Problem Set 1 (R)

1. The number of particles Y emitted from a substance depends on the unknown proportion θ of the substance that is radioactive. Suppose that for each possible value of θ,

$$Pr(Y = y|\theta) = f(y|\theta) = \frac{(5\theta)^y e^{-5\theta}}{y!}$$

Suppose that the substance is monitored and it is observed that $Y = 3$.

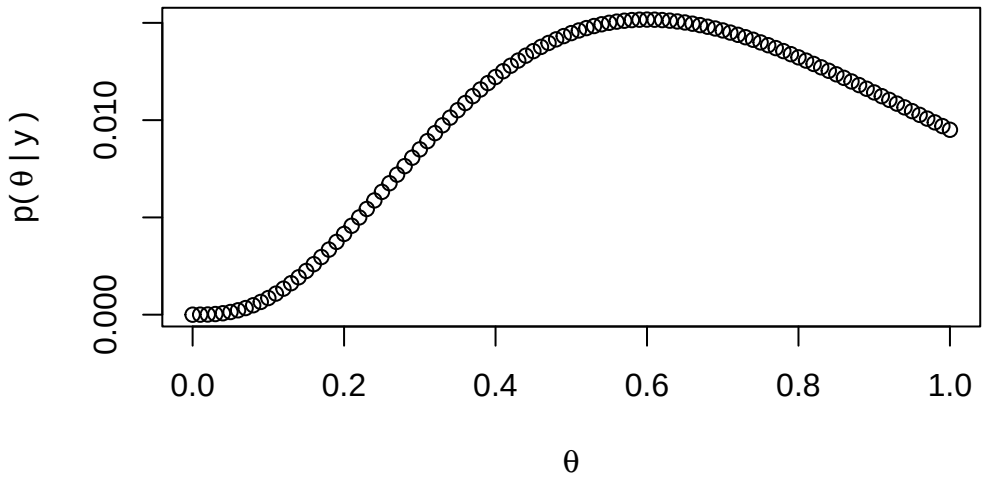
a) Make a graph of $f(\theta|y)$ for $y = 3$ in the case that $f(\theta) = \frac{1}{101}$ for each $\theta \in \{0, \frac{1}{100}, \frac{2}{100}, \ldots, 1\}$ (and 0 otherwise).

```
y <- 3
theta <- seq(0, 1, 0.01)

like <- ((5*theta)^y) * exp(-5*theta) / factorial(y)
prior <- rep(1/101, 101)
marg <- sum(like * prior)
post <- like * prior / marg
sum(post) ## sanity check
```

```
[1] 1
```

```
plot(theta, post, xlab = expression(theta), ylab = expression("p("~ theta ~ "|"~ y ~ ")"))
```



b) What are the mean, median, and mode of θ under our prior? Using code and/or your graph, what are the posterior mean, median, and mode?

```
# prior mode DNE

# prior median
m1 <- theta[max(which(cumsum(rep(1/101, 101)) <= 0.5))]
m2 <- theta[min(which(cumsum(rep(1/101, 101)) >= 0.5))]
prior_med <- mean(c(m1, m2))

# prior mean
prior_mean <- sum(theta * prior)
```

The prior mode DNE, the prior median is 0.495, and the prior mean is 0.5.

```
# posterior mode
post_mode <- theta[which(post == max(post))]

# posterior median
m1 <- theta[max(which(cumsum(post) <= 0.5))]
m2 <- theta[min(which(cumsum(post) >= 0.5))]
post_med <- mean(c(m1, m2))

# posterior mean
post_mean <- sum(theta * post)
```

The prior mode 0.6, the prior median is 0.605, and the prior mean is 0.6108682.