Normal Distribution

1. Let $X \sim N(1, 4)$. Find the exact value of each of the following probabilities. This usually involves expressing quantities in terms of Φ. Then, if possible, approximate these probabilities.

   (a) $P(X \leq 3)$

   (b) $P(-3 \leq X \leq 5)$

2. Assume the following result is true (we will show later why): if X_1 and X_2 are independent with $X_i \sim N(\mu_i, \sigma_i^2)$ for $i \in \{1, 2\}$, then $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. Using this result and properties of Normals, find an expression for $P(X < Y)$ when $X \sim N(a, b)$ independent of $Y \sim N(c, d)$ using Φ notation. Does your answer make sense in the special case when X and Y are i.i.d?

3. Prove the theorem about linear transformations of Normals: Let $X \sim N(\mu, \sigma^2)$, and define $Y = aX + b$ for $a > 0, b \in \mathbb{R}$. Then $Y \sim N(a\mu + b, a^2\sigma^2)$.