This problem set covers material from Week 11, dates 4/28-5/02. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding chapter.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Monday 4/28

1. 9.12

2. Let $X \sim \text{Exp}(\lambda)$. We will find $\mathbb{E}[X|X < 1]$ in the following two ways. Your answers should agree, and confirm for yourself why the solution "makes sense" when compared to $\mathbb{E}[X]$.

   (a) By definition, by first finding the conditional PDF of $X|X < 1$. Don't forget the support! *Recall in class how we found the conditional PDF of $X|X < c$ when $X \sim$ Unif$(0, 1)$ and $c \in (0, 1)$.*

   (b) Using Law of Total Expectation and a nice property of the Exponential distribution.

Wednesday 4/30

3. 9.33

4. 9.27 part a only. *Hint: Normalize the time to go from 0 to 1 units of time (i.e. 1 $\equiv$ 240 minutes after 8:00pm $\equiv$ midnight).*

Friday 5/02

5. (Adapted from textbook 9.53) Suppose that in the US, the distribution of heights of women in inches is Normal with mean 64 and variance 9. A woman is chosen at random from this population and we measure her height, but suppose our measuring tape only has tick marks for whole inches! So even if someone's height is between 53 and 54, we will have to round to either 53 or 54. Because there's some rounding error, the recorded height may be correct on average but has a Normal measurement error with variance 1. Let X be the person's true height, and let Y be the measured height that we recorded. Then we have

$$\begin{aligned} Y|X = x &\sim N(x, 1) \\ X &\sim N(64, 9) \end{aligned}$$

   (a) Find the unconditional mean and variance of Y.

(b) Find the covariance $\text{Cov}(X, Y)$.

(c) Find the marginal distribution of Y. One way is via MGF. *As a check: your answer here should agree with part (a)*

6. 9.48 (part (a) is probably a bit counter-intuitive, and part (d) will require you to dig back through your notes!)

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.