

# CSCI 570 - Fall 2021 - HW 2

Due September 9th

## 1 Graded Problems

1. What is the worst-case runtime performance of the procedure below?

```
c = 0
i = n
while i > 1 do
  for j = 1 to i do
    c = c + 1
  end for
  i = floor(i/2)
end while
return c
```

2. Arrange these functions under the  $O$  notation using only  $=$  (equivalent) or  $\subset$  (strict subset of):

- (a)  $2^{\log n}$
- (b)  $2^{3n}$
- (c)  $n^{n \log n}$
- (d)  $\log n$
- (e)  $n \log(n^2)$
- (f)  $n^{n^2}$
- (g)  $\log(\log(n^n))$

E.g. for the function  $n$ ,  $n + 1$ ,  $n^2$ , the answer should be

$$O(n + 1) = O(n) \subset O(n^2).$$

3. Given functions  $f_1, f_2, g_1, g_2$  such that  $f_1(n) = O(g_1(n))$  and  $f_2(n) = O(g_2(n))$ . For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.
- (a)  $f_1(n) \cdot f_2(n) = O(g_1(n) \cdot g_2(n))$
  - (b)  $f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$
  - (c)  $f_1(n)^2 = O(g_1(n)^2)$
  - (d)  $\log_2 f_1(n) = O(\log_2 g_1(n))$
4. Given an undirected graph  $G$  with  $n$  nodes and  $m$  edges, design an  $O(m + n)$  algorithm to detect whether  $G$  contains a cycle. Your algorithm should output a cycle if  $G$  contains one.

## 2 Practice Problems

1. Solve Kleinberg and Tardos, **Chapter 2, Exercise 6**.
2. Solve Kleinberg and Tardos, **Chapter 3, Exercise 6**.