Properties of covariance

Let X, Y, Z be random variables (not necessarily independent). Let $a \in \mathbb{R}$.

1. $\text{Cov}(X, X) = \text{Var}(X)$
2. Symmetry: $\text{Cov}(X, Y) = \text{Cov}(Y, X)$
3. Scaling: $\text{Cov}(aX, Y) = a\text{Cov}(X, Y)$
4. Invariance to shift: $\text{Cov}(X + a, Y) = \text{Cov}(X, Y)$
5. Bilinearity: $\text{Cov}(X + Y, Z) = \text{Cov}(X, Z) + \text{Cov}(Y, Z)$
6. If X and Y independent, then $\text{Cov}(X, Y) = 0$

Properties of Normals

Let $X \sim N(\mu_X, \sigma_X^2)$ be independent of $Y \sim N(\mu_Y, \sigma_Y^2)$. Let $a, b \in \mathbb{R}$.

1. Location-scale: define $W = a + bX$. Then $W \sim N(a + b\mu_X, b^2\sigma_X^2)$.
2. Sums of independent Normals: $X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$
3. Linear combination of independent normals: by properties 1 and 2, any linear combination of independent normal random variables is itself a normal random variable.
4. If X_1 and X_2 are two Normal random variables, then $\text{Cov}(X_1, X_2) = 0 \iff X_1$ independent X_2.