

322 Coin Change problem

denominations = $\{1, 2, 5\}$

amount = 11

idea

$\text{opt}[i] \rightarrow$ min coins for amount i

$$\text{opt}[i] = \min \left[\begin{array}{l} \text{opt}[i-1] + 1, \\ \text{opt}[i-2] + 1, \\ \text{opt}[i-5] + 1 \end{array} \right] \quad \text{Rec Formula } \textcircled{1}$$

For $i = 5$ to n

use Rec Formula $\textcircled{1}$

But we have to initialize $\text{opt}[0], \dots, \text{opt}[5]$ to be able to use this formula which is not possible

Approach #2

We consider denomination by denomination

Say we only have 1 cent coins

$$\text{opt}[i] = \min(\text{opt}[i], \text{opt}[i-1] + 1)$$

$$\text{opt}[0] = 0$$

0	1	2	3	4	5	6	7	8	9	10	11
0	1	2	3	4	5	6	7	8	9	10	11

Now considering denomination 2

$$\text{opt}[i] = \min(\text{opt}[i], \text{opt}[i-2] + 1)$$

0	1	2	3	4	5	6	7	8	9	10	11
0	1	$\min(2, 0+1)$	$\min(3, 1+1)$	2	3	3	4	4	5	5	6
0	1	1	2	2	3	3	4	4	5	5	6

Now considering denomination 5

$$\text{opt}[i] = \min(\text{opt}[i], \text{opt}[i-5] + 1)$$

0	1	1	2	2	3	3	4	4	5	5	6
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0	1	1	2	2	1	2	2	3	3	2	3
0	1	2	3	4	5	6	7	8	9	10	11

return 3

Algo

For x in coins :

For y in $(x, \text{amount} + 1)$:

$\text{opt}[y] = \min(\text{opt}[y], \text{opt}[y - a] + 1)$

return $\text{opt}[\text{amount}]$