Bayes Estimators

1. Suppose we have a single observation $X \sim \text{Unif}(0, \theta)$ where $\theta > 0$ is the unknown parameter of interest. Let the prior distribution for θ be

$$p(\theta) = \begin{cases} \theta e^{-\theta} & \theta > 0 \\ 0 & o.w. \end{cases}$$

Under this model, the posterior for θ is

$$p(\theta|x) = \begin{cases} e^{x-\theta} & x < \theta < \infty \\ 0 & o.w. \end{cases}$$

(a) Find the Bayes estimator under absolute loss.

(b) Find the Bayes estimator under squared loss.

(c) Great practice: Derive the posterior distribution of θ given $X = x$. Be careful with the supports! (Do your recognize the posterior up to proportionality? If not, we will need to calculate $f(\mathbf{x})$ to find the exact posterior).