Conditional expectation given a RV

1. Let X_1, X_2 be iid random variables. Define $\bar{X} = \frac{1}{2}(X_1 + X_2)$ as the sample mean. Find $\mathbb{E}[wX_1 + (1-w)X_2|\bar{X}]$ where $w \in [0,1]$ is constant.

2. Let $\mathbf{X} \sim \text{Multinom}_5(n, \mathbf{p})$. Recall this notation means that both $\mathbf{X}$ and $\mathbf{p}$ are vectors of length 5. Using the story of the multinomial, find $\mathbb{E}[X_1|X_2]$.

3. Show that for any random variables X and Y,

$$\mathbb{E}[Y|\mathbb{E}[Y|X]] = \mathbb{E}[Y|X]$$

Hint: use Adam's law with extra conditioning.