This problem set covers material from Week 6, dates 10/17- 10/20. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding chapter. "AP" stands for additional problems not found in the book.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Tuesday 10/17

- **Chapter 4**: 53, 86
- **AP 1**: Show that $\text{Var}(X + c) = \text{Var}(X)$, where $c \in \mathbb{R}$ is a constant.
- **AP 2**: Show that $\text{Var}(cX) = c^2\text{Var}(X)$, where $c \in \mathbb{R}$ is a constant.

Thursday 10/19

- **Chapter 4**: 30, 69, 70
- **AP 3**: If a random variable X has a discrete distribution with PMF $p(x)$, then the value of $x = x^* \in S_X$ that maximizes $p(x)$ is called the *mode* of the distribution. If this same maximum $f(x^*)$ is attained at more than one value of x, then all such values of x are called the modes of the distribution. If $X \sim \text{Poisson}(\lambda)$, find the mode or modes of this distribution. . *Hint 1: consider the ratio* $\frac{p_X(k)}{p_X(k-1)}$, *and try to understand why this ratio is useful for solving this problem. Hint 2: consider the cases when* λ *is an integer versus when it is not.*

Friday 10/20

- **Chapter 5**: 1, 4, 8
- **AP 4**: **(Change-of-Variables Formula)** Suppose X is a random variable with support S_X and CDF $F_X(x)$, and let $Y = g(X)$. If g is increasing (and hence, invertible) on S_X, then the CDF $H_Y(y)$ of Y is given by

  $$H_Y(y) = F_X(g^{-1}(y)).$$

  Moreover, if X is a continuous random variable with PDF $f_X(x)$, then the PDF $h_Y(y)$ of Y is

  $$h_Y(y) = \frac{f_X(g^{-1}(y))}{g'(g^{-1}(y))}.$$

  where $S_Y = \{g(x) : x \in S_X\}$

1. Prove the change-of-variables formula above (note, there are two statements to prove: one about the CDF and one about the PDF). *Hint: Use the chain-rule from calculus, along with the formula for the derivative of an inverse function.*

2. Use the Change-of-Variable formula from part (a), along with u-substitution from calculus, to prove LOTUS for continuous random variables (at least in the case when g is an increasing function):

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

where X is a continuous random variable with density $f_X(x)$.

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.