Hypothesis Testing

UC Berkeley, MIDS w203

Statistics for Data Science

February 18, 2022

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Introducing the Two-Sample t-Test

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Motivating the Two-Sample t-Test

Comparing Two Metric Variables

Is group A different from group B in expectation?

Examples:

- Are customers who get a birthday gift less likely to leave than those who don't?
- Do patients who take Vitamin W get over the flu faster than patients who don't?
- Do democracies or autocracies start more wars?

Motivating the Two-Sample t-Test
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• Do democracies or autocracies start more wars?

1. The two-sample t-test is one of the most common procedures in inferential statistics. What is it for? As a data scientist you will often face a question about two groups - group A and group B. Someone will ask you if one is greater or less than than the other.
2. Questions like this are everywhere. In business, you have A-B testing.
3. In medicine, you may want to test a group of patients taking a vitamin against a control group
4. In political science, you may want to test whether democracies or autocracies start more wars.
5. What do these questions have in common? They all have

Example Scenario

From the Journal of Empirical Fashion		
	New Yorkers	San Franciscans
black outfits	12.1	13.3
sample size	50	50

Is this evidence that San Franciscans have more black outfits than New Yorkers *in general*?

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Is this evidence that San Franciscans have more black outfits than New Yorkers *in general*?

1. Here's a specific example, with data.
2. write on slide: 13.3-12.1 = 1.2 outfits
3. Is this a big difference? is it just noise?
4. To be more precise, is this evidence that SF'ans have more black outfits than NYers in general?
5. To analyze this question, we need a model...

Two-Sample Model Framework

Basic Model Setup

Suppose $(X_1, .., X_{n_1})$ are i.i.d. with mean μ_X.

Suppose $(Y_1, .., Y_{n_2})$ are i.i.d. with mean μ_Y.

Null Hypothesis

$H_0 : \mu_X = \mu_Y$ (The two groups' means are equal)

Alternative hypotheses

$H_1 : \mu_X \neq \mu_Y$ (best choice in most cases)

$H_2 : \mu_X > \mu_Y$

$H_3 : \mu_X < \mu_Y$

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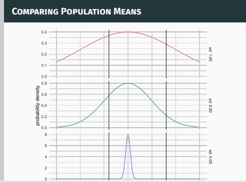
1. A model is a representation of the world built of RVs. for a two-sample t-test, the model begins like this.
2. Let $X_1..X_{n_1}$ be RVs representing group A. Similarly for Y.
3. Let's assume that the X's are iid with mean μ_X. same for Y
4. We'll need more assumptions, but this is a start.
5. The null is that our two means are the same.
6. Our usual alternative is that the means are different.
7. It is also possible to run a one-sided test in special circumstances. alt is one mean is greater than other.
8. Given this model, how plausible is the null hypothesis?

Comparing Population Means



1. One important piece of information is how much does the number of black outfits vary in the population.
2. Here's a picture with three different distributions for our sample averages.
3. The top distribution has very high standard deviation. You can see that it doesn't seem unusual to get two sample averages that are 1.2 outfits apart.
4. The middle distribution has medium deviation. Now it starts to look a little more surprising that our two sample averages were 1.2 apart.
5. The bottom distribution has low standard deviation. Now it seems quite unlikely to get two sample averages 1.2 apart. The data seems inconsistent with this hypothesis

Comparing Population Means

Two-Sample t-Test

$$t = \frac{\bar{X} - \bar{Y}}{\text{Estimate of Standard Deviation}}$$

Assess whether two populations have the same expectation **while accounting for variability**

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1. Putting these ideas together, we can make a statistic by taking the difference in sample averages, and dividing by an estimate of standard deviation - that’s exactly the idea behind the two- sample t-test

The Two-Sample z-Test

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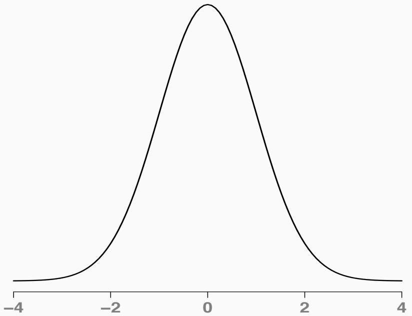
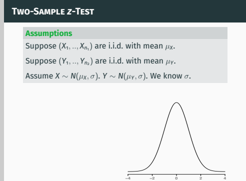
Assumptions

Suppose $(X_1, .., X_{n_1})$ are i.i.d. with mean μ_X.

Suppose $(Y_1, .., Y_{n_2})$ are i.i.d. with mean μ_Y.

Assume $X \sim N(\mu_X, \sigma)$. $Y \sim N(\mu_Y, \sigma)$. We know σ.





1. Before we tackle the two-sample t-test, let's begin with the simpler 2 samp. z test. unrealistic, but build intuition.
2. Same assumptions as before, but add assump. of equal var σ which we know.
3. How do we create a test statistic?
4. What is the distribution of $\bar{X} - \bar{Y}$? Let's use fact that a difference of normal RVs is normal. but which normal?
5. $V[\bar{X} - \bar{Y}] = V[\bar{X}] + V[\bar{Y}] = \sigma^2/n_1 + \sigma^2/n_2$
6. $\bar{X} - \bar{Y} \sim N(0, \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}})$
7. Can use the standardized difference in means. $z = \frac{\bar{X}-\bar{Y}}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim N(0, 1)$
8. Here's the standard normal. As usual, we can create

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Assume $\mathrm{V}[X] = \mathrm{V}[Y] = 4$.

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1. Here's an exercise to build more intuition.
2. $z = \frac{\bar{X}-\bar{Y}}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{12.1-13.3}{2\sqrt{\frac{1}{50}+\frac{1}{50}}} = \frac{-1.2}{2\sqrt{\frac{1}{25}}} = -3.0$
3. $z > 1.96$ REJECT
4. We can also compute p-value 2*pnorm(-3) = .0027

General Procedure for Testing

Three steps:

1. Specify model and null hypothesis
2. Calculate z or t statistic
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1. From this example, I hope you see that the two-sample t-test is not all that different from the one-sample tests we did earlier.
2. No matter what the test is, there's a general pattern that you follow...

The Two-Sample t-Test

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Types of Two-Sample t-Tests

1. Student's t-Test
2. Welch's t-Test

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1. There are actually two versions of the two-sample t test you should be aware of.
2. Student's t-test is the original t-test. it's simpler, but requires strong assumptions
3. Welch's t-test is more general, and this is really the modern t-test.

Student's t-Test

$$t = \frac{\mu_1 - \mu_2}{S_{\mu_1 - \mu_2}}$$

$$S_{\mu_1 - \mu_2} = \sqrt{\left(\frac{(N_1 - 1)S_1^2 + (N_2 - 1)S_2^2}{N_1 + N_2 - 2}\right)\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}$$

$$t = \frac{(mean\ of\ group\ 1) - (mean\ of\ group\ 2)}{standard\ error\ of\ difference\ between\ means}$$

t value

Difference between group means (mean difference), divided by the **variability of the two groups** (standard error of the differences)

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Degrees of Freedom

degrees of freedom *(df)*

Number of independent pieces of information that vary given model parameters

- **One sample t-test**
  - $df = n - 1$ (only uses one known quantity)
  - Tests whether one sample's mean is significantly different from some hypothesized mean
- **Student's two-sample t-test**
  - Uses two known quantities (the two group means)
  - $df = n_1 + n_2 - 2$

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Practical Significance of the T-Test

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After using a t-test to assess statistical significance, it is important to assess practical significance.

> *Your main goal is to explain to your audience why they should or should not care about the effect.*

Three common effect size measures:

1. Difference in means
2. Cohen's d
3. Correlation r

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$$\overline{X}_A - \overline{X}_B$$

- Answers the question “*How different are these groups?*”
- Often makes great headlines and is a good choice if units are familiar
- But lacks context in its calculation
- People who eat chocolate live 1.5 years longer than those who do not each chocolate

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Cohen's d

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Cohen's d is a measure of difference of means standardized by the variance in the data.

$$\frac{\overline{X}_A - \overline{X}_B}{s}$$

Where s is a pooled standard deviation: $\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2}}$

- Answers the question "*How many standard deviations apart are the groups?*"
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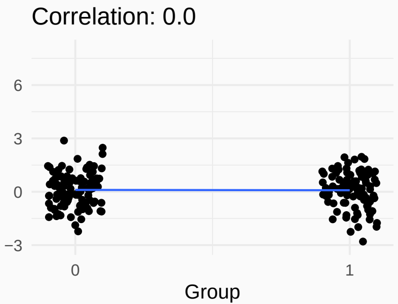
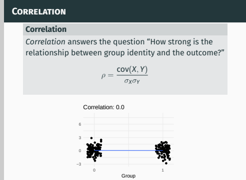
Correlation

Correlation answers the question “How strong is the relationship between group identity and the outcome?”

$$\rho = \frac{\mathrm{cov}(X, Y)}{\sigma_X \sigma_Y}$$





1. Notice the similarity in the form between Cohen’s d and correlation – Cohen’s d divides by the pooled standard deviation; correlation divides by the product of two group standard deviations.

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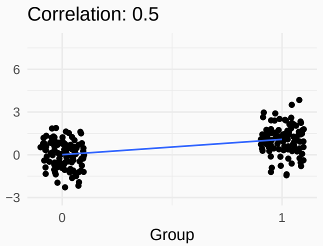

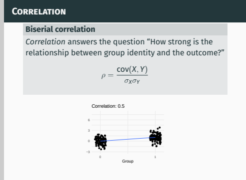

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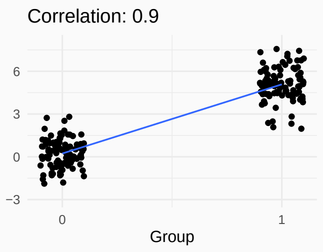

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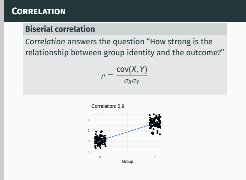

Practical Significance is about Context

- How strong is the same relationship between *different* groups?
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Paired t-Test

Climbing grip

Suppose you randomly sample 30 Berkeley students. For each student i, you measure right-hand strength (R_i) and left-hand strength (L_i).

- You conduct a t-test with $H_0 : \mathrm{E}[R] = \mathrm{E}[L]$
- **Problem**: Grip strength varies a lot person-to-person, $\Rightarrow$ t-test has low power.

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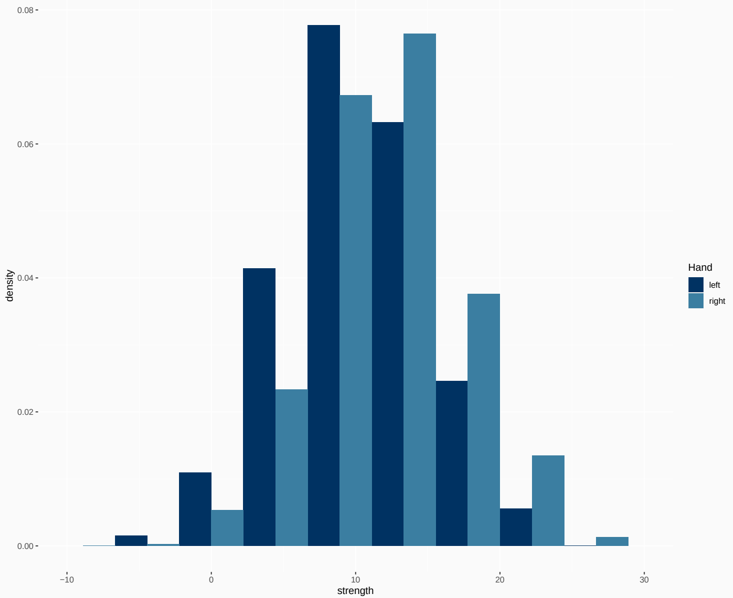

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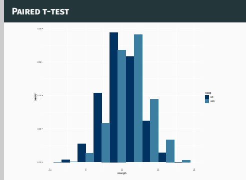

Paired t-Test

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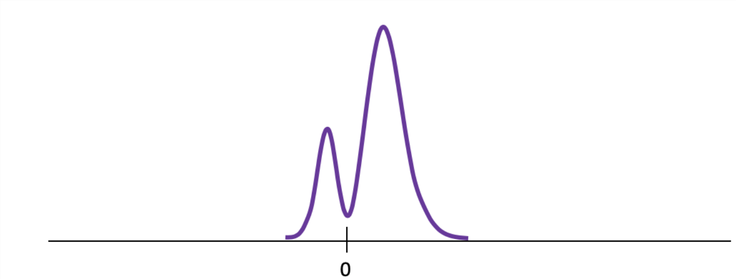
- Within-person variation is small.



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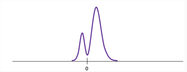

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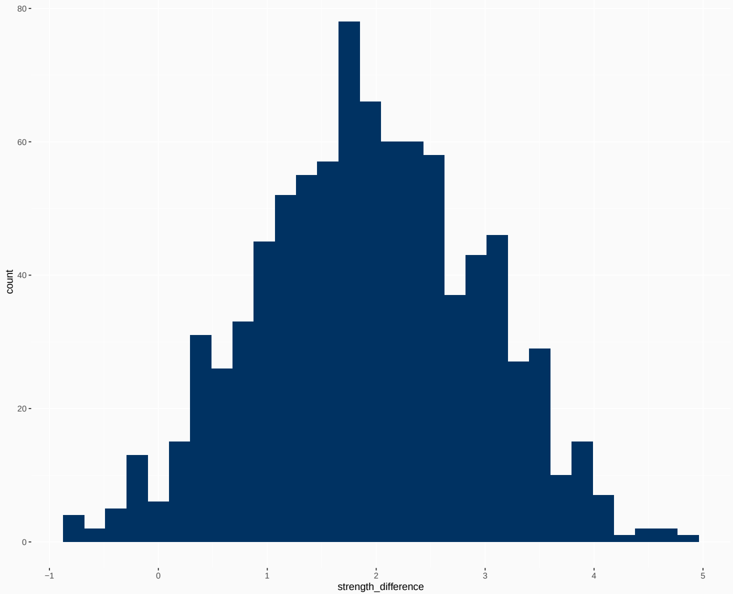

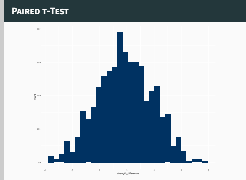

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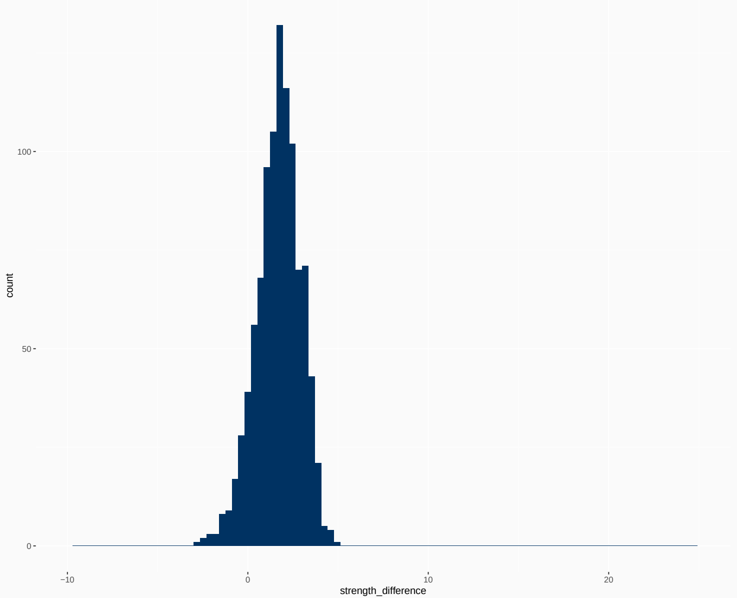

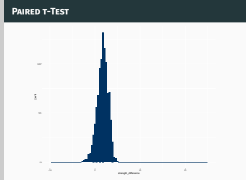

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Unpaired vs. Paired t-Test

Unpaired

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Paired

- $t = \frac{\overline{A}-\overline{B}}{\sigma_{(A-B)}}$

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Paired t-Test Assumptions

- A and B have a metric scale with the same units.
- There is a natural pairing between observations for A and for B.
    - pre-test and post-test for same individual
    - response to two types of stimulus for same mouse
    - responses for a pair of spouses
- Each pair (A_i, B_i) is drawn i.i.d.
- The distribution of $A - B$ is sufficiently normal given the sample size.

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Non-parametric Tests (cont.)

Large sample

- No Problem
- central limit theorem tells us that the sampling distribution of the mean will be approximately normal, so t-tests are valid
- Parametric tests are generally valid for large samples

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- *t*-test is fairly robust to deviations from normality, but you should look at your distribution and see how non-normal it is
- Suppose you have a small sample and you suspect you have a major deviation from normality
- You might be able to transform the variable to make it more normal, but that can alter the meaning and make results harder to interpret

An alternative is to use a *non-parametric* test

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Non-parametric Test Details

- Non-parametric tests can be also called **distribution- free tests**
  - Still involve assumptions, but they are less restrictive than those of parametric tests
- Many tests work on principle of ranking data
  - List the scores from lowest to highest – each score gets a rank, so higher scores have higher ranks
  - Only consider ranks instead of looking at the metric value of the variable
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- Rule of thumb: if you throw away information, you lose statistical pwoer

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Rank-Based Tests for Ordinal Variables

- Rank-based tests are especially useful when we have an ordinal variable
  - eg. a Likert variable such as "how do you feel about a presidential campaign?"
  - Neutral, support, strongly support, etc.
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Love Tester Example

Do you trust that the difference between harmless and mild is the same as the difference between burning and passionate?

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If you run a *t*-test in these cases, you impose a linear structure on your variable, treating it as metric

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- Coye is going to tell you more about the specifics

Wilcoxon Rank-sum Test for Independent Groups

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Parametric and Non-parametric Tests for Comparing Only Two Groups

Type of Design	Parametric Tests	Non-parametric Tests
Two independent samples	Independent samples t test	Wilcoxon rank-sum test (Mann-Whitney test)
Two dependent Samples	Dependent samples t test	Wilcoxon signed-rank test

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Comparing Two Independent Conditions: Wilcoxon Rank-Sum Test

- Data are ranked from lowest to highest across groups
- This provides **potential rank** scores
- If the same score occurs more than once then all scores of the same value receive the average of the potential ranks for those scores

ID	Group	Score	Potential Rank	Final Rank
1	A	10	1	1
2	A	11	2	2.5
3	B	11	3	2.5
4	B	12	4	4
5	A	20	5	6
6	B	20	6	6
7	B	20	7	6
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Calculating the Wilcoxon Rand-Sum Test

- After assigning final ranks, add up all the final ranks for each of the two groups
- Subtract the mean rank for a group of the same size as our groups
  - Otherwise, larger groups would always have larger values
  - For example, the mean group for a group of four = 1 + 2 + 3 + 4 = 10
- Our final calculation in therefore:
  - W = sum of ranks - mean rank

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Interpretation of the Wilcoxon Rank-Sum Test

Default is a two-sided test, like a *t* test

Null hypothesis: There is no difference in ranks
Alternative hypothesis: There is a difference in ranks

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- For small sample sizes ($N < 40$), R calculates the p value with the Monte Carlo methods
  - ie. simulated data are used to estimate the statistic
- For larger samples, R calculates the p value with a normal approximation method
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Effect Size Correlation

$$r = \frac{Z}{\sqrt{N}}$$

Divide the z statistic by the square root of the total sample size

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