

# LC Oscillators

# LC Oscillators

Oscillators that uses LC network as Feedback:

- i. **Tuned Collector Oscillator**
- ii. **Colpitts Oscillator**
- iii. **Clapp Oscillator**
- iv. **Hartley Oscillator**
- v. **Armstrong Oscillator**
- vi. **Crystal Oscillator**

# Tuned Collector Oscillator

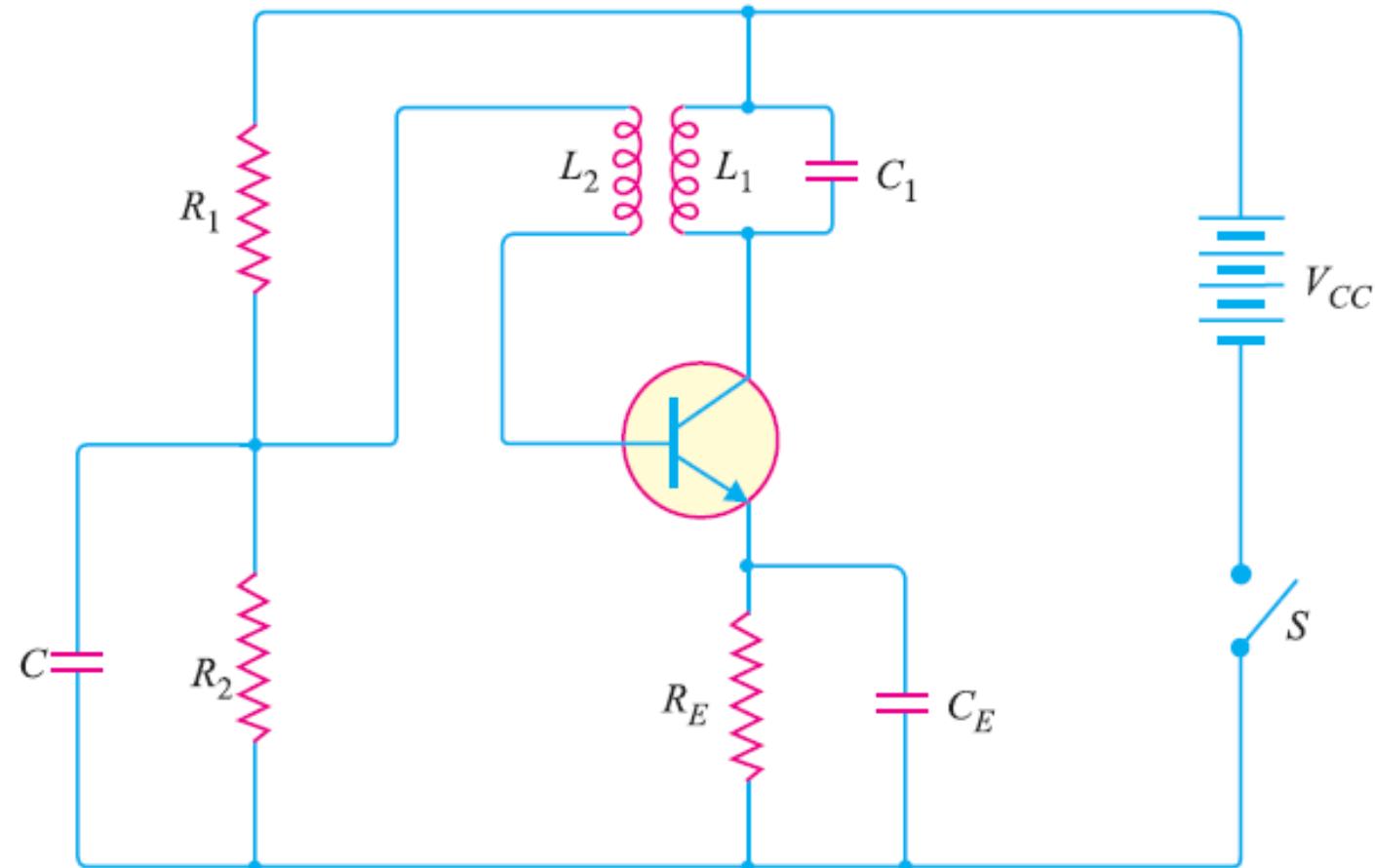
- It contains tuned circuit  $L_1-C_1$  in the collector and hence the name.

Frequency of Oscillation:

$$f = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

\*The LC circuit is often called **tuned circuit** or **tank circuit**.

\*Transformer provides **180° phase shift**.



# Circuit Operation

- The feedback coil  $L_2$  in the base circuit is magnetically coupled to the tank circuit coil  $L_1$ .
- In practice, **L1 and L2 form the primary and secondary** of the transformer respectively.
- The biasing is provided by potential divider arrangement. The capacitor  $C$  connected in the base circuit provides low reactance path to the oscillations.
- When switch  $S$  is closed, collector current starts increasing and **charges the capacitor  $C_1$** . When this capacitor is fully charged, it **discharges through coil  $L_1$** , setting up **oscillations of frequency**.
- These oscillations induce some voltage in coil  $L_2$  by mutual induction. The frequency of voltage in coil  $L_2$  is the same as that of tank circuit but its magnitude depends upon the number of turns of  $L_2$  and coupling between  $L_1$  and  $L_2$ .
- The voltage across  $L_2$  is applied between base and emitter and appears in the amplified form in the collector circuit, thus overcoming the losses occurring in the tank circuit.
- The number of turns of  $L_2$  and coupling between  $L_1$  and  $L_2$  are so adjusted that oscillations across  $L_2$  are amplified to a level just sufficient to supply losses to the tank circuit.

# Circuit Operation

- It may be noted that the phase of feedback is correct i.e. energy supplied to the tank circuit is in phase with the generated oscillations.
- A **phase shift** of  $180^\circ$  is created between the voltages of L1 and L2 due to **transformer \*action**.
- A further **phase shift** of  $180^\circ$  takes place between **base-emitter and collector** circuit due to transistor properties. As a result, the energy feedback to the tank circuit is in phase with the generated oscillations.

# Sample Problem #1

The tuned collector oscillator circuit used in the local oscillator of a radio receiver makes use of an LC tuned circuit with  $L_1 = 58.6 \mu\text{H}$  and  $C_1 = 300 \text{ pF}$ . Calculate the frequency of oscillations.

Ans.

1.2 MHz

## Sample Problem #2

Find the capacitance of the capacitor required to build an LC oscillator that uses an inductance of  $L_1 = 1 \text{ mH}$  to produce a sine wave of frequency 1000 GHz.

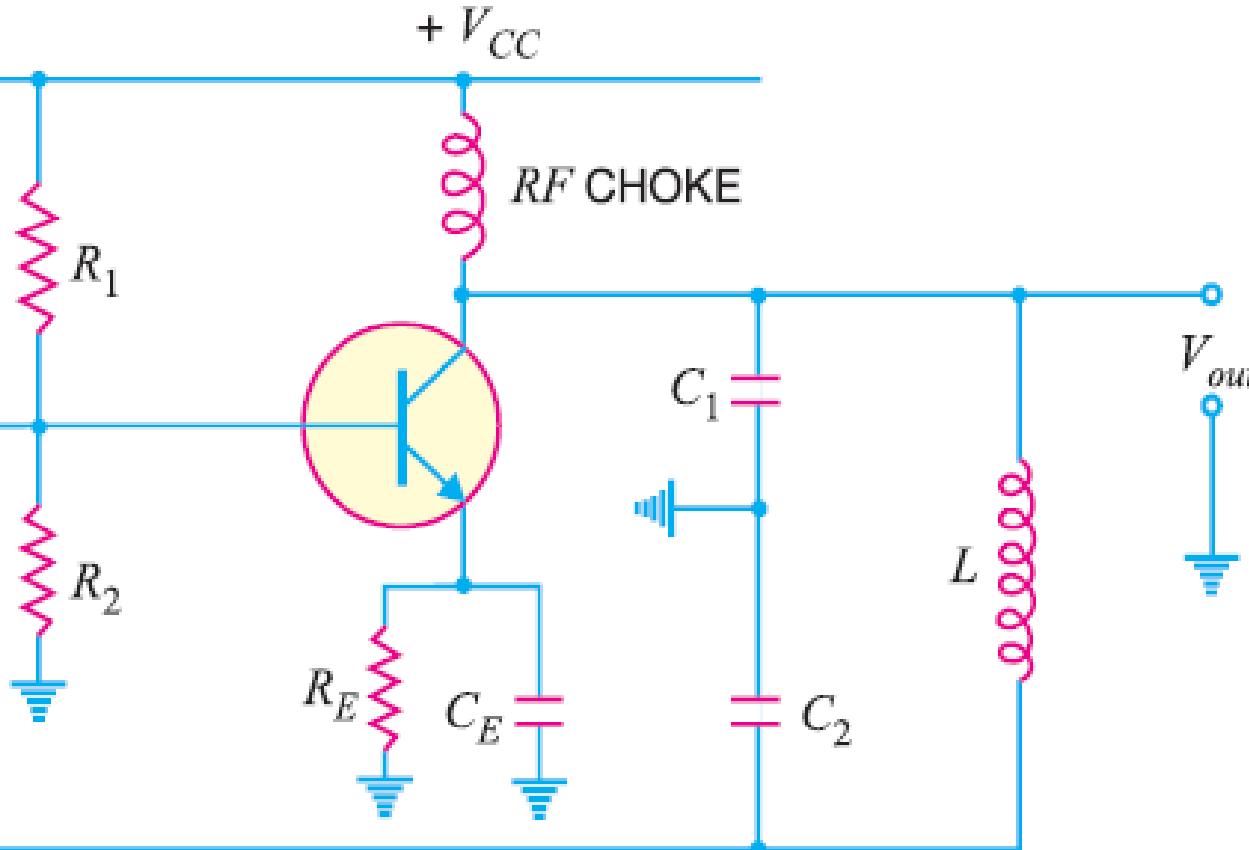
Ans.

$$25.33 \times 10^{-24} \text{ F}$$

# Colpitts Oscillator

- It is one basic types of resonant circuit feedback oscillators.
- Named after its inventor, **Edwin Henry Colpitts**.
- Uses an LC circuit in the feedback loop to provide the necessary **phase shift** and to act as a **resonant filter** that **passes only the desired frequency of oscillation**.
- It uses **two capacitors** and placed **across a common inductor L** and the center of the two capacitors is **tapped**.
- The tank circuit is made up of C1, C2 and L.

# Colpitts Oscillator



Frequency of Oscillation:

$$f = \frac{1}{2\pi\sqrt{LC_T}}$$

Where:

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

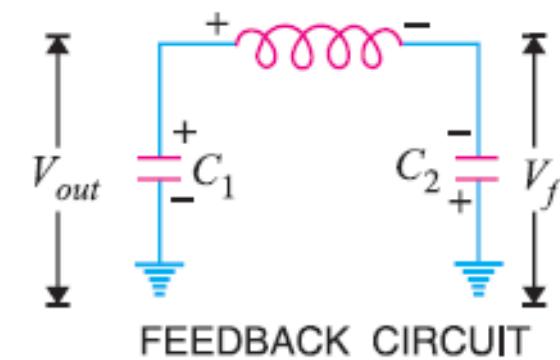
$C_T$  = total capacitance

\*Note that  $C_1 - C_2 - L$  is also the feedback circuit that produces a phase shift of  $180^\circ$ .

\*The RF choke decouples any ac signal on the power lines from affecting the output signal.

# Circuit Operation

- When the circuit is turned on, the capacitors  **$C_1$  and  $C_2$  are charged.**
- The capacitors discharge through  $L$ , setting up oscillations of frequency determined by the formula.
- The output voltage of the amplifier appears across  $C_1$  and feedback voltage is developed across  $C_2$ .
- The voltage across it is  $180^\circ$  out of phase with the voltage developed across  $C_1$  ( $V_{out}$ ) as shown in figure.
- It is easy to see that voltage feedback (voltage across  $C_2$ ) to the transistor provides positive feedback.
- A phase shift of  $180^\circ$  is produced by the transistor and a further phase shift of  $180^\circ$  is produced by  $C_1 - C_2$  voltage divider.
- In this way, feedback is properly phased to produce continuous **undamped** oscillation.



# Feedback Fraction ( $\beta$ )

The amount of feedback voltage in Colpitt's oscillator depends upon feedback fraction  $\beta$  of the circuit.

Feedback fraction ( $\beta$ ):

$$\beta = \frac{V_f}{V_{out}} = \frac{X_{c2}}{X_{c1}} = \frac{C_1}{C_2}$$

Or

$$\beta = \frac{C_1}{C_2}$$

# From Module

$$B = \frac{V_f}{V_{out}} \cong \frac{IX_{C1}}{IX_{C2}} = \frac{X_{C1}}{X_{C2}} = \frac{1/(2\pi f_r C_1)}{1/(2\pi f_r C_2)}$$

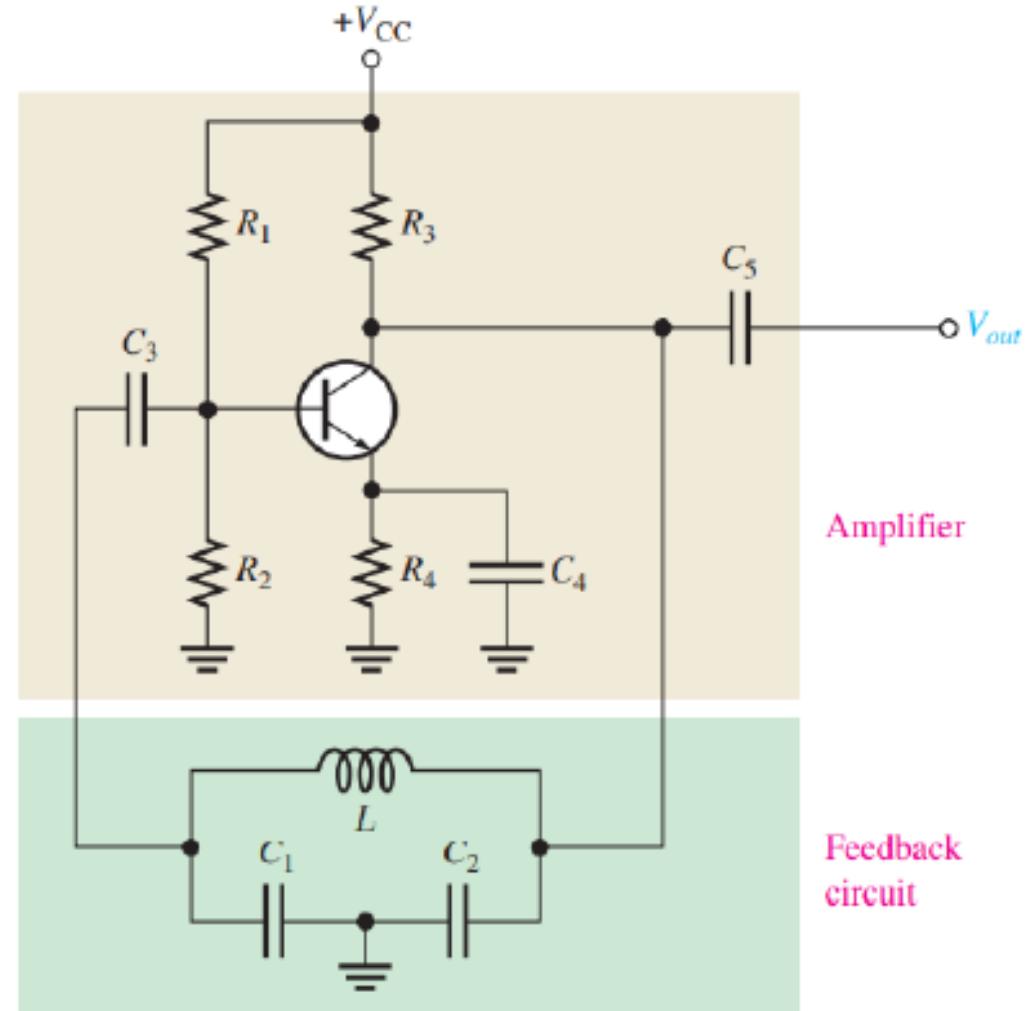
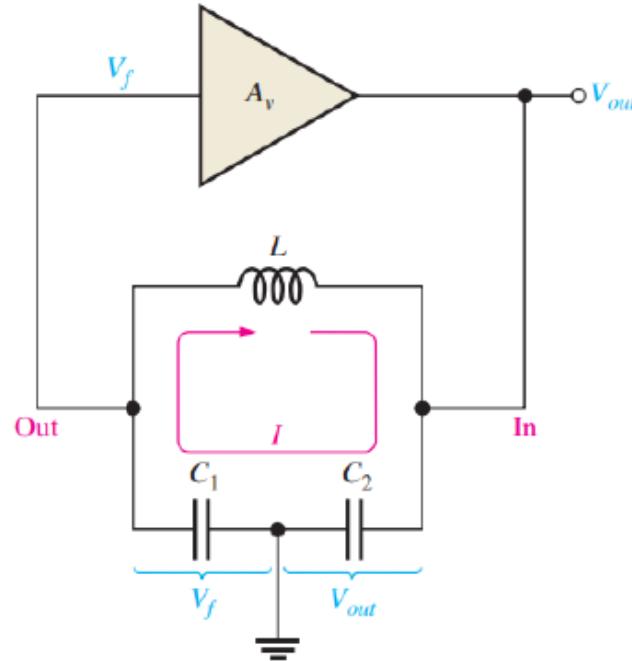
Cancelling the  $2\pi f_r$  terms gives

$$B = \frac{C_2}{C_1}$$

As you know, a condition for oscillation is Since  $A_V B = 1$ . Since  $B = C_2/C_1$

$$A_V = \frac{C_1}{C_2}$$

$$A_V > \frac{C_1}{C_2}$$

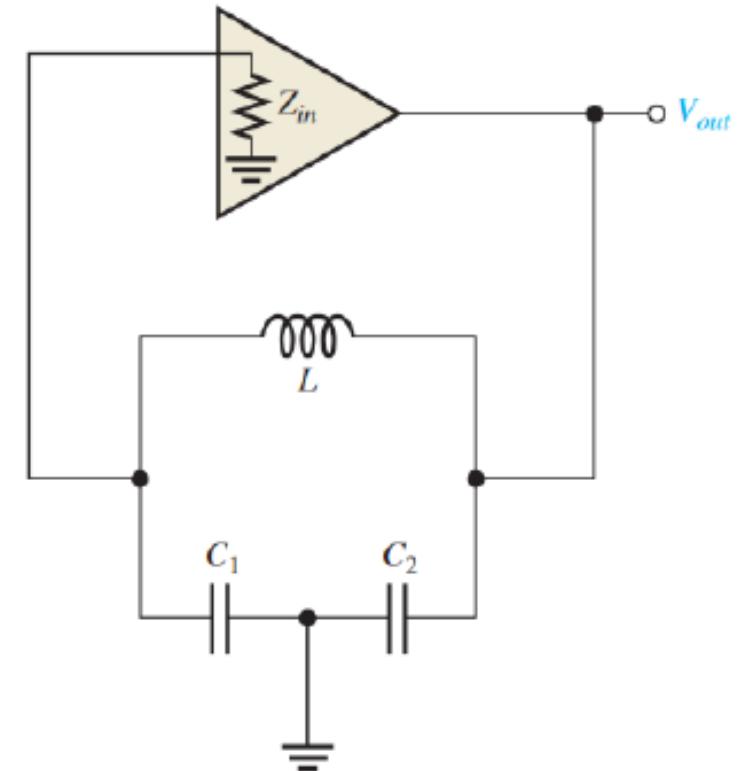


# Loading of the Feedback Circuit Affects the Frequency of Oscillation

The input impedance of the amplifier acts as a load on the resonant feedback circuit and reduces the Q of the circuit.

The resonant frequency of a parallel resonant circuit depends on the Q, according to the following formula:

$$f_r = \frac{1}{2\pi\sqrt{LC_T}} \left( \sqrt{\frac{Q^2}{Q^2 + 1}} \right)$$



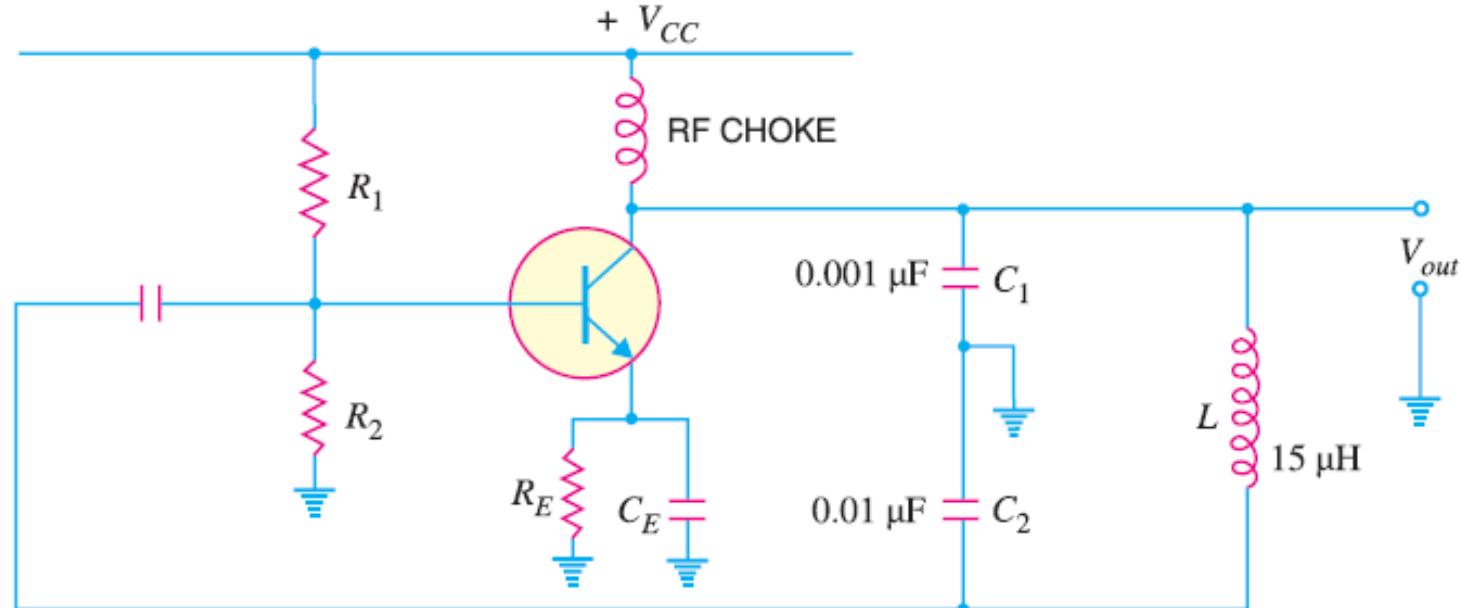
# Problem #3

Determine the

- (i) Operating frequency and
- (ii) Feedback fraction for Colpitt's oscillator figure.

Ans.

- (i) 1.363 MHz
- (ii) 1/10 or 0.1



## Problem #4

A 1 mH inductor is available. Choose the capacitor values in a Colpitts oscillator so that  $f = 1 \text{ MHz}$  and  $\beta = 0.25$ .

Ans.

$$C_1 = 31.66 \text{ pF}$$

$$C_2 = 126.65 \text{ pF}$$

# Clapp Oscillator

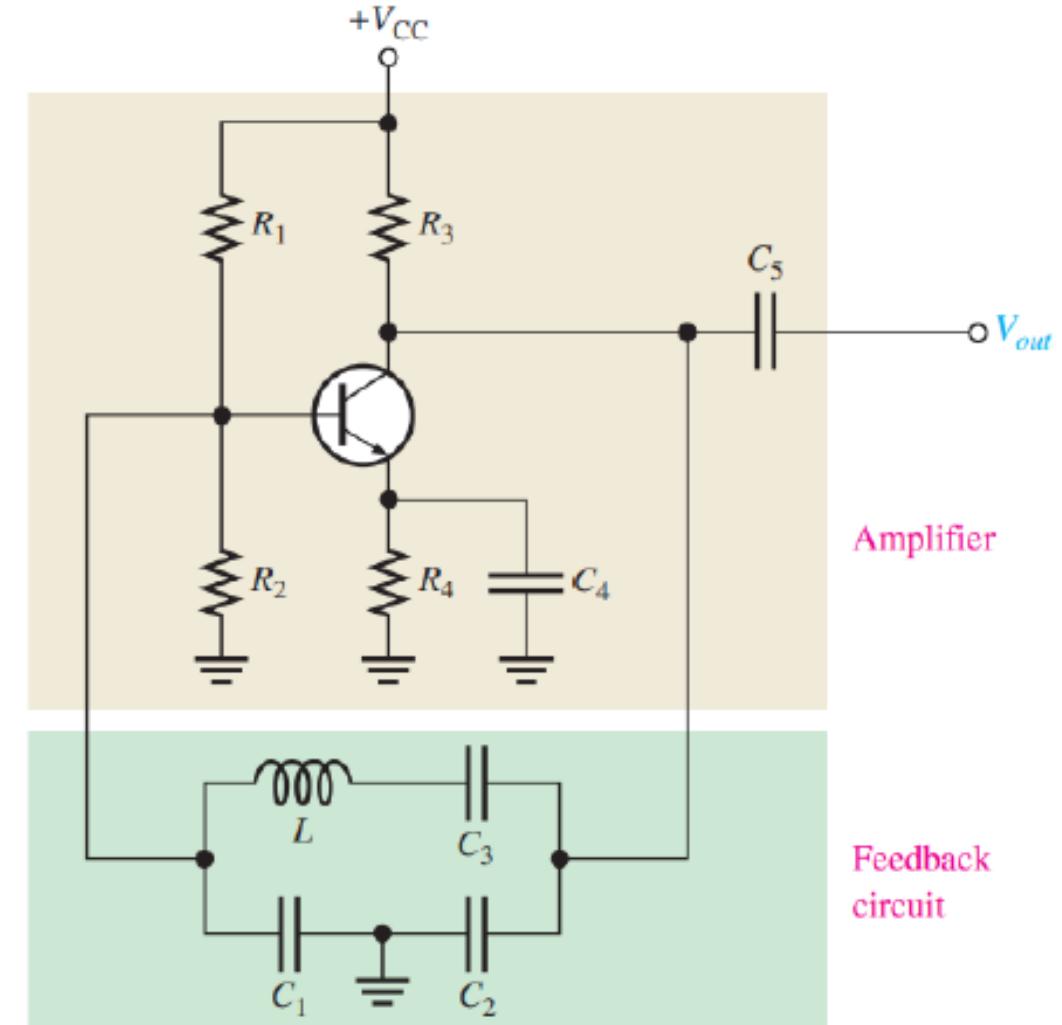
- By James Kilton Clapp
- The Clapp oscillator is a variation of the Colpitts
- The basic difference is an additional capacitor, **C<sub>3</sub>** in series with the inductor in the resonant feedback circuit.

Frequency of Oscillation: ( $Q > 10$ )

$$f \cong \frac{1}{2\pi\sqrt{L_1 C_T}}$$

Where:

$$C_T = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



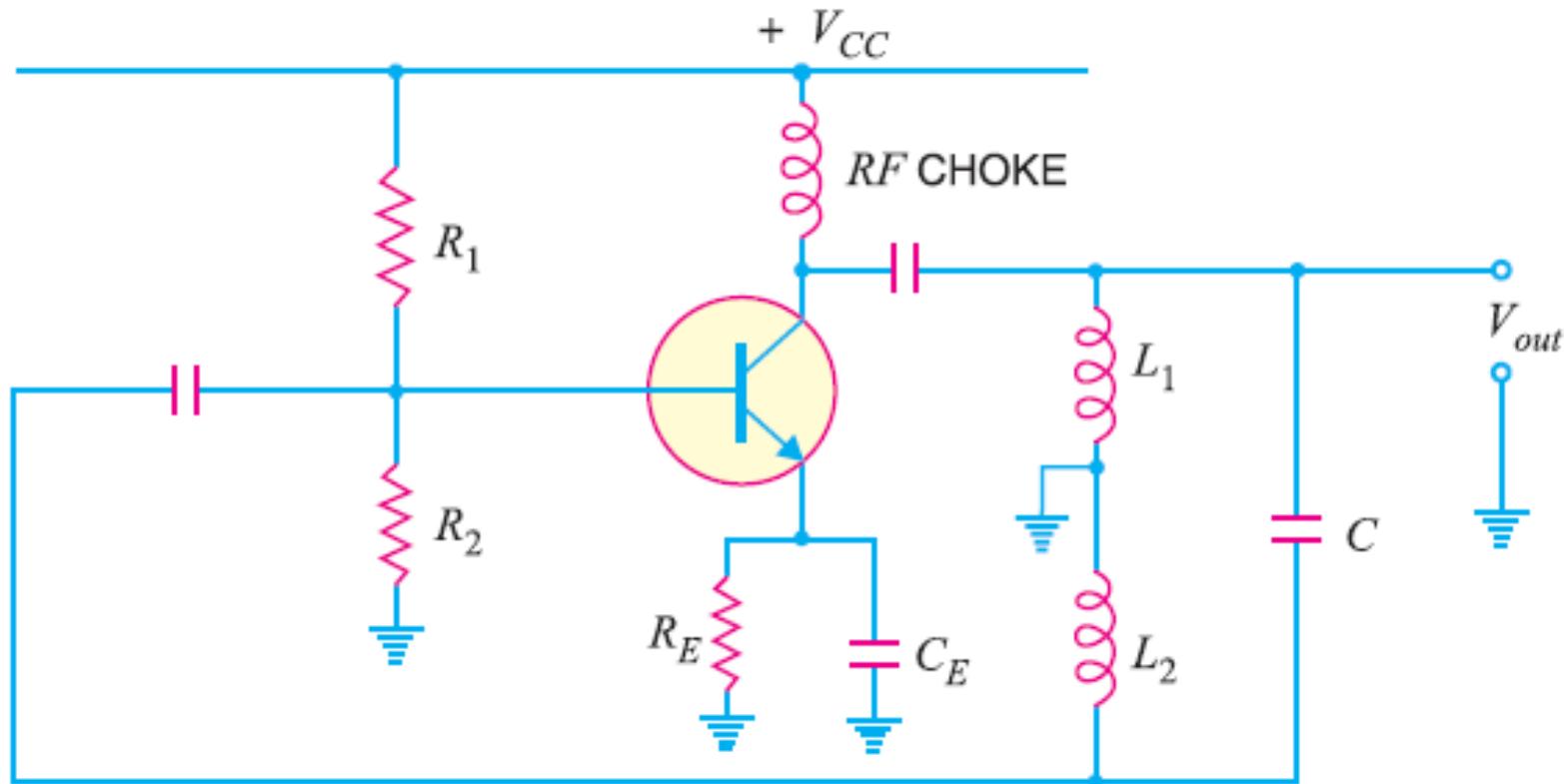
# Clapp Oscillator

- If  $C_3$  is much smaller than  $C_1$  and  $C_2$ , then  $C_3$  almost entirely controls the resonant frequency ( $f_r \approx 1/(2\pi\sqrt{LC_3})$ ).
- Since  $C_1$  and  $C_2$  are both connected to ground at one end, the junction capacitance of the transistor and other stray capacitances appear in parallel with  $C_1$  and  $C_2$  to ground, altering their effective values.
- $C_3$  is not affected, however, and thus provides a more accurate and stable frequency of oscillation.

# Hartley Oscillator

- The Hartley oscillator is similar to Colpitt's oscillator with minor modifications.
- Instead of using tapped capacitors, two inductors  $L_1$  and  $L_2$  are placed across a common capacitor  $C$  and the center of the inductors is tapped.
- It was named after **Ralph Hartley**.

# Hartley Oscillator



*Frequency of Oscillation:*

$$f = \frac{1}{2\pi\sqrt{L_T C}}$$

*Where:*

$$L_T = L_1 + L_2 \pm 2M$$

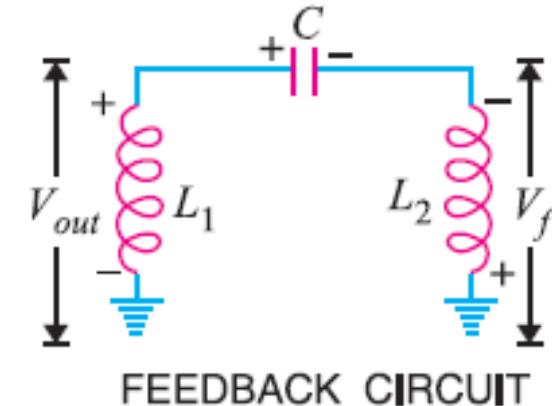
$M$  = mutual inductance

$C_T$  = total capacitance

\*Note that  $L_1 - L_2 - C$  is also the feedback circuit that produces a phase shift of  $180^\circ$ .

# Circuit Operation

- When the circuit is turned on, the **capacitor is charged**.
- When this capacitor is fully charged, it **discharges through coils  $L_1$  and  $L_2$  setting up oscillations of frequency** determined by the formula.
- The output voltage of the amplifier appears across  $L_1$  and feedback voltage across  $L_2$ .
- The voltage across  $L_2$  is **180° out of phase** with the voltage developed across  $L_1$  ( $V_{out}$ ) as shown in the figure.
- It is easy to see that voltage feedback (i.e., voltage across  $L_2$ ) to the transistor provides positive feedback.
- A phase shift of 180° is produced by the transistor and a further phase shift of 180° is produced by  $L_1 - L_2$  voltage divider.
- In this way, feedback is properly phased to produce continuous undamped oscillations.



# Feedback Fraction ( $\beta$ )

In Hartley oscillator, the feedback voltage is across L2 and output voltage is across L1.

Feedback fraction ( $\beta$ ):

$$\beta = \frac{V_f}{V_{out}} = \frac{X_{L2}}{X_{L1}} = \frac{L_2}{L_1}$$

Or

$$\beta = \frac{L_2}{L_1}$$

# From Module

In this circuit, the frequency of oscillation for  $Q > 10$  is

$$f_r \cong \frac{1}{2\pi\sqrt{L_T C}}$$

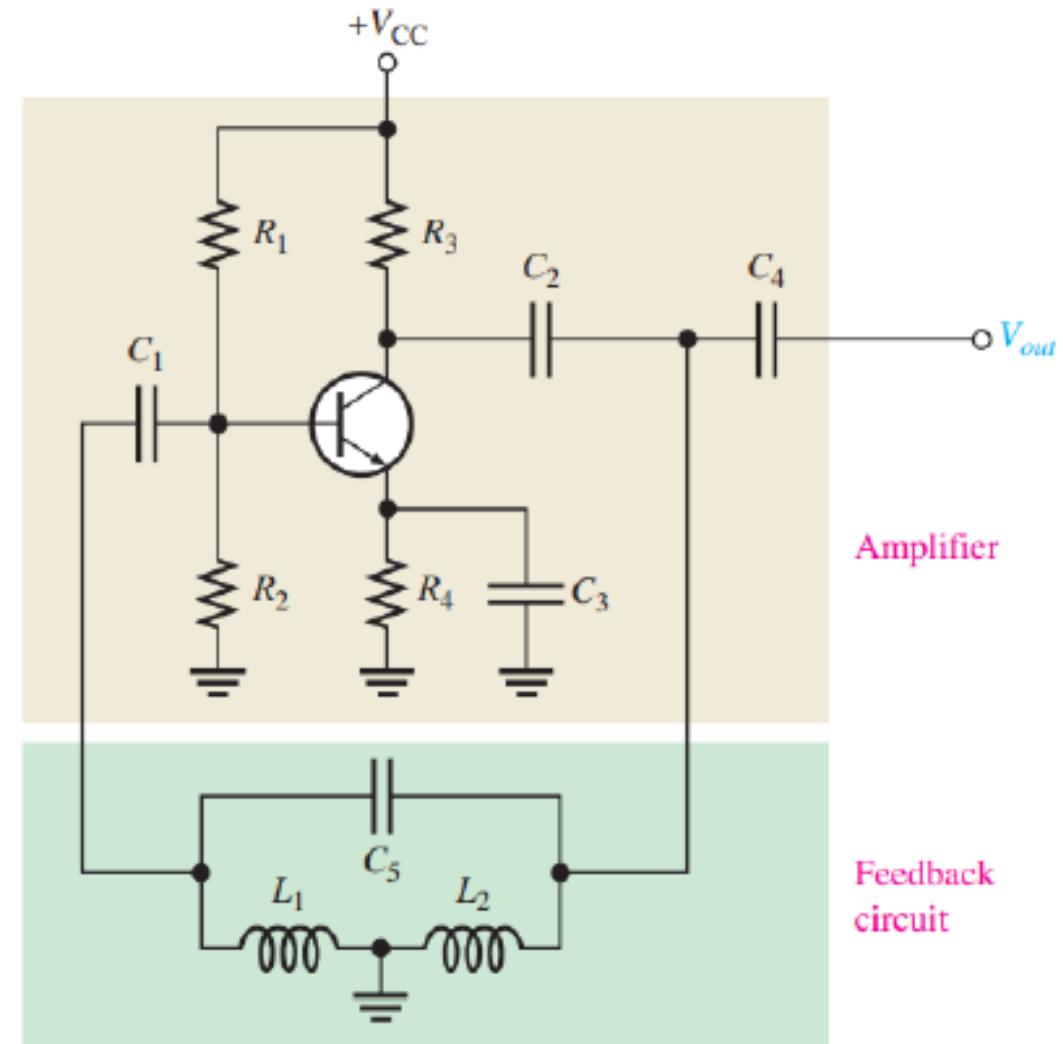
where  $L_T = L_1 + L_2$ . The inductors act in a role similar to  $C_1$  and  $C_2$  in the Colpitts to determine the attenuation,  $B$ , of the feedback circuit.

$$B \cong \frac{L_1}{L_2}$$

To assure start-up of oscillation,  $A$  must be greater than  $1/B$ .

$$A_v \cong \frac{L_2}{L_1}$$

Loading of the tank circuit has the same effect in the Hartley as in the Colpitts; that is, the  $Q$  is decreased and thus  $f_r$  decreases.



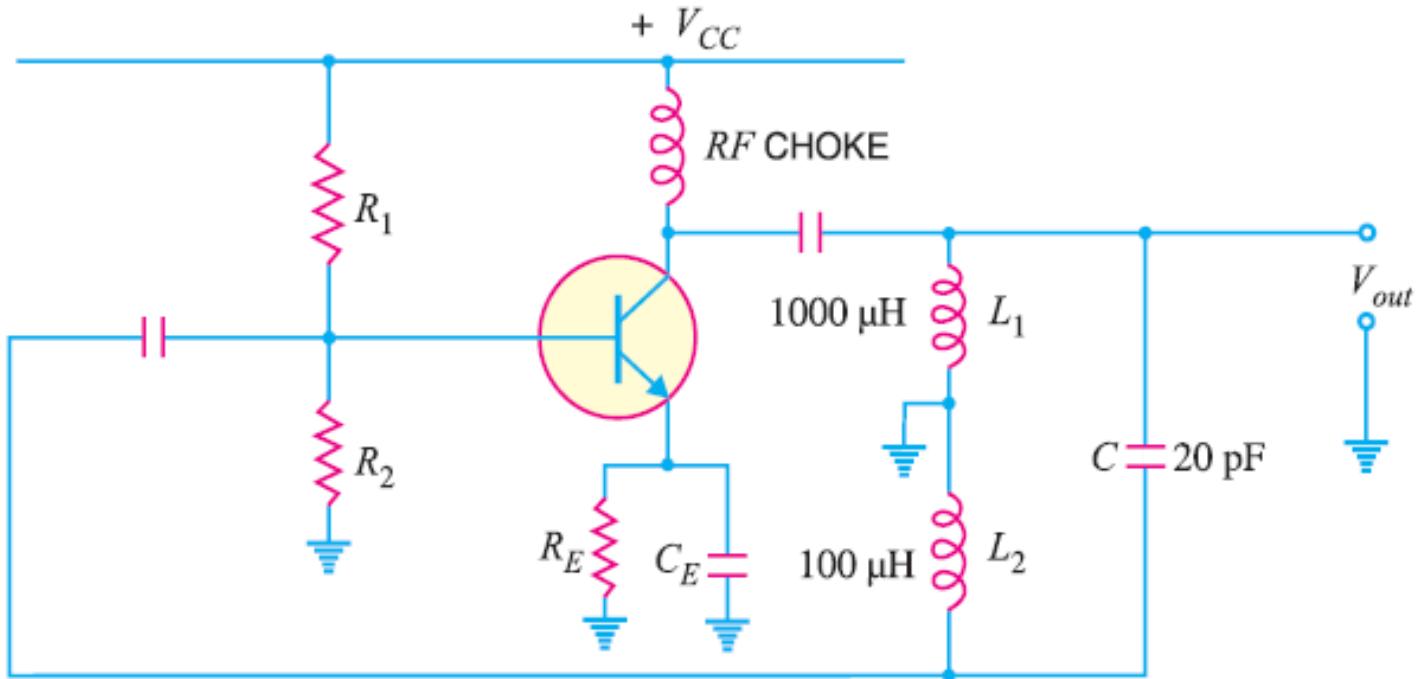
# Problem #5

Calculate the

- (i) Operating frequency and
- (ii) Feedback fraction for Hartley oscillator shown in the figure. The mutual inductance between the coils,  $M = 20 \mu\text{H}$ . (Assume aiding inductance)

Ans.

- (i) 1.054 MHz
- (ii) 1/10 or 0.1



## Problem #6

A 1 pF capacitor is available. Choose the inductor values in a Hartley oscillator so that  $f = 1 \text{ MHz}$  and  $\beta = 0.2$ .

Ans.

$$L_1 = 21.11 \text{ mH}$$

$$L_2 = 4.22 \text{ mH}$$

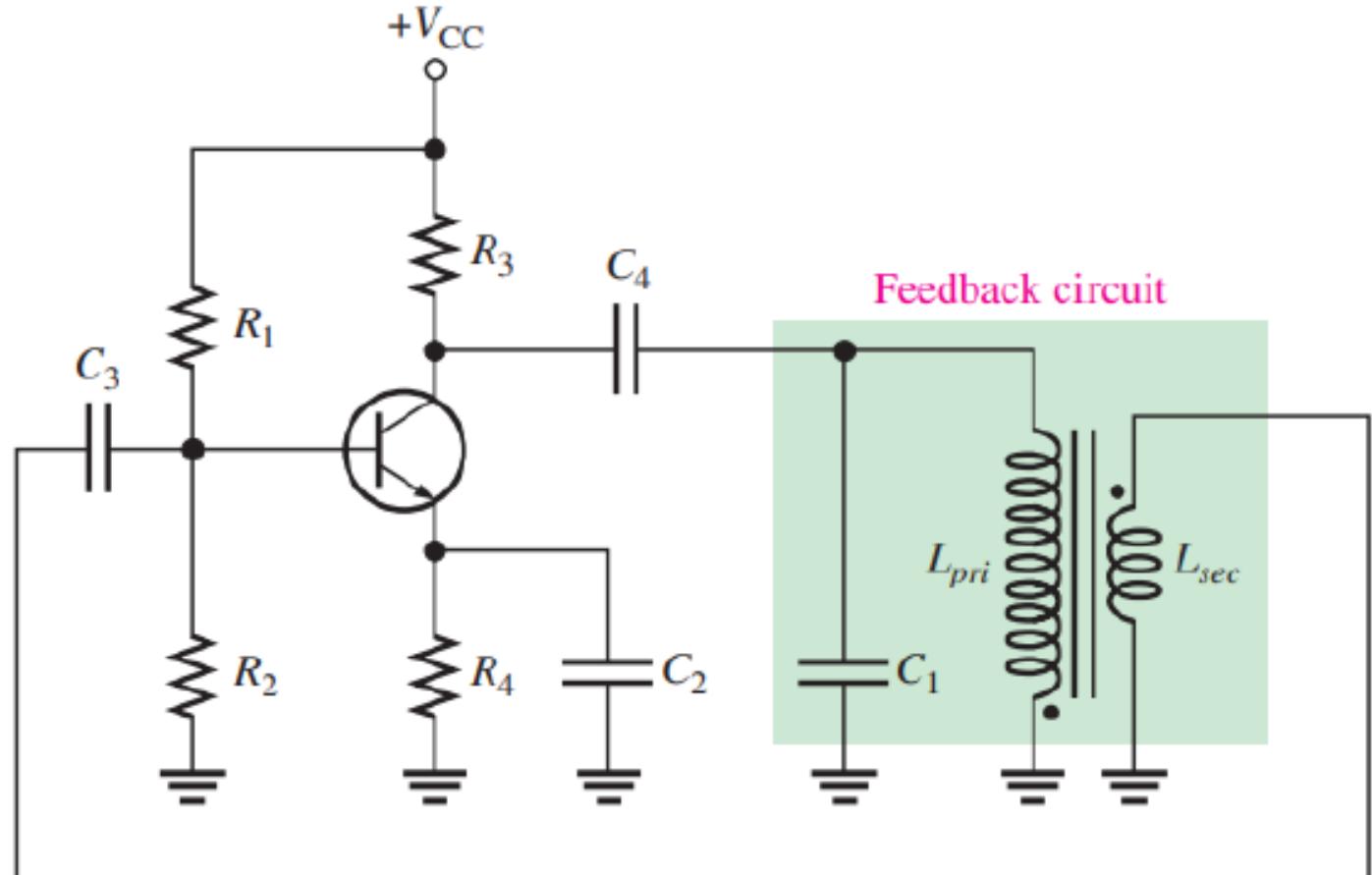
# Armstrong Oscillator

- This type of LC feedback oscillator uses transformer coupling to feed back a portion of the signal voltage.
- It is sometimes called a “**tickler**” oscillator in reference to the transformer secondary or “**tickler coil**” that provides the feedback to keep the oscillation going.
- The Armstrong is **less common** than the Colpitts, Clapp, and Hartley, mainly because of the **disadvantage** of **transformer size and cost**.
- The frequency of oscillation is set by the inductance of the primary winding (***L<sub>pri</sub>***) in parallel with ***C<sub>1</sub>***.

# Armstrong Oscillator

Frequency of Oscillation:

$$f = \frac{1}{2\pi\sqrt{L_T C}}$$

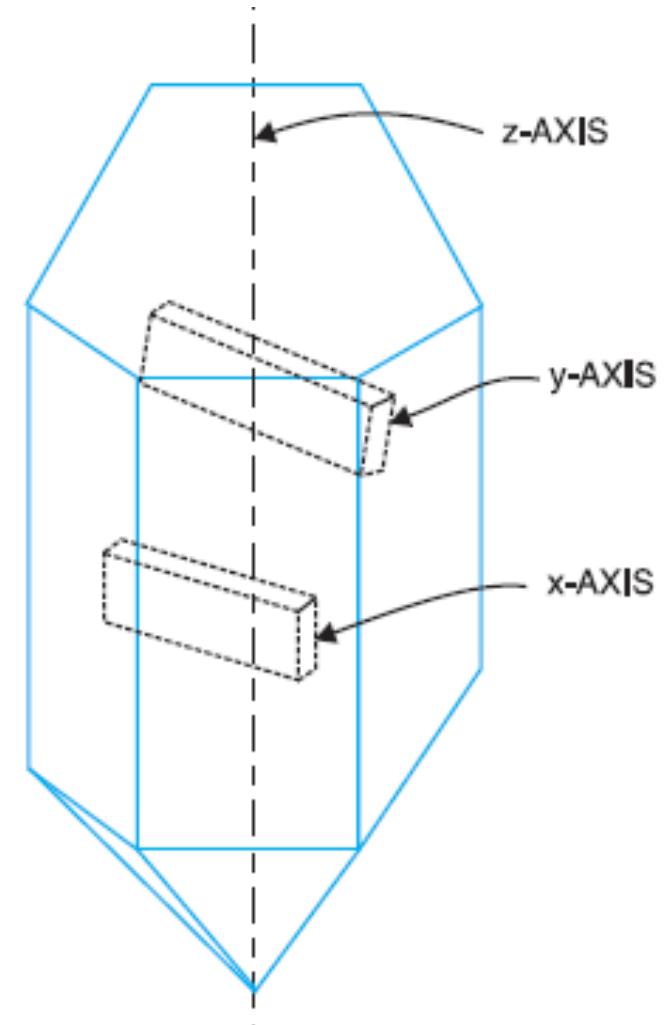


# Crystal-Controlled Oscillator

- Certain crystalline materials, namely, **Rochelle salt, quartz** and **tourmaline** exhibit the *piezoelectric effect* i.e., when we apply an a.c. voltage across them, they vibrate at the frequency of the applied voltage.
- Conversely, when they are compressed or placed under mechanical strain to vibrate, they produce an a.c. voltage.
- Such crystals which exhibit piezoelectric effect are called **piezoelectric crystals**.
- Of the various piezoelectric crystals, **quartz** is most commonly used because it is **inexpensive** and **readily available in nature**.

# Quartz Crystal

- Quartz crystals are generally used in crystal oscillators because of their great mechanical strength and simplicity of manufacture.
- The natural shape of quartz crystal is **hexagonal** as shown in the figure.
- The three axes are shown : the **z-axis** is called the **optical axis**, the **x-axis** is called the **electrical axis** and **y-axis** is called the **mechanical axis**.
- Quartz crystal can be cut in different ways.
- Crystal cut **perpendicular to the x-axis** is called **x-cut crystal** whereas that **cut perpendicular to y-axis** is called **y-cut crystal**. The piezoelectric properties of a crystal depend upon its cut.



# Frequency of Crystal

Each crystal has a natural frequency like a pendulum.

The natural frequency  $f$  of a crystal is given by :

$$f = \frac{K}{t}$$

Where

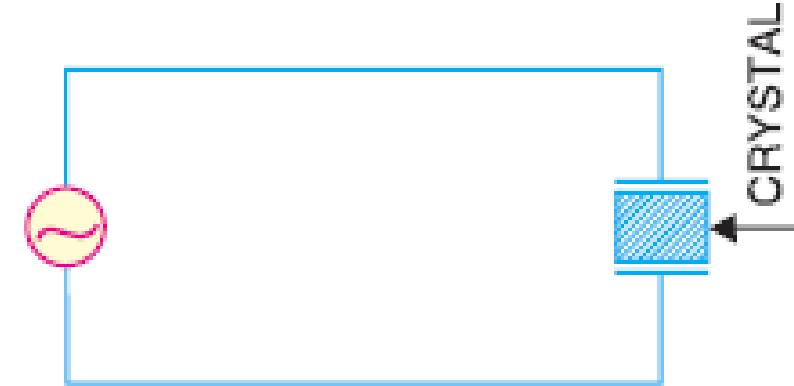
$K$  is a constant that depends upon the cut and

$t$  is the thickness of the crystal.

- It is clear that **frequency is inversely proportional to crystal thickness**.
- The thinner the crystal, the greater is its natural frequency and *vice-versa*.
- However, extremely thin crystal may break because of vibrations. This puts a limit to the frequency obtainable.
- In practice, frequencies between  $25\text{ kHz}$  to  $5\text{ MHz}$  have been obtained with crystals.

# Working of Quartz Crystal

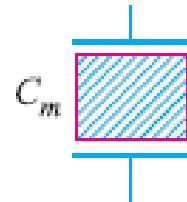
- In order to use crystal in an electronic circuit, it is placed between two metal plates.
- The arrangement then forms a capacitor with crystal as the dielectric as shown in the figure.
- If an a.c. voltage is applied across the plates, the crystal will start vibrating at the frequency of applied voltage.
- However, if the frequency of the applied voltage is made equal to the natural frequency of the crystal, resonance takes place and crystal vibrations reach a maximum value.
- This natural frequency is almost constant.
- Effects of temperature change can be eliminated by mounting the crystal in a temperature-controlled oven as in radio and television transmitters.



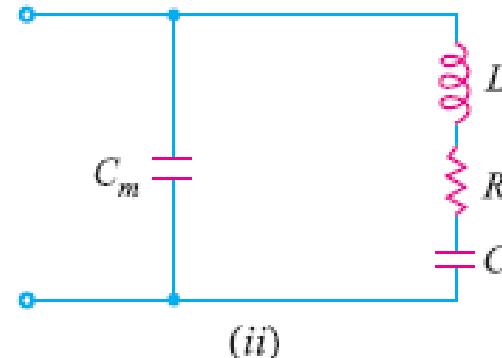
# Equivalent Circuit of Crystal

Although the crystal has electromechanical resonance, we can represent the crystal action by an equivalent electrical circuit.

- i. When the crystal is **not vibrating**, it is equivalent to capacitance  $C_m$  because it has two metal plates separated by a **dielectric**. This capacitance is known as *mounting capacitance*.
- ii. When a crystal **vibrates**, it is equivalent to **R – L – C series circuit**. Therefore, the equivalent circuit of a vibrating crystal is **R – L – C series circuit shunted by the mounting capacitance  $C_m$** .

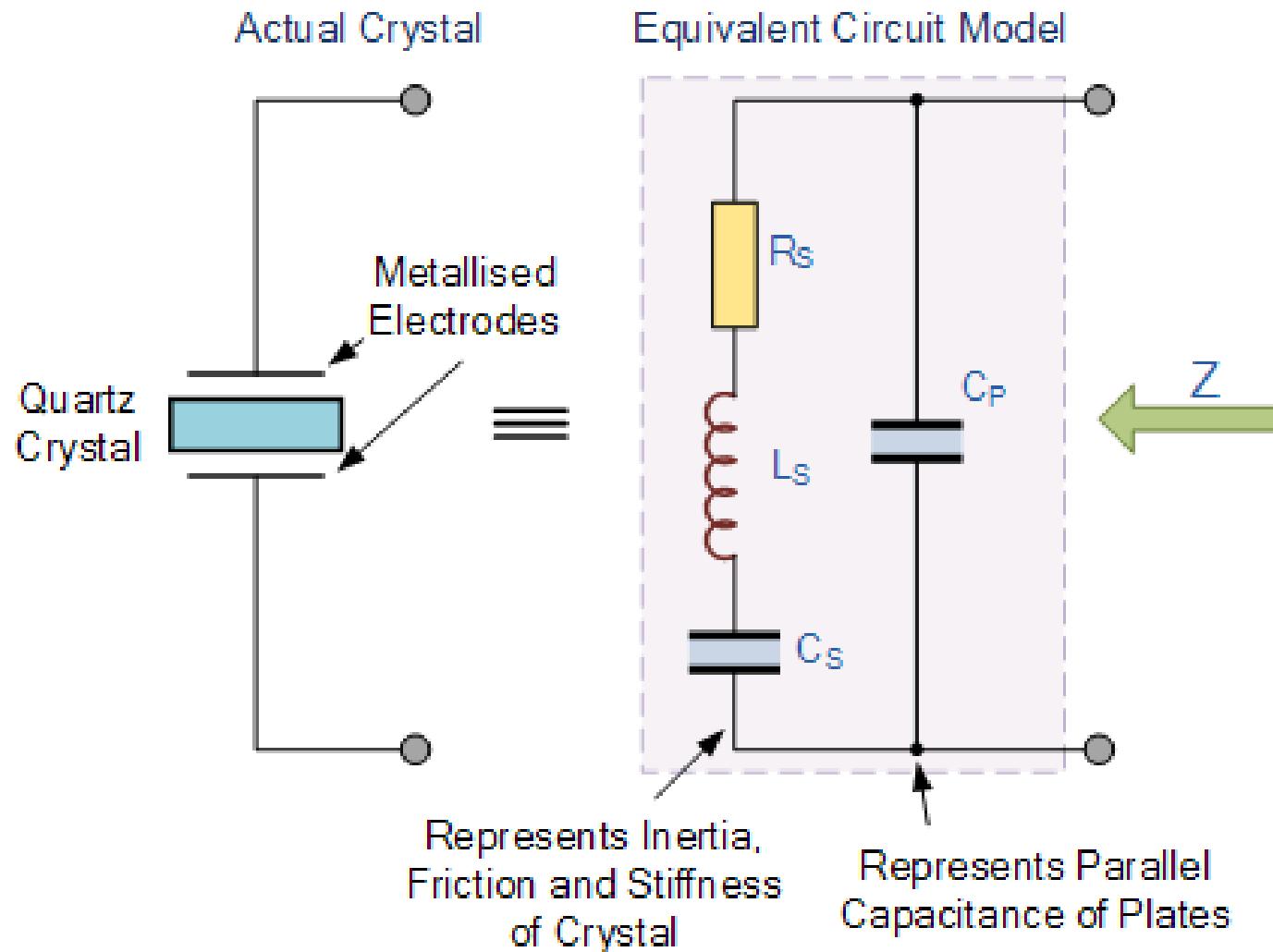


(i)



(ii)

# Equivalent Circuit of Crystal



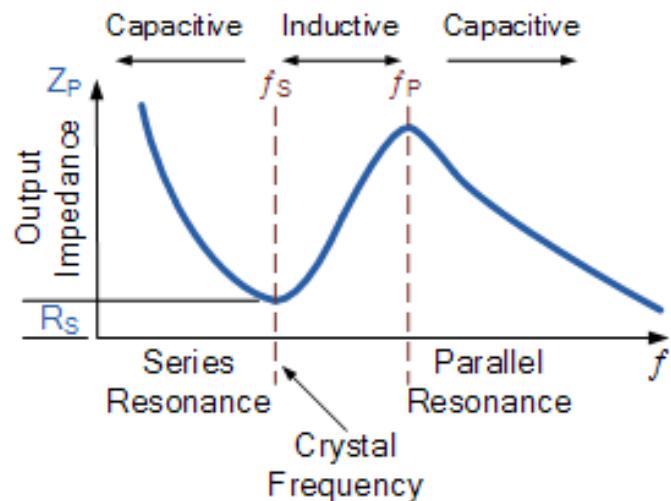
# Frequency of Oscillation

- Quartz crystal oscillators tend to operate towards their “**series resonance**”.
- The equivalent impedance of the crystal has a series resonance where  $C_s$  resonates with inductance,  $L_s$  at the crystals operating frequency. This frequency is called the crystals **series frequency**,  $f_s$ .
- There is a **second frequency** point established as a result of the parallel resonance created when  $L_s$  and  $C_s$  resonates with the parallel capacitor  $C_p$ .

# Frequency of Oscillation

- Crystal Impedance against Frequency

$$R = R_s \text{ and } X_{LS} = 2\pi f L_s$$



$$X_{CS} = \frac{1}{2\pi f C_s} \text{ and } X_{CP} = \frac{1}{2\pi f C_p}$$

$$Z_s = \sqrt{R_s^2 + (X_{LS} - X_{CS})^2}$$

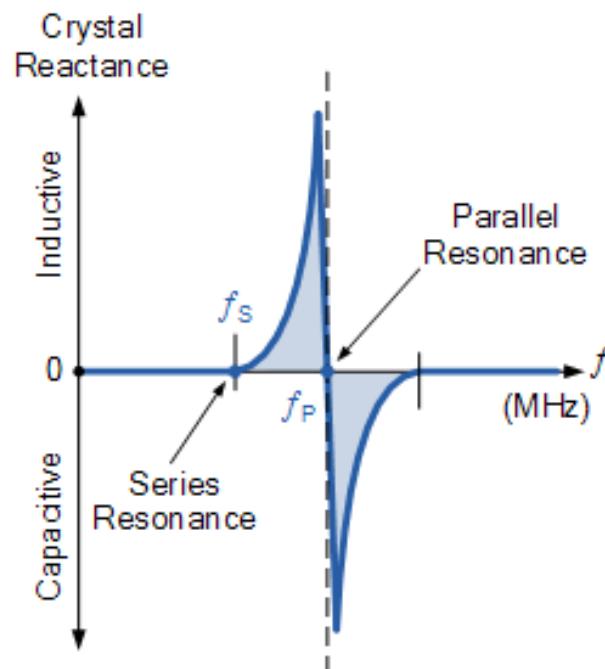
$$\therefore Z_p = \frac{Z_s \times X_{CP}}{Z_s + X_{CP}}$$

As the **frequency increases** above the series resonance point, the crystal behaves like an **inductor** until the frequency reaches its parallel resonant frequency  $f_p$ .

At this frequency point the interaction between the series inductor,  $L_s$  and parallel capacitor,  $C_p$  creates a parallel **tuned LC tank circuit** and as such the impedance across the crystal reaches its **maximum value**.

# Frequency of Oscillation

- Crystal Reactance against Frequency



$$X_s = R^2 + (X_{LS} - X_{CS})^2$$

$$X_{CP} = \frac{-1}{2\pi f C_p}$$

$$X_p = \frac{X_s \times X_{CP}}{X_s + X_{CP}}$$

The slope of the reactance against frequency above, shows that the **series reactance at frequency  $f_s$  is inversely proportional to  $C_s$  because below  $f_s$  and above  $f_p$  the crystal appears capacitive.**  
Between frequencies  $f_s$  and  $f_p$ , the crystal appears inductive as the **two parallel capacitances cancel out**.

# Frequency of Oscillation

- Series Resonant Frequency

$$f_s = \frac{1}{2\pi\sqrt{L_s C_s}}$$

- Parallel Resonant Frequency

The parallel resonance frequency,  $f_p$  occurs when the reactance of the series LC leg equals the reactance of the parallel capacitor,  $C_p$  and is given as:

$$f_p = \frac{1}{2\pi\sqrt{L_s \left( \frac{C_s C_p}{C_s + C_p} \right)}}$$

## Problem #7

- A quartz crystal has the following values:  $R_s = 6.4\Omega$ ,  $C_s = 0.09972\text{pF}$  and  $L_s = 2.546\text{mH}$ . If the capacitance across its terminal,  $C_p$  is measured at  $28.68\text{pF}$ , Calculate the fundamental oscillating frequency of the crystal and its secondary resonance frequency.

Ans.

$$f_s = 9.988 \text{ MHz}$$

$$f_p = 10.006 \text{ MHz}$$

## Problem #8

- Calculate the Q-factor of the previous problem (#7).

Ans.

$Q = 24965$  or  $25000$

## Problem #9

- A quartz crystal has the following values after being cut,  $R_s = 1\text{k}\Omega$ ,  $C_s = 0.05\text{pF}$ ,  $L_s = 3\text{H}$  and  $C_p = 10\text{pF}$ . Calculate the crystals series and parallel oscillating frequencies.

Ans.

$$f_s = 410.94 \text{ kHz}$$

$$f_p = 411.96 \text{ kHz}$$