min MSE as a function of c

$$\min_c MSE = E[(x-c)^2]$$

$$E[(x-c)(x-c)]$$

$$E[x^2 - 2xc - c^2]$$

$$\frac{\partial MSE}{\partial c} = E[x^2] - 2c'E[x] - c^2$$

$$2E[x] - 2c' = 0$$

$$\frac{2}{2}E[x] - \frac{2}{2}c = 0$$

$$E[x] - c = 0$$

$$c = E[x]$$

$$MSE = V[x] + (E[x] - c)^2$$