# Decision Rules

# Hypothesis Test Example

**Mad data science**

Suppose that your lab has synthesized a new compound, *Vitamin W*.

Let random variable $B$ represent the change in blood pressure that results from taking *Vitamin W*.

Let $\mu = \mathrm{E}[B]$.

You need to make a decision, to invest resources in Vitamin W or not.

# Two Possible States of the World

**Goal:** Begin with a resonable default supposition; leave this supposition behind if data provides compelling evidence

**Null hypothesis**

- Default assumption, status quo, statement that data might overturn
- $H_\varnothing$ : Usually $\mu = 0$
- No effect

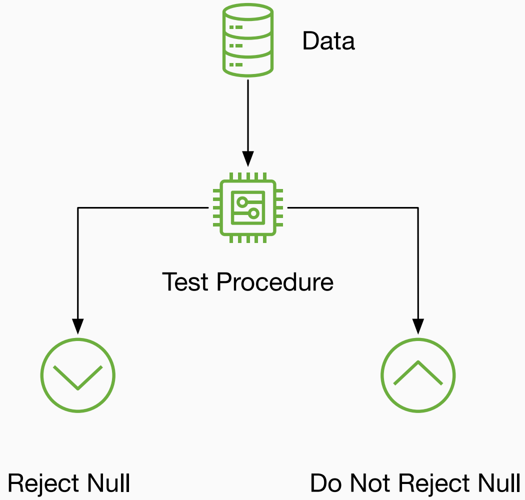
**Alternative hypothesis**

- Idea or alternative to status quo
- $H_a$ : Usually $\mu \neq 0$
- Some effect exists

With compelling evidence, we leave the specific null hypothesis ($H_\varnothing$) for the alternative ($H_a$)

# A Hypothesis Test

A *hypothesis test* is a procedure.



# False Positive and False Negative Errors

| | **True state of the world** | |
|---|---|---|
| | *The null is true* | *The null is false* |
| *Reject the null* | False Positive (Type I Error) | |
| *Do not reject the null* | | False Negative (Type II Error) |

# False Positive and False Negative Errors (cont.)

**False Positive Errors**

- Typically the most destructive
- Error rate, denoted $\alpha$, is the probability of rejecting the null hypothesis when we should not; $P(\text{Reject } H_\varnothing | H_\varnothing)$
- Starting with Ronald Fisher: set $\alpha = 0.05$

A hypothesis test is a procedure for rejecting or not rejecting a null, such that the false positive error rate is controlled ($\alpha = 0.05$).

# Breaking Down a Test Procedure

**A test statistic**

- A function of our sample
- Measures deviations from the null hypothesis
- Distribution must be completely determined by the null

**A rejection region**

- A set of values for which we will reject the null
- Chosen to be contrary to the null
- Total probability must be $\alpha = 0.05$

# What a Hypothesis Test Doesn't Do

**A hypothesis test does not prove the null hypothesis.**

- We control Type 1 error rates
- We cannot control Type 2 error rates
- How can you be sure the real B is not 0.01? Or 0.00001?

**Never accept the null hypothesis.**

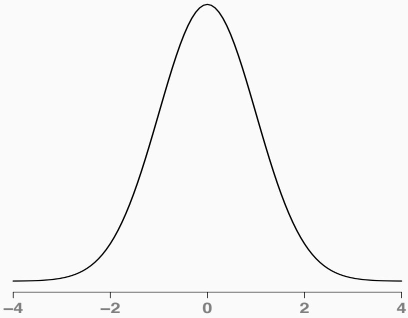
- The valid decisions are reject and fail to reject.

# The One-Sample z-Test

## Vitamin W Example

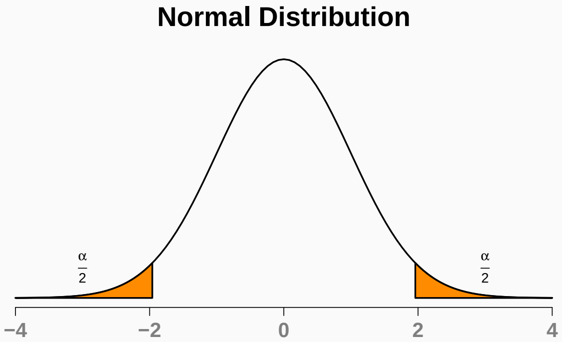
Suppose $(B_1, .., B_{100})$ are i.i.d. random variables with mean $\mu = \mathrm{E}[B]$, representing changes in blood pressure.

Assume $B \sim N(\mu, \sigma)$. Assume we know $\sigma[B] = 20$.



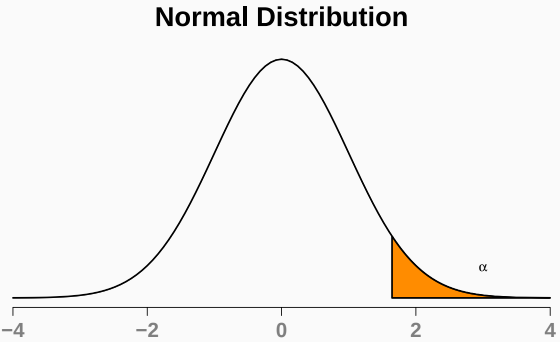
# One- and Two-Tailed Tests

# The Two-Tailed z-Test



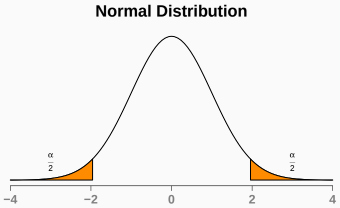

- **Null hypothesis**: $\mu = 0$
- **Alternative hypothesis**: $\mu \neq 0$

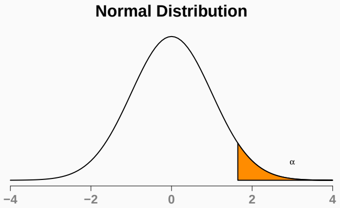
# The One-Tailed z-Test




- **Null hypothesis**: $\mu = 0$
- **Alternative hypothesis 1**: $\mu > 0$
- **Alternative hypothesis 2**: $\mu < 0$

# Choosing One or Two Tails




Switching your test after you see the statistic is cheating.

# One-Tailed Test: Things to Consider

Before using a one-tailed test, ask yourself these questions:

1. Will the audience believe that I started with one tail before I saw the data?
2. Will the audience share my opinion of which tail is interesting?
3. Am I really 100% committed to only this tail?
    - What if the effect turns out to be huge, but in the other direction?
    - Would I be willing to call that a negative result?
    - Can I convince my audience I have this much commitment?

# T-Test Assumptions

# T-Test Assumptions, Part I

**Assumptions of t-test**

The textbook assumptions

- $X$ is a metric variable.
- $\{X_1, X_2, \ldots, X_n\}$ is a random sample.
- $X$ has a normal distribution.

Variables are almost never normal.

# T-Test Assumptions, Part II

But, in the large sample case, this is more plausible.

**Large sample t-test assumptions**

**If**:

- $X$ is a metric variable
- $\{X_1, X_2, \ldots, X_n\}$ is a random sample
- $n$ is large enough that the CLT implies a normal distribution of mean

**Then**: The t-test is asymptotically valid

# T-Test Assumptions, Part III

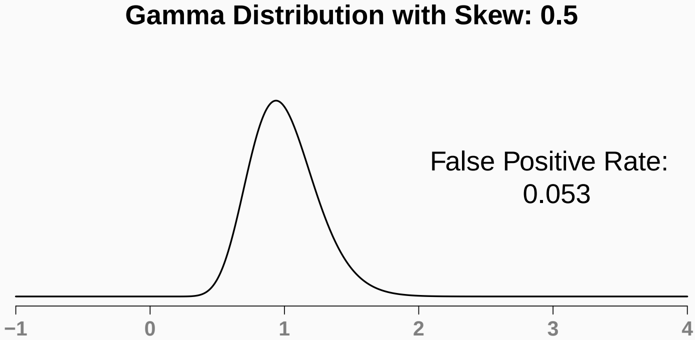
# T-Test Assumptions, Part IV

The t-test is considered "reasonably robust," even when $n < 30$, as long as deviations from normality are moderate.

However, watch out for strong skewness, especially when $n < 30$.

# Gamma With Increasing Skew

Twenty draws from gamma distributions



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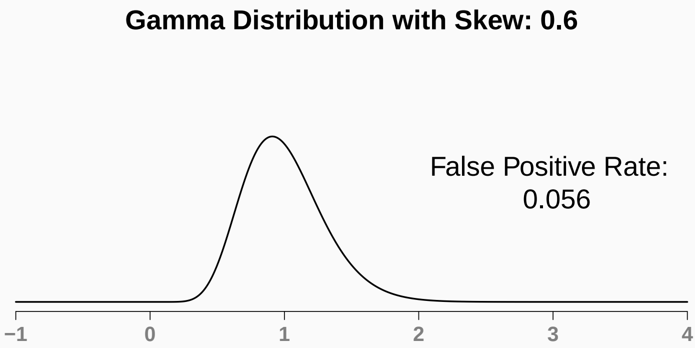

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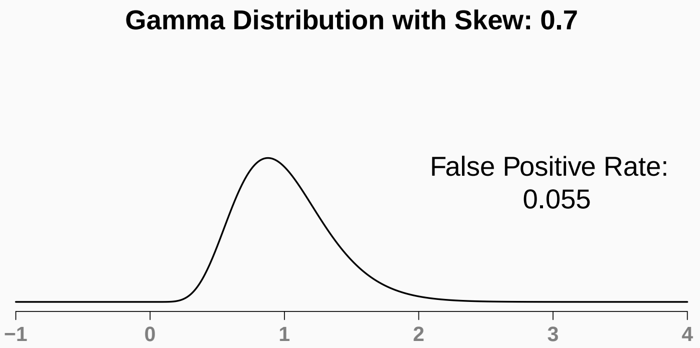

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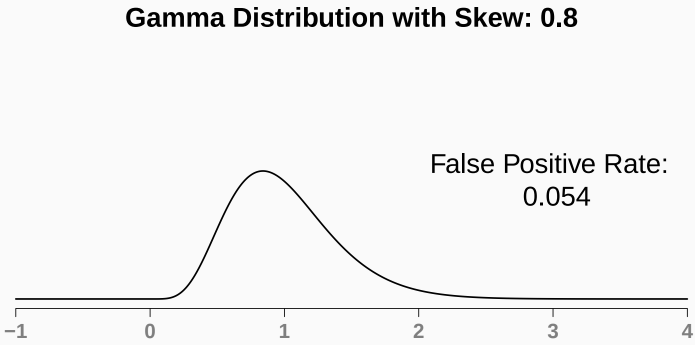

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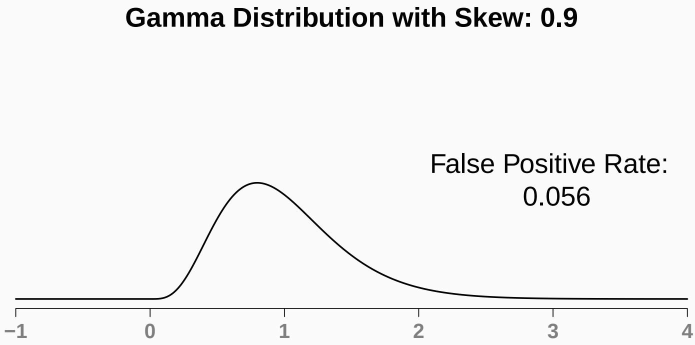

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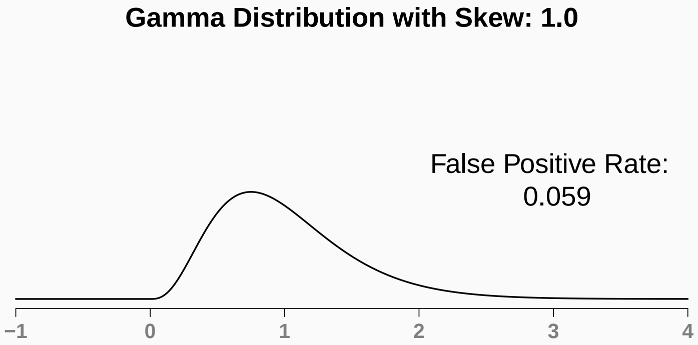

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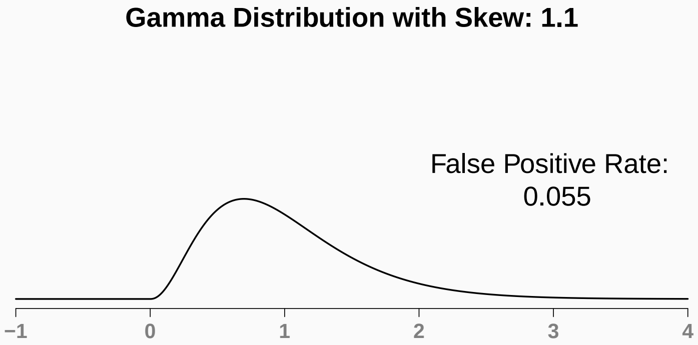

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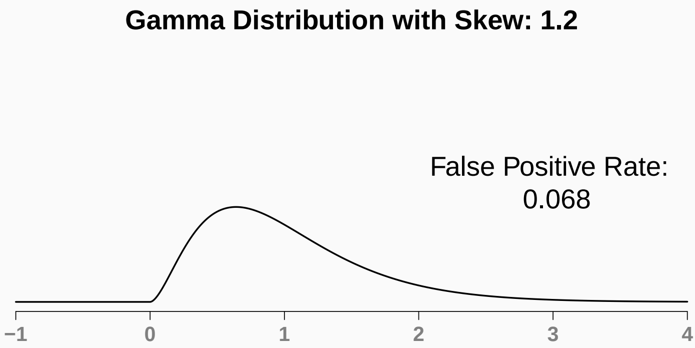

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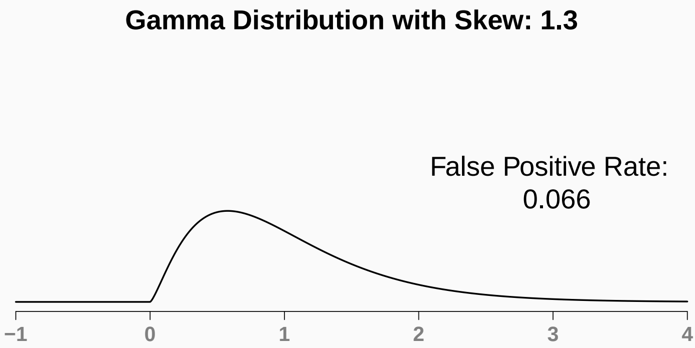

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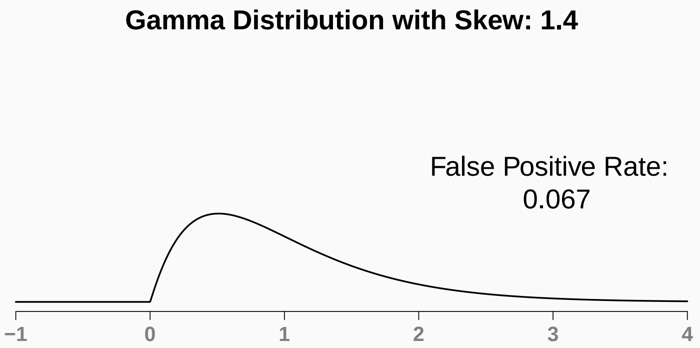

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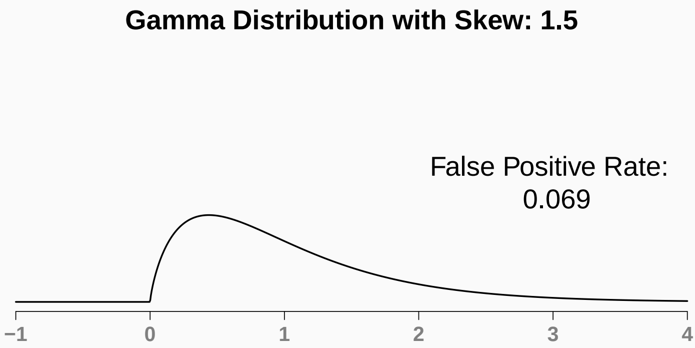

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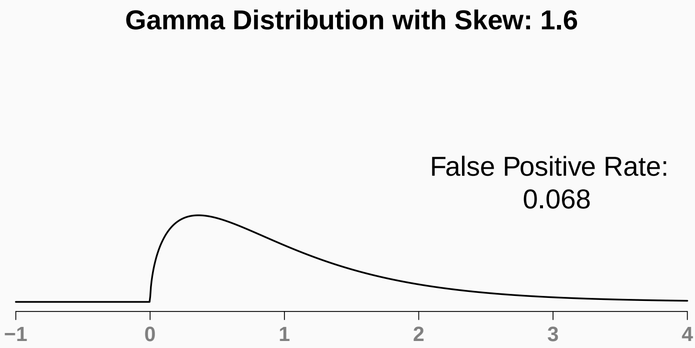

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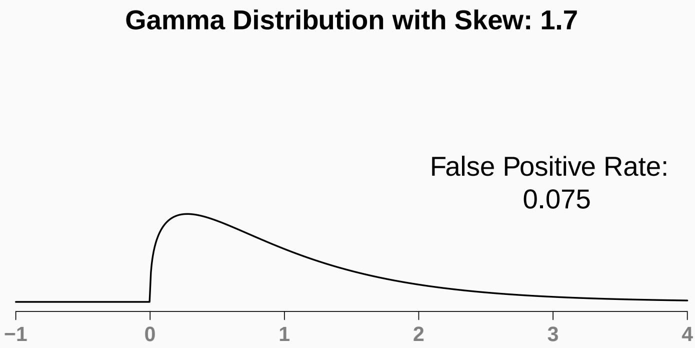

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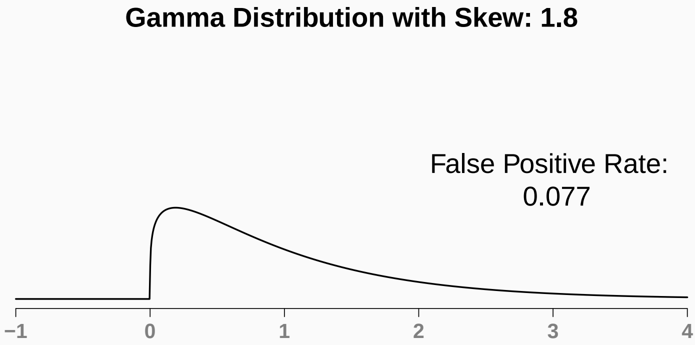

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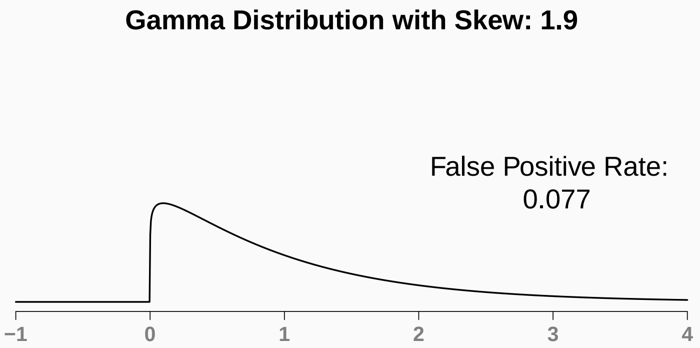

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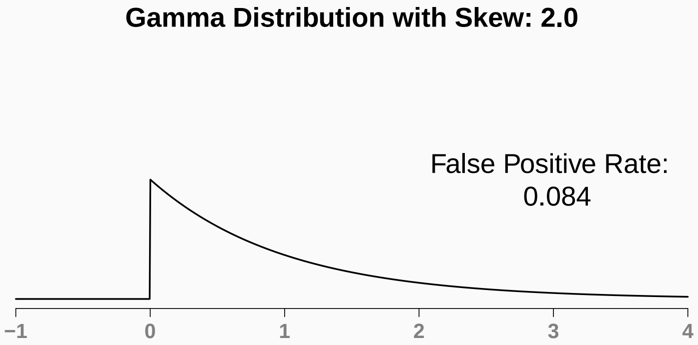

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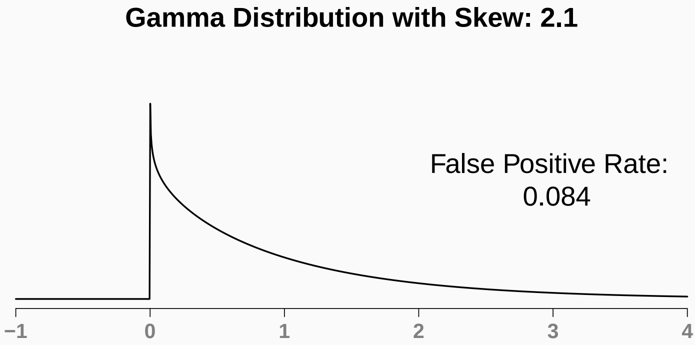

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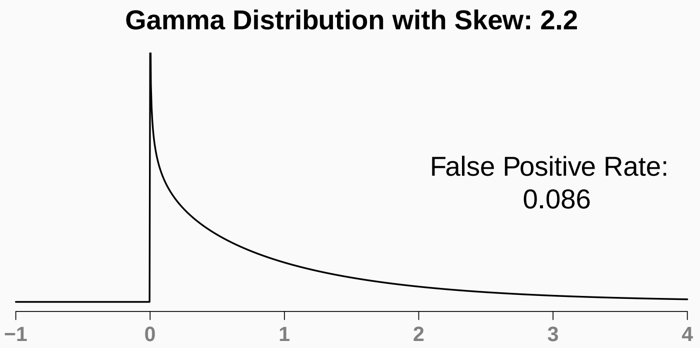

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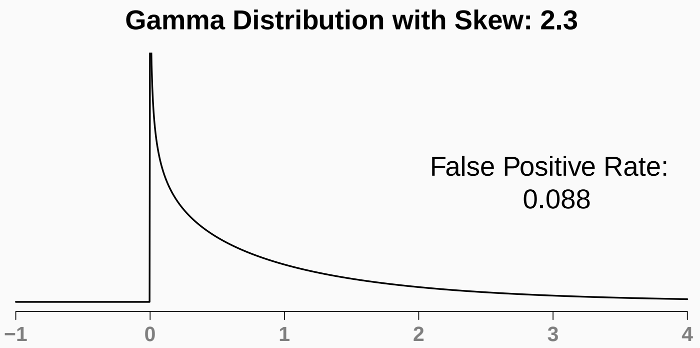

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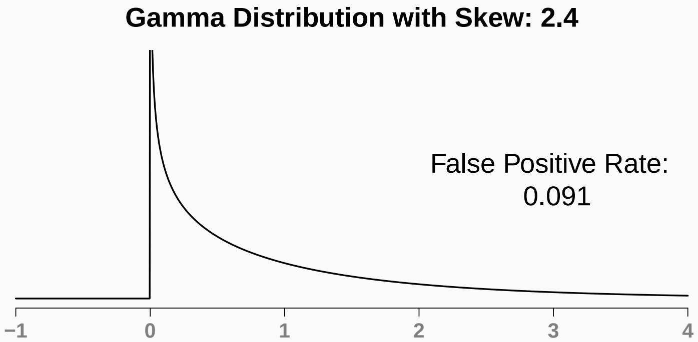

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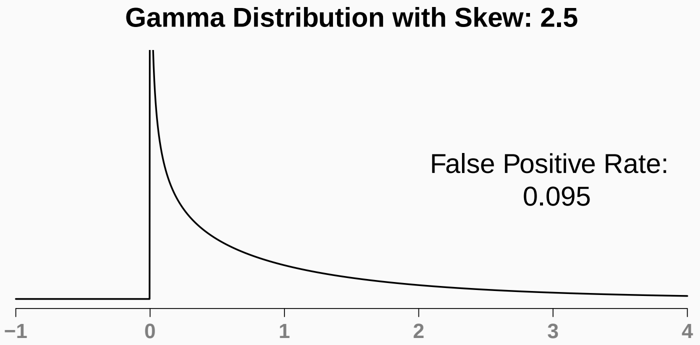

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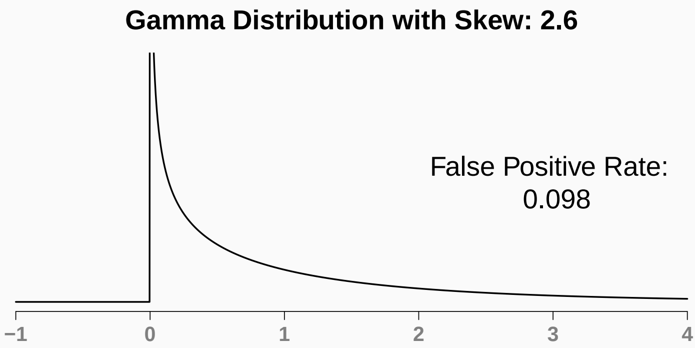

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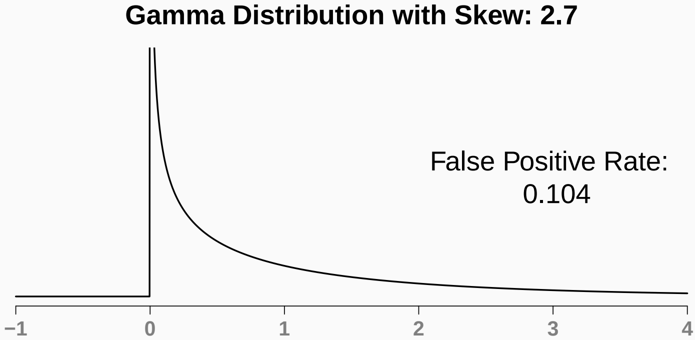

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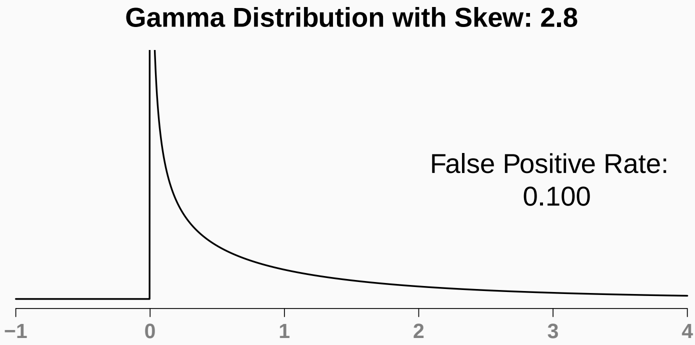

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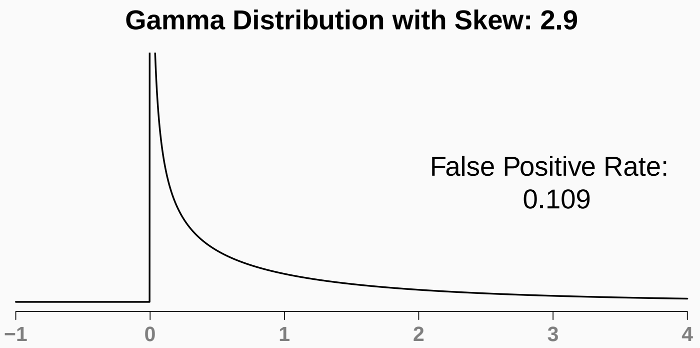

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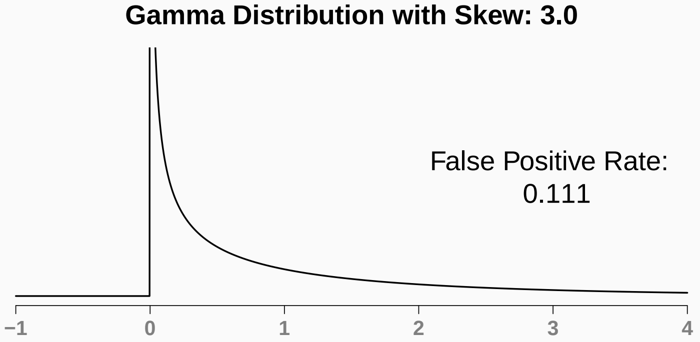

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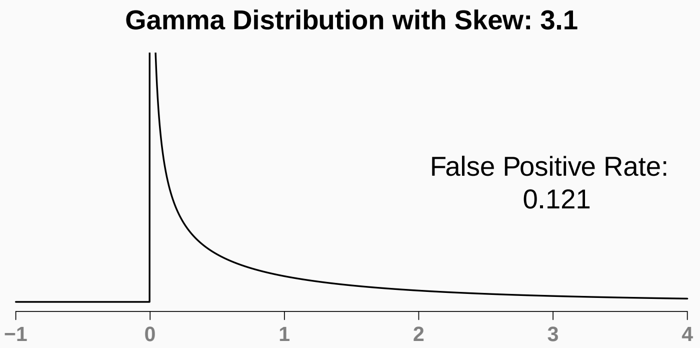

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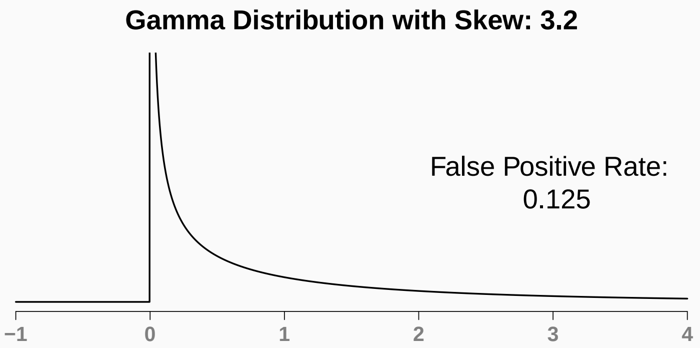

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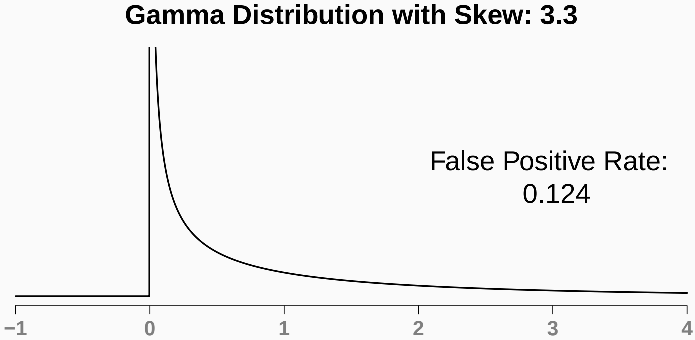

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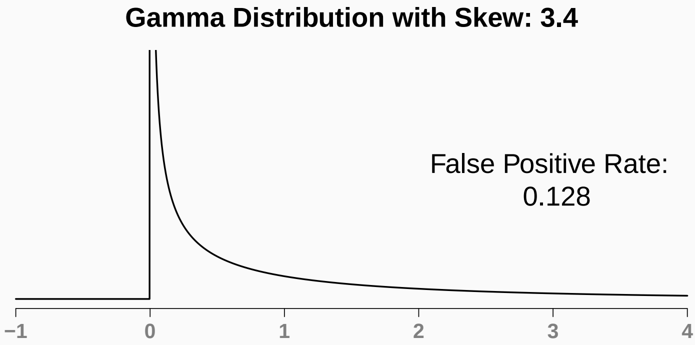

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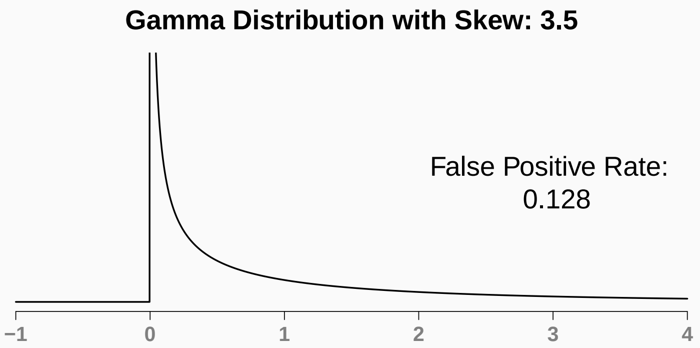

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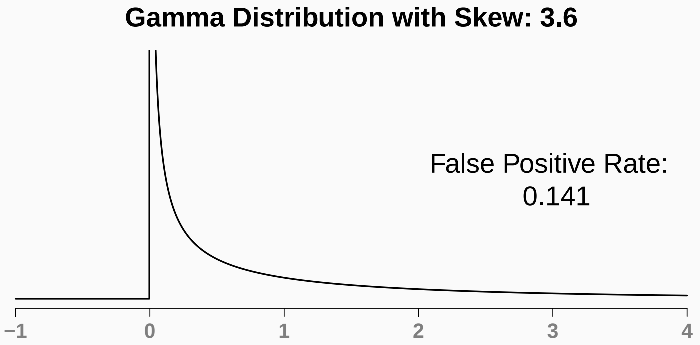

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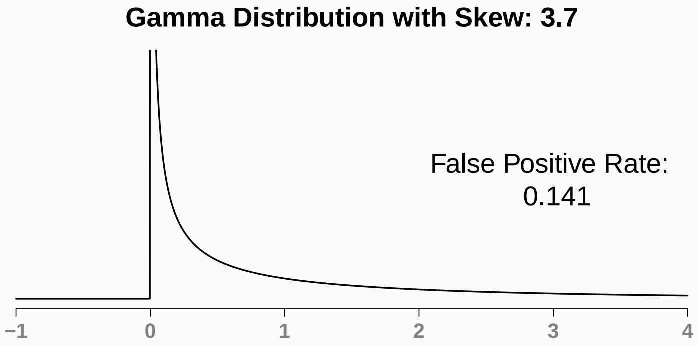

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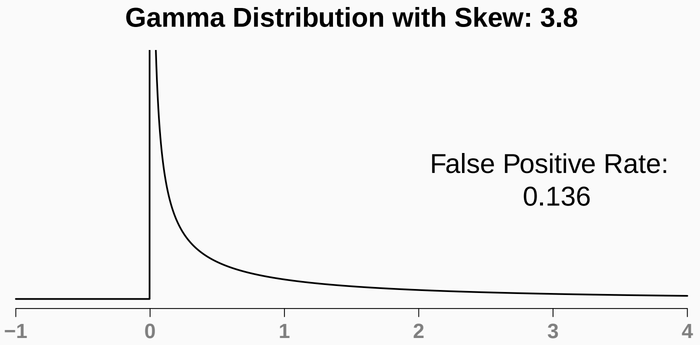

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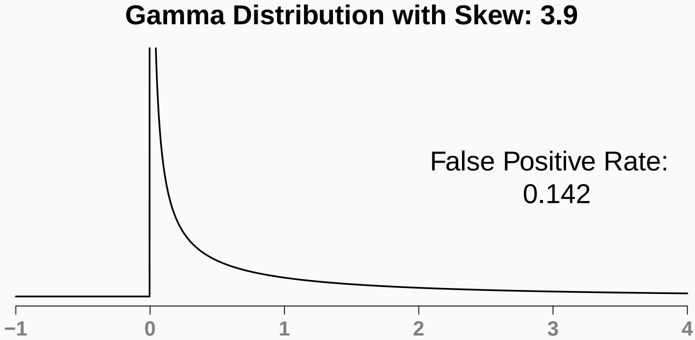

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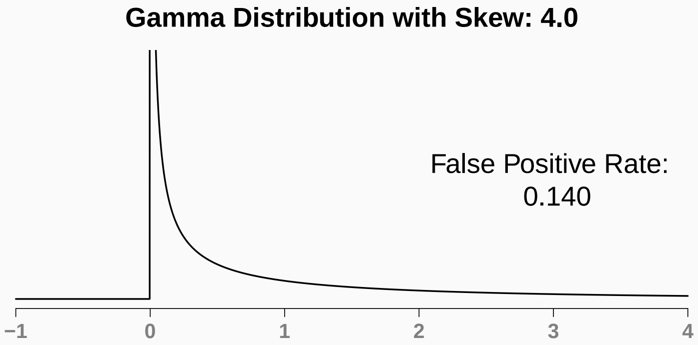


# T-Test Assumptions

More practical guidance:

- $X$ is a metric variable.
- $\{X_1, X_2, ..., X_n\}$ is a random sample.
- The distribution is not too non-normal, considering $n$.

When the t-test is not valid, consider using a non-parametric test instead.

# Introduction to P-Values

# Introducing P-Values

*The p-value is the probability, calculated assuming that the null hypothesis is true, of obtaining a value of the test statistic at least as contradictory to $H_0$ as the value calculated from the available sample.*

*Jay L. Devore (2015)*

# Z-Distribution

**Vitamin W**

You measure the effects of Vitamin W on blood pressure (measured in *mmHg*) for 100 patients and get $\bar{X} = 3$.

Assume $X \sim N(\mu, 20)$.

- $H_0 : \mu = 0$
- $z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$

# The P-Value and Decision Rules, Part I

Neyman-Pearson hypothesis testing: rules to make a decision and usually be right ($\alpha = 0.05$)

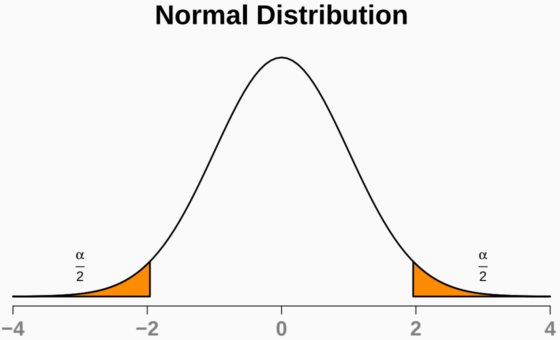
**A classic z-test**

- z=1 $\rightarrow$ Do not reject null.
- z=2 $\rightarrow$ Reject null.
- z=10 $\rightarrow$ Reject null.

- Strict frequentist with a dichotomous decision rule: treat $z = 2$ and $z = 10$ identically.
- But is there value in knowing *how contrary* the data is to the null?

# The P-Value and Decision Rules, Part II

$$|z| > \text{critical value} \Rightarrow \text{reject } H_0$$
$$|z| < \text{critical value} \Rightarrow \text{fail to reject } H_0$$



# The P-Value and Decision Rules, Part III

$$|z| > \text{critical value} \Rightarrow \text{reject } H_0$$
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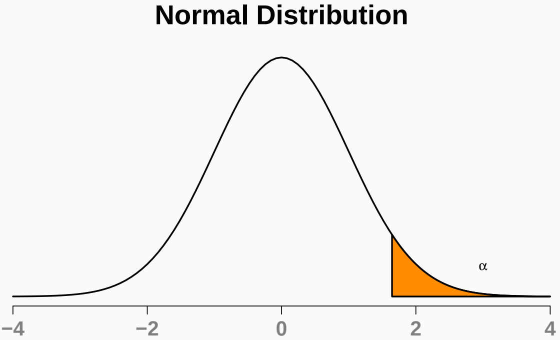


# An Equivalent Decision Procedure

Compute p-value.

- If $p < .05 \Rightarrow$ reject $H_0$
- If $p \geq .05 \Rightarrow$ do not reject $H_0$

But, can you justify making such a bright-line statement after reducing information so much?

1. Concept
2. Measurement
3. Statistic
4. Assumptions about distribution
5. **p-value**
6. Reject/fail to reject

# t-Test and p-Values

# P-Value Convention

| p-value range | Convention | Symbol |
|---|---|---|
| $p > 0.10$ | Non-significant | |
| $0.10 > p > 0.05$ | Marginally-significant | . |
| $p < 0.05$ | Significant | * |
| $p < 0.01$ | Highly significant | ** |
| $p < 0.001$ | Very highly significant | *** |

# Reporting Test Results

- A t-test for the effect of Vitamin W on blood pressure was highly significant ($t = 3.1$, $p = .008$).
- We found evidence that Vitamin W decreases blood pressure ($t = 2.3$, $p = .04$).
- The effect of Vitamin X on blood pressure was not statistically significant ($t = 1.2$, $p = .23$).

| Vitamin W | Vitamin X |
|---|---|
| 2.2 ** | 1.2 |
| (0.6) | (0.8) |

This is half the story; next, you'll need to describe practical significance.

# Variable Importance and P-Values

Does a small p-value mean that a variable is "important"?

- Statistical significance
- Practical significance

# A Warning

A very common mistake is to assume a p-value is the chance the null hypothesis is true.

Frequentist statistics cannot tell you the probability of a hypothesis!

# A Warning (cont.)

**Example**

I test whether Vitamin X decreases blood pressure: $p = 0.03$.

However, you know that Vitamin X is secretly cornstarch because you created it yourself.

My test will not convince you that there is a 97% chance Vitamin X decreases blood pressure.

# Statistical Power

# False Positive and False Negative Errors

| | The null is true | The null is false |
|---|---|---|
| Reject the null | False Positive (I) | |
| Do not reject the null | | False Negative (II) |

- False Positive (I) errors are jumping without cause
- False Negative (II) errors are failing to jump when you should
    - Failing to detect a real effect
    - Missed opportunity to create a product, publish a paper, or advance knowledge

# Statistical Power, Part I

**Much Vitamin W**

Consider a *specific* alternate hypothesis:

- $H_a$ : Vitamin W decreases blood pressure by 20 mmHg

- False Negative Error Rate: $\beta = P(\text{not rejecting } H_0 | H_a)$
- Statistical power: $1 - \beta$
- Statistical power is the probability of supporting the alternate hypothesis, assuming it is true

# Statistical Power, Part II

# Statistical Power, Part II

# Statistical Power, Part III

How to increase power

- Increase sample size.
- Choose a powerful test (if you can justify its assumptions).

# Practical Significance

# Practical significance

**Statistical significance**

- How much does the data support the existence of an effect?

**Practical significance**

- Is the size of this effect important?
- What is the magnitude of the effect?
- Should we care about this effect?

# Example

## Productivity supplements

### Vitamin W

$$n = 30$$
$$\mu_{treat} = 12.6$$
$$\mu_{control} = 6.1$$
$$p = 0.11$$

*"The difference between groups was not statistically significant, ($t = 1.34$, $p = 0.11$)."*

### Vitamin Q

$$n = 30,000$$
$$\mu_{treat} = 6.25$$
$$\mu_{control} = 6.21$$
$$p = 0.0005$$

*"The difference between the two groups was highly significant, ($t = 3.34$, $p < 0.001$)."*

# Practical Significance: Context

**Primary goal**: Provide context for your audience to reason about results.

- Who is your audience?
- What action might be taken based on these results?
- How does this result alter how you would run the business?
- What is the cost-benefit for implementing a change based on this result?
- How does this result "stack up" to other effects?

# Practical Significance: Model Explainability

- Some tasks require *explainable* models.
- Finance, healthcare, insurance, and other regulated industries stipulate specific model forms .
- Humans reason in linear hypotheses—higher-dimensional and conditional hypotheses are too much to keep in mind.

# Practical Significance: Effect Sizes

**Effect sizes**

- Single-number metrics that characterize the magnitude of an effect
- Population parameters that we estimate—*do not vary based on sample size*

**Invalid effect size metrics**

- t-stat
- p-value

**Valid effect size metrics**

- Mean values
- Difference in means between groups

# Standard Effect Size Measures

Standardized effect sizes are designed to be flexible and apply in many scenarios:

- Cohen's *d*
- Correlation $\rho$
- Cramer's *V*

General metrics ignore the specific context around your research or business question.

# Cohen's d

Sometimes, a mean (or difference in means) is hard to assess because the units are unfamiliar.

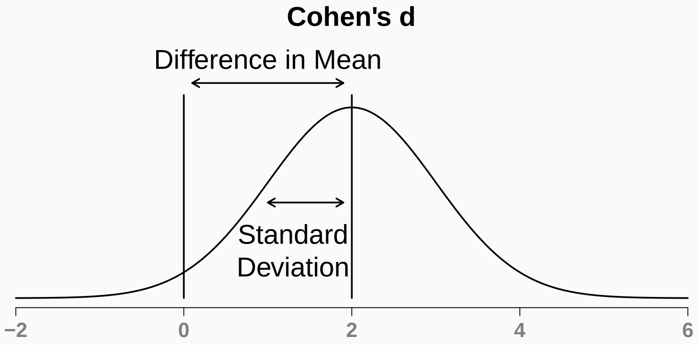
- **Example**: The effect of angled bristles on tooth decay is 5 millicaviparsecs per brushstroke

**Cohen's d**

Compare effect size relative to the underlying natural variation in the outcome.

$$\text{Cohen's } d = \frac{\text{mean difference}}{\text{standard deviation}}$$

# Cohen's d (cont.)



## Rules of thumb (according to Cohen)

| | |
|---|---|
| Small effect | $d = 0.2$ |
| Medium effect | $d = 0.5$ |
| Large effect | $d = 0.8$ |

- Applicable across a huge number of contexts
- Ignores any important differences between context
- Saving dollars or saving lives are the same to Cohen's d

# Takeaways

- After a statistical test, it's important to assess both statistical significance and practical significance.
- Standard effect size measures can help in a wide variety of situations.
- But don't get carried away and reach for them automatically.
- The main objective is to clearly explain how important the magnitude of the effect is.