This problem set covers material from Week 9, dates 4/16- 4/19. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding subsection.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

General thoughts to keep in mind:

- It can seem scary to find $\sup_{\theta \in \Omega_0} g(\theta)$ for a function $g(\theta)$ when Ω_0 contains more than a single value. One thing that can be helpful is to ask yourself if $g(\theta)$ is an increasing or decreasing function in θ. If $g(\theta)$ is increasing in θ, then the supremum occurs at the largest possible value of $\theta \in \Omega_0$, and vice versa for decreasing $g(\theta)$.
- Size/power functions/p-values/sups are not easy concepts!! Go slowly, and think carefully about the definitions to logic your way through finding these quantities.

You will need to make some plots in R, but I will not be providing a template this time! Make sure the problem number is clearly indicated on your .Rmd file. Slightly different rubric for this homework since all problems have multiple parts.

Tuesday 4/16

1. Suppose a sample $X_1, \dots, X_n) \overset{\text{iid}}{\sim} N(\mu, \sigma^2)$ where μ is unknown but σ^2 is known. Consider the following hypotheses testing procedure δ_c:

$$H_0 : \mu = \mu_0 \quad \text{vs.} \quad H_1 : \mu \neq \mu_0$$

where μ_0 is some number in $\mathbb{R}$ (e.g. 3). We will use the test statistic $r(\mathbf{X}) = \frac{|\bar{X} - \mu_0|}{\sigma/\sqrt{n}}$, and let the rejection region for $r(\mathbf{X})$ be $R = [c, \infty)$ for some $c > 0$.

   (a) In your own words, briefly describe when we would reject H_0.

   (b) Find the power function for δ in terms of the standard normal CDF Φ. (It's good practice to write the probability statements conditioning on the parameter.)

   (c) Now suppose $\mu_0 = 0$, $n = 16$, and $\sigma^2 = 1$. Now consider δ_c for $c = \{1, 2, 3\}$, specifically. What is the probability of a Type I error for each of the three values of c? You may use `R` to help you evaluate the probabilities. If you do, write/include the code you used.

   (d) (`R`) Still suppose $\mu_0 = 0$, $n = 16$, and $\sigma^2 = 1$. Using `R`, plot in the same graph the three power functions as a function of μ for when $c = \{1, 2, 3\}$. (These are called power curves.) Interpret what you see. *For plotting purposes, you can restrict your range of μ values to $(-2, 2)$.*

Thursday 4/18

TBD

Friday 4/19

None!

General rubric

Points	Criteria
10	The solution is correct *and* well-written. The author leaves no doubt as to why the solution is valid.
9	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
8	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
6	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
4	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
2	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.