Conjugate priors

1. Let $X_1, \ldots, X_n | \theta \overset{\text{iid}}{\sim} N(\mu, \theta)$ where μ is known and the *variance* $\theta > 0$ is the unknown parameter of interest. (Note, the variance is θ, not θ^2.) We will assume (i.e. choose the prior) $\theta \sim$ Inv. Gamma(α, β) where

$$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\beta/\theta}, \qquad \theta > 0$$

   (a) Find the posterior distribution of θ given the data.

   (b) Is the Inverse Gamma a conjugate family of prior distributions for the variance when the data follow a normal distribution and the mean is known? Justify your answer.