This problem set covers material from Week 2, dates 2/17- 2/21. Textbook problems (if assigned) can be found at the end of the corresponding chapter.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Monday 2/17

1. We are continuously flipping a fair coin. We stop when the absolute value of the difference between the number of Heads flips and the number of Tails flipped is 3 (e.g. TTT or THTHTTT). Let's find the probability p that we stop in **at most 6 tosses**.

   Define the event A_i as the event that we stop on the i-th toss, for $i \geq 1$.

   (a) For each i of interest, find $P(A_i)$ (provide some brief justification/reasoning for each).

   (b) Using part (a), find p. Provide justification where necessary.

2. 1.49 (Hint: define some events whose unions or intersections probabilities will be easy to find!)

3. 1.55 (No need to simplify)

Wednesday 2/19

Parts of these two problems will need to be done in the associated `.Rmd` file.

4. The classic birthday problem asks: in a room of k people, what is the probability that at least two people share the same birthday (assuming no leap year, no twins, and all days equally likely).

   (a) What is the theoretical probability that in our classroom of 25 people (Prof. Tang included), at least one person has the same birthday *as you*? Obtain this in closed form (on paper); not need evaluate yet. Then evaluate your probability in `R`.

   (b) Verify your answer in (a) using simulation in `R`.

   (c) How does the probability you found in this problem compare to the probability of at least one match in the usual birthday problem (with $k = 25$ people)? Explain intuitively why this difference might be. *You can either hand-write your response here, or type your response in the corresponding space in the .Rmd.*

5. Suppose each of n balls is independently placed into one of n boxes at random, with all boxes equally likely for each ball.

   (a) What is the theoretical probability that *exactly one* box will be empty for general n? Then evaluate this probability in `R` for $n = 9$.

   (b) Write a function called `sim_one_empty` that estimates the probability of interest in (a) using simulation. Your function should take in as input the number of balls/boxes n and the number of simulations/iterations to run B. Your function should report back the probability.

   *Hint: it may be useful to use the **`unique()`** function, which returns the subset of unique entries of a vector, along with the **`length()`** function, which returns the length of a vector.*

   (c) Verify your theoretical probability from (a) by using your function `sim_one_empty` for $n = 9$ and some large number of iterations.

Friday 2/21

Please define events!
Note: you can do some of these problem parts through "reasoning"/thinking conditionally or by using formulas/theorems. Both are fine, and I encourage you to think through multiple ways of solving the problem.

6. 2.20
   Depending on how you approach this problem, it might be helpful to note that intersections distribute over unions: for any events A, B, C: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

7. 2.21

8. **Game of Craps**. Craps is a casino dice rolling game where two dice are rolled. There are many bets that can be made, but we will focus on betting on the "Pass" line. The rules are as follows:

   - Roll two dice and sum the two values.
   - If the outcome is 7 or 11: you win!
   - If the outcome is 2, 3, or 12: you lose!
   - If the outcome is any other number (e.g. 5), call this number "the point". Then:
     - Roll the pair of dice repeatedly until you roll a 7 or the point.
     - If you roll the point before a 7: you win!
     - Otherwise if you roll a 7 before the point: you lose!

   What is the probability of winning the game (round to thousandths place)? *Hint: it might be helpful to define some events, e.g. W = event that we win.*

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.