

Name	Notation	Parameters	PDF/PMF	Support	Mean	Variance
Bernoulli	$\text{Bern}(p)$	$p \in (0, 1)$	$f(x) = p^x(1 - p)^{1-x}$	$x \in \{0, 1\}$	p	$p(1 - p)$
Beta	$\text{Beta}(a, b)$	$a, b > 0$	$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1 - x)^{b-1}$	$x \in (0, 1)$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$
Binomial	$\text{Binom}(n, p)$	$n \in \mathbb{Z}^+, p \in (0, 1)$	$f(x) = \binom{n}{x}p^x(1 - p)^{n-x}$	$x \in \{0, 1, \dots, n\}$	np	$np(1 - p)$
Exponential	$\text{Exp}(\lambda)$	$\lambda > 0$	$f(x) = \lambda e^{-\lambda x}$	$x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Dirichlet	$\text{Dirichlet}(\boldsymbol{\alpha})$ $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_K)$	$\alpha_k > 0, k = 1, \dots, K$	$f(\mathbf{x}) = \frac{\prod_{k=1}^K \Gamma(\alpha_k)}{\Gamma(\sum_{k=1}^K \alpha_k)} \prod_{k=1}^K x_k^{\alpha_k-1}$	$x_k \geq 0, \sum_{k=1}^K x_k = 1$	$\mu_k = \alpha_k$	—
First Success	$\text{FS}(p)$	$p \in (0, 1)$	$f(x) = p(1 - p)^{x-1}$	$x = 1, 2, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Gamma	$\text{Gamma}(a, b)$	$a, b > 0$	$f(x) = \frac{b^a}{\Gamma(a)}x^{a-1}e^{-bx}$	$x > 0$	$\frac{a}{b}$	$\frac{a}{b^2}$
Geometric	$\text{Geom}(p)$	$p \in (0, 1)$	$f(x) = p(1 - p)^x$	$x = 0, 1, 2, \dots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$\text{HGeom}(w, b, n)$	$w, b, n = 0, 1, \dots$	$f(x) = \frac{\binom{w}{x}\binom{b}{n-x}}{\binom{w+b}{n}}$	$x \in \{0, \dots, w\},$ $(n - x) \in \{0, \dots, b\}$	$\frac{nw}{w+b}$	$\frac{w+b-n}{w+b-1} \left(\frac{nw}{(w+b)^2} \right)$
Inverse Gamma	$\text{Inv.Gamma}(a, b)$	$a, b > 0$	$f(x) = \frac{b^a}{\Gamma(a)}x^{-a-1}e^{-\frac{b}{x}}$	$x > 0$	$\frac{b}{a-1}$	$\frac{b^2}{(a-1)^2(a-2)}$
Multinomial	$\text{Multinom}_k(n, \mathbf{p})$ $\mathbf{p} = (p_1, \dots, p_k)$	$n \in \mathbb{Z}^+, p_j \geq 0$ $\sum_{j=1}^k p_j = 1$	$f(\mathbf{x}) = \frac{n!}{x_1! \dots x_k!} p_1^{x_1} \dots p_k^{x_k}$	$\sum_{j=1}^k x_j = n$		
Negative Binomial	$\text{NegBinom}(r, p)$	$n = 0, 1, \dots$ $p \in (0, 1)$	$f(x) = \binom{x+r-1}{x} p^r (1 - p)^x$	$x = 0, 1, 2, \dots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$
Normal	$N(\mu, \sigma^2)$	$\mu \in \mathbb{R}, \sigma^2 > 0$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	$x \in \mathbb{R}$	μ	σ^2
Poisson	$\text{Poisson}(\lambda)$	$\lambda > 0$	$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0, 1, 2, \dots$	λ	λ
Uniform (cont.)	$\text{Unif}(a, b)$	$a, b \in \mathbb{R}, a < b$	$f(x) = \frac{1}{b-a}$	$x \in (a, b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Uniform (disc.)	$\text{DiscUnif}(A)$ $A = \{a_1, \dots, a_n\}$	$a_i \in \mathbb{R}$	$f(x) = \frac{1}{ A }$	$x \in A$	$\frac{\sum a_i}{ A }$	—

Other helpful things

- For any positive value z , $\Gamma(z+1) = z\Gamma(z)$. If z is a positive integer, $\Gamma(z) = (z-1)!$
- Fisher Information for θ from $f(y|\theta)$ is: $-\mathbb{E}_{Y|\theta} \left[\frac{d^2}{d\theta^2} \log f(y|\theta) \right]$

Conjugate priors

- $\theta \sim \text{Beta}(a, b)$ for $Y_i|\theta \stackrel{\text{iid}}{\sim} \text{Bernoulli}(\theta)$
 - Posterior: $\theta|\mathbf{y} \sim \text{Beta}(a + \sum y_i, b + n - \sum y_i)$
 - Posterior predictive: $\text{Bernoulli} \left(\frac{a + \sum y_i}{a + b + n} \right)$
- $\theta \sim \text{Gamma}(a, b)$ for $Y_i|\theta \sim \text{Poisson}(\theta)$
 - Posterior: $\theta|\mathbf{y} \sim \text{Gamma}(a + \sum y_i, b + n)$
 - Posterior predictive: $\text{NegBinom} \left(a + \sum y_i, \frac{b+n}{b+n+1} \right)$
- $\boldsymbol{\theta} \sim \text{Dirichlet}(a_1, \dots, a_K)$ for $Y_i|\boldsymbol{\theta} \sim \text{Categorical}(\boldsymbol{\theta})$
 - Posterior: $\boldsymbol{\theta}|\mathbf{y} \sim \text{Dirichlet}(a_1 + n_1, \dots, a_K + n_K)$ where $n_k = \sum_{i=1}^n \mathbf{1}_{\{Y_i=k\}}$
 - Posterior predictive: $\text{Categorical} \left(\frac{a_1 + n_1}{n + \sum a_k}, \dots, \frac{a_K + n_K}{n + \sum a_k} \right)$
- $\theta \sim N(\mu_0, \sigma_0^2)$ for $Y_i|\theta, \sigma^2 \sim N(\theta, \sigma^2)$ where σ^2 known
 - Posterior: $\theta|\mathbf{y}, \sigma^2 \sim N \left(\left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2} \right)^{-1} \left(\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2} \right), \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2} \right)^{-1} \right)$
 - Posterior predictive: $N \left(\left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2} \right)^{-1} \left(\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2} \right), \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2} \right)^{-1} + \sigma^2 \right)$