AOA Notes week 1

PAGE No.
DATE / /

Stable Matching:

'N' men : $\{M_1, M_2, \ldots M_n\}$

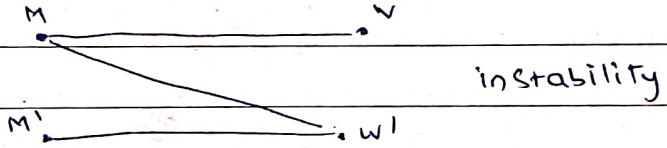
'N' women : $\{w_1, w_2, \ldots w_n\}$

Matching is a set of ordered pairs

perfect Matching: each member of M and each member of W only exist in one pair

All men rank women acc to their pref

Similarly all women rank men acc to their pref



we need perfect and stable matching

Gale Shaplay Algorithm

$M = \{m_1, m_2, m_3\}$ $W = \{w_1, w_2, w_3\}$

men proposing

$M_1 : \{w_1\ w_2\ w_3\}$ $w_1\ \{m_2\ m_1\ m_3\}$

$M_2\ \{w_1\ w_3\ w_2\}$ $w_2\ \{m_1\ m_2\ m_3\}$

$M_3\ \{w_2\ w_1\ w_3\}$ $w_3\ \{m_3\ m_2\ m_1\}$

STABLE MATCHING

w_1	~~M_1~~ M_2
w_2	~~M_3~~ M_1
w_3	M_3

side that does proposing in G-S algo ends with the best possible stable matching (from their perspective).

From womens perspective, she starts single, and once she gets engaged, she can only get into better engagements.

From mans perspective, he start single, gets engaged, and may get dropped repeatedly only to settle for lower ranking woman.

Soln terminates in atmost (n^2) iterations

Proof of correctness:

proof by contradiction

Assume instability exists in our soln involving two pairs (m, w) (m', w')

say (mw') is instability

Q: Did m propose to w' at some point in the execution.

A: ~~No~~ If no, w must be higher than w' on his list → contradiction.

IF yes, he must have been rejected in favor of m'' and due to ① either $m'' = m'$ or m' is better than m'' ↯ contradiction.

Ⓐ Identify highest ranked woman to whom m has not yet proposed O(1)

Next [1 . . . N]

Next [m]

↳ Next women 'm' will be proposing to

ManPref [1.. n, 1.. n]

ManPref [m, i] denotes i^{th} women on man 'm' list

(A) To find next woman w to whom m will be proposing to — O(1)

w = ManPreF [m, Next [m]]

(A) Determine w's status — O(1)

Current [1.. N]

initially current [1.. N] = null

once male 'm' assigned to women w3

current [3] = m

(B) Determine which ~~man~~ is preferred by w — O(1)

women ranking

man5	man10	. . ..
1	2	

Prep before entering GS Algorithm.

create a ranking array where ranking [w, m] contains the rank of mem m based on w's pref

preparation $O(n^2)$ + GS iteration $O(n^2)$

$O(n^2)$

Women w is a valid partner of a man m if there is a stable matching that contains the pair (m, w)

Highest women is best valid partner of man m

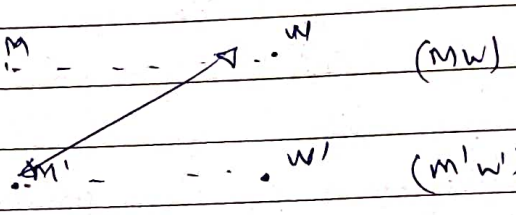
Every exec of GS algo (men proposing) results in same stable matching regardless the order in which men propose.

(Best valid partner are invariant. with respect to the Algorithm)

Proof by contradiction.

Say M is the first man rejected by a valid partner W in Favor of M'

IF M and W are valid partners in some pairing S' they are paired



But W prefers M' over M as ~~assumed~~ W rejected M for M'

But M' proposed W over W Hence pair (M'W) cant be formed.

IF M' proposed to W and M was first to get rejected (M'W) would be paired but M'W' paired

Hence Contradiction.

1924

$T(k) = 2T(k-1) + O(1)$

$T(k-1) = 2T(k-2)$

T

$T(1) = 2T(0)$

PAGE NO.

DATE / /

Discussion Week 1

$T(1) = 2T(0)$

$T(2) = 2T(1)$

$T(3) = 2T(2)$

$T(k) = 2T(k-1) + O(1)$

$O(k)$