Power and Size

1. Suppose we observe a single $X|\theta \sim \text{Exp}(\theta)$, so $F_X(x) = 1 - e^{-\theta x}$ for $x > 0$. We have the following hypothesis testing procedure δ:

$$H_0 : \theta \geq 1 \qquad \text{vs. } H_1 : \theta < 1$$

Our rule is reject H_0 if $X \geq 1$.

   (a) Why does this rule seem appropriate for the given set of hypotheses?

   (b) What is the power function $\pi(\theta|\delta)$ of this test? Is it increasing or decreasing in θ?

   (c) What is the probability of a Type II error? What is the interpretation as θ increases? Why does this "make sense"?

   (d) For after next class: What is the size of this test?

2. Suppose we observe a single $X|\theta \sim \text{Poisson}(\theta)$. We have the following hypothesis testing procedure δ:

$$H_0 : \theta \leq 1 \qquad \text{vs. } H_1 : \theta > 1$$

Our rule is reject H_0 if $X \geq c$ for some c.

   (a) Why does this rule seem appropriate for the given set of hypotheses?

   (b) What is the power function $\pi(\theta|\delta)$ of this test? For fixed c, is it increasing or decreasing in θ?

   (c) For after next class (and good practice for Midterm 2): Find c to make the size of this test δ as close as possible to 0.1, without exceeding 0.1. R may be helpful.