NOTE: these are not exhaustive!

1. Let $Z \sim N(0, 1)$ and $Y = |Z|$. We say that Y has the *folded Normal* distribution. Find **two expressions** for the MGF of Y as unsimplified integrals, one integral based on the PDF of Y and one based on the PDF of Z.

2. Suppose that X is a discrete random variable whose MGF is as follows:

$$M_X(t) = \frac{1}{5}e^t + \frac{2}{5}e^{4t} + \frac{2}{5}e^{8t} \qquad t \in \mathbb{R}$$

   (a) Find the PMF of X.

   (b) Find $\mathbb{E}[X]$ in two ways: using the definition of expectation and using the MGF.

3. A variable X is said to have the *arcsine* distribution if its CDF F is given by

$$F(x) = \frac{2}{\pi} \sin^{-1}(\sqrt{x}) \qquad \text{for } 0 < x < 1$$

   and $F(x) = 0$ for $x \leq 0$ and $F(x) = 1$ for $x \geq 1$.

   (a) Check that F is indeed a valid CDF.

   (b) Find the corresponding PDF f. (Recall that the derivative of $\sin^{-1}(x)$ is $\frac{1}{\sqrt{1-x^2}}$)

   (c) Find a formula for the quantile function F^{-1} of X.

   (d) Suppose $U \sim \text{Unif}(0, 1)$ and let F^{-1} be the quantile function of X. What is the name of the distribution of $F^{-1}(U)$?

4. Let X_1, X_2, X_3 be lifetimes of memory chips. Suppose that $X_i \overset{\text{iid}}{\sim} N(300, 100)$ for $i = 1, 2, 3$. Find an exact expression for the probability that at least one of the three chips lasts for more than 290 hours.

5. A painting process consists of two stages. In stage one, the paint is applied. In stage two, a protective coat is added. Let X be the time spent on the first stage, and Y the time spent on the second stage. The first stage involves an inspection; if the paint fails the inspection, one must wait three minutes and apply the paint again. After a second application of paint, there is no further inspection. The joint PDF of X and Y is

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{3} & 1 < x < 3,\ 0 < y < 1 \\ \frac{1}{6} & 6 < x < 8,\ 0 < y < 1 \end{cases}$$

   (a) Sketch the support of the joint PDF.

   (b) Find the marginals of X and Y.

   (c) Are X and Y independent of each other?

6. Suppose X and Y have the following joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} \frac{3}{2}y^2 & 0 < x < 2,\ 0 < y < 1 \\ 0 & o.w. \end{cases}$$

   (a) Determine the marginal PDFs of X and Y.

   (b) Are X and Y independent?

   (c) Are the event $\{X < 1\}$ and the event $\{Y \geq \frac{1}{2}\}$ independent? Demonstrate mathematically and intuitively why or why not.

7. Suppose we have three continuous random variables X_1, X_2, X_3 with the following joint PDF: Suppose X and Y have the following joint PDF:

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = \begin{cases} ce^{-(x_1+2x_2+3x_3)} & x_1 > 0, x_2 > 0, x_3 > 0 \\ 0 & o.w. \end{cases}$$

   (a) Determine the value of the constant c.

   (b) Obtain the *marginal joint* PDF $f_{X_1,X_3}(x_1,x_3)$.

   (c) Are the three random variables independent? How can you tell?

   (d) Obtain the marginal PDFs f_{X_1}, f_{X_2}, and f_{X_3}. Do your marginals "agree" with the marginal joint PDF of X_1 and X_3 in part (b)?

   (e) Obtain $P(X_1 < 1 | X_2 = 2, X_3 = 1)$.

8. Re-prove the theorem: If X has MGF $M_X(t)$, then $\mathbb{E}[X^n] = M_X^{(n)}(0)$.

9. Let (X, Y) be a uniformly random point in the triangle in the $2D$ plane with vertices $(0,0)$, $(0,1)$, $(1,0)$. Find $\mathbb{E}[X]$, $\mathbb{E}[Y]$, and $\mathbb{E}[XY]$.