CSCI 570 - Fall 2021 - HW 10

Due November 18th

Graded

1. (20 pts) Consider the partial satisfiability problem, denoted as 3-Sat(α). We are given a collection of k clauses, each of which contains exactly three literals, and we are asked to determine whether there is an assignment of true/false values to the literals such that at least αk clauses will be true. Note that 3-Sat(1) is exactly the 3-SAT problem from lecture.

   Prove that 3-Sat(15/16)is NP-complete.

   Hint: If x, y, and z are literals, there are eight possible clauses containing them: $(x \vee y \vee z)$, $(!x \vee y \vee z)$, $(x \vee !y \vee z)$, $(x \vee y \vee !z)$, $(!x \vee !y \vee z)$, $(!x \vee y \vee !z)$, $(x \vee !y \vee !z)$, $(!x \vee !y \vee !z)$

2. (20 pts) Consider modified SAT problem, SAT' in which given a CNF formula having m clauses in n variables $x_1, x_2, \dots, x_n$, the output is YES if there is an assignment to the variables such that exactly $m - 2$ clauses are satisfied, and NO otherwise. Prove that SAT' is also NP-Complete.

3. (20 pts) Given a graph G=(V,E) and two integers k, m, the *Dense Subgraph Problem* is to find a subset V' of V, whose size is at most k and are connected by at least m edges. Prove that the *Dense Subgraph Problem is* NP-Complete.

Ungraded

4. (20 pts) (Modified from Textbook 8.16) Consider the problem of reasoning about the identity of a set from the size of its intersections with other sets. You are given a finite set U of size n, and a collection $A_1 \dots A_m$ of subsets of U. You are also given numbers $c_1 \dots c_m$, and numbers $d_1 \dots d_m$. The question is:

   Does there exist a set $X \subset U$ so that for each i = 1…m, the cardinality of $X \cap A$ is larger than c_i but smaller than d_i? We will call this an instance of the *Intersection Inference Problem, with* input U, $\{A_i\}$, and $\{c_i\}$, $\{d_i\}$. Prove that Intersection Inference is NP-complete.

5. (20 pts) (Textbook 8.28) The following is a version of the independent Set Problem. you are given a graph G = (V, E) and an integer k. For this problem, we will call a set $I \subset V$ *strongly independent if,* for any two nodes v, u ∈ I, the edge (v, u) does not belong to E, and there is also no path of two edges from u to v, that is, there is no node w such that both (u, w) ∈ E and (w, v) ∈ E. The Strongly independent Set Problem is to decide whether G has a strongly independent set of size at least k.

   Prove that the Strongly independent Set Problem is NP-complete.