Properties of MLEs

1. Let $X_1, \ldots, X_n | \theta \overset{\text{iid}}{\sim} N(\theta, \sigma^2)$ where σ^2 is known and the mean θ is unknown. Show that $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ is consistent for θ using the definition of consistency (and not just the LLN). *There are at least two ways to do this!*

2. Suppose $X_1, \ldots, X_n$ form a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Find the MLE of the 0.95 quantile of the distribution. It will be helpful to know that the 0.95 quantile of a *standard normal* (i.e. $N(0.1)$) is 1.645.