Hypothesis Testing

UC Berkeley, MIDS w203

Statistics for Data Science

March 4, 2022

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Introducing the Two-Sample t-Test

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Motivating the Two-Sample t-Test

An Important Data Science Question

Is group A different from group B?

Examples:

- Are customers who get a birthday gift less likely to leave than those who don't?
- Do patients who take Vitamin W get over the flu faster than patients who don't?
- Do democracies or autocracies start more wars?

Motivating the Two-Sample t-Test
An Important Data Science Question
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• Do democracies or autocracies start more wars?

1. The two-sample t-test is one of the most common procedures in inferential statistics. What is it for? As a data scientist you will often face a question about two groups - group A and group B. Someone will ask you if one is greater or less than than the other.
2. Questions like this are everywhere. In business, you have A-B testing.
3. In medicine, you may want to test a group of patients taking a vitamin against a control group
4. In political science, you may want to test whether democracies or autocracies start more wars.
5. What do these questions have in common? They all have

Example Scenario

From the Journal of Empirical Fashion

	New Yorkers	San Franciscans
black outfits	12.1	13.3
sample size	50	50

Is this evidence that San Franciscans have more black outfits than New Yorkers *in expectation*?

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1. Here's a specific example, with data.
2. write on slide: 13.3-12.1 = 1.2 outfits
3. Is this a big difference? is it just noise?
4. To be more precise, is this evidence that SF'ans have more black outfits than NYers in expectation?
5. To analyze this question, we need a model...

Two-Sample Model Framework

Basic Model Setup

Suppose $(X_1, .., X_{n_1})$ are i.i.d. with mean μ_X.

Suppose $(Y_1, .., Y_{n_2})$ are i.i.d. with mean μ_Y.

Null Hypothesis

$H_0 : \mu_X = \mu_Y$ (The two population means are equal)

Alternative hypotheses

$H_1 : \mu_X \neq \mu_Y$ (best choice in most cases)
$H_2 : \mu_X > \mu_Y$
$H_3 : \mu_X < \mu_Y$

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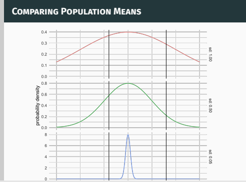
1. A model is a representation of the world built of RVs. for a two-sample t-test, the model begins like this.
2. Let $X_1..X_{n_1}$ be RVs representing group A. Similarly for Y.
3. Let's assume that the X's are iid with mean μ_X. same for Y
4. We'll need more assumptions, but this is a start.
5. The null is that our two means are the same.
6. Our usual alternative is that the means are different.
7. It is also possible to run a one-sided test in special circumstances. alt is one mean is greater than other.
8. Given this model, how plausible is the null hypothesis?

Comparing Population Means



1. One important piece of information is how much does the number of black outfits vary in the population.
2. Here's a picture with three different distributions for our sample averages.
3. The top distribution has very high standard deviation. You can see that it doesn't seem unusual to get two sample averages that are 1.2 outfits apart.
4. The middle distribution has medium deviation. Now it starts to look a little more surprising that our two sample averages were 1.2 apart.
5. The bottom distribution has low standard deviation. Now it seems quite unlikely to get two sample averages 1.2 apart. The data seems inconsistent with this hypothesis

Comparing Population Means

Two-Sample t-Test

$$t = \frac{\bar{X} - \bar{Y}}{\text{Estimate of Standard Deviation}}$$

Assess whether two populations have the same expectation **while accounting for variability**

Comparing Population Means
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1. Putting these ideas together, we can make a statistic by taking the difference in sample averages, and dividing by an estimate of standard deviation - that's exactly the idea behind the two- sample t-test

The Two-Sample z-Test

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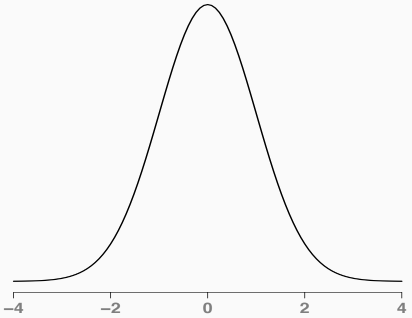
Two-Sample z-Test

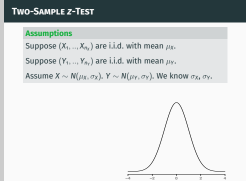
Assumptions

Suppose $(X_1, .., X_{n_X})$ are i.i.d. with mean μ_X.

Suppose $(Y_1, .., Y_{n_Y})$ are i.i.d. with mean μ_Y.

Assume $X \sim N(\mu_X, \sigma_X)$. $Y \sim N(\mu_Y, \sigma_Y)$. We know σ_X, σ_Y.





1. Before we tackle the two-sample t-test, let's begin with the simpler 2 samp. z test. unrealistic, but build intuition.
2. Same assumptions as before, but add assump. of equal var σ which we know.
3. How do we create a test statistic?
4. What is the distribution of $\bar{X} - \bar{Y}$? Let's use fact that a difference of normal RVs is normal. but which normal?
5. $V[\bar{X} - \bar{Y}] = V[\bar{X}] + V[\bar{Y}] = \sigma_X^2/n_X + \sigma_Y^2/n_Y$
6. $\bar{X} - \bar{Y} \sim N(0, \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}})$
7. We can standardize by dividing. $z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}} \sim N(0, 1)$
8. Here's the standard normal. We can plot our statistic on

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1. Here's an example with actual data.
2. $z = \frac{\bar{X}-\bar{Y}}{\sqrt{\frac{\sigma_X^2}{n_X}+\frac{\sigma_Y^2}{n_T}}} = \frac{12.1-13.3}{\sqrt{\frac{4}{50}+\frac{4}{50}}} = \frac{-1.2}{\sqrt{\frac{4}{25}}} = -3.0$
3. $z < -1.96$ REJECT
4. We can also compute p-value 2*pnorm(-3) = .0027

General Procedure for Testing

Three steps:

1. Specify model, null hypothesis, rejection criterion
2. Calculate statistic
3. Plot statistic on the null distribution to get the p value.

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1. From this example, I hope you see that the two-sample z-test is not all that different from the one-sample tests we did earlier.
2. No matter what the test is, there's a general pattern that you follow...

The Two-Sample t-Test

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Types of Two-Sample t-Tests

1. Student's t-Test
2. Welch's t-Test

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1. There are actually two versions of the two-sample t test you should be aware of.
2. Student's t-test is the original t-test. it's simpler, but requires strong assumptions
3. Welch's t-test is more general, and this is really the modern t-test.

The Two-Sample z-Test

$$z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}$$

- Problem : we don't know σ_X or σ_Y

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1. To explain these tests, let's remember our equation for the z test
2. The big problem with this test is we're using the standard deviations σ_X and σ_Y - but we don't know what they are
3. We're going to have to estimate these in some way.

Student's t-Test

- Estimate a single "pooled" standard deviation, s.
- Substitute s for both s_X and s_Y.

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{s^2}{n_X} + \frac{s^2}{n_Y}}}$$

Theorem

if $\sigma_X = \sigma_Y$, t has a T distribution with $n_X + n_Y - 2$ degrees of freedom.

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1. Here's what Student did..
2. Student's idea is to create a single estimate for standard deviation - let's call it s.
3. I'm not including the equation for s - i don't think it's very important.
4. You plug s in both both unknown quantities
5. Why would you do this? Well, it turns out that Student got a really clean, elegant result
6. Theorem: this statistic follows a t dist. with $n_X + n_Y - 2$ degrees of freedom.

degrees of freedom (*df*)

Number of independent pieces of information that vary given estimated parameters

- **One sample t-test**
  - Model has one parameter (the mean)
  - Given the sample mean, and $n-1$ observations, can compute the last one.
  - $df = n-1$
- **Student's two-sample t-test**
  - Model has two parameters (μ_X and μ_Y)
  - Given the sample means, $n_X - 1$ observations for X and $n_Y - 1$ observations for Y, can compute the rest
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1. Why is there a -2 in degrees of freedom?
2. There's a pattern. Think of degrees of freedom in this way. (read def)
3. For the one-sample test, our model had one parameter - the mean. it's the mean we wanted to test.
4. If I gave you the sample mean, and n-1 rows of data, you could compute the last row.
5. There's no more information in the last row of data. it's already locked in.
6. so df is n minus 1 for the estimated parameter.
7. For the two-sample test, we now have a model with two parameters...

Student's t-Test Summary

- Tests if mean of X equals mean of Y.
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- **Major disadvantage:** Only valid if $\sigma_X = \sigma_Y$.

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1. Let's take a moment to summarize Student's t-Test.
2. We use the t-Test to see if $\mu_X = \mu_Y$.
3. Student used a pooled estimate for standard deviation.
4. But there's one problematic assumption: the test is only valid if X and Y have the same standard dev.

Welch's t-Test

- Compute two sample standard deviations: s_X and s_Y.
- Substitute s_X for σ_X and s_Y for σ_Y.

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}}$$

Theorem

t has *approximately* a T distribution. The degrees of freedom are given by

$$\nu = \frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(s_X^2/n_X\right)^2}{n_X-1} + \frac{\left(s_Y^2/n_Y\right)^2}{n_Y-1}}$$

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1. Let's move on to Welch's test.
2. Unlike Student, Welch computed two different sample standard deviations.
3. Then substitute in
4. Unfortunately, this method just isn't as clean mathematically.
5. You do get a similar theorem, but (1) the distribution is only approximately t. and (2) there's this complicated equation for degrees of freedom. the degrees of freedom you get are usually not an integer.

Choosing a Two-Sample t-Test

Some authors recommend a two step process:

1. Use Levine's test for equal variances ($H_0 : \sigma_X = \sigma_Y$)
2. If non-significant, proceed with Student's t-Test

Our advice: always use Welch's t-Test

- Power is almost as high as for Student's test
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• Power is almost as high as for Student's test
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1. Why is this our advice? 3 reasons
2. First, the only reason you'd ever want more assumptions is to get more power. It's true that Student's test is more powerful - but studies have shown that the difference is too small to worry about
3. Second, you never know if the variances are equal - you can use a test, but a test is just a test - it's not proof. and the result really depends on the sample size. for small samples in particular, it's really hard to know.
4. Finally, this is the default setting in most software. not really a reason, but it is convenient.

Welch's Two-Sample t-Test Assumptions

- **Metric Scale:** $X_1, X_2, ..X_{n_X}$ and $Y_1, Y_2, ..., Y_{n_Y}$ are random variables measured on a metric scale.
- **Independence:** X's are iid, Y's are iid, and X's and Y's are mutually independent.
- **Normality:** The distribution of the X's is normal and the distribution of the Y's is normal
  - The CLT guarantees normality for large samples
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1. Let's summarize the assumptions in a more user-friendly form.

Practical Significance of the T-Test

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After using a t-test to assess statistical significance, it is important to assess practical significance.

> *Your main goal is to explain to your audience why they should or should not care about the effect.*

Three common effect size measures:

1. Difference in means
2. Cohen's d
3. Correlation r

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Difference in Means

Difference in means

$$\overline{X}_A - \overline{X}_B$$

- Answers the question "*How different are these groups?*"
- Often makes great headlines and is a good choice if units are familiar
- But lacks context in its calculation
- People who eat chocolate live 1.5 years longer than those who do not each chocolate

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Cohen's d

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Cohen's d is a measure of difference of means standardized by the variance in the data.

$$\frac{\overline{X}_A - \overline{X}_B}{s}$$

Where s is a pooled standard deviation: $\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2}}$

- Answers the question "*How many standard deviations apart are the groups?*"
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Correlation

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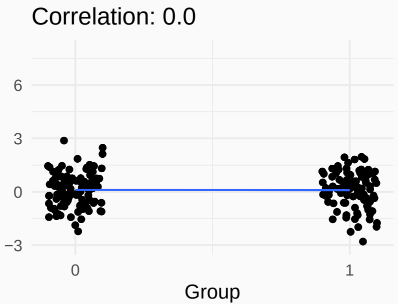
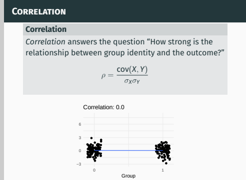
Correlation answers the question “How strong is the relationship between group identity and the outcome?”

$$\rho = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}$$





1. Notice the similarity in the form between Cohen’s d and correlation – Cohen’s d divides by the pooled standard deviation; correlation divides by the product of two group standard deviations.

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Biserial correlation

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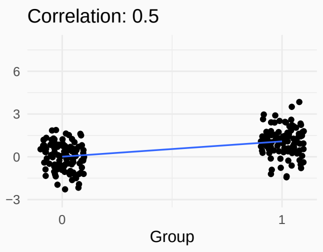

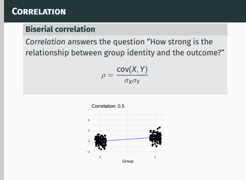

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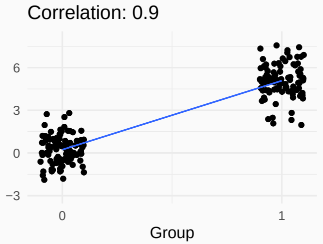

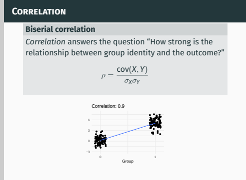

Practical Significance is about Context

- How strong is the same relationship between *different* groups?
- How strong is a *different* relationship between the same group?
- What is the underlying dispersion in the data?
- What is a meaningful anchor or reference point that you can use for context?

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The Paired t-Test

Paired t-Test

Climbing grip

Suppose you randomly sample 30 Berkeley students. For each student i, you measure right-hand strength (R_i) and left-hand strength (L_i).

- You conduct a t-test with $H_0 : \mathrm{E}[R] = \mathrm{E}[L]$
- **Problem**: Grip strength varies a lot person-to-person, $\Rightarrow$ t-test has low power.

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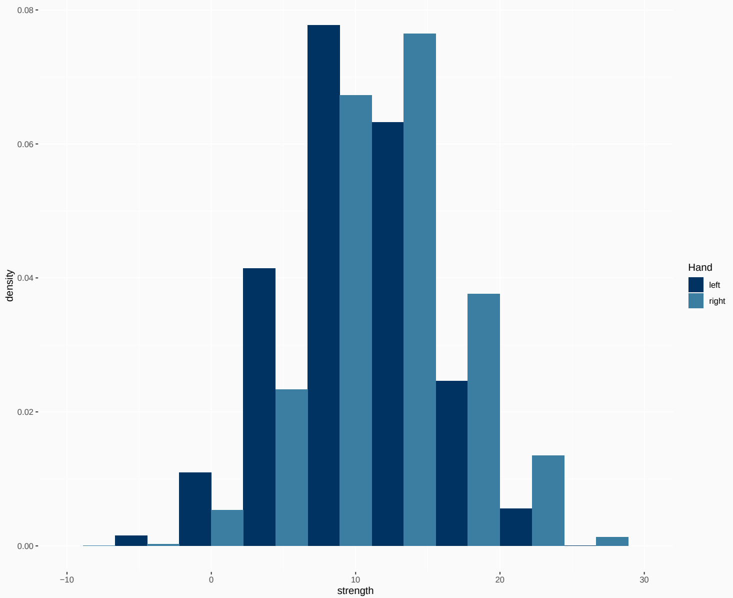
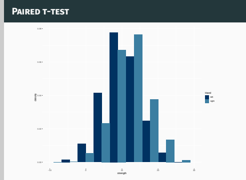
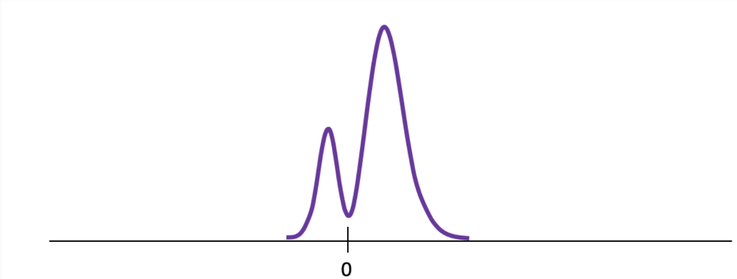




Paired t-Test

- **Idea:** For any *particular* subject i, the difference between right-hand strength and left-hand stregth, $R_i - L_i$, will usually be small.
- Within-person variation is small.



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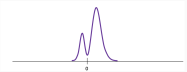

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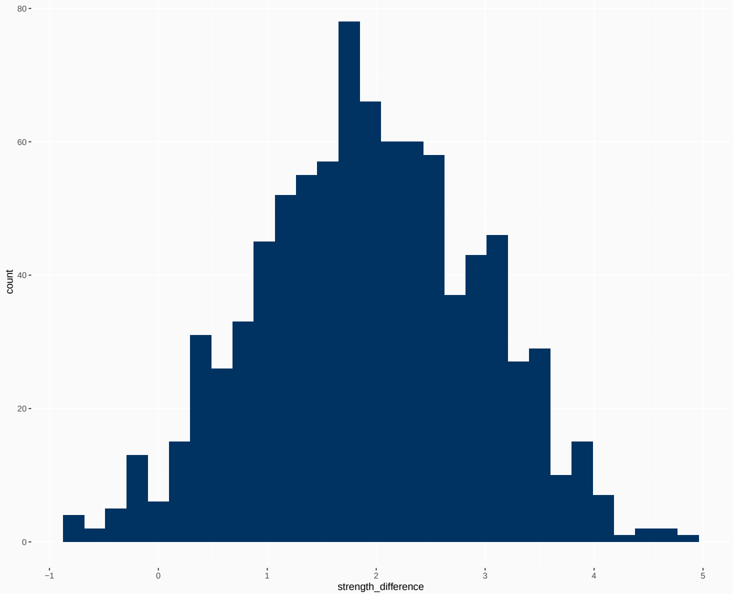

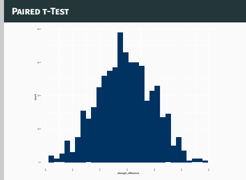

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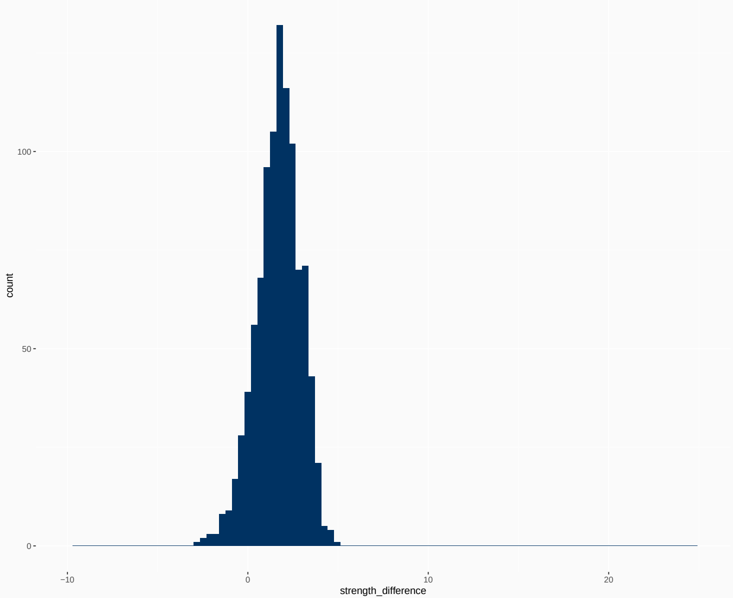

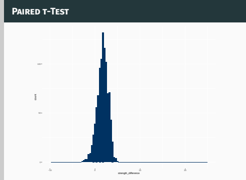

Paired t-Test

Paired t-test

A *paired t-test*, sometimes called a *dependent t-test*, builds an explicit dependency between data. Instead, perform a one-sample t-test with $H_0 : \mathrm{E}[R_i - L_i] = 0$.

- This dependency must actually exist
- Cannot simply change the test

Paired t-Test

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A *paired t-test*, sometimes called a *dependent t-test*, builds an explicit dependency between data. Instead, perform a one-sample t-test with $H_0 : \mathrm{E}[R_i - L_i] = 0$.

- This dependency must actually exist
- Cannot simply change the test

Unpaired vs. Paired t-Test

Unpaired

- $t = \frac{\overline{A} - \overline{B}}{\sigma_{A\&B}}$

Paired

- $t = \frac{\overline{A} - \overline{B}}{\sigma_{(A-B)}}$

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Paired t-Test Assumptions

- A and B have a metric scale with the same units.
- There is a natural pairing between observations for A and for B.
  - pre-test and post-test for same individual
  - response to two types of stimulus for same mouse
  - responses for a pair of spouses
- Each pair (A_i, B_i) is drawn i.i.d.
- The distribution of $A - B$ is sufficiently normal given the sample size.

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- No Problem
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Small sample

- *t*-test is fairly robust to deviations from normality, but you should look at your distribution and see how non-normal it is
- Suppose you have a small sample and you suspect you have a major deviation from normality
- You might be able to transform the variable to make it more normal, but that can alter the meaning and make results harder to interpret

An alternative is to use a *non-parametric* test

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Non-parametric Test Details

- Non-parametric tests can be also called **distribution- free tests**
  - Still involve assumptions, but they are less restrictive than those of parametric tests
- Many tests work on principle of ranking data
  - List the scores from lowest to highest – each score gets a rank, so higher scores have higher ranks
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Rank-Based Tests for Ordinal Variables

- Rank-based tests are especially useful when we have an ordinal variable
  - eg. a Likert variable such as "how do you feel about a presidential campaign?"
  - Neutral, support, strongly support, etc.
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Wilcoxon Rank-sum Test for Independent Groups

2022-03-04

Hypothesis Testing

└─Wilcoxon Rank-sum Test for Independent Groups

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Parametric and Non-parametric Tests for Comparing Only Two Groups

Type of Design	Parametric Tests	Non-parametric Tests
Two independent samples	Independent samples t test	Wilcoxon rank-sum test (Mann-Whitney test)
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Comparing Two Independent Conditions: Wilcoxon Rank-Sum Test

- Data are ranked from lowest to highest across groups
- This provides **potential rank** scores
- If the same score occurs more than once then all scores of the same value receive the average of the potential ranks for those scores

ID	Group	Score	Potential Rank	Final Rank
1	A	10	1	1
2	A	11	2	2.5
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4	B	12	4	4
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Calculating the Wilcoxon Rand-Sum Test

- After assigning final ranks, add up all the final ranks for each of the two groups
- Subtract the mean rank for a group of the same size as our groups
    - Otherwise, larger groups would always have larger values
    - For example, the mean group for a group of four = 1 + 2 + 3 + 4 = 10
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Null hypothesis: There is no difference in ranks
Alternative hypothesis: There is a difference in ranks

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  - ie. simulated data are used to estimate the statistic
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Effect Size for the Wilcoxon Rank-Sum Test

Effect Size Correlation

$$r = \frac{Z}{\sqrt{N}}$$

Divide the z statistic by the square root of the total sample size

r	Effect Size
0.10	Small
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