STAT 412 Reading Guide for *Explaining the Gibbs Sampler*

General tips for reading papers:

- Do not rush the process or necessarily try to finish the paper in one sitting.

- Make sure you feel really solid about Section x before moving to Section $x + 1$.

- Do not ignore figures!

- You should be constantly flipping back and forth between pages, visiting previous sections as you move forward with new sections. This may be facilitated by printing out the paper!

Reading guide for Casella, G. and George, E. I. (1992), "Explaining the Gibbs Sampler", *The American Statistician*, **46**(3), 167-174. The following questions are provided to give you some guidance in understanding the concepts in the paper. Answer the following questions, as we will discuss them in class as a large group!

1. [**Section 2**] How does Gelfand and Smith (1990) suggest to obtain an approximate sample from $f(x)$? How is it different from or similar to the approach we talked about in class? What are the advantages and disadvantages of each approach?

2. [**Section 2**] The authors claim "Gibbs sampling can be used to estimate the density itself by averaging the final conditional densities from each Gibbs sequence." What is the theory behind this claim? How does Figure 3 support this claim?

3. [**Section 2**] There are two different simulations shown in Figure 1 and Figure 3. What are the similarities and differences? Why does the conditional carry more information than the marginal?

4. [**Section 3**] Following the example provided in the paper, write down the marginal distribution of y, and verify the conditional probabilities $A_{y|x}$ and $A_{x|y}$. Also verify the following:

   - $A_{x|x} = A_{y|x} A_{x|y}$
   - $f_x A_{x|x} = f_x$

5. [**Section 4**] What is a fixed point integral equation in the bivariate case? How does it help illustrate how sampling from conditionals produces a marginal distribution? Hint: check equations (3.5), (4.1), and (4.2).

6. [**Section 4**] The authors claimed "a defining characteristic of the Gibbs sampler is that it always uses the full set of univariate conditionals to define the iteration." Explain this claim by illustrating how a Gibbs sampler works with k parameters $(\theta_1, \theta_2, \ldots, \theta_k)$.

7. [**Section 5**] Summarize different approaches to sampling the Gibbs sequence.