Suppose a bowl contains 700 pieces of candy, with 100 pieces of each color (Red, Orange, Yellow, Green, Blue, Indigo, Violet). Moreover, suppose you scoop out 20 pieces of candy at random, so that every subset of size 20 is equally likely. Let R, O, Y, G, B, I, V denote the number of candies of a specific color in a scoop, respectively.

   (a) Explain why the distribution of (R, O, Y, G, B, I, V) is **not** multinomial.

   (b) Explain how to modify the sampling procedure so the resulting joint distribution **IS** Multinomial.