Birthday problem

Suppose we have k people in a room. Define event A as the event at least two people in the room share the same birthday. What is $P(A)$?

Assumptions:

- "same birthday" here means same month and day
- We don't have twins/triplets/etc.
- No one born on February 29 (leap year)
- Each day of the year is equally likely to be a birthday → let's use naive prob.

$$P_{naive}(A) = \frac{|A|}{|S|} = \frac{|A|}{365^k}$$

A^c = no one has same birthday = choosing k birthdays without replacement from 365 total

$$|A^c| = P_{365,k} = \frac{365!}{(365-k)!}$$

365 364 363 × ... —

$$P_{naive}(A) = 1 - P_{naive}(A^c) = 1 - \frac{365!/(365-k)!}{365^k}$$