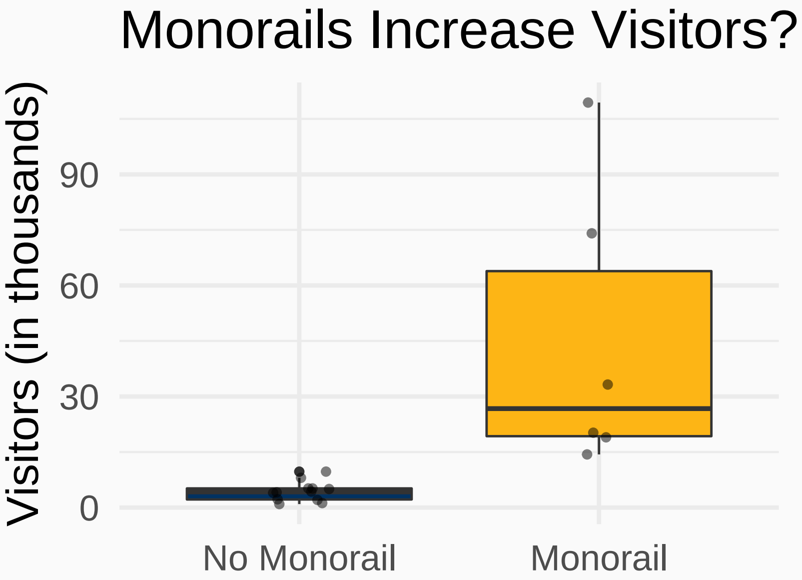
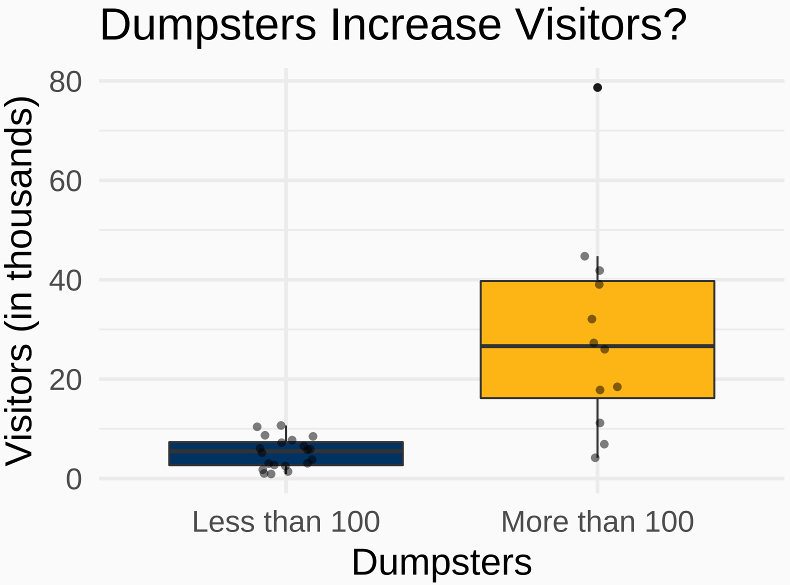
# Questions of Causation

# Some Business Questions

- What will happen to coffee sales if we buy a new roaster?
- Will profits be higher if we design a new jet or upgrade our existing one?
- Will more people visit our amusement park if we add a monorail?


Monorails Increase Visitors?
Visitors (in thousands)
0
30
60
90
No Monorail
Monorail

# Amusement Park Daily Visitors (con't)



# Correlation $\neq$ Causation

**Explanatory modeling:** How can we test or estimate an effect in a causal theory?

# Unit Plan

# Plan for the Week

Three sections

1. What is explanatory modeling?
2. The one-equation structural model
3. Common violations of the one-equation model
    - Confounding, omitted variable bias
    - Outcome on the RHS
    - Simultaneity bias

# Plan for the Week (cont.)

At the end of this week, you will be able to:

- Recognize major strategies for estimating causal effects
- Understand the assumptions behind the one-equation structural model
- Reason about common violations of the one-equation structural model

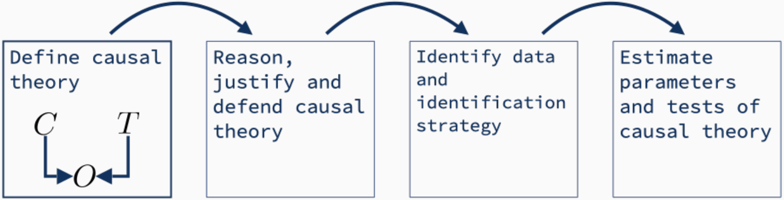
# What Is Explanatory Modeling?

# Toward Explanation

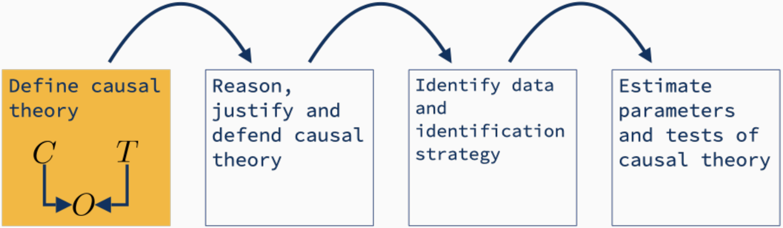
What extra assumptions are needed for OLS regression coefficients to be causal?*

* Misleading question

# The Explanatory Modeling Workflow



# The Explanatory Modeling Workflow



# What Is Causal Theory?

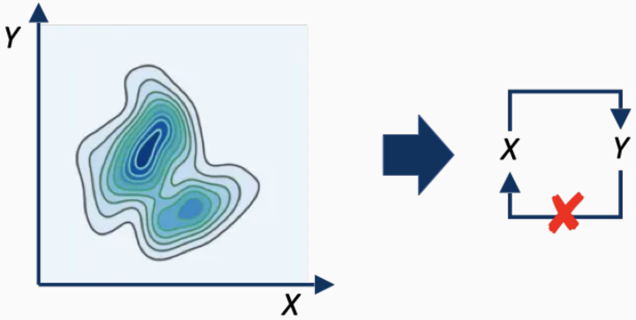
**Causal theory**

A *causal theory* is a statement of beliefs about what concepts *do* and what concepts *do not* cause other concepts.

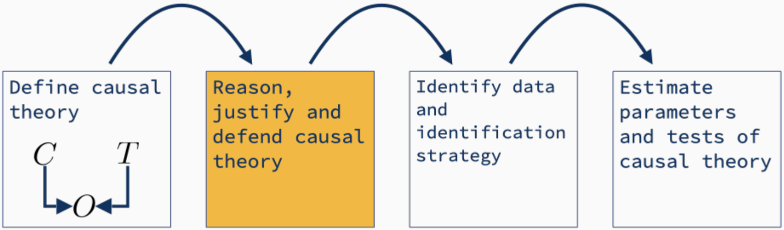
- Objective: narrow the range of causal explanations for associations we find in data.

# What is Causal Theory? (cont.)

- Joint distributions and cumulative density functions cannot identify causal information
- If we begin with causal statements, we can use logic to reach causal conclusions



# How to Reason About a Causal Theory

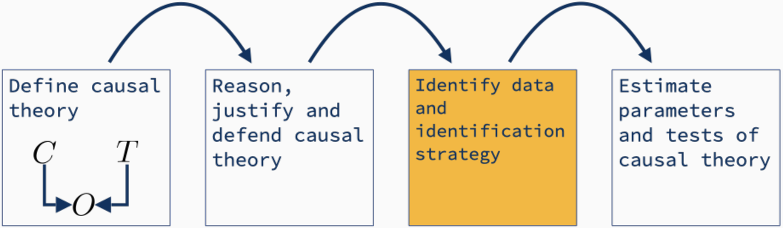


# How to Reason About A Causal Theory (cont.)

Creating and eliminating possible causal paths

- *Time structure*
    - If $X$ happens after $Y$, then $X$ cannot have caused $Y$.
- *Domain Knowledge*
    - Germ theory of infections disease
    - Often formed through past experiments
- *Effectively "random" events*
    - Coin flips
    - Tropical storms
    - pseudorandom generators

# How to Identify an Identification Strategy, Part I



# How to Identify an Identification Strategy, Part II

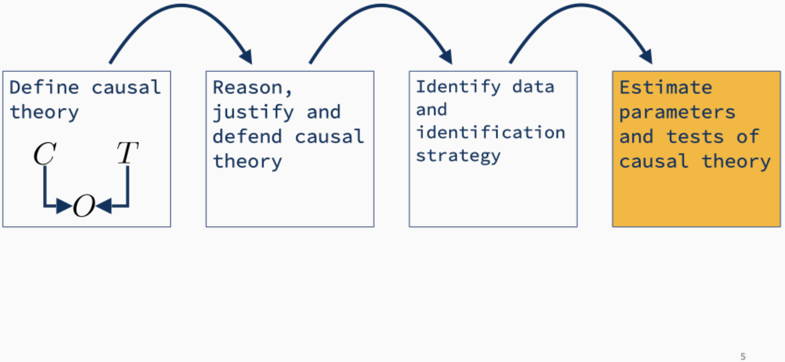
**Goal**: Produce a consistent estimate of the strength of the causal relationship given:

1. Causal theory
2. Data

No estimator provides estimates that *always* have a causal interpretation

- OLS Regression
- Diff-in-Diff
- Regression discontinuity
- Two-Stage Least Squares

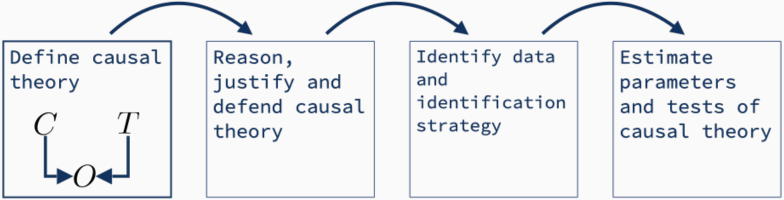
# How to Identify an Identification Strategy, Part III



## How to Estimate Parameters

- Estimate model and interpret coefficients
- Return to reasoning about the causal model and possible violations

# The Explanatory Modeling Workflow



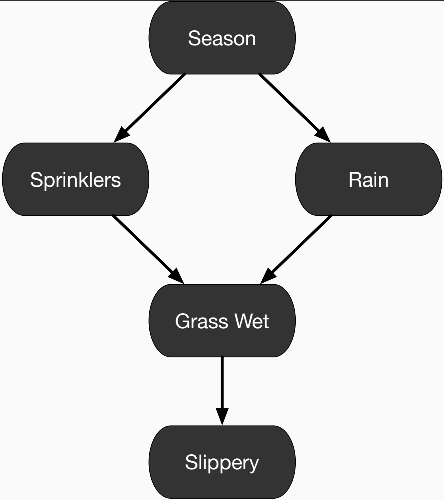
# WE SHOULD HAVE AN ACTIVITY HERE

**Note: We should either have a reading, or applied activity here.**

- Reading activity

# Pearl and Structural Equation Models

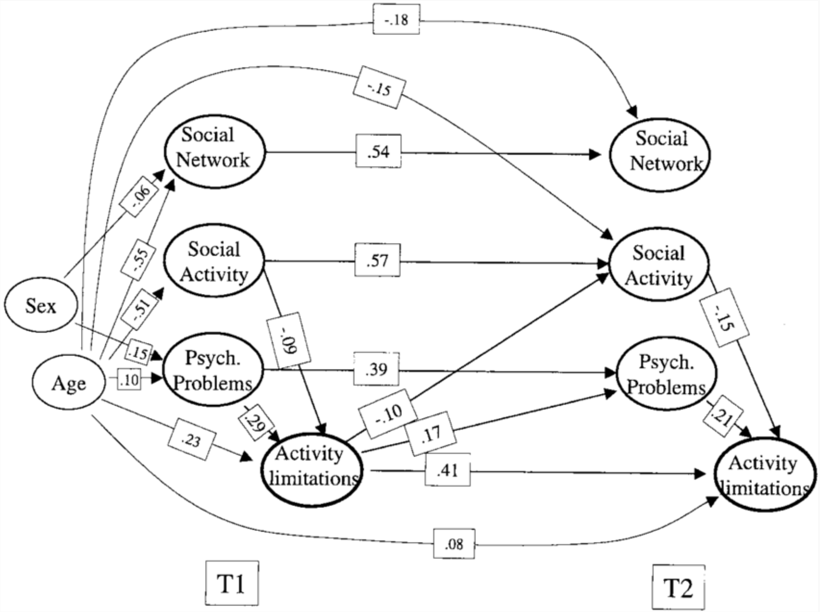
# Toward a Flexible Causal Framework




*Causality* (2000, 2009)

Judea Pearl

# How to Reason About a Causal Theory



# Structural Equation Model (SEM) Basics

**Endogenous variables**

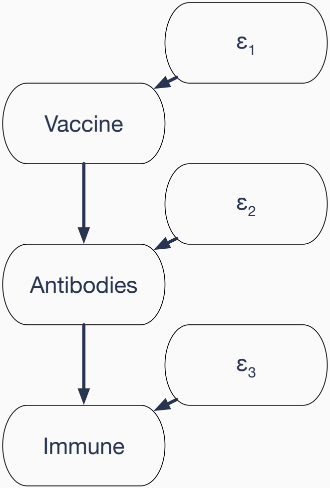
- $V$: Vaccine
- $A$: Antibodies
- $I$: Infection

**Background variables**

- $\epsilon$: Outside causes

**Structural equations**

- $V = f_V(\epsilon_1)$
- $A = f_A(V, \epsilon_2)$
- $I = f_I(A, \epsilon_3)$



# Pearl and Structural Equation Models

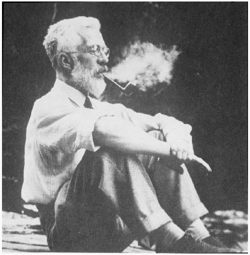
## Reading: Alleged Dangers of Cigarette Smoking

# Reading: Alleged Dangers of Cigarette Smoking

**Note: This is a reading call. We're just placing it here for organization.**

Read the two-page article, published in the BMJ in 1957 written by Ronald A. Fisher. Some context. R.A. Fisher is

- Perhaps the most influential statistician *of all time*
- At the very least, up there with Bayes, Neyman, and the canon.
- The student interested in a longer-form profile of this content can read the following article written by Pricenomics. [Link here].



Of course, smoking causes lung cancer – Fisher was dogmatic.

# Pearl and Structural Equation Models

## Evaluation and Execution of a Structural Equation Model

# Example: Execution of an SEM

**Note: This is a whiteboard, we're just placing it here for organization.**

- What causes lung cancer?
- Coffee $\rightarrow$ Alertness $\rightarrow$ Work
- Interest $\rightarrow$ Awareness $\rightarrow$ Purchase

# Execution of an SEM

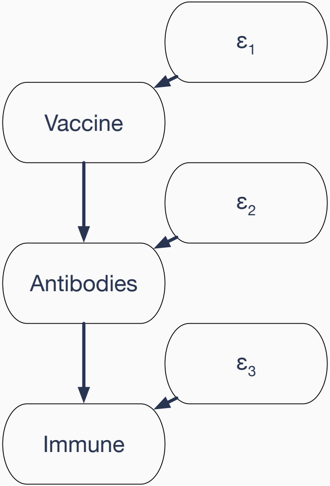
**Step one**

- Draw values of $\epsilon_1, \epsilon_2, \epsilon_3$.
- Assume $\epsilon_1 \dots \epsilon_k$ are independent

**Step two**

- Assign endogenous variables their values

$$V = f_V(\epsilon_1)$$
$$A = f_A(V, \epsilon_2)$$
$$I = F_I(A, \epsilon_3)$$



# Learnosity Check

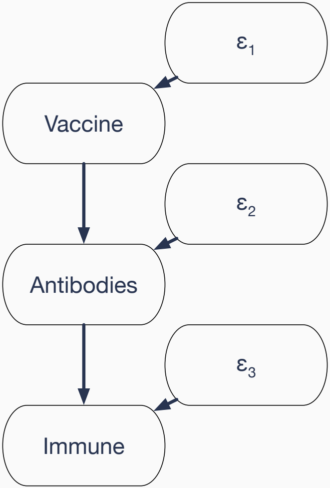
# Executing An SEM

In this SEM, assume the following:

$\epsilon_1, \epsilon_2, \epsilon_3$ are Bernoulli variables.

- $P(\epsilon_1 = 1) = 2/3$
- $P(\epsilon_2 = 1) = 3/4$
- $P(\epsilon_3 = 1) = 1/2$
- $V = f_V(\epsilon_1) = \epsilon_1$
- $A = f_A(V, \epsilon_2) = V \cdot \epsilon_2$
- $I = f_I(A, \epsilon_3) = (1 - A) \cdot \epsilon_3$

What is $P(I = 1)$, representing an infection? (Answer: 1/4)



# Statistical Implications of a Causal Graph

# How to Apply an SEM

Causal models require that we write down, clearly, the assumptions about the causal process *in the data generating process*.

- Evaluate how closely our data matches our *theory* about the world
- Choose an appropriate estimator for the data and theory

Currently, this slide doesn't seem to motivate causal graphs?

# Causal Graph Definition

**Causal graph**

A **causal graph** is a graph that describes the causal pathways among a subset of all variables.

Causal graphs encode our theory about the causal structure:

1. If there is an arrow from $X$ to $Y$, then $X$ has a direct causal effect on $Y$.
2. If there is **no** arrow from $X$ to $Y$, $X$ has **no** direct causal effect on $Y$.
3. If $X$ and $Y$ have a common cause $Z$, then $Z$ must be in the diagram, even if we cannot measure it.

# Examples of Causal Graphs

**Example: Direct and indirect effects**

- $V \rightarrow A \rightarrow I$. Vaccines have a direct causal effect on Antibodies, but no direct effect on Infection.

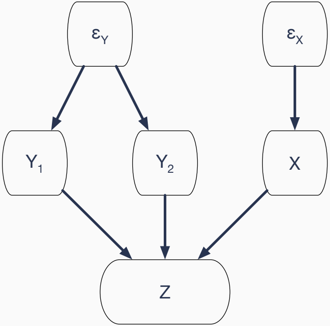
**Example: Common Causes**

- Education $\rightarrow$ Wage. Do we need to include motivation?

# Statistical Implications of a Causal Graph

**Theorem: independence**

If $X$ and $Y$ have no common ancestors in an acyclic SEM, they are independent.

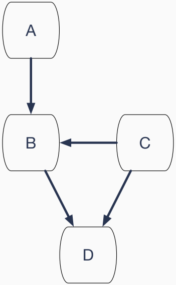


# Learnosity Check

# Independence in Causal Graphs

**Note: This is a learnosity activity. We're placing it here for organization.**

In this causal graph, which pairs of variables are independent?




Answer: A and D

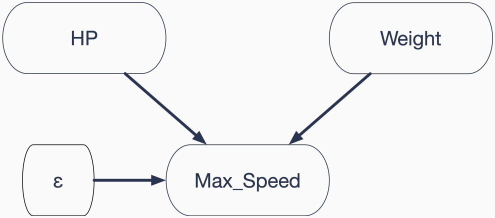
# The One-Equation Structural Model

# The Simplest Causal Graph

One causal relationship:

- A single outcome: *Max_Speed*
- A set of background variables that have a causal effect on the outcome: *HP*, *Weight*
- An error term that also has a causal effect on the outcome: $\epsilon$

# The One-Equation Structural Model




$$Max\_Speed = \beta_0 + \beta_1 HP + \beta_2 Weight + \epsilon \qquad (S)$$

Where $E[\epsilon] = 0$

# The Error Term in Structural Equations

**To a statistician:** $\epsilon$ is the difference between the target and the prediction

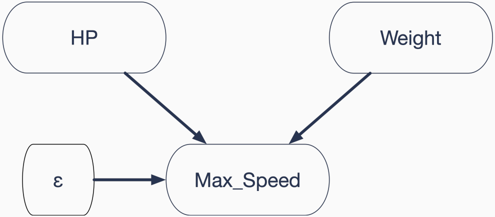
**To an explanatory modeler:** $\epsilon$ is unmeasured factors that have a causal effect on the outcome

# The Error Term in Structural Equations (cont.)

**Thought experiment:** Write down any missing variable that can affect the outcome.

$$\begin{aligned} \mathit{Max\_Speed} =& \beta_0 + \beta_1 \mathit{HP} + \beta_2 \mathit{Weight} \\ & + \underbrace{\beta_3 \mathit{Air\_Resistance} + \beta_4 \mathit{Tires} + \ldots}_{\epsilon} \end{aligned}$$

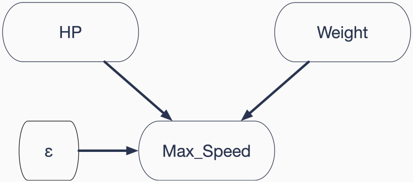
# Assessing the Causal Graph




Two things we look for:

1. Are there any causal pathways back from *Max_Speed* to *HP* and *Weight*?
2. Are there any common ancestors of *HP* and *Max_Speed* or of *Weight* and *Max_Speed*?

# Estimation in the One-Equation Model

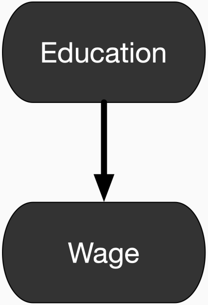


# Applications for the One-Equation Model

## An Important Question

**When is the one-equation structural model valid?**

# Confounding Variables in Observational Data

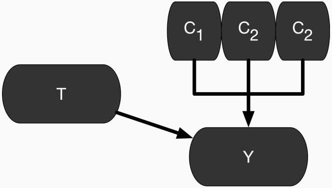


# When Is the One-Equation Model Credible?

- True experiments
- Some natural experiments
- Differenced panels

# The True Experiment

- Treatment $T$ is randomly assigned (e.g. coin flip) $\implies$ no incoming paths *other than the coin*.
- Controls $C_1, C_2, C_3$ are either measured or determined before treatment
    - No paths from $T$ to controls, or controls to $T$
- Outcome $Y$ measured after $T$



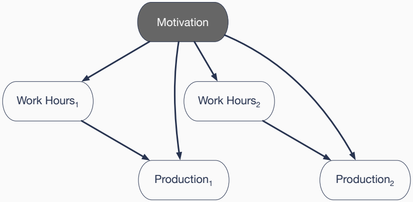

$\implies$ OLS consistent estimates effect of $T$ on $Y$.

# Some Natural Experiments

**Natural experiment:** A scenario in which we can exploit naturally occurring variation to estimate structural parameters

- Often through instrumental variables, regression discontinuity, or other advanced techniques
- May enable OLS to *identify* causal quantities if treatment is random
    - The Vietnam War lottery
    - Tropical cyclones
    - Forest fires
    - Network outages

# Differenced Panels




$$
\begin{array}{r}
Production_1 = \beta_0 + \beta_1 Work\_Hours_1 + \beta_2 Motivation + \epsilon_1 \\
-\left[ Production_2 = \beta_0 + \beta_1 Work\_Hours_2 + \beta_2 Motivation + \epsilon_2 \right] \\
\hline
\Delta Production = \qquad \beta_1 \Delta Work\_Hours \qquad\qquad + (\epsilon_1 - \epsilon_2)
\end{array}
$$

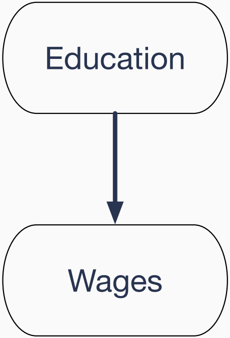
# Violations of the One-Equation Structural Model

# Omitted Variables

# Omitted Variables

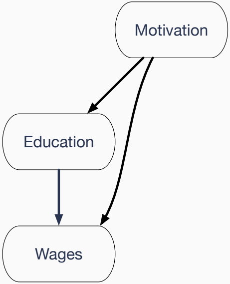
**Assumed Model**

Education causes wages




**True Model**

Motivation causes both education and wages



# Omitted Variables

**We fit**

$W = \tilde{\beta}_0 + \tilde{\beta}_1 E + \tilde{\epsilon}$

**True structural equation**

$W = \beta_0 + \beta_1 E + \beta_2 M + \epsilon$

- We are interested in $\beta_1$.
- What is the bias, $E[\tilde{\beta}_1 - \beta_1]$?

# Omitted Variable Bias in Simple Regression

| We fit | True structural equation |
| --- | --- |
| $W = \tilde{\beta}_0 + \tilde{\beta}_1 E + \tilde{\epsilon}$ | $W = \beta_0 + \beta_1 E + \beta_2 M + \epsilon$ |

Regress $M$ on $E$:

$$M = \delta_0 + \delta_1 E + \nu$$

Consider two quantities

- $\beta_2$ is the effect of $M$ on $W$.
- $\delta_1$ represents how related $M$ and $E$ are.

**Omitted Variable Bias:** $\tilde{\beta}_1 - \beta_1 = \beta_2 \delta_1$

# Omitted Variable Bias in Multiple Regression

| We fit | True structural equation |
|---|---|
| $Y = \tilde{\beta}_0 + \tilde{\beta}_1 X_1 + \tilde{\beta}_2 X_2 + \ldots + \tilde{\beta}_{k-1} X_{k-1} + \tilde{\epsilon}$ | $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \tilde{\beta}_{k-1} X_{k-1} + \beta_k X_k + \epsilon$ |

Regress $X_k$ on other $X$'s:

$$X_k = \delta_0 + \delta_1 X_1 + \ldots + \delta_{k-1} X_{k-1} + \nu$$

**Omitted Variable Bias:** $\tilde{\beta}_1 - \beta_1 = \beta_k \delta_1$

# Estimating Omitted Variable Bias

$$\text{Omitted Variable Bias} = \beta_2 \delta_1$$

How much does omitted variable affect outcome?

How related are measured and omitted variables?

We fit: $\widehat{Wage} = \tilde{\beta}_0 + \tilde{\beta}_1 Education$; Omitted: *Motivation*

# Assessing Omitted Variable Bias

Which is worse: Bias toward zero or bias away from zero?

# Proof of Omitted Variable Bias

# The Omitted Variable Bias in Simple Regression

**We fit**

$W = \tilde{\beta}_0 + \tilde{\beta}_1 E + \tilde{\epsilon}$

**True structural equation**

$W = \beta_0 + \beta_1 E + \beta_2 M + \epsilon$

# Learnosity Check

# Estimating Omitted Variable Bias

In the following equation, estimate whether the omitted variable bias is towards zero or away from zero.

$\widehat{Air\_Purity} = .97 - .00034\ Bicycles\_per\_Square\_Mile$

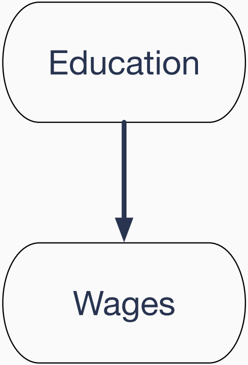
Omitted: People per Square Mile

# Reverse Causality

# Reverse Causality

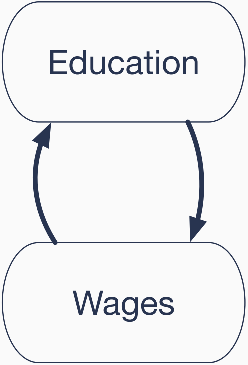
**Assumed model**

Education causes wages




**True model**

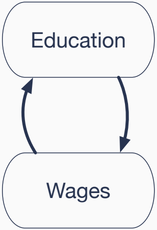
Education causes wages *and* wages cause education



# An SEM Version

True structural equations:

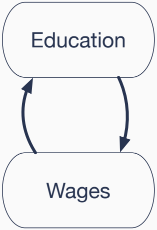
$$W = \beta_0 + \beta_1 E + \epsilon_1 \qquad (1)$$
$$E = \gamma_0 + \gamma_1 W + \epsilon_2 \qquad (2)$$




Observations

- E is a descendant of $\epsilon_1$.
- $\implies$ $E$ and $\epsilon_1$ are dependent.
- $\implies$ (1) is not the BLP.

Since OLS estimates the BLP, it can't estimate (1).



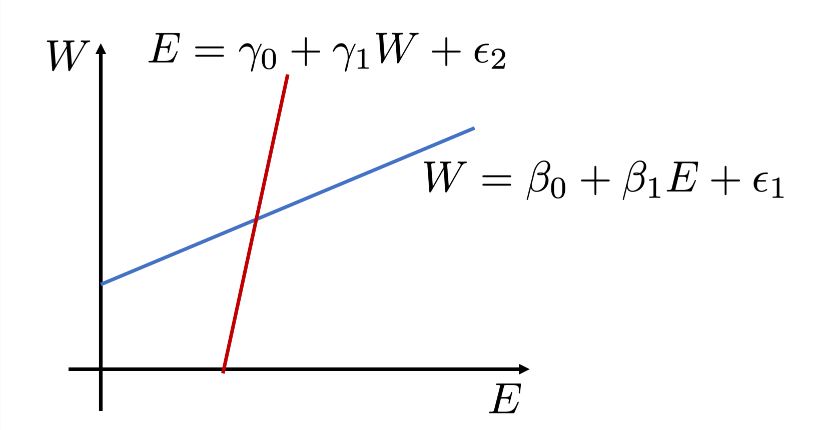

True structural equations:

$$W = \beta_0 + \beta_1 E + \epsilon_1$$
$$E = \gamma_0 + \gamma_1 W + \epsilon_2$$

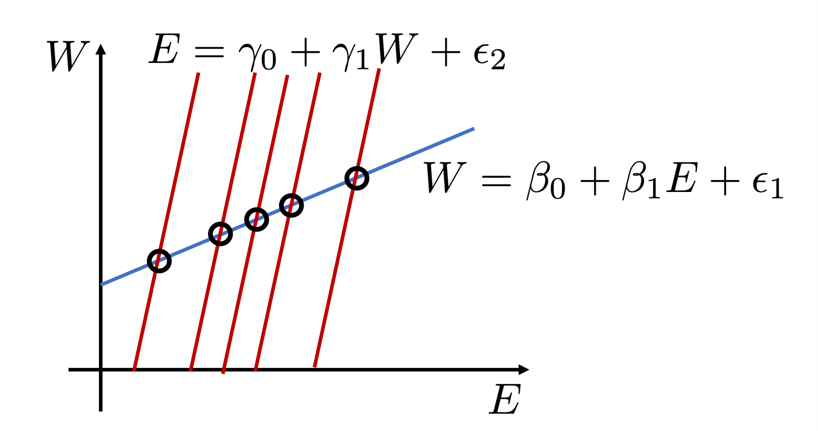
Suppose $\beta_1 > 0$

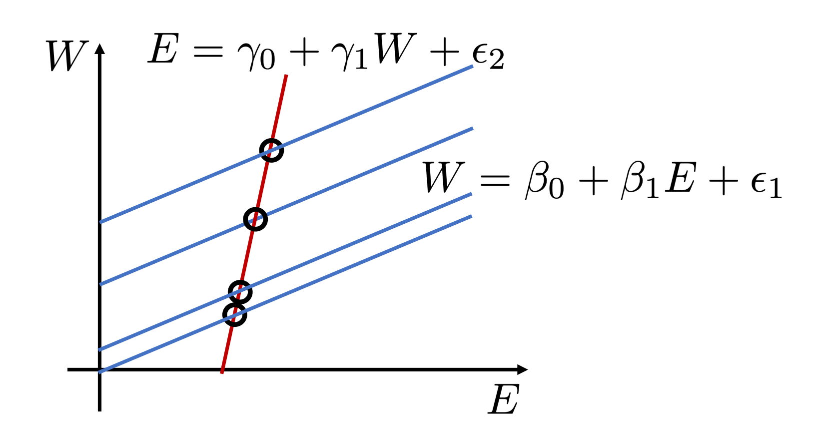
- Positive feedback $\gamma_1 > 0$
  - $\tilde{\beta}_1 > \beta_1$
- Negative feedback $\gamma_1 < 0$
  - $\tilde{\beta}_1 < \beta_1$

# Understanding the SEM Equilibrium



# Understanding the SEM Equilibrium




$W$
$E = \gamma_0 + \gamma_1 W + \epsilon_2$
$W = \beta_0 + \beta_1 E + \epsilon_1$
$E$

# Learnosity Check

# Direction of Reverse Causality Budget

**Note: This is a Learnosity Activity. We are just placing it here for organization.**

In this causal graph, $P$ is number of police, $C$ is number of crimes. You run a regression of crime on police. is the direction of bias due to reverse causality positive or negaitve?

# Outcome Variables on Right-Hand Side

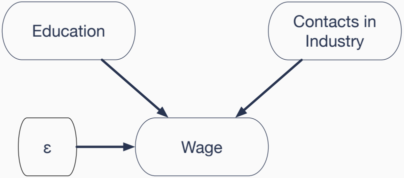
# Outcome Variables on the Right-Hand Side

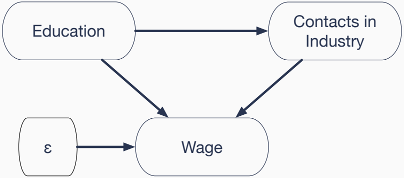
**Assumed model**

- Education causes wages
- Contacts in industry cause wages

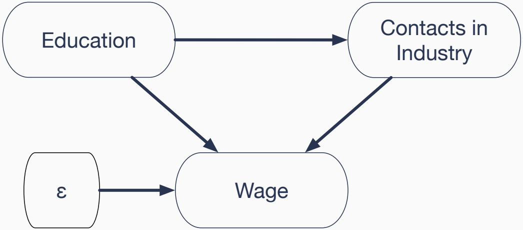
**True Model**

- Education causes wages
- Contacts in industry cause wages
- Education creates contacts in industry



# Estimating the Structural Parameters




Structural Equation: $W = \beta_0 + \beta_1 E + \beta_2 C + \epsilon \qquad (S)$

$\epsilon$ and $E$ have no common ancestors.

$\implies$ $\epsilon$ and $E$ are independent.

$\implies$ $cov(E, \epsilon) = 0$

$\implies$ OLS is consistent for $\beta_1$

# Interpreting the Structural Coefficient

$\beta_1$ - The expected increase in Wage, from getting an extra year of eduction, holding the number of industry contacts constant.

# Take Away

**Do not put outcome variables on the right hand side.**

# Explanatory Modeling Wrap-Up

# Take Aways

- Explanatory modeling takes place inside a causal theory.
- The one-equation structural model is usually wrong.
- In special circumstances, advanced techniques can overcome omitted variables and reverse causality.
    - To learn more, try the instrumental variables and simultaneous equations chapters in *Introductory Econometrics* (Wooldridge).