Priors and Posteriors

1. A box contains 5 tickets, where each ticket is either pink or blue. Let θ be the unknown parameter representing the proportion of pink tickets in the box. We will assume as our prior that each of the six following values of θ are equally likely: $\{0, \frac{1}{5}, \frac{2}{5}, \ldots, \frac{5}{5}\}$ (i.e. our prior for θ is a discrete uniform). Suppose we draw one ticket from the box and observe its color. Let $X = 1$ if the ticket is pink, and 0 if the ticket is blue.

   (a) What is the implied statistical model?

   (b) Fill out the first column of prior probabilities $p(\theta)$ in the table below.

   (c) Based on the data we observed, fill out the second column representing the likelihood function for θ.

   (d) Find the marginal probability of the observed data, $f(x) = P(X = x)$.

   (e) Fill out the final column in the table with the posterior probability of each value of θ given the observed data, using the first two columns and your answer in (d). Verify that the posterior is indeed a valid distribution!

$\tilde{\theta}$	$p(\theta) \equiv P(\theta = \tilde{\theta})$	$f(x\|\theta) \equiv P(X = x\|\tilde{\theta})$	$p(\theta\|x) \equiv P(\theta = \tilde{\theta}\|X = x)$
0			
$\frac{1}{5}$			
$\frac{2}{5}$			
$\frac{3}{5}$			
$\frac{4}{5}$			
1			

2. Suppose that the prior distribution of some parameter θ is a Gamma distribution, for which the mean is 10 and the variance is 5. Specify the prior hyper-parameters (i.e. the specific parameters of this Gamma prior).