

1. A group of 42 voters is voting on whether or not to pass a law. Every voter must vote, and no voter can vote more than once. 15 voted to pass the law, and the remaining voted against. If you randomly select 10 voters after the vote, what is the probability that at most three were in favor of passing the law?
2. A box has three coins in it. One has heads on both sides, one has tails on both sides, and one is a fair coin. A coin is selected at random and flipped twice. If the result of the first flip is heads, what is the probability that the coin is two-headed?
3. Suppose we flip a fair coin  $n$  times. What is the probability that the  $k$ -th Head lands on the  $n$ -th flip, for  $0 < k \leq n$ ?
4. Suppose  $A$  and  $B$  are events with  $0 < P(B) < 1$ . Show that if  $P(A|B) > P(A|B^c)$ , then

$$P(A|B) > P(A) > P(A|B^c).$$

5. Suppose that there are six light bulbs arranged in a row that are used to emit messages. Each bulb can flash the colors red or white. The specific bulbs and the color that they flash correspond to a specific message. In a given message, each bulb can flash at most once, emitting red or white. Further suppose that all flashes must occur at the same time, and messages are comprised of a minimum of one flash and a maximum of six flashes. How many different signals are possible?

*Examples of messages include ( \_ R W \_ R W ) and ( W \_ \_ \_ \_ )*

6. A family has  $n$  children. We pick one of them at random and find out that she is a girl. Assuming gender binary and boys/girls are equally likely, what is the probability that all the children of this family are girls?
7. **2/27/25: Sorry, this problem was originally poorly worded! It's a slightly different set-up now, which will hopefully make things easier.** A bag has four marbles, each of a different color: red, blue, yellow, and green. 10 people take turns removing a random marble from the bag, noting its color, and then putting the marble back into the bag. Find the probability that all four colors have been selected among the 10 people.
8. Consider the five vowels 'a', 'e', 'i', 'o', and 'u', along with the four consonants 'b', 'c', 'd', and 'f'. What is the probability that if we randomly re-arrange all of the nine letters, the resulting word does not have any two vowels next to each other?