Platforms & Tools

D2L (DEN) : Syllabus ✓
Lecture Notes ✓
Lecture Videos ✓
HW Assignments ✓
HW Submissions ✓
Any other reference material ✓
online Exams ✓

Piazza : Discussion Board ✓

Roles & Responsibilities

- Instructor — Lectures & Discussions
- TAs — HWs & Exams
- Graders — HW grading
- Course Producers — *
- CS Dept. Advisors
- DEN Support

Textbooks

- Algorithms Design by Jon Kleinberg & Eva Tardos

- Supplemental Textbook:
  Introduction to Algorithms, 3rd edition, by Cormen et al.

Your Responsibilities

1. - Attending lectures & Discussions
2. - Completing reading assignments
3. - Doing HW problems
4. - Doing as many other problems from textbook as possible

Your Grade

Exam 1	30%	Oct. 1
Exam 2	30%	Nov. 5
Exam 3	35%	Dec. 3
Final Proj	5%	Dec. 8
	100%	

Grading Scale

90 – 100	A	60 – 64.99	C+
86 – 89.99	A-	55 – 59.99	C
80 – 85.99	B+	50 – 54.99	C-
70 – 79.99	B	45 – 49.99	D
65 – 69.99	B-	Below 45.99	F

- Scale will be adjusted if median falls below 75.

- At least the top 20% of the class will receive an A.

- At least the next 10% of the class (between top 20% and top 30%) will receive an A-.

Prerequisites

- Discrete Math – Mathematical Induction
- Sorting methods
- Basic data structures: Arrays, stacks, queues, linked lists
- Basics of graphs: Trees, cycles, DAG, adjacency list/matrix, etc.
- Graph search algorithms: BFS, DFS

High level Syllabus

Today!

- Introduction
- Review of some prereqs + asymptotic notation
- Major algorithmic techniques
  - Greedy
  - Divide & Conquer
  - Dynamic Programming

Exam 1

- Network Flow
- Computational Complexity Theory
- Approximation Algorithms
- Linear Programming

Exam 2

covers only 2 topics

Exam 3

comprehensive

Corrections

1- ~~An algorithm is a set of instructions in machine language.~~

2- ...~~Algorithmic science advanced on Wall Street...~~

3- ... ~~Invite 6 million algorithms for a listen...~~

When studying a problem, we go through the following steps:

1- Come up with a concise problem statement

2- Present a solution

3- Prove correctness

4- Perform complexity analysis

Stable Matching

Stable Matching Example

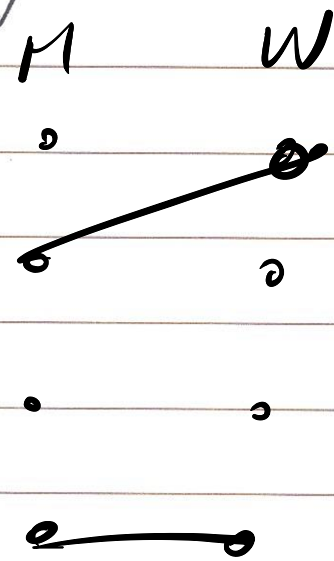
Problem: We are interested in matching $\underline{n}$ men with $\underline{n}$ women so that they could stay happily married ever after.

Step 1: Come up with a concise problem statement.

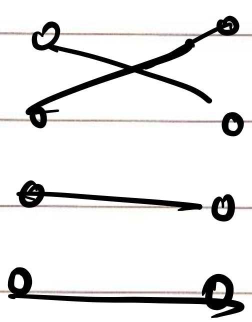
We have a set of $\underline{n}$ men, $M = \{m_1, \ldots m_n\}$

We have a set of $\underline{n}$ women, $W = \{w_1, \ldots w_n\}$

Def. A __Matching__ S is a set of ordered pairs.



Def. A __perfect matching__ S' is a matching with the property that each member of M and each member of W appear in exactly one pair in S'.



A perfect matching

Add notion of preferences

Each man $m \in M$ ranks all women

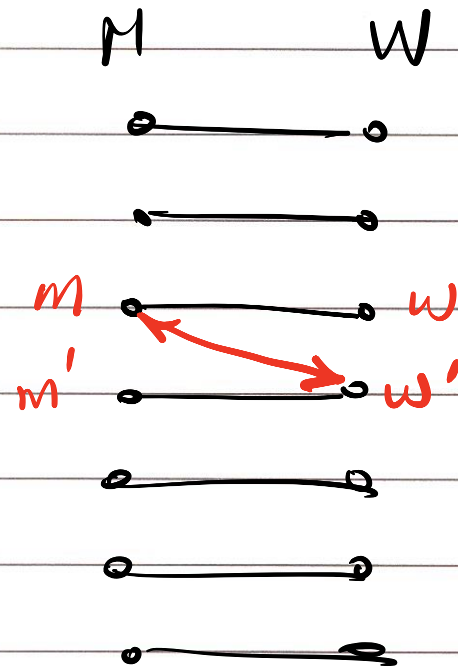
- $\underline{m}$ prefers $\underline{w}$ to $\underline{w'}$ if $\underline{m}$ ranks $\underline{w}$ higher than w'.

- Ordered ranking of $\underline{m}$ is his preference list

$\underline{P_{m_i}} = \{ w_{i_1}, w_{i_2}, \ldots w_{i_n} \}$

Same for women, i.e. each woman $w \in W$ ranks all men...

S



Such a pair (m, w') is called an instability

Def. Matching S is stable if

1. It is perfect

2. There are no instabilities WRT S

Step 1: Input: Preference lists for a set of $\underline{n}$ men & $\underline{n}$ women.

Output: Set of $\underline{n}$ marriages w/ no instabilities

Step 2: Gale-Shapley Alg.

Step 3 Proof of Correctness

① From the woman's perspective, she starts single, and once she gets engaged and she can only get into better engagements

② From the man's perspective, he starts single, gets engaged, and may get dropped repeatedly only to settle for a lower ranking woman.

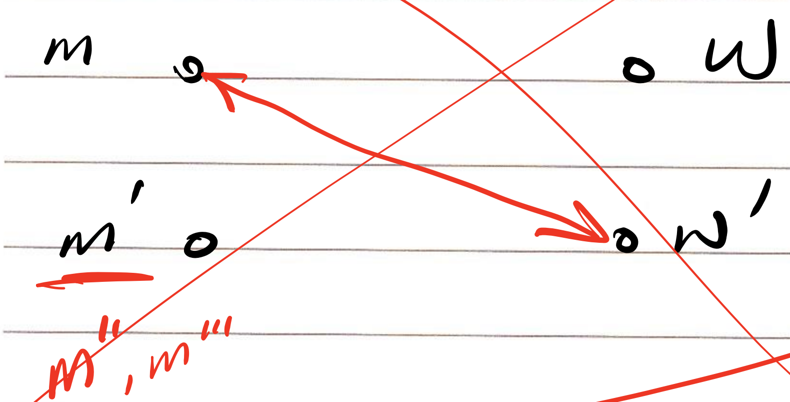
③ Solution will terminate in at most n^2 iterations

④ Solution is a perfect matching

⑤ Solution is a stable "

Proof by Contradiction

Assume an instability exists in our solution involving two pairs $(m, w), (m', w')$



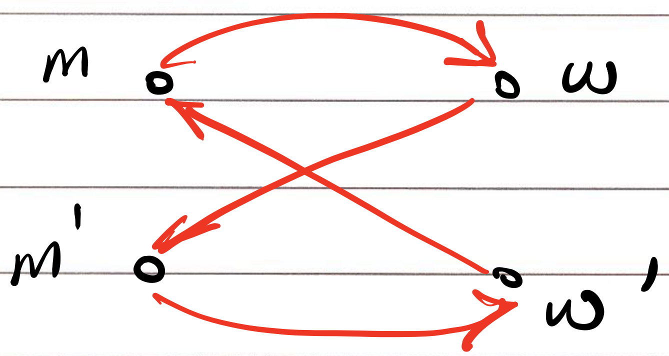
Say (m, w') is an instability

Q: Did $\underline{m}$ propose to w' at some point in the execution?

If no, then $\underline{w}$ must be higher than $\underline{w'}$ on his list $\rightarrow$ contradiction!

If yes, he must have been rejected in favor of m'' and due to (1) either $m'' = m'$ or m' is better than m''

$\Longrightarrow$ contradiction!



$(m, w), (m', w')$

$(m, w'), (m', w)$

Step 4 Complexity Analysis

1- Identify a free man $O(1)$

2- For a man <u>m</u>, identify the highest ranked woman to whom he has not yet proposed. $O(1)$

3- For a woman <u>w</u>, decide if <u>w</u> is engaged, and if so to whom $O(1)$

4- For a woman <u>w</u> and two men <u>m</u> & <u>m'</u>, decide which man is preferred by <u>w</u> $O(1)$

5- Place a man back in the list of free men. $O(1)$

1. Identify a free man

	get	put
Array	O(1)	O(1)
linked list	O(1)	O(1)
queue	O(1)	O(1)
stack	O(1)	O(1)

2. Identify the highest ranked woman to whom <u>m</u> has not yet proposed.

Keep an array Next[1..n], where Next[m] points to the position of the next woman he will be proposing to on his pref. list.

Men's preference list: ManPref[1..n, 1..n],
where
ManPref[m, i] denotes the i^{th} woman on man m's preference list.

To find next woman w to whom m will be proposing to:

$\underline{w}$ = ManPref[m, Next[m]]

takes O(1)

3_ Determine woman $\underline{w}$'s status

Keep an array called Current[1..n] where Current[w] is Null if w is single & set to $\underline{m}$ if $\underline{w}$ is engaged to $\underline{m}$.

takes $O(1)$

4_ Determine which man is preferred by $\underline{w}$.

WomanPref$_i$

3	8	4	32	1	.	.	.
1	2	3	4	5			

Woman Ranking

5		1	3			
1						

Preparation before entering GS iterations

Create a Ranking array where Ranking [w,m] contains the rank of man m based on w's preference

$$\underbrace{\text{Preparation}}_{O(n^2)} + \underbrace{\text{GS iterations}}_{O(n^2)}$$

Overall Complexity = $O(n^2)$

highest

$$m_i \quad \left\{ \begin{array}{l} w_{i_1} \\ w_{i_2} \\ w_{i_3} \\ w_{i_4} \\ \vdots \\ w_{i_{23}} \\ w_{i_n} \end{array} \right.$$

Mi's best valid partner

lowest

Def. Woman w is a valid partner of a man m if there is a stable matching that contains the pair (m, w)

Def. m's best valid partner is

Claim: Every execution of the G-S algorithm (When men propose) results in the same stable matching regardless of the order in which men propose.

Plan: to prove this, we will show that when men propose, they always end up with their best valid partner.

Proof by contradiction:

Say <u>m</u> is the first man rejected by a valid partner w. in favor of m'

Claim: When men propose, women end up with their worst valid partners

Proof: By contradiction . . .

Suppose we end up with a matching S where for a pair (m, w) in S, m is not w's worst valid partner.
So there must be another matching S' where w is paired with a man m' whom she likes less.