

## Background

*People v. Collins* (1968) is an example where conditional probability and independence were used in probability calculations to aid in a court case in the Supreme Court of California. The case involved a purse being stolen. Witnesses claimed to see a young woman with blond hair in a ponytail running away from the scene in a yellow car that was driven by a black man with a beard. A few days later, a couple was arrested because they matched these descriptions. However, there was no physical evidence on them.

A mathematician was hired, and they calculated the probability that a randomly selected couple in this California area would possess these characteristics. They found the probability to be  $8.3 \times 10^{-8}$ , or about 1 in 12 million. The jury found this probability extremely compelling and convicted the couple. However, the Supreme Court thought, in the light of no evidence, that a different probability might be more useful: Given that there is one couple who meets the descriptions provided by the witness, what is the probability that a second couple also has those same characteristics?

## Set-up

Let's define the following:

- $p$  is the probability that a randomly selected couple from a population of  $n$  couples has the aforementioned characteristics. The mathematician found  $p = 8.3 \times 10^{-8}$ .
- **We will assume that the  $n$  couples are mutually independent**
- $A$  is the event that *at least one* couple in the population has the characteristics
- $B$  as the event that *at least two* couples in the population have the characteristics
- $C$  be the event that *exactly one* couple has the characteristics
- $A_i$  is the event that couple  $i$  has the characteristics described by the witness, for  $i = 1, \dots, n$ .

In terms of these events, the probability the Court is interested in is:

What are some relationships between some or all the events  $A, B, C$ ?

1.

2.

**Calculations**

Let's begin by finding  $P(A)$ . Made easier by re-writing  $A$  in terms of the  $A_i$

Now let's find  $P(B)$ .

Realize that we need  $P(C)$ ! Once again made easier by re-writing  $C$  in terms of the  $A_i$

**Results**

Because the crime occurred in a heavily populated area of California, they estimated  $n$  to be in the millions. Letting  $p = 8.3 \times 10^{-8}$  based off the mathematician's calculations and  $n = 8,000,000$  couples (according to California statistics):