

Properties of MLEs

1. Suppose X_1, \dots, X_n form a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Find the MLE of the 0.95 quantile of the distribution. It may be helpful to know that the 0.95 quantile of a *standard normal* distribution is 1.645.
2. Let $X_1, \dots, X_n | \theta \stackrel{\text{iid}}{\sim} N(\theta, \sigma^2)$ where σ^2 is known and the mean θ is unknown. Show that $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$ is consistent for θ using the definition of consistency (and not just the LLN). *There are at least two ways to do this!*