This problem set covers material from Week 7, dates 4/02- 4/05, with content that builds on material from Week 6. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding subsection.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Tuesday 4/02 (and some material from week 6)

1. 8.3: Problem 6 (write the exact probability in terms of the appropriate CDF; then use R to obtain a decimal approximation. Please provide the code (hand-written is fine) that you used!)

2. 8.4: Problem 2 (use R to obtain a decimal approximation. Please provide the code (hand-written is fine) that you used!)

Thursday 4/04

3. 8.5: Problem 4

4. R problem 1

5. R problem 2

Friday 4/05

6. Suppose $X_1, \ldots, X_n | \lambda \overset{\text{iid}}{\sim} \text{Exp}(\lambda)$, where $\lambda > 0$ is unknown. Our ultimate goal is to obtain an exact γ-coefficient confidence interval for λ. In order to do so, we need a pivot!

   (a) Show that $\sum_{i=1}^{n} X_i \sim \text{Gamma}(n, \lambda)$ (i.e. sum of IID Exponentials is Gamma). *Hint: how have we found distributions of sums of random variables before?*

   (b) Secure your statistical hat firmly in place and suggest a pivotal quantity based off of $\sum_{i=1}^{n} X_i$ and state its distribution.

   (c) Suppose $n = 15$ and we want to obtain a 90% coefficient exact confidence interval for λ where the bounds are symmetric/equal-tailed (i.e. there is 0.05 probability below and 0.05 above the bounds of the interval for the pivotal's distribution). Using R software where necessary, obtain the bounds/form of the interval estimator.

7. We return to our beloved case of $X_1, X_2, \ldots, X_n|\theta \overset{\text{iid}}{\sim} \text{Unif}[0, \theta]$, where $\theta > 0$ is the unknown parameter. In this problem, we want to obtain a γ-coefficient confidence interval for θ. We have seen that for this data, $T(\mathbf{X}) = Y = \max\{X_i\}$ is the MLE for θ, so maybe we should use Y to construct a confidence interval! Here we will obtain one possible γ-coefficient confidence interval for θ.

   (a) Consider the following random variable: $U = \frac{Y}{\theta}$. Find the PDF (don't forget the support!) and quantile function for U.

   (b) Clearly eplain why U is pivotal but Y is not.

   (c) Obtain a γ-coefficient confidence interval estimator for θ of the form $[A(\mathbf{X}), \infty)$. You should express $A(\mathbf{X})$ either in terms of the observed data $\mathbf{X}$ or Y (i.e. not U).

   (d) Now suppose that I've changed my mind and no longer want a one-sided interval. Obtain the form of a γ-coefficient confidence interval for θ where both bounds are finite.

   (e) Suppose we observed the following data from this statistical model,

   $$X_1 = 3, X_2 = 1, X_3 = 0.25, X_4 = 2.75, X_5 = 2.2$$

   What is an equal-tailed 90% observed confidence interval for θ based on this data? What is a one-sided 90% confidence interval for θ?

General rubric

Points	Criteria
5	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.