MGF refresh

Recall that the MGF (it if exists) of a random variable X is defined as

$$M_X(t) = \mathbb{E}[e^{tX}]$$

for t in some open interval about 0. The MGF uniquely defines a distribution, and is especially useful for finding the distribution of a sum of independent random variables (why?).

Another nice property about MGFs that you proved in 310 is as follows: Let X have MGF $M_X(t)$. If $Y = a + bX$ for $a, b \in \mathbb{R}$, then $M_Y(t) = e^{at} M_X(bt)$.

Problems

If $X \sim \text{Gamma}(\alpha, \beta)$, then its MGF is $M_X(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha}$ for $t < \beta$.

1. **Scaling property of Gammas**: Let $X \sim \text{Gamma}(\alpha, \beta)$ and define $Y = cX$ for any constant $c > 0$. Show $Y \sim \text{Gamma}(\alpha, \frac{\beta}{c})$.

2. **Sums of independent Gammas**: Let $X_i \overset{\text{indep}}{\sim} \text{Gamma}(\alpha_i, \beta)$ for $i = 1, \ldots, n$, and define $Y = \sum_{i=1}^{n} X_i$. Find the distribution of Y.