Poisson Distribution and Paradigm

1. For each of the following situations, determine whether it would be appropriate to use the Poisson Paradigm to approximate the variable N. If it is appropriate, briefly explain why the conditions for the paradigm are satisfied and write down the approximate distribution. If it isn't appropriate, explain what condition(s) isn't satisfied.

   (a) Suppose $A_1, A_2, \ldots$ is an infinite sequence of independent events, each with probability $p = 10^{-1000}$ of occurring. Let N be the number of events that occur.

   (b) A particular data file is stored as a sequence of 10^6 binary digits. When the data file is copied, each term in the sequence has probability $p = 10^{-4}$ of having an error, independent of other terms. Let N be the number of errors.

   (c) Suppose two copies of each of 52 labeled cards are throughly shuffled (104 cards total, 2 each labeled 1, 2, …, 52). Cards are drawn from the deck two at a time. Let N be the number of cards that are paired with their other copy.

   (d) Let $X \sim \text{Binomial}(100, \frac{1}{100})$. For each j in $0, 1, 2, \ldots, 100$ let $\mathbf{1}_j$ be the indicator for the event "$X = j$". Let $N = \mathbf{1}_0 + \mathbf{1}_1 + \cdots + \mathbf{1}_{100}$.

   (e) Each of 20 students in a statistics classroom reveals the last 3 digits of their phone number (hereforth referred to as 'phone number'). Let N be the number of phone numbers that are repeated at least once.

2. The number of fish in a certain lake X is a Poisson(λ) random variable. Worried that there might be no fish at all, a statistician adds one fish to the lake. Let Y be the resulting number of fish (so $Y = X + 1$).

   (a) Find $\mathbb{E}[Y^2]$.

   (b) Find $\mathbb{E}[\frac{1}{Y}]$.