Conditional Variance

1. Suppose you see a total of $N \sim \text{Geom}(s)$ movies in your lifetime. Further suppose that for each movie, you have a probability p of liking the movie, independently of other movies and of N. Let T be the number of movies you like in your lifetime.

   (a) Find the mean of T.

   (b) Find the variance of T.

2. Emails arrive one at a time in an inbox. Let T_j be the time at which the j-th email arrives. Suppose the waiting times between emails are iid $\text{Exp}(\lambda)$ random variables (i.e. $T_1, T_2 - T_1, T_3 - T_2, \ldots$ are are iid $\text{Exp}(\lambda)$).

   Each email is non-spam with probability p and spam with probability $1 - p$, independently of other emails and of the waiting times. Let X be the time at which the first non-spam email arrives (so X is a continuous random variable). For example, $X = T_1$ is the first email is non-spam, $X = T_2$ if the first email is spam and the second email is non-spam, etc.

   Hint for both parts: Let N be the number of emails we require to see the first non-spam (including that non-spam email) and write X as a sum of N terms. Hopefully, we believe N is independent of when a particular email arrives.

   (a) Find the mean of X.

   (b) Find the variance of X.