

This homework is graded as participation, and is mainly for you (and me) to get a sense of the level of preparation of students going into this course. As such, please complete this assignment on your own (assistance from Prof. Tang is allowed!). For full credit, show all of your work (if you have no idea how to solve one, writing a brief explanation about your thought process in attempting the problem counts as showing your work).

1. Suppose that a precinct contains 350 voters, of which 250 are Democrats and 100 are Republicans. If 30 voters are chosen at random from the precinct (without replacement), what is the probability that exactly 18 Democrats will be selected?
2. Derive/re-derive the moment generating function (MGF) of a $\text{Poisson}(\lambda)$ random variable. *Remember, the MGF of a random variable X is $M_X(t) = \mathbb{E}[e^{tX}]$. Also, remember that $e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!}$.*
3. Suppose $X \sim \text{Poisson}(\lambda_1)$ independent of $Y \sim \text{Poisson}(\lambda_2)$. Find the probability mass function (PMF) of $N = X + Y$. *Use the previous problem to help you!*
4. Let $U \sim \text{Unif}(0, 1)$. Find the probability density function (PDF) of $X = \log \frac{U}{1-U}$. (In this class/stats in general, we use $\log()$ to denote the natural log $\ln()$.)
5. Two friends plan to meet at noon. In reality, each arrives independently at a time uniformly distributed between 12:00p.m. and 1:00 p.m.
 - (a) What is the probability the first to show up has to wait more than 10 minutes for the second person to arrive? *Hint: draw a picture!*
 - (b) What time would we expect the earlier friend to arrive?
6. Suppose $X \sim \text{Poisson}(\lambda_1)$ independent of $Y \sim \text{Poisson}(\lambda_2)$, and define $N = X + Y$.
 - (a) Find $\mathbb{E}[X|N = n]$. *Hint: try finding a useful conditional distribution using definition of conditional probability. And remember the formula of the binomial coefficient: for $x, n \in \mathbb{Z}^*$ with $n \geq x$, $\binom{n}{x} = \frac{n!}{x!(n-x)!}$*
 - (b) Use the Law of Total Expectation to confirm $\mathbb{E}[X] = \lambda_1$.