The following exercises are designed to be review from MATH/STAT 310 and to help you become familiar with the notation that I use. I understand that it's been awhile since you've seen this material, so don't hesitate to look through old notes or ask me/others for some help! The important thing is that these concepts are within easy reach *after* doing the review.

This assignment will count towards participation!

1. What is the support of a probability distribution?
2. For a r.v. Y with distribution f_Y, when does $f_Y(y) = \Pr(Y = y)$?
3. Let $\mathbf{1}_A$ denote the indicator random variable for event A. What is its distribution?
4. Recall that the expectation of a discrete random variable Y with PMF f_Y is
$$\mathbb{E}[Y] = \sum_{y \in \mathcal{S}} y f_Y(y)$$
where $\mathcal{S}$ is the support of Y. Suppose $Y \sim \text{Binomial}(2, \frac{1}{4})$.
   (a) Compute $\mathbb{E}[Y]$ and $\mathbb{E}[Y^2]$ by hand.
   (b) Compute $\text{Var}(Y)$ from the expectations obtained in (a).
5. Suppose the outcome Y of an experiment has a $\text{Binomial}(3, p)$ distribution, with unknown parameter p. However, it is known that p is either $1/2$ or $1/4$, and we have no reason to believe one possibility over the other, i.e. $\Pr(p = 1/2) = \Pr(p = 1/4) = 1/2$. Compute $\Pr(p = 1/2|Y = 2)$. *Maybe think about Bayes' Rule...*
6. Suppose we have two continuous random variables Y_1, Y_2 with joint PDF
$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} \frac{1}{2} & 0 < y_1 < y_2 < 2 \\ 0 & o.w. \end{cases}$$
   (a) Draw a picture that demonstrates where the joint PDF is positive.
   (b) What is $P(Y_1 + Y_2 > 3)$? *You can do this geometrically or via calculus.*
   (c) Obtain the marginal distribution of Y_1.
   (d) Obtain the conditional distribution $f_{Y_2|Y_1}(y_2|y_1)$.
   (e) Using your solution from the previous answer, what is the distribution of $f_{Y_2|Y_1}(y_2|1.5)$? Can you name it?
   (f) What is the expected value of Y_2 if you know that $Y_1 = 1.5$? That is, what is $\mathbb{E}[Y_2|Y_1 = 1.5]$? Do this by hand, and confirm your solution using your named distribution from the previous question.
7. Demonstrate (with just a small amount of work) that
$$\int_{-\infty}^{\infty} \exp\left\{-\frac{x^2}{2}\right\} dx = \sqrt{2\pi}$$
Hint: can you spot a known distribution?