Linear Algebra: wk3 rref

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```
library(far)
```

```
## Loading required package: nlme

## far library : Modelization for Functional AutoRegressive processes

## version 0.6-4 (2014-12-07)
```

```
library(MASS)
library(pracma)
```

Concepts

Space, subspace

- Domain, codomain (Range, C($mathbbA$))

Domain, codomain, Range

- You will see the following notation from time to time

$$T : R^n \to R^m$$

- the above notation is saying that `matrix T will be used to multiply vector with size of n and the resulting vector will have size m`
- And we will get into the details later.
- Vector resize within a space which consist of so many subspaces.
- When you put vectors into a matrix, you get two space, I call them `input` and `output space`. Input space can be divided into `row space` and `nullspace`, and `output space` can be divided into `column space` and `left null space`
- Think of domain as `row space` and `codomain` as `output space` and `range` as `column space`

Rank nullity theorem

If $\mathbb{A}$ has n columns, then $\text{Rank}(\mathbb{A}) + \dim \text{Nul}(\mathbb{A}) = n$

- see page 156 for the invertible matrix theorem (continued)

Invertible Linear Transformation

- A linear transformation $\mathbb{T} : R^n \rightarrow R^n$ is said to be `invertible` if there exists a funciton $\mathbb{S} : R^n \rightarrow R^n$ such that

$$\mathbb{S}(\mathbb{T}(\vec{x})) = \vec{x} \text{ for all } \vec{x} \text{ in } R^n$$
$$\mathbb{T}(\mathbb{S}(\vec{x})) = \vec{x} \text{ for all } \vec{x} \text{ in } R^n$$

where $\dim(\mathbb{A})$ = n by n, $\vec{x}, \vec{b} \in R^n$ - $\mathbb{A}$ is the standard matrix for $\mathbb{T}$

$$\mathbb{A}\vec{x} = \vec{b}$$

- $\mathbb{A}^{-1}$ is the standard matrix for $\mathbb{S}$

$$\mathbb{A}^{-1}\vec{b} = \vec{x}$$

```
r1 <- c(0,1,-4)
r2 <- c(2,-3,2)
r3 <- c(5,-8,9)
A <- rbind(r1,r2,r3)
print(rref(A))
```

```
##    [,1] [,2] [,3]
## r1    1    0    0
## r2    0    1    0
## r3    0    0    1
```

- Above matrix is invertible matrix based on `rref()`
- Inverse transformation `undo` the transformation

```
x <- c(3,6,9)

#to use the same notation
T <- A
b<- T%*%x
print("Before the transformation")
```

```
## [1] "Before the transformation"
```

```
print(x)
```

```
## [1] 3 6 9
```

```r
print("After the transformaiton")
```

```
## [1] "After the transformaiton"
```

```r
print(T%*%x)
```

```
##    [,1]
## r1  -30
## r2    6
## r3   48
```

When $\mathbb{A}$ is not a square matrix

- With respect to $\mathbb{A}$, $\vec{b}$ is in your `range` and $\vec{x}$ is in `domain`.
- $\mathbb{A}$ transform vectors in domain to range.
- $\mathbb{A}^{-1}$ can transform values in range back to domain, when the $\mathbb{A}$ involved 1-1 transformation.

```r
#chapter 1.1 example 3

r1 <- c(0,3,-6,6,4,-5)
r2 <- c(3,-7,8,-5,8,9)
r3 <- c(3,-9,12,-9,6,15)

A <- rbind(r1,r2,r3)

rref(A)
```

```
##    [,1] [,2] [,3] [,4] [,5] [,6]
## r1    1    0   -2    3    0  -24
## r2    0    1   -2    2    0   -7
## r3    0    0    0    0    1    4
```

Exercise.

- Get 5 matrices from class
- Given a matrix and a vector
    - Show different ways of spanning the given vector