Matrix Chain Multiplication

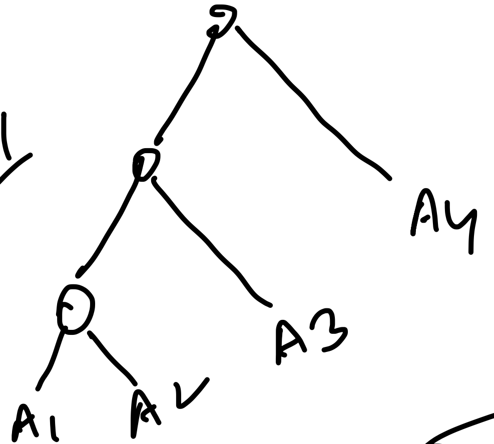
$$A_1 \cdot A_2 \cdot A_3 \cdot A_4$$
$$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$$

possible ways

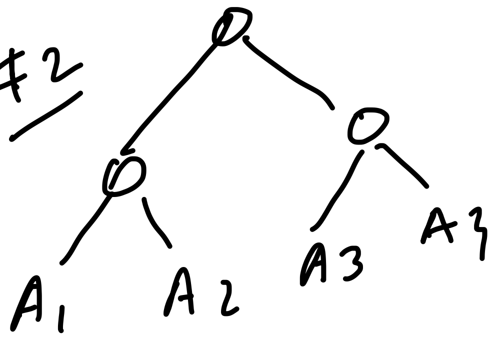
#1 $((A_1 \cdot A_2) \cdot A_3) \cdot A_4$

#2 $(A_1 \cdot A_2) \cdot (A_3 \cdot A_4)$

#1



#2



$$T(n) = \frac{{}^{2n}C_n}{(n+1)}$$ → no. of binary trees possible

no. of nodes

∴ for tree with 'n' nodes there are $T(n)$ different possible trees

$$T(3) = \frac{{}^{6}C_3}{4} = \frac{6!}{3!\,3! \times 4} = \frac{\cancel{6} \times 5 \times \cancel{4}}{\cancel{6} \times \cancel{4}}$$

$$\boxed{T(3) = 5}$$

∴ 1 way to find combination that gives minimum cost is to try out all combinations

Dynamic Programming approach.

$A_1 \cdot A_2 \cdot A_3 \cdot A_4$

$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$

m	1	2	3	4
1				
2				
3				
4				

s	1	2	3	4
1				
2				
3				
4				

m

	1	2	3	4
1	0	120	88	158
2		0	48	104
3			0	84
4				0

s

	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

Initialization

$m[1,1]$ A_1 $\rightarrow$ 0 — only 1 matrix, no multiplication cost

$m[2,2]$ A_2 $\rightarrow$ 0

$m[3,3]$ A_3 $\rightarrow$ 0

$m[4,4]$ A_4 $\rightarrow$ 0

$m[1,2]$ $\quad A_1 \cdot A_2 \rightarrow 5\times4\times6 = 120$

$5\times4 \quad 4\times6$

$m[2,3]$ $\quad A_2 \cdot A_3 \rightarrow 4\times6\times2 = 48$

$4\times6 \quad 6\times2$

$m[3,4]$ $\quad A_3 \cdot A_4 \rightarrow 6\times2\times7 = 84$

$6\times2 \quad 2\times7$

$m[1,3]$

$A_1 \cdot (\underline{A_2 \cdot A_3})$ OR $(A_1 \cdot A_2) \cdot A_3$

$5\times4 \quad 4\times6 \quad 6\times2$ | $5\times4 \quad 4\times6 \quad 6\times2$

$m[1,1] + m[2,3] + 5\times4\times2$ | $m[1,2] + m[3] + 5*6*2$

multiplication cost of $A_1 * A_2 \cdot A_3$

$= \underline{88}$ | $\underline{180}$

$m[2,4]$

$A_2 \cdot (A_3 \cdot A_4)$ | $(A_2 \cdot A_3) \cdot A_4$

252 | 104

$$m[1,4] = \min \begin{cases} m[1,1] + m[2,4] + 5*4*7, \\ m[1,2] + m[3,4] + 5*6*7, \\ m[1,3] + m[4,4] + 5*2*7 \end{cases}$$

min

$= 158$

5//

	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

position of split

$s(1,2)$ $\quad A_1 \cdot A_2$

$s(2,3)$ $\quad A_2 \cdot A_3$

$s(3,4)$ $\quad A_3 \cdot A_4$

$s(1,3)$ $\quad A_1 \cdot (A_2 \cdot A_3)$

$s(2,4)$ $\quad (A_2 \cdot A_3) \cdot A_4$

$s(1,4)$ $\quad (A_1 \; A_2 \; A_3)(A_4)$

S

	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

from S finding paranthesis

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4$$

From $s(1,4)$

$$(A_1 \cdot A_2 \cdot A_3) \cdot A_4$$

From $S(1,3)$

$$\boxed{(A_1 \cdot (A_2 \cdot A_3)) \cdot A_4}$$

This chain multiplication gives min value

Formula

$$M[i,j] = \min_{i \le k \le j} \left\{ \min[i,k] + m[k+1, j] + R_i C_k C_j \right\}$$

For $i = 1$ to n:
    $opt(i, i) = 0$

$O(n)$

For $j = 2$ to n:
    For $i = j-1$ to 1 dec by -1:

k

$O(n)$

Formula

$O(n)$

$$\boxed{O(n^3)}$$