

This problem set covers material from Week 3, dates 2/27- 3/01. Unless otherwise noted, all problems are taken from the textbook. Problems can be found at the end of the corresponding subsection.

Instructions: Write or type complete solutions to the following problems and submit answers to the corresponding Canvas assignment. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A general rubric for homework problems appears on the final page of this assignment.

Unless otherwise stated, you must confirm that your critical point is indeed a maximum for full credit!

Tuesday 2/27

1. 7.10: Problem 7(a).
2. Let $X_1, \dots, X_n | \theta \stackrel{\text{iid}}{\sim} \text{Poisson}(\theta)$. In class, we showed $\hat{\theta}_{MLE} = \bar{X}$ given X_1, \dots, X_n . Now define the following random variable:

$$Y_i = \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases}$$

We seek to find an MLE for θ using the Y_1, \dots, Y_n .

- (a) Find the PMF for Y_i and the likelihood function for θ given Y_1, Y_2, \dots, Y_n . (*Hint: it should have a similar structure to the Bernoulli likelihood function.*)
- (b) Show that

$$\hat{\theta}_{MLE} = \log \left(\frac{n}{\sum (1 - y_i)} \right)$$

is the MLE estimate for θ given y_1, \dots, y_n and provided that not all the y_i 's are equal to 1.

- (c) What does the likelihood function equal if $y_i = 1$ for $i = 1, \dots, n$? Use the likelihood to explain why the MLE for θ does not exist if all of the y_i 's are equal to 1.
 - (d) For fixed n , what is the probability that all of the y_i 's are equal to 1? What happens to that probability as $\theta \rightarrow \infty$? What does that say about the probability that MLE does not exist?
 - (e) For a fixed θ , what happens to the probability that the MLE does not exist as $n \rightarrow \infty$?
3. 7.5: Problem 12 *Note: the implied statistical model is a Multinomial. The sum-to-one constraint on the proportions is important! Dig deep into your Calculus classes and try*

to re-learn the method of LaGrange Multipliers. Also, you do not need to verify that the critical point is indeed a maximum; believe me that it is!

General rubric

Points	Criteria
5	The solution is correct <i>and</i> well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect.
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification.
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information).
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.
Notes:	For problems with code, well-written means only having lines of code that are necessary to solving the problem, as well as presenting the solution for the reader to easily see. It might also be worth adding comments to your code.