Pre-class preparation

Please read the following textbook sections from Blitzstein and Hwang's *Introduction to Probability* (second edition) OR watched the indicated video from Blitzstein's Math 110 YouTube channel:

- Textbook: Appendix A.1 (Set Theory review) and Section 1.6
- Video: Lecture 2: Story proofs, Axioms of Probability from 39:00 onward, and Lecture 3: Birthday problem, Properties of probability

Objectives

By the end of the day's class, students should be able to do the following:

- Perform and interpret operations (unions, intersections, complements) on sets.
- State the general axiomatic definition of a probability space, and interpret the axioms in everyday language.
- Use the axioms to derive set-theoretic properties of probability, and be able to apply these properties.
- Explain why we require axiomatic probability in addition to the "naive" understanding of probability.

Reflection Questions

Please submit your answers to the following questions to the corresponding Canvas assignment by 7:45AM:

1. Show that for any events A and B, the following is true: $P(A)+P(B)-1 \leq P(A \cap B)$. When is this inequality ($\leq$) an exact equals ($=$)?

   We know $P(A \cup B) = P(A) + P(B) - P(A \cup B)$. Since $P(A \cup B) \leq 1$, it must be the case that $P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$, with equality holding only when $P(A \cup B) = 1$.

2. You go mushroom foraging and happen to find two mushrooms. You are not a mushroom expert (yet), so you do not know with 100% uncertainty whether either mushroom is toxic. But suppose that the probability that the first mushroom is toxic is $\frac{2}{3}$. The probability that the second mushroom is toxic is $\frac{1}{2}$. The probability that both mushrooms are toxic is $\frac{1}{3}$. What is the probability that neither mushroom is toxic?

   Let A be the event that mushroom 1 is toxic and B the event that mushroom 2 is toxic. $P(A) = 2/3, P(B) = 1/2$, and $P(A \cap B) = 1/3$. We would like $P(A^c \cap B^c) = P(A \cup B)^c$ by DeMorgan's Law. $P(A \cup B) = 2/3 + 1/2 - 1/3 = 5/6$, so $P(A \cup B)^c = 1/6$.

3. (Optional) Is there anything from the pre-class preparation that you have questions about? What topics would you like would you like some more clarification on?