Math 2144 Worksheet 4: Derivatives September 18, 2015

1. (a) Write down the limit definition of the derivative of a function $f(x)$ at the point $x = a$.

   (b) Use this definition to explain why $f(x) = x^{1/3}$ is not differentiable at $a = 0$ (actually use the limit definition - don't find $f'(x)$ and just say you can't plug in 0).

2. (a) Given a function $f(x)$, write down the limit definition of the function $f'(x)$.

   (b) Let $f(x) = \sqrt{x+1}$. Have we learned a rule to compute $f'(x)$ yet? Using the definition of the derivative, find $f'(x)$.

3. Write down the product and quotient rules. Find the derivatives of the following functions:

   (a) $f(x) = (x+2)(2x+1)$

   (b) $f(x) = x(2x+2)(2x-2)$

   (c) $f(x) = \dfrac{x}{1+x^2}$

4. Find the tangent line to $f(x) = \dfrac{2x}{e^x}$ at the point $(0, f(0))$.

5. Let $C(x)$ be the cost of producing x units of a particular product. The **production level** is the number of units x manufactured.

   (a) Company data suggests $C(x) = .0005x^3 - .38x^2 + 120x$, where x is the number of units produced of a certain product. When the production level is 150 units, find the instantaneous rate of change of $C(x)$.

   (b) The **marginal cost** at production level x_0 is the cost of producing one more unit, i.e.,

   $$\text{Marginal Cost} = C(x_0 + 1) - C(x_0)$$

   At production level 150, what is the marginal cost?

   (c) The tangent line to $C(x)$ at $x = 150$ gives a linear function which is a pretty good approximation to $C(x)$ near $x = 150$. Use this linear function to approximate the marginal cost, and compare your answer to (b).

6. Find equations of the tangent lines to the curve $y = \dfrac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$.

7. A car is following a road shaped like the Witch of Agnesi, which is a curve with equation $f(x) = \frac{1}{1+x^2}$.

   (a) If the car drives off to $+\infty$ or $-\infty$, what horizontal line will it get close to?

   (b) Oddly enough, there is a witch standing at $(0, 1)$. Assuming the car starts pretty far back along the road ($x < 0$) and is driving in the positive x direction, at what point do the headlights of the car first shine on the witch?